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BEAMSTRAHLUNG AND PAIR PRODUCTION IN THE DEEP QUANTUM REGIME

CLIC Note *)

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ABSTRACT

The general properties of beamstrahlung and of pair production are presented, focusing on the simplifications which occur in the deep quantum regime. A Feynman graph approach yields simple results which are accurate in the low disruption approximation. Both photon radiation by electrons and positrons in the colliding bunches, and pair formation by a photon traversing a bunch are considerable. This is an introduction to recently published works.

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*) Written after a CLIC seminar presented on the 7th April 1989.

**) Work done in collaboration with T.T. Wu.

1. - INTRODUCTION

This is the second CLIC note written on this question and, like the first one, 104-87, it reports on work done at CERN in collaboration with T.T. Wu. The linear colliders under consideration are in the TeV range. Reaching such energies is difficult; it is, however, pointless if one does not also reach a large enough luminosity. The luminosity should increase as E^2, where E is the beam energy, if one wishes to collect a decent enough rate for the bench mark cross-section $e^+e^- \to \mu^+\mu^-$. A luminosity of 10^{33} is a must for $E \sim 1$ TeV.

The luminosity is practically determined by four parameters:

$$L \sim \frac{N^2 f H}{R^2} \tag{1}$$

Increasing the number of particles per bunch N and the bunch crossing frequency f costs power. The possible gain through H, the pinch parameter, is limited. One has to play on R and one is talking about R at the level of 10^{-8}m!

With large N and small R, the field inside the bunch (~N/R) is very high. Incident particles radiate as they cross such a high field region. One is driven into the deep quantum regime where

$$\Upsilon = \frac{\gamma^2}{m\rho_c} >> 1 \tag{2}$$

where ρ_c is the radius of curvature.

Two questions are considered in this note, namely radiation by an electron traversing a positron bunch and pair formation by a photon traversing a positron (electron) bunch. The kinematics is defined in Figs. 1a and 1b, respectively. These pictures correspond to a classical approximation and the space location of the process is defined as the point of the stationary phase. The process is actually "spread" over a distance ℓ_c around that point, where ℓ_c will be defined later.

The quantum mechanical calculation corresponds to the Feynman graphs of Figs. 2a and 2b respectively. The calculation is done to lowest order in α and thus corresponds to a distorted wave Born approximation. The amplitude reads in both cases:

$$\bar{\psi}_f \gamma_\mu \psi_i \varepsilon^\mu \qquad (3)$$

where ϕ_i and ϕ_f are the wave functions of the charged particles in the field of the bunch and ε is the polarization vector of the photon. The crosses on the graph correspond to interaction with the field in the bunch.

From (3) one calculates the radiation intensity I(x), normalized to unit incident flux, where x is the fraction of the electron energy taken by the photon (beamstrahlung) or the fraction of the photon energy taken by the electron (pair formation). The spectrum found are rather hard, with a rather flat distribution in x. The fractional energy loss:

$$\delta = \int_0^1 x\, I(x)\, dx \qquad (4)$$

is considerable. Values at the level of 20% can be considered as typical.

The classical regime is well known. We concentrate here on the deep quantum regime, emphasizing its specific simplicity which this regime also possesses. It is clear that there is also an interesting intermediate regime which deserves attention.

2. - MAIN RESULTS

Our interest in this problem started with the remark that the respective dominance of two important lengths is inverted as one goes from the classical regime to the deep quantum regime. In the classical regime, the coherent radiation length L_c:

$$L_c \sim \left(\frac{N}{L_b}\right)^{-1} \left(\frac{\alpha}{Rm}\right)^{-1} \qquad (5)$$

is much larger than the quantum mechanical radiative length L_e

$$L_e \sim \frac{\gamma}{m} \qquad (6)$$

In the deep quantum regime, into which one is naturally driven with a high energy, high luminosity machine, it is the converse which is true

$$L_e >> L_c \tag{7}$$

The radiation process changes in nature. We found that simplicity still prevails when one introduces a new coherent radiation length

$$\ell_c = \left(L_c^2 L_e\right)^{1/3} \tag{8}$$

In the deep quantum regime $\ell_c >> L_c$, and one indeed finds that

$$\Upsilon = \left(\frac{\ell_c}{L_c}\right)^3 \tag{9}$$

In practice the quantity

$$\Lambda = \frac{L_b}{\ell_c} \tag{10}$$

remains large (10 to 100) $\Lambda \sim N\alpha D$, where D is the disruption parameter. To a first approximation, one can consider that different sections of the bunch, each one of them ℓ_c long, radiate incoherently. One may remark that ℓ_c is that particular combination of L_c and L_e which eliminates any explicit reference to the electron mass. This is natural since we are in a regime where the transverse momentum collected by the electron as it crosses the bunch is much larger than the electron mass

$$\Delta_T >> m \tag{11}$$

The canonical parameter of the classical approach α/m has to drop out entirely!

More specifically, one has

$$\ell_c = \left(\frac{R^2 L_b^2 E}{(N\kappa)^2}\right)^{1/3} \tag{12}$$

This new regime leads naturally to a calculation based on Feynman graphs. With it we found two things:

(i) simplicity prevails once one introduces ℓ_c;

(ii) there are typical quantum effects, which one may refer to as radiation "before and after" bunch crossing, since they do not involve the bunch length L_b. Their leading contribution is proportional to $\alpha \ln L_e \ell_c$. This is connected with a Weizsächer-Williams fragmentation.

Our approach was first presented and exploited in four papers (M.J. and T.T.W.):

(i) Quantum approach to beamstrahlung, Phys. Lett. B197 (1987) 253;
(ii) Quantum calculation of beamstrahlung: the spinless case, Nucl. Phys. B303 (1988) 373;
(iii) Quantum calculation of beamstrahlung: the Dirac case, Nucl. Phys. B303 (1988) 389.
(iv) Beamstrahlung in the multi-TeV (a general review), Proceedings of the Kazimierz Conference (88), WSPC.

The previous CLIC note (87-104) was written in between the completion of papers (ii) and (iii).

Our attitude has been to go as far as possible analytically. For that reason we worked in the low D approximation. Our results are accurate provided that a linear expansion in D is acceptable. Radiation effects at large D can a priori only be larger than those found for low D.

To be more specific we start with a high energy approximation of the type

$$A = A_0 + \frac{A_1}{E} \tag{13}$$

for both the modulus and the non-trivial part of the phase of the wave functions. This sounds a priori very accurate as E is very large, but it turns out to be only a low D (linear in D) approximation.

The leading contributions to δ are then particularly simple. For beamstrahlung they read:

$$\delta = K_e \frac{\alpha}{\pi} \ln \frac{L_e}{\ell_c} + K_i \frac{\alpha}{\pi} \frac{L_b}{\ell_c} \tag{14}$$

where K_e and K_i are both numerical coefficients of order one. Relation (14) refers to a uniform bunch for which ℓ_c is defined by (12). The radiation spectrum is rather hard. It has also a very simple expression. For the most important term, proportional to L_b, one finds

$$I(x) \sim \left(\frac{1-x}{x}\right)^{2/3} \frac{1+(1-x)^2}{1-x} \tag{15}$$

The first factor results from the intrinsic properties of radiation in the deep quantum regime. It is the one found in the spinless (Klein-Gordon) case. The second one is mere spinology and corresponds to the complications met when dealing with a Dirac particle.

In the regime considered ($\Delta_T >> m$), only one helicity configuration matters, once parity conservation has been imposed. The neglected one is proportional to L_b/L_e, which is small in practice. For that reason, one can work in the (simpler) Klein-Gordon case and merely include a Dirac spin factor in the spectrum, at the end. The same applies for pair formation. In that case the shape of the spectrum is (Dirac)

$$I(x) \sim \frac{(1-x)^2 + x^2}{(x(1-x))^{1/3}} \tag{16}$$

when the production intensity reads

$$I(x) = k_p \frac{\alpha}{\pi} \frac{L_b}{\ell_c} \frac{(1-x)^2 + x^2}{(x(1-x))^{1/3}} \tag{17}$$

thus picking up only the leading terms proportional to L_b. Here K_p is also a numerical factor of order one and ℓ_c is the same as in the case of beamstrahlung (12). The calculation of pair production, prompted by a remark by W. Schnell, is reported in TH.5274/89, to be published in Phys. Lett. B.

We see that, in all cases, a small factor α/π is to a large extent compensated by a large factor Λ (10). Radiation losses and pair production are therefore both considerable effects. The deep quantum regime is characterized by an intense radiation with a rather hard spectrum.

This has some drawbacks:

(i) a loss in effective power;
(ii) important background and a hazard to the machine and the detector;
(iii) an erosion of the resonance peaks which one may wish to find.

However:

(i) for heavy Higgs search ($WW \to H$) an important radiation is tolerable;

(ii) an electron-positron linear collider is also an intense photon-photon collider. This is also very interesting in heavy Higgs search ($\gamma\gamma \to H$). Yet, the important hard photon flux turns partly into hazardous electron-positron pairs in the intense field of the bunch!

One may remark at this stage that ℓ_c includes a factor R^2/N^2, inversely proportional to the luminosity and a factor s, corresponding to the centre-of-mass energy square. Since the two factors should compensate each other, ℓ_c varies as $L_b^{-1/3}$.

The deep quantum regime calls for compact bunches. As emphasized by Blankenbecler and Drell, ribbon bunches should help in keeping the luminosity up while decreasing the field inside the bunch and hence the beamstrahlung losses. This however raises further complications for the machine.

Our latter work on that topic, in 1988, mainly concentrated on the effect of varying density in order to go beyond the poorly realistic constant density bunch considered in our former work. This has been reported in three papers:

(i) Beamstrahlung with fluctuating charge density, to be published in Nucl. Phys. B; TH.5133/88;

(ii) Beamstrahlung for longitudinally non-uniform bunch, to be published in Nucl. Phys. B; TH.5192/88;

(iii) Beamstrahlung in high energy electron-positron linear colliders with non-uniform bunches, Phys. Lett. B216 (1989) 442.

The approach now consists of an expansion in Λ (10) which is a large parameter (10 to 100 in practice). One takes the Mellin transform of the radiation intensity

$$I(x) = \frac{\alpha}{\pi} K(x,\Lambda)$$

$$\bar{K}(\xi) = \int_0^\infty K(x,\Lambda)\, \Lambda^{-1-\xi}\, d\Lambda \tag{18}$$

and calculates the residues at the poles met at $\Lambda = 1$ (leading term proportional to L_b), $\Lambda = 0$ (which includes the $\ell n L_e$ edge effects) and $\Lambda = -1$ [which is sensitive to rapid density variation within the bunch, proportional to $(\rho')^2$ and $\rho\rho"$]. This can be done though a series of successive analytic continuations as the different pole terms can be isolated one by one. The result is a rigorous treatment of edge effects and varying density for arbitrary bunch shapes.

The edge effects are fully calculated and include the $\ell n L_e$ term previously focused upon.

The terms corresponding to an integral over the bunch length (roughly speaking now proportional to L_b) are of two kinds:

(i) a contribution proportional to Λ (large) which merely multiplies K_i in (14) by the extra factor

$$\frac{\int \tilde{\rho}^{2/3}(z)\,dz}{\int \tilde{\rho}(z)\,dz} \tag{19}$$

where $\tilde{\rho}$ is the normalized varying density

$$\int \tilde{\rho}(z)\,dz = L_e \tag{20}$$

(ii) A contribution of relative order Λ^{-2} (hence small) which involves an integral over z of

$$R(z) = \tilde{\rho}^{-2/3}(z) \left\{ \frac{3\,\tilde{\rho}(z)\,\tilde{\rho}''(z) - 4\,\tilde{\rho}'^2(z)}{\tilde{\rho}^2(z)} \right\} \tag{21}$$

A proper analytic continuation has to be made to deal with edge effects, when $\tilde{\rho}(z) \to 0$. An actual numerical value (a relatively small contribution) can thus easily be obtained for realistic bunch shapes.

As the density varies, so does $\ell_c(z)$ which has now to be defined locally. From (12) one sees that $\ell_c(z) \sim \tilde{\rho}^{-2/3}(z)$, hence extra technical complication at the edges when this quantity becomes large.

3. - CALCULATION

The reader is referred to the different papers previously itemized for a detailed presentation. We here merely stress a few general points.

The calculation proceeds as follows:

(i) calculate the wave functions. This is done in the high E (but actually low D) approximation, as previously said. One solves the Klein-Gordon (Dirac) equation in the field of the bunch.

(ii) Study the conditions for the stationary phase for the production amplitude. In practice, most of the radiation originates from a limited zone corresponding to a limited stationary variation of the phase. The location of this zone is determined by the kinematics and the bunch properties.

Two points are worth emphasizing:

a) The phase stationarity conditions are very different in the beamstrahlung case and in the pair production case. This is due to the fact that the main additive effect, associated with the phase of the electron wave function before and after radiation, hence throughout the whole bunch in beamstrahlung, cancels in the production case, where an electron and a positron propagate with almost collinear paths, after formation only. To be more specific the transverse co-ordinate for the stationary phase corresponds to

$$\frac{\vec{r}}{R} = -\left(\frac{2N\alpha}{R}\right)^{-1}\left(\vec{k}_{f\perp} + \vec{k}_{\gamma T}\right) \tag{22}$$

for beamstrahlung (Fig. 1a), and to

$$\frac{\vec{r}}{R} = -\left(\frac{2N\alpha}{R}\right)^{-1}\left(\frac{3}{1-\tilde{\xi}}\right)\left(\frac{4x(1-x)}{(1-\tilde{\xi})^2}\,\frac{\vec{K}_\perp}{\varepsilon} + \vec{k}_\perp\right) \tag{23}$$

for pair production (Fig. 1b).

Here $2N\alpha/R$ is the maximum bending momentum which can be collected through bunch-crossing. In the second case, we have introduced

$$\mathcal{E} = \frac{N\alpha}{R^2}\frac{L\ell}{E} \sim D$$
$$\tilde{z} = \frac{L\ell}{2} z \tag{24}$$
$$\vec{K}_\perp = \vec{k}_\perp + \vec{k}_\perp' \sim \Sigma\, \vec{k}_\perp$$

Such a difference is understandable. We are however very far from a mere crossing relation between the two processes.

b) Once we have fixed $\vec{r}$ by (22) or (23), we find that there is no real point of stationary phase in z. One actually finds that

$$\frac{\partial \phi}{\partial z} = \frac{x}{8(1-x)E}\left(|\mathcal{M}|^2 + 4m^2\right) \tag{25}$$

where $|\mathcal{M}|^2$ is the modulus square of the radiation (production) amplitude average or summed over polarizations. This relation turns out to be very useful!

There are nevertheless two nearly (separation of order Λ^{-1}) complex points of stationary phase and, in between, a real point where $\partial\phi^2/\partial z^2 = 0$. It is therefore natural to expand around that point z_0, defined as the point of the stationary phase. The phase then varies as an odd cubic in $(z-z_0)$ around that point. The characteristic length for its variation defines the proper coherent length. It is ℓ_c (12), for both beamstrahlung and pair production. The deep quantum regime corresponds to values of ℓ_c significantly larger than L_c (9).

(iii) The next step is to compute the relevant matrix elements in the neighbourhood of the stationary point, $\mathcal{M}$, the values of which were anticipated in writing (25). Because of the different conditions on r (22) and (23), the radiation matrix elements and the pair production matrix elements are very different. One is again far from a simple crossing relation. The same relation (25) however holds!

(iv) At this stage one can proceed in two different directions.

a) One calculates the full amplitude integrating the production amplitude calculated through steps (i), (ii) and (iii) over all space. The integral over the radial co-ordinate is done through the stationary phase method. The integral over z, with its odd cubic dependence of the phase, naturally leads to Airy functions. One then integrates the modulus square of the amplitude over phase space to reach the production rate I(x).

b) One integrates over the radial co-ordinates and over phase space, reaching I(x) as a double integral over z and z'. This may a priori seem to be more difficult but one can make great use of (25) to extract the actual rate from a much simpler integral, through the calculation of partial derivatives. The proportionality of the derivative of the phase in (25) to the matrix element square which appears in the rate (m is in most cases negligible in front of the matrix element which is proportional to bending momenta) clearly brings much simplification. One therefore shortcuts entirely Airy function and picks up with increasing simplicity terms of higher order in Λ (the leading ones!)

While we followed path (a) in our earlier approach, we switched to path (b) (it looked like a must) when dealing with varying densities. Insofar as the complicated z dependence of the production amplitude in pair formation can be formally compared to a varying density effect, the new method is particularly efficient in that case. For the leading term, proportional to Λ, the most complicated functions encountered are merely Γ functions. We thus shortcut the integration over Bessel functions of fractional orders met along path (a).

In the case of beamstrahlung studied along path (b), one finds an integral over z-z' with a rapidly changing phase which limits its range in practice to ℓ_c. One is left with a trivial integral over z+z' which simply gives an overall factor Λ.

In the case of pair production, the integrand has, as previously stressed, a rather complicated z, z' dependence. However, the phase space limits are also z, z' dependent, since bending occurs only after the pair is formed (beyond z_0). It turns out that the z, z' dependence brought by the phase space conditions magnificently cancels the z, z' dependence met with the computation of the matrix element, once the integral over z-z' has imposed the strong conditions associated with the variation of the phase ($z-z' \lesssim \ell_c$).

While crossing simplicity is totally lost in the intermediate steps, both when dealing with the matrix element and the phase space limits, it beautifully reappears at the end with the results for beamstrahlung (14) and for pair production (17) which show a very strong similarity. Only once the rate is obtained do we find for both terms of order $(\alpha/\pi)\Lambda$ and, in both cases, rather hard spectra.

The $\ell n L_e$ term has a pure quantum origin. It can be considered as resulting from the quantum dissociation of the electron into a photon and an electron, or of the photon into an electron-positron pair before entering the bunch (and after leaving the bunch in the former case), mass-shell conditions being imposed over a length ℓ_c at the edge of the bunch. It becomes quite sizeable as one increases the energy, but is still far from leading in the TeV range, where the term linear in Λ dominates.

This is where we are. We have still clearly to try to push our method to larger D and extend it to the intermediate regime ($\ell_c \sim L_c$) where a good fraction of its simplicity should of course go away. The question of multi-photon production and of pair production with incident electrons clearly calls for further calculations.

ACKNOWLEDGEMENTS

In this note we have presented our own work. It is an introduction to our published work on that subject. It is clear however that the study of beamstrahlung has already a long history and that previous work could extract a lot from semi-classical approaches. The subject has received much attention recently and we feel that a new approach which starts with Feynman graphs has much appeal in the deep quantum regime. We have of course interacted with many of our colleagues engaged in similar calculations, R. Blankenbecler and S. Drell and P. Chen and K. Yokoga, at SLAC; J.S. Bell and M. Bell at CERN and V. Baier, V.M. Katkov and V.M. Strakharenko at Novosibirsk. Everyone follows a somewhat different approach and there is now a good consensus on most results. T.T. Wu acknowledges the support of the United States Department of Energy under grant DE-FG02-84ER40158.

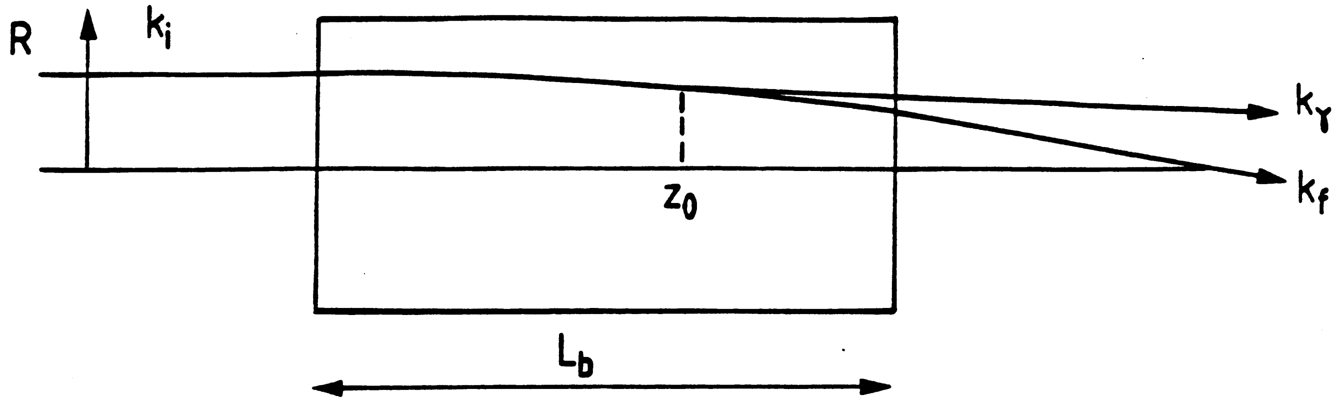

FIGURE 1a

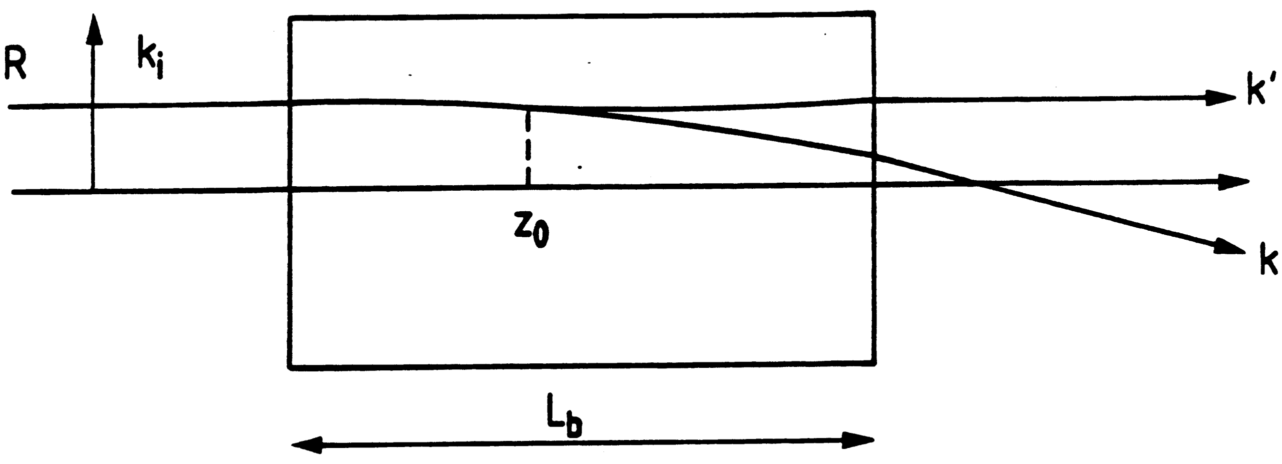

FIGURE 1b

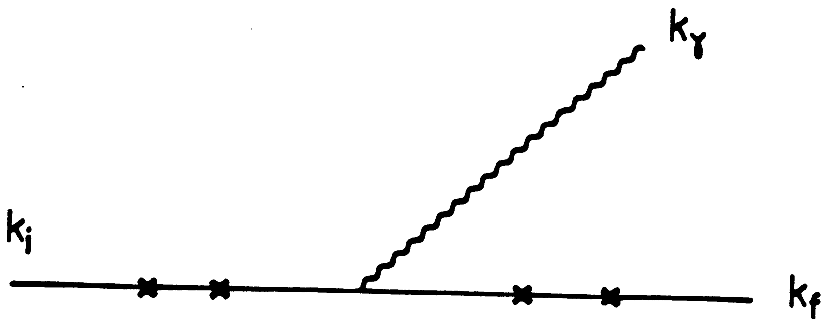

FIGURE 2a

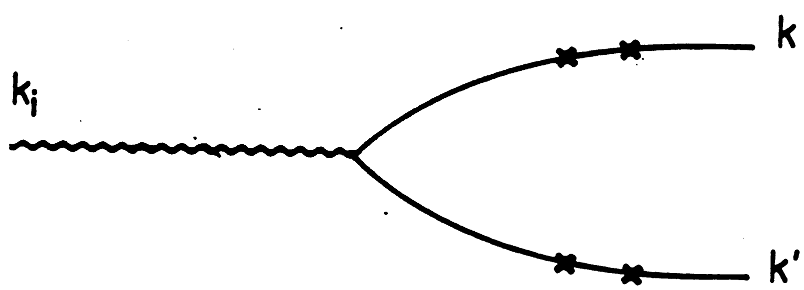

FIGURE 2b