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Description of Possible CP Effects in $b\bar{b}\gamma$ Events at LEP

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Abstract

In this letter we investigate the possibilities of using events at LEP with $b\bar{b}\gamma$ in the final state to study CP Violation. We describe a very convenient set of CP sensitive observables and show how they receive contributions from the most general $Zb\bar{b}\gamma$ form factors.

In the last few years all four LEP experiments have extensively studied Z decays into b pairs. The decay $Z \to b\bar{b}$ not only provides a means for a high precision measurement of $\sin^2\theta_w$, but is also a useful source of information on the fragmentation function of the b quark as well as $B-\overline{B}$ mixing, which can be studied at LEP in a clean environment unlike at hadron colliders. Furthermore, since the b quark is the heaviest quark discovered to date, there are compelling reasons to believe that new physics beyond the standard model might show up most clearly in b couplings. A particularly appealing possibility is that of CP violating contributions to the Zbb vertex beyond those arising from the CKM Matrix, which we will consider in detail.

In earlier work, [1],[2],[3] CP sensitive observables have been discussed in connection with the processes $e^+e^- \to \tau^+\tau-$ and $e^+e^- \to W^+W^-$; exploiting the fact that the spin of the final state particles can be experimentally determined. The b quark polarisation however, is in general washed out by fragmentation effects; so additional information is needed in order to construct CP odd observables. (Polarizing the incoming beams will not serve our purpose as this leads only to increased sensitivity to non-standard Zee couplings [4]) This additional information is available for example when final state photon radiation either from the b or $\bar{b}$ is present; hence we suggest using the process $e^+e^- \to Z \to b\bar{b}\gamma$ as a probe for CP Violation beyond the Standard Model. As we will see in detail later, CP Violating form factors contribute terms to the matrix element $\sim \varepsilon(e^+e^-b\bar{b}) = \varepsilon_{\alpha\beta\mu\nu}p^\alpha(e^+)p^\beta(e^-)p^\mu(b)p^\nu(\bar{b})$ which are strongly suppressed in the Standard Model, making this an ideal channel to search for non standard effects. Furthermore, there is no problem with statistics, at the time of writing the OPAL collaboration alone has observed $\mathcal{O}$ (10^3) $b\bar{b}\gamma$ events [5] .At the end of LEP1 all four experiments together are expected to have collected $\mathcal{O}(10^4)\bar{b}b\gamma$ events.

The only real background to the CP studies we are interested in comes from initial state radiation. However, the effects of initial state radiation can easily be brought under control by suitable experimental cuts which can be optimised by Monte Carlo studies. In order to avoid soft and mass singularities we require in addition that the photons be isolated from the b jets. More precisely, we fix a certain y_{cut} and demand that $y, \overline{y} > y_{cut}$, where $y, \overline{y}$ are defined as follows,

$$y(\bar{y}) = \frac{2b(\bar{b}).p}{s}, \tag{1}$$

using b,$\bar{b}$ and p to denote the 4 momenta of the b,$\bar{b}$, and photon respectively, and $s = M_Z^2$.

The most popular framework for describing the effects of physics beyond the standard model at presently accessible energies is the effective Lagrangian approach, where one typically begins with a Lagrangian of the form

$$L = L_{SM} + \frac{1}{\Lambda}L_5 + \frac{1}{\Lambda^2}L_6 + \mathcal{O}(\frac{1}{\Lambda^3}) + \cdots \qquad (2)$$

All the effects of new physics characterised by the scale Λ are contained in the non-renormalisable terms L_5, L_6 etc. . This leads, in addition to the Standard Model contribution shown in Fig. 1, to the diagrams shown in Fig. 2. In [3], L_5 and L_6 were choosen just requiring $U(1)$ invariance and not the full $SU(2)_L \otimes U(1)$ invariance of the Standard Model. This was later objected to in [6], where the authors argued that after imposing $SU(2)_L \otimes U(1)$ invariance, the only permissible CP Violating terms arise first in L_6 and are further m_b supressed and therefore unobservably small. In the following we will give up the effective Lagrangian Formalism approach all together. Instead we will parameterise all CP Violating effects by looking at the most general U(1) invariant form factors which can lead to observable effects at the Z resonance. With this approach, all the various diagrams in Fig.2 are compactified to Fig. 3, which may be schematically denoted by $\overline{u}\Gamma_{\mu\alpha}Z^\mu A^\alpha v$, where $\Gamma_{\mu\alpha}$ is the new CP Violating $\bar{b}bZ\gamma$ vertex. Our main motivation is that the CP scale Λ which can realistically be probed by existing experiments may not be much larger than the Fermi scale; the validity of an expansion in $\frac{m_Z}{\Lambda}$ which is the basis of the effective Lagrangian approach is then questionable. The first step in the construction of $\Gamma_{\mu\alpha}$ is to choose the Dirac structure in such a way that the interference amplitude between the new terms we introduce and the Standard Model contribution is non vanishing, even in the limit that the b quark mass vanishes. After a quick glance at Fig. 1, it is easy to see that only form factors with γ^μ or $\gamma^\mu\gamma^5$ lead to non-vanishing interference terms. This leads to the following expression

$$\begin{aligned} \Gamma_{\mu\alpha} &= i\{\gamma_\mu(F_1 b_\alpha + F_2\bar{b}_\alpha) + \gamma_\alpha(F_3 b_\mu + F_4\bar{b}_\mu + F_5 p_\mu) + F_6 g_{\alpha\mu}\not{p} + \\ &\quad \not{p}[b_\alpha(F_7 p_\mu + F_8 b_\mu + F_9\bar{b}_\mu) + \bar{b}_\alpha(F_{10}p_\mu + F_{11}b_\mu + F_{12}\bar{b}_\mu)]\} \qquad (3) \end{aligned}$$

where the (complex) form factors $F_i(y,\overline{y})$ may be decomposed into $F_i = f_i + g_i\gamma_5$. All form factors need not, and in practise will not, originate from operators of a fixed dimension and must in principle be determined from experiment.

For the sake of reference, the standard model vertex is given by

$$\Gamma^{SM}_{\mu\alpha} = -ie^2 Q_b\{\gamma_\alpha \frac{\not{b}+\not{p}}{2b.p}(v_b + a_b\gamma_5)\gamma_\mu - (v_b + a_b\gamma_5)\gamma_\mu \frac{\not{\bar{b}}+\not{p}}{2\bar{b}.p}\gamma_\alpha\} \tag{4}$$

where a_b and v_b are the vector and axial couplings of the b quark to the Z.

It is convenient to rewrite the expression for $\Gamma_{\mu\alpha}$ so only gauge invariant combinations of form factors appear. This leads to the following result

$$\begin{aligned}\Gamma_{\mu\alpha} = i\{(v_1 + a_1\gamma_5)\gamma_\mu(2b_\alpha\bar{b}.p - 2\bar{b}_\alpha b.p) + \\ ((v_2 + a_2\gamma_5)b_\mu + (v_3 + a_3\gamma_5)\bar{b}_\mu)(\gamma_\alpha 2b.p - 2b_\alpha\not{p}) + \\ ((v_4 + a_4\gamma_5)b_\mu + (v_5 + a_5\gamma_5)\bar{b}_\mu)(\gamma_\alpha 2\bar{b}.p - 2\bar{b}_\alpha\not{p}) + \\ (v_6 + a_6\gamma_5)(\gamma_\alpha(b_\mu + \bar{b}_\mu) + \not{p} g_{\alpha\mu})\}\end{aligned} \tag{5}$$

Once again, all the form factors v_i and a_i are complex functions of y and $\overline{y}$.

One can calculate the interference between the Standard Model amplitude and the new form factors we have introduced. The result may be presented in the following form

$$\begin{aligned}&\mid \mathcal{M} \mid^2_{\mathcal{CP}}= \\ &\varepsilon(e^+, e^-, b, \bar{b}) \\ &\{a_b(a_e^2 + v_e^2)\{\tilde{v}_1(b.(e^+ - e^-) + \bar{b}.(e^+ - e^-)) \\ &-(\tilde{v}_2 b.(e^+ - e^-) + \tilde{v}_3\bar{b}.(e^+ - e^-))(1 + \frac{b.\bar{b}}{\bar{b}.p}) \\ &+(\tilde{v}_4 b.(e^+ - e^-) + \tilde{v}_5\bar{b}.(e^+ - e^-)(1 + \frac{b.\bar{b}}{b.p}) \\ &+\tilde{v}_6(\frac{-b.(e^+ - e^-)}{b.p} + \frac{\bar{b}.(e^+ - e^-)}{\bar{b}.p})/2\} \\ &+a_b v_e a_e\{2\tilde{a}_1(\bar{b}.p - b.p) \\ &+2\tilde{a}_2((b.p)(-1 + \frac{\bar{b}.b}{\bar{b}.p}) + b.\bar{b}(1 + \frac{\bar{b}.b}{b.p}) \\ &-2\tilde{a}_3\frac{(\bar{b}.p + \bar{b}.b)^2}{\bar{b}.p}\end{aligned}$$

$$-2\tilde{a_4}\frac{(b.p+\bar{b}.b)^2}{b.p}$$
$$+2\tilde{a_5}((\bar{b}.p)(-1+\frac{\bar{b}.b}{\bar{b}.p})+b.\bar{b}(1+\frac{\bar{b}.b}{b.p})$$
$$+\tilde{a_6}(2+\frac{\bar{b}.b}{b.p}+\frac{\bar{b}.b}{\bar{b}.p}))\}+(a_b\leftrightarrow v_b,\tilde{a}_i\leftrightarrow\tilde{v}_i)\}\frac{1}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} \quad (6)$$

where $\sim$ denotes the imaginary part of the corresponding form factor. We have been deliberately somewhat cavalier with colour factors, quark charges and other overall constants as they will divide out in the definition of the CP sensitive asymmetries which we will later define.

In order to cast this result in a more usable form it is useful to parameterise the momenta as follows

$$\begin{aligned} b &= \frac{\sqrt{s}}{2}x(1,0,0,1) \\ e^- &= \frac{\sqrt{s}}{2}(1,0,\sin\theta,\cos\theta) \\ e^+ &= \frac{\sqrt{s}}{2}(1,0,-\sin\theta,-\cos\theta) \\ \bar{b} &= \frac{\sqrt{s}}{2}\bar{x}(1,\sin\phi\sin\beta,\sin\beta\cos\phi,\cos\beta) \end{aligned}$$

With this choice of momenta we have

$$\varepsilon(e^+,e^-,b,\bar{b})=\frac{s^2}{4}x\bar{x}\sin\theta\sin\beta\sin\phi.$$

where $\cos\beta$ is given by

$$\cos\beta=2\frac{1-x}{x}\frac{1-\bar{x}}{\bar{x}}-1$$

Observe that all scalar products constructed with the above momenta are independent of $\sin\phi$. $\sin\phi$ appears in the matrix element only via the ϵ tensor which in turn arises only as a result of CP Violation. Hence, any experimental evidence of a $\sin\phi$ dependence of the cross-section is a signal of CP Violation. To see the physical significance of the angle ϕ recall that the angular part of the massless three body phase space integral may be written as

$$\int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi$$

It is then clear that if there is indeed a $\sin\phi$ dependence then there exists a non-vanishing azimuthal asymmetry defined by

$$\mathcal{A} = \frac{\sigma(\phi < \pi) - \sigma(\phi > \pi)}{\sigma_{\rm total}}$$

A non-vanishing value of $\mathcal{A}$ means a difference in the number of photons observed on different sides of a plane defined by the beam axis and the outgoing b jet. We have tacitly assumed that it is experimentally possible to distinguish between b and $\bar{b}$ jets. In practice this can be achieved by identifying the charge of the muon from semi-leptonic decays. There are complications due to $B - \overline{B}$ mixing, but this is a well understood effect which can be corrected for.

A closer look at $\mid \mathcal{M} \mid^2_{CP}$ reveals that there are forward backward as well as azimuthal asymmetries. More precisely, in addition to the terms proportional to $\varepsilon(e^+, e^-, b, \bar{b})$ already mentioned there are also terms proportional to $\varepsilon(e^+, e^-, b, \bar{b})\cos\theta$ and $\varepsilon(e^+, e^-, b, \bar{b})\cos\bar{\theta}$ where $\bar{\theta}$ is the angle between the $\bar{b}$ quark and the beam axis. These give rise to the following additional asymmetries,

$$\mathcal{A}_{FB} = \frac{\sigma\left\{\begin{array}{c}\phi < \pi \\ \cos\theta > 0\end{array}\right\} - \sigma\left\{\begin{array}{c}\phi < \pi \\ \cos\theta < 0\end{array}\right\} - \sigma\left\{\begin{array}{c}\phi > \pi \\ \cos\theta > 0\end{array}\right\} + \sigma\left\{\begin{array}{c}\phi > \pi \\ \cos\theta < 0\end{array}\right\}}{\sigma_{\rm total}}$$

and

$$\overline{\mathcal{A}_{FB}} = \frac{\sigma\left\{\begin{array}{c}\phi < \pi \\ \cos\bar{\theta} > 0\end{array}\right\} - \sigma\left\{\begin{array}{c}\phi < \pi \\ \cos\bar{\theta} < 0\end{array}\right\} - \sigma\left\{\begin{array}{c}\phi > \pi \\ \cos\bar{\theta} > 0\end{array}\right\} + \sigma\left\{\begin{array}{c}\phi > \pi \\ \cos\bar{\theta} < 0\end{array}\right\}}{\sigma_{\rm total}}$$

which differ in definition from the better known b quark forward-backward asymmetries due to the azimuthal angle dependence. Note that $\mathcal{A}$, $\mathcal{A}_{FB}$ and $\overline{\mathcal{A}_{FB}}$ consitute the *complete* set CP of sensitive observables which can be observed in the process $e^+e^- \rightarrow \bar{b}b\gamma$. (CP Violating asymmetries in the energy distribution of b and $\bar{b}$ require absorbtive parts and are furthermore difficult to measure [3].) Each of the three asymmetries we have discussed can be studied either at fixed values of (anti)quark energies (x and $\overline{x}$) or integrated over energies to give information about different linear combinations of form factors. In any case, the angular dependence of the asymmetries may be expressed through three universal integrals, J_0, J and $\overline{J}$. These are given by

$$J_0 = \int_0^1 d\cos\theta \int_0^\pi d\phi \sin\theta \sin\beta \sin\phi = \frac{\pi}{4}\sqrt{1-r^2} \tag{7}$$

$$J = \int_0^1 d\cos\theta \int_0^\pi d\phi \cos\theta \sin\theta \sin\beta \sin\phi = \frac{4}{3\pi} J_0 \tag{8}$$

$$\overline{J} = \int_{-1}^1 d\cos\theta \int_0^\pi d\phi \cos\overline{\theta} \sin\theta \sin\beta \sin\phi = r J_0 \tag{9}$$

where we have used

$$\cos\overline{\theta} = \sin\theta \sin\beta \cos\phi + \cos\theta \cos\beta$$

and $\cos\beta = r$.

What we have done in effect is to define the phase space in such a way that angular asymmetries generated by CP Violating terms in the matrix element can easily be highlighted. It is interesting to note that our CP sensitive variables $\varepsilon(e^+,e^-,b,\overline{b})$, $\varepsilon(e^+,e^-,b,\overline{b})\cos\theta$, and $\varepsilon(e^+,e^-,b,\overline{b})\cos\overline{\theta}$ are essentially the same as the CP sensitive variables b_3, c_4, and c_5 of [3].

As a practical application and as a pedagogical exercise let us consider the case where all form factors vanish except $\tilde{v}_3$. We can then express the asymmetries as functions of the photon isolation y_{cut} as follows;

$$\mathcal{A} = \frac{v_e a_e v_b \int_{y_{cut}}^1 dy \int_{y_{cut}}^{1-y} d\overline{y} \tilde{v}_3 m_Z^4 \sqrt{1-r^2}\frac{x\overline{x}^3}{\overline{y}}}{4Q_b e^2(v_e^2+a_e^2)(v_b^2+a_b^2)\int_{y_{cut}}^1 dy \int_{y_{cut}}^{1-y} d\overline{y}\frac{(x^2+\overline{x}^2)}{y\overline{y}}} \tag{10}$$

$$\mathcal{A}_{FB} = \frac{a_b \int_{y_{cut}}^1 dy \int_{y_{cut}}^{1-y} d\overline{y} \tilde{v}_3 m_Z^4 r \sqrt{1-r^2}\frac{x\overline{x}^3}{\overline{y}}}{8Q_b e^2(v_b^2+a_b^2)\int_{y_{cut}}^1 dy \int_{y_{cut}}^{1-y} d\overline{y}\frac{(x^2+\overline{x}^2)}{y\overline{y}}} \tag{11}$$

$$\overline{\mathcal{A}_{FB}} = \frac{a_b \int_{y_{cut}}^1 dy \int_{y_{cut}}^{1-y} d\overline{y} \frac{4}{3\pi} \tilde{v}_3 m_Z^4 \sqrt{1-r^2}\frac{x\overline{x}^3}{\overline{y}}}{8Q_b e^2(v_b^2+a_b^2)\int_{y_{cut}}^1 dy \int_{y_{cut}}^{1-y} d\overline{y}\frac{(x^2+\overline{x}^2)}{y\overline{y}}} \tag{12}$$

(13)

where $x = 1-\overline{y}$, $\overline{x} = 1-y$, $a_b = \frac{-1}{2\sin\theta_w \cos\theta_w}$ etc.

The denominator of $\mathcal{A}$ is essentially the standard model cross-section. We see that $\mathcal{A}$ is sensitive only to the combination $\sim v_b \tilde{v}_3$ whereas the other asymmetries are sensitive to terms proportional to $a_b \tilde{v}_3$. Note that $\mathcal{A}_{FB} \sim \overline{J}$, while $\overline{\mathcal{A}}_{FB} \sim J$. This is because the contribution $\sim a_b \tilde{v}_3$ is $\sim \cos\theta$ (c.f. eq. 6). $\mathcal{A}$ is always $\sim J_0$. The factor m_Z^4 in the above equations arises because dim $\tilde{v}_3 = -4$.For definiteness we have chosen $\tilde{v}_3 m_Z^4 = const.(y, \overline{y}) = 1$. Then one obtains the following values for the asymmetries (as function of y_{cut})

y_{cut}	$\mathcal{A}$	$\mathcal{A}_{FB}$	$\overline{\mathcal{A}}_{FB}$
0.01	0.0049	-0.033	0.019
0.05	0.011	-0.068	0.043
0.1	0.017	-0.088	0.067
0.2	0.027	-0.076	0.101
0.3	0.030	-0.016	0.117

For suitable values of y_{cut} asymmetries of the order one to ten per cent arise.

To conclude, we have constructed the most general $U(1)$ invariant CP Violating $Z\bar{b}b\gamma$ vertex and parameterised it in terms of twelve independent form factors. Non-vanishing values of these form factors are shown to lead to various asymmetries whose physical significance is transparent. In particular, we have introduced a parameterisation of momenta in the lab frame which facilitates detection of the asymmetries we have defined. All of the formalism which we have developed may be carried over to describe events where the final state photon is replaced by a gluon. However, we have choosen to restrict our attention to $b\bar{b}\gamma$ final states where we anyway expect any signals of CP violation in the electro-weak sector to show up more strongly.

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Fig. 1

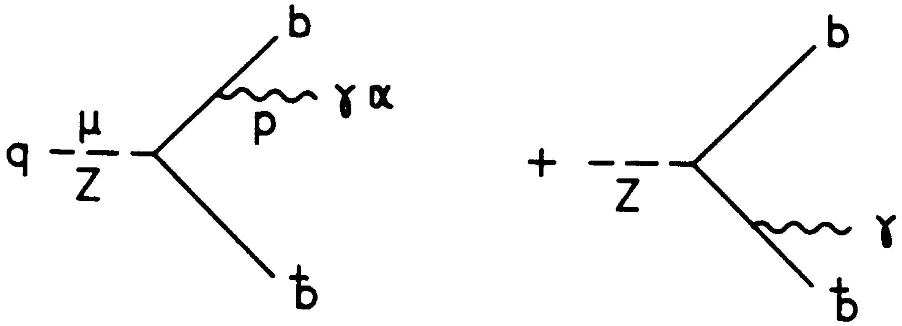

Fig. 2

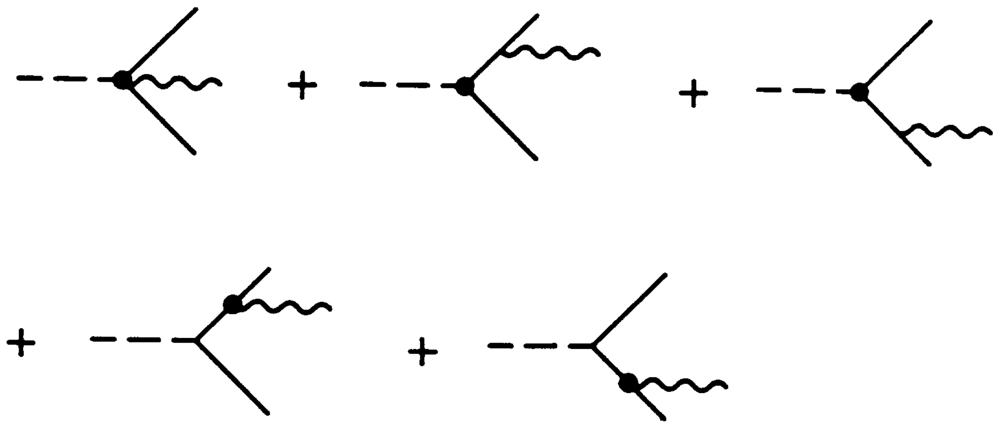

Fig. 3