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Quark and Lepton Generations: CP-Violation and Rare Processes in $SUSY$ $SU(3)_{HV}$- Gauge Horizontal Model

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Abstract

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In this paper, the current state of the generation mixing and CP-violation problems is discussed from the standpoint of the hypothetical existence of $SUSY - SU(3)_{HV}$-gauge horizontal symmetry. Underlying the analysis of the expected greatness of the symmetry breaking scale are the experimental limitations on the probabilities of typical rare processes. The behavior of the estimates for the horizontal symmetry breaking scale showed certain regularities depending on particular symmetry breaking schemes and generation mixing mechanisms (different ansatzes for quark and lepton mass matrices with 3 and 4 generations have been discussed). As is noted, the current understanding of quark and lepton mass spectra leaves room for the existence of an unusually low mass scale of horizontal bosons $\sim someTeV$ (independent experiments for verifying the relevant hypotheses have been proposed). Supersymmetry is supposed to be important for: the hierarchy problem, certain useful constraints on Yukawa couplings, the super-Higgs effect in the case of a broken local horizontal symmetry, the estimates of the gauge coupling constant g_H and the restrictions on the horizontal gaugino masses.

Аннотация

А.Н. Амаглобели, Г.Г. Волков, А.Г. Липартелиани и А.А. Масликов. Кварковые и лептонные поколения: CP-нарушение и редкие процессы в $SUSY$ $SU(3)_{HV}$ "горизонтальной" модели: Препринт ИФВЭ 91–185. — Протвино, 1991. — 54 с., библиогр.: 24.

В данной работе рассматривается современное состояние проблемы смешивания поколений и CP- нарушения с точки зрения возможного существования $SUSY - SU(3)_{HV}$-калибровочной горизонтальной симметрии. Прослеживаются закономерности в поведении оценок на масштаб нарушения горизонтальной симметрии в зависимости от схем нарушения, а также механизмов смешивания поколений (рассмотрены разные анзацы для массовых матриц кварков и лептонов в случае 3-х и 4-х поколений). Роль суперсимметрии обозначена в следующих проблемах: проблема иерархии, определнные связи на Юкавские константы, суперхиггсовский эффект при нарушении $SU(3)_{HV}$, оценки константы g_H, ограничения на массы $\tilde{H}^a$- калибрино.

1. Introduction

There are no experimental indications which would impel one to go beyond the framework of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model (SM) with three generations of quarks and leptons. None of the up-to-date experiments contradict, within the limits of accuracy, the validity of the SM predictions for low energy phenomena.

One has ten parameters in the quark sector of the SM with three generations: six quark masses, three mixing angles and the single phase - the Kobayashi-Maskawa (KM) phase ($0 < \delta^{KM} < \pi$) .

Recently, the interest in the CP-violation problem was excited again due to the yet contradictory data on the search for the direct CP-violation effects in neutral K-mesons [1] :

$$Re\left(\frac{\varepsilon'}{\varepsilon}\right) = (2.3 \pm 0.7) \times 10^{-3}$$

NA31 (CERN,1986+1988+1989,1991)

$$Re\left(\frac{\varepsilon'}{\varepsilon}\right) = (6.0 \pm 6.9) \times 10^{-4}$$

E731 (FNAL,1987-1988,1991)

At the same time, the t-quark mass is known to be bound: $89GeV < m_t < 200GeV$ [2]. The lower bound is an experimental limit resulting from the search for t-quarks at FNAL collider. The upper bound comes from NC and LEP data. Moreover, the last estimates obtained give the following range for m_t : $m_t = 125 \pm 30$ GeV [3].

The major contribution to the CP-violation parameters ε_K and ε'_K (K^0-decays), as well as to the $B_d^0 - \bar{B}_d^0$ mixing parameter $x_d = \frac{\Delta m_{(B_d)}}{\Gamma_{(B_d)}}$, is due to the large t-quark mass contribution. The same statement holds also for the amplitudes of K- and B-meson rare decays. The combined analysis of the data on the $B_d^0 - \bar{B}_d^0$ mixing parameter $x_d = 0.67 \pm 0.10$ and $\varepsilon_K = (2.26 \pm 0.02) \times 10^{-3} exp(i43.7^o)$ indicated that the top quark mass should lie in the range (135 ± 35)GeV, although a very massive top quark in the range $m_t \sim 200GeV$ is not excluded, either [4]. Note that for $m_t \approx O(100GeV)$ the SM predicts $\left(\frac{\varepsilon'}{\varepsilon}\right)_K \approx (1.0 \pm 0.5) \times 10^{-3}$, and the value of $\delta_{13} = argV_{ub}^*$ ($V_{ub} = s_{13} \times exp(-\delta_{13})$) in the second quadrant ($\delta_{13} > \frac{\pi}{2}$) of unitary triangle (see Fig. 1)is favored. For $m_t \sim 200GeV$ one gets the superweak behaviour predicted in the SM, i.e. $\left(\frac{\varepsilon'}{\varepsilon}\right)$ is close to zero. In this case the value of δ_{13} is likely to be in the first quadrant ($0 < \delta_{13} < \frac{\pi}{2}$).

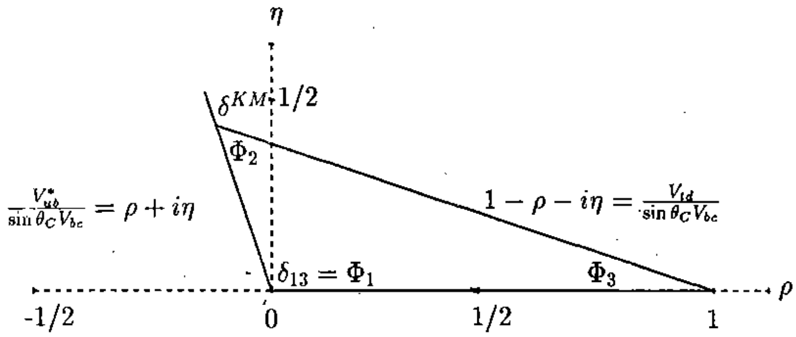

Fig.1 Swedish triangle. $V_{ub}^* + V_{td} \approx \sin\theta_C V_{bc}$; $V_{ud} \approx V_{tb} \approx 1$

It is worthwhile to note that the above conclusions depend strongly on the values of the hadronic matrix elements $< Q_6 >$,f_K,f_B, as well as on the KM mixing angles s_{ij}, which have not been measured with good enough accuracy so far. For example,the determination of $V_{cb} = s_{23} = 0.046 \pm 0.006$ from the $\Gamma(b \longrightarrow c)$ decay rate and the determination of the ratio $q = \left|\frac{V_{ub}}{V_{cb}}\right|$ by ARGUS and CLEO lead to the model dependent results

$$0.07 \leq \left(q = \frac{s_{13}}{s_{23}}\right) \leq 0.12.$$

On the other hand, the unitarity triangle [5] constraint yields for $|V_{td}|$, that $|V_{td}| = |s_{23}(s_{12} - q \times exp(i\delta_{13}))| = (0.0035 - 0.0200)$. The value of V_{td} depends crucially on the δ_{13} phase. For the small values of this matrix element, δ_{13} tends to be in the first quadrant, whereas the experimental values for $\varepsilon_K, \frac{\varepsilon'_K}{\varepsilon_K}, x_d$ favor larger m_t values.

Another interesting possibility to check the sign of $\cos\delta_{13}$ comes from the experimental observation of the $B_s^0 - \bar{B}_s^0$ mixing. Thanks to the following relation between the parameters of the $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings:

$$\frac{x_s}{x_d} = \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{1}{s_{12}^2 - 2qs_{12}cos\delta_{13} + q^2}$$

for $\delta_{13} \approx \pi$ we get $\frac{x_s}{x_d} = 10 - 14$ and $x_s = 7 - 11$, whereas for $\delta_{13} \approx 0$ we get $\frac{x_s}{x_d} = 36 - 100$ and $x_s = 27 - 70$.

The case of the symmetric form for the CKM-matrix [6], which leads to $|V_{td}| = |V_{ub}| = |\frac{1}{2}V_{us}V_{cb}|$ (q $\simeq$ 0.1 in this case), corresponds to δ_{13}-phase $\simeq 0$, $x_s/x_d \simeq 100$ and $x_s \simeq 70$.

From the above discussion one can see that there are several experimental quantities, which are sensitive to the values of the unknown parameters m_t and δ^{KM}. Therefore, we need additional experimental information to prove the validity of the SM with three generations and to get convinced that there are no additional contributions to the amplitudes of rare processes due to new hypothetical forces beyond the SM.

Contrary to the above-mentioned experimental uncertainties, which might signify some new physics beyond the SM but do not so far, there are quite a number of theoretical problems indicative of the existence of such physics. Why the number of generations is three? What is the origin of the δ^{KM}-phase, or what is the nature of CP-violation? Why in the quark sector of the SM there are too many unconnected parameters (mixing angles, phase and quark masses)?

Several models beyond the SM suggest special forms for the mass matrix of "up" and "down" quarks (Fritzsch ansatz, "Democratic" ansatz, etc.) [7]. These mass matrices have less than ten independent parameters, giving thus some relations between the quark masses, mixing angles and the phase. These relations could be checked experimentally. The construction of these mass matrices has to be based on a new symmetry of quark and lepton generations. The fact that the upper bound on m_t is close to the electroweak symmetry breaking scale motivates us to make new efforts towards a deeper understanding of nature of the CKM matrix by relating mixing angles to the ratios of quark masses.

The generation mixing problem is one of the most exciting puzzles [8]. We hope that in the nearest future the horizontal gauge models could shed light upon this problem, at least in the quark sector. So, these models might lead, in the "up" and "down" quark sectors, to a certain mass mixing already known (like Fritzsch ansatz ," democratic", or some other ansatz), which could result in observed charged currents consistent with the experimental shape of the CKM matrix. In this connection, the possibility to look autonomously at the mixing angles in the "up" and "down" sectors seems to be one of the specific features of these new hypothetical interactions. The situation is quite different in the electroweak model, where we can study only the resulting mixing matrix $V_{CKM} = UD^{+}$. So this feature could help us to understand the hierarchy of "up" and "down" quarks ($m_u \ll m_c \ll m_t$; $m_d \ll m_s \ll m_b$), and probably the mass lepton hierarchy. Also, it seems to be impossible to set up the correspondence between quark and lepton generations without including horizontal interactions. It is more than natural that some authors try to achieve understanding of the mixing problem, as well as the hierarchy of the quark and lepton mass problem, by searching for some new physics [8]. It seems evident that the mixing problem and the hierarchy of fermion masses are closely connected with the local symmetry of quark-lepton generations.

A very interesting question is the origin of the KM phase. In the SM, this parameter can be obtained via complex Yukawa constants. However, one knows other, more natural, mechanisms of spontaneous CP-violation. In the framework of spontaneous mechanisms, the EW Lagrangian is CP-invariant, but the vacuum is not. In this case we have to consider the extension of the SM in which the δ^{KM}-phase comes from the spontaneous mechanism of CP-violation on the high energy scale of new gauge interactions. That is why it is more than interesting to consider, first of all, the local gauge symmetry of quark-lepton families. Now we have to extend the fermionic sector, where in addition to ordinary fermions one can introduce into each generation new heavy quarks, singlet under the $SU(2)_L$ group, ("U_i" and "D_i" quarks with $Q_{U_i} = \frac{2}{3}$, $Q_{D_i} = -\frac{1}{3}$, i=1,2,3) [8]. Besides, we introduce the $SU(2)_L - SU(3)_H$-singlet complex scalar fields. In this approach it is not so difficult to get the vacuum breaking the CP-invariance spontaneously due to the complex vacuum expectation values of new scalar fields. In this model, the δ^{KM}-phase might appear in the observable sector of the SM.

Underlying another mechanism to get CP-violation is the suggestion that CP-invariance ($\delta^{KM} = 0$) is conserved in the SM and CP-violation is due to only gauge horizontal interactions [9]. Then, thanks to the horizontal interactions, the superweak nature of CP-violation allows to estimate the upper bound on the lightest horizontal gauge boson mass: $M_H \leq (2-3) \times 10^3 TeV$[11].

The lowest bound on M_H can be obtained from the analysis of the branching ratios of μ, π, K, D, B, ... rare decays (Br$\geq 10^{-15-17}$) [9][10][11].

For all the beauty of this approach, simulating the superweak mechanism of CP-violation for the K_L^0-decays ($10^{-7} \leq \left(\frac{\varepsilon'}{\varepsilon}\right)_K \leq 10^{-3-4}$ [11]), a serious disadvantage of these models is the large number of unknown parameters, i.e. masses of H_μ^a-gauge bosons (a=1,2, ... ,8), "up" and "down" quark mixing angles.

The space-time structure of horizontal interactions depends on the $SU(3)_H$ quantum numbers of quark and lepton superfields, and their C- conjugate superfields [9–14]. One can obtain vector-like horizontal interactions if the particle quantum numbers G_H are conjugate to those of antiparticles. In this case, it is also possible to construct the GUTs proceeding from the direct product of simple Lie groups. The question arising in these theories is how such horizontal interactions are related with strong and electroweak ones. All these interactions can be unified within one gauge group, which would allow to calculate the value of the coupling constant of horizontal interactions. The point is that the unification of strong, electroweak and horizontal interactions can be based on the group $G = SU(4)_C \times SU(4)_W \times SU(4)_H$ with the discrete symmetry $C \leftrightarrow W \leftrightarrow H$ (i.e., $g_C = g_W = g_H$). An alternative unification of horizontal, strong and electroweak interactions might rest on the GUTs $\tilde{G} \equiv G \times SU(3)_H$ (where, for example, $\tilde{G} \equiv E(8)$, $G \equiv SU(5), SO(10)$ or E_6), which may be further broken down to $SU(3)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$. In the alternative case we have to introduce "mirror" superfields. Speaking more definitely, if we want to construct GUTs of the type $\tilde{G} \equiv G \times SU(3)_H$, each generation must encompass double G-matter supermultiplets, mutually conjugate under the $SU(3)_H$−group. In this approach the first supermultiplet consists of the superfields f and $f_m^c \in 3_H$, while the second is constructed with the help of the supermultiplets f^c and $f_m \in \bar{3}_H$. In this scheme, proton decays are possible only if there is a mixing between ordinary and mirror fermions. In turn, this mixing must, in particular, be related with the $SU(3)_H$−symmetry breaking. The $SU(5)$−scale of unification might be considerably lower in this case.

The analysis of modern experimental data on search for rare processes in terms of the horizontal gauge symmetry approach allows, in some particular models, the existence of rather light H-bosons: $m_H \geq (1-10)TeV$ [14]. These estimates imply that the horizontal fine structure constant is $\alpha_H \sim or < \alpha_{EM}$ [11] (Appendix 7.1).

Nevertheless, till now nonabelian gauge models of horizontal interactions have not enjoyed much attention for comparison with experimental data. The reason was the low predictive force of the nonabelian approach because of the unknown mass spectrum of horizontal gauge bosons. In this paper we will analyze, in the framework of the "minimal" horizontal supersymmetric gauge model, the possibilities for obtaining a satisfactory hierarchy for quark masses and connecting it with the splitting of horizontal gauge boson masses. We expect that due to this approach the horizontal model will become more definite allowing thus to study the amplitudes of rare processes and the CP-violation mechanism more thoroughly. In this way we hope to get a deeper insight into the nature of interdependence between the generation mixing mechanism and the local horizontal symmetry breaking scale.

As the fundamental model, we have to consider the $SU(3)_{HV}$− vector-like model in its supersymmetric realization. Before getting down to the studies of this model, we ought to make some general statements:

(i) A horizontal interaction scale might be rather large (for instance, larger than that of left-right symmetry restoration), but today it does not seem unreasonable that it might lie in the " TeV"- region, attainable at the accelerators of new generation like LHC or SSC. The last possibility may be realized in a gauge horizontal model, provided it is taken into account that the correlation between the lepton (L_i)- and quark Q_j- family mixings cannot be defined in the frames of the standard model. This is a consequence of quark - lepton universality in the SM of electroweak interactions.

(ii) The predictions of this model depends, to a considerable degree, on our knowledge of the nature of quark and lepton masses. So, what we face today is the unsatisfactory understanding of the difference in the origin of up- (or down) quarks and charged lepton mass matrices. The vital question arising here is the nature of the ν mass. All our predictions for the gauge horizontal breaking symmetry scale, especially those resulting from the experimental searches for quark-lepton rare processes, are very sensitive to the origin of quark and lepton mass spectra and their corresponding mixings. It seems- at first glance, at least- that the known quark and lepton mass spectra might imply the existence of new heavy quarks and leptons.

(iii) The vector character of horizontal interactions is crucial for the construction of GUTs. The reasons have already been elucidated. Here we just underline that the insertion of horizontal gauge models into the GUT-structure based on a simple Lie group results, on the one hand, in doubling the matter spectrum and, on the other, can lead to a decrease of the Grand Unification scale. Also, if we compare this approach with the ordinary GUTs, we have to mention the distinctive character of the main proton decay channels in these models. In this case the proton can mainly decay in a multi-particle way: $p \to m \cdot l + n \cdot \nu + (\pi \, and/or \, K - mesons)$; $m + n = 2k + 1$.

(iv) The N=1 SUSY character. Considering the SUSY version of the $SU(3)_{HV}$−model, it is natural to ask: why do we need to supersymmetrize the model? Proceeding from our present-day knowledge of the nature of supersymmetry, the answer will be:

(1) First, it is necessary to preserve the hierarchy of the scales: $M_{EW} < M_{SUSY} < M_H < \cdots ? \cdots < M_{GUT}$ Breaking the horizontal gauge symmetry, one has to preserve SUSY on that scale. Another sample of hierarchy to be considered is : $M_{EW} < M_{SUSY} \sim M_H$. In this case, the scale M_H should be pretty low ($M_H \leq$ a few TeV).

(2) Using the SUSY $U(1)_R$ degrees of freedom, one can construct the superpotential and forbid undesired Yukawa couplings.

(3) Super-Higgs mechanism - the possibility of describing Higgs bosons by means of massive gauge superfields [15].

(4) The connection between the vector- like character of the $SU(3)_H$-gauge horizontal model and $N = 2$ SUSY.

Also, the existence of horizontal interactions might be closely connected with the CP-violation problem. Here noteworthy are the following two points:

(a) The appearance of the phase in the CKM mixing matrix may be due to new dynamics working at short distances ($r \ll \frac{1}{M_W}$). The horizontal forces may be the source of this new dynamics [9]. As result of this approach we might have the CP- violation effects- both due to electroweak and horizontal interactions.

(b) The CP is conserved in the electroweak sector ($\delta^{KM} = 0$), and its breaking is provided by the structure of the horizontal interactions [1] In particular, this model gives rise to a rather natural mechanism of superweak CP-violation due to the $(CP = -1)$ part of the effective Lagrangian of horizontal interactions- $(\epsilon'/\epsilon)_K \leq 10^{-4}$. That part of $\mathcal{L}_{\text{eff}}$ includes the product of the $SU(3)_H$-currents $I_{\mu i}$ $I_{\mu j}$ (i=1,4,6,3,8; j=2,5,7 or, vice versa, i$\longleftrightarrow$j) [9]. In the case of vector- like $SU(3)_H$- gauge model CP- violation could be only due to charge symmetry breaking.

In the present paper we will investigate the samples of different scenarios of $SU(3)_H$ breakings up to the $SU(2)_H \times U(1)_H$, $U(1)_H \times U(1)_H$ and $U(1)_H$ subgroups [13], as well as the mechanism of the complete breaking of the base group $SU(3)_H$ [14]. We will try to realize this program conserving $SUSY$ on the scales where the relevant gauge symmetry is broken. In the framework of these versions of the gauge symmetry breaking, we will search for the spectra of horizontal gauge bosons and gauginos and calculate the amplitudes of some typical rare processes. The theoretical estimates for the branching ratios of some rare processes obtained as a result of these calculations will be compared with the experimental data on the corresponding values. After that we will get some bounds on the masses of H_μ-bosons and the appropriate H-gauginos. Of particular interest is the case of the $SU(3)_H$-group which breaks completely on the scale M_{H_0}. We calculate the splitting of eight H-boson masses in a model dependent fashion. This splitting, depending on the quark mass spectrum, allows us to reduce considerably the predictive ambiguity of the model. In our studies of the splitting of eight gauge H-boson masses we were guided by three approaches to constructing the quark mixing matrix : (1) the Fritzsch ansatz with (3+1)- generations [6a]; (2) the modified Fritzsch ansatz [6c], and the so-called "democratic" ansatz [6b] with 3- generations.

The main thing we want to understand is the mechanism of the unitary compensation for the contributions of horizontal forces to rare processes [9]. In particular, we are interested in the dependence of this unitary compensation on different versions of the $SU(3_H)$- symmetry breaking. The investigation of this dependence allows, first, to understand how low the horizontal symmetry breaking scale M_H may be, and, second, how this scale is determined by a particular choice of a mass matrix ansatz both for quarks and leptons. We will analyze the possibilities of the existence of a very small scale

[1] Let us think of the situation when $\delta^{KM} = 0$. In the SM, such a case might be realized just accidentally. The vanishing phase of the electroweak sector ($\delta^{KM} = 0$) might arise spontaneously due to some additional symmetry. Again, such a situation might occur within the horizontal extension of the electroweak model.

($\sim someTeV$) for the horizontal gauge symmetry breaking. We will try to establish the important rare processes in which these interactions are most pronounced.

2. Construction of the model

2.1. The $SU(3)_{HV}$ Lagrangian

We will consider the supersymmetric version of the Standard Model extended by the family (horizontal) gauge symmetry. The supersymmetric Lagrangian of strong, electroweak and horizontal interactions based on the $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_H$-gauge group has the general form:

$$\begin{aligned}
\mathcal{L} &= (\sum_k \int d^2\theta \; Tr \; (W^k \, W^k) + \; h.c.) \\
&+ \; \sum_I \int d^4\theta \; S_I^+ \; e^{\sum_k 2g_k \hat{V}_k} \; S_I \\
&+ \; \int d^4\theta \; Tr(\Phi^+ \; e^{2g_H \hat{V}_H} \; \Phi \; e^{-2g_H \hat{V}_H}) \\
&+ \; \int d^4\theta \; Tr(H_y^+ \; e^{2g_2 \hat{V}_2 + y 2 g_1 \hat{V}_1} \; e^{2g_H \hat{V}_H} \; H_y \; e^{-2g_H \hat{V}_H}) \\
&+ \; (\int d^2\theta \; P(S_i, \Phi, H_y, \eta, \xi, ...) + \; h.c.) \qquad (2.1)
\end{aligned}$$

In formula (2.1) index k runs over all gauge groups: $SU(3)_C, SU(2)_L, U(1)_Y$, $SU(3)_H$, $\hat{V} = \mathbf{T}^a V^a$, where V^a are the real vector superfields , and $\mathbf{T}^a$ are the generators of the $SU(3)_C, SU(2)_L, U(1)_Y, SU(3)_H$ -groups; S_I are left-chiral superfields transforming in fundamental representations, and $I = i, 1, 2$; $S_i = Q, u^c, d^c, L, e^c, \nu^c$- are matter superfields, $S_1 = \eta, S_2 = \xi$- are Higgs fundamental superfields; the Higgs left chiral superfield Φ transforms according to the adjoint representation of $SU(3)_H-$group, the Higgs left chiral superfields H_y : $H_{Y=+\frac{1}{2}} = H$, $H_{Y=-\frac{1}{2}} = h$ transform nontrivially under the horizontal $SU(3)_H-$ and electroweak $SU(2)_L$ - symmetries (see Table 1). P in formula (2.1) is a superpotential to be specified below. To construct it, we use the internal $U(1)_R-$symmetry which is habitual for a simple N=1 supersymmetry.

2.2. Supersymmetry breaking

It is known to be impossible to achieve simultaneously in models with a global supersymmetry a realistic $SUSY$ breaking and vanishing of a cosmological term. The reason is the semipositive definition of the scalar potential in the rigid supersymmetry approach (in particular, in the case of a broken $SUSY$ we have $V_{min} > 0$). The problem of supersymmetry breaking, with the cosmological term $\Lambda = 0$ vanishing, is solved in the framework of the $N = 1$ $SUGRA$ models. This could be done under the appropriate choice of the Kaehler potential, in particular, in the frames of "mini-maxi"- or "maxi" type models[16].

In such approaches the spontaneous breaking of the local $SUSY$ is due to the possibility to get nonvanishing VEVs for the scalar fields from the "hidden" sector of $SUGRA$[16]. The appearance of the so-called soft breaking terms in the observable sector comes as a consequence of this effect.

In the "flat" limit, i.e. neglecting gravity, one is left with lagrangian (2.1) and soft SUSY breaking terms, which on the scales $\mu << M_{Pl}$ have the form:

$$\begin{aligned} \mathcal{L}_{SB} &= \frac{1}{2}\sum_i m_i^2|\phi_i|^2 + \frac{1}{2}m_1^2 Tr|h|^2 + \frac{1}{2}m_2^2 Tr|H|^2 + \\ &+ \frac{1}{2}\mu_1^2|\eta|^2 + \frac{1}{2}\mu_2^2|\xi|^2 + \frac{1}{2}M^2 Tr|\Phi|^2 + \\ &+ \frac{1}{2}\sum_k M_k \lambda_k^a \lambda_k^a + h.c. + \text{trilinear terms}, \end{aligned} \tag{2.2}$$

where i runs over all the scalar matter fields $\tilde{Q}$, $\tilde{u}^c$, $\tilde{d}^c$,$\tilde{L}$, $\tilde{e}^c$, $\tilde{\nu}^c$ and k - over all gauge groups: $SU(3)_H$, $SU(3)_C$, $SU(2)_L$, $U(1)_Y$. At the energies close to the Plank scale all the masses, as well as the gauge coupling, are correspondingly equal (this is true if the analytic kinetic function satisfies $f_{\alpha\beta} \sim \delta_{\alpha\beta}$)[11], but at low energies they have different values depending on the corresponding renormgroup equation (RGE). The squares of some masses may be negative, which permits the spontaneous gauge symmetry breaking (like in Ref.[11]).

2.3. The spontaneous breaking of $SUSY$ $SU(3)_{HV}$ horizontal gauge symmetry.

Since the expected scale of the horizontal symmetry breaking is sufficiently large: $M_H >> M_{EW}$, $M_H >> M_{SUSY}$ (where M_{EW} is the scale of the electroweak symmetry breaking, and M_{SUSY} is the value of the splitting into ordinary particles and their superpartners), it is reasonable to search for the SUSY-preserving stationary vacuum solutions.

Let us construct the gauge invariant superpotential P of Lagrangian (2.1). With the fields given in Table 1, the most general superpotential will have the form

$$\begin{aligned} P &= \lambda_0\Big[\frac{1}{3}Tr(\hat{\Phi}^3)+\frac{1}{2}M_I Tr(\hat{\Phi}^2)\Big]+\lambda_1\Big[\eta\hat{\Phi}\xi+M'\eta\xi\Big]+\lambda_2 Tr(\hat{h}\hat{\Phi}\hat{H})+ \\ &+\lambda_3 Q\hat{H}d^c+\lambda_4 L\hat{H}e^c+\lambda_5 Q\hat{h}u^c+\ (\text{ Majorana terms } \nu^c\ ). \end{aligned} \quad (2.3)$$

Table 1. The Matter and Higgs Superfields with their Relative $SU(3)_H$, $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ and $U(1)_H$ Quantum Numbers

	H	C	L	Y	Y_H		H	C	L	Y	Y_H
Q	3	3	2	1/6	1	Φ	8	1	1	0	0
u^c	$\bar{3}$	$\bar{3}$	1	−2/3	−2	H	8	1	2	−1/2	−1
d^c	$\bar{3}$	$\bar{3}$	1	1/3	0	h	8	1	2	1/2	1
L	3	1	2	−1/2	1	ξ	$\bar{3}$	1	1	0	−4
ν^c	$\bar{3}$	1	1	0	−2	η	3	1	1	0	4
e^c	$\bar{3}$	1	1	1	0						

The spontaneous breaking of the horizontal symmetry can be constructed by different scenarios -both with intermediate scale, and without it:

$$\begin{aligned} &(i) \quad SU(3)_H \xrightarrow{M_I} SU(2)_H \times U(1)_H \xrightarrow{M_H} \text{completely breaking} \\ &(ii) \quad SU(3)_H \xrightarrow{M_I} U(1)_H \times U(1)_H \xrightarrow{M_H} \text{c.b.} \\ &(iii) \quad SU(3)_H \xrightarrow{M_I} U(1)_H \xrightarrow{M_H} \text{c.b.} \\ &(iv) \quad SU(3)_H \xrightarrow{M_H} \text{c.b.} \end{aligned} \quad (2.4)$$

If we assume that the soft breaking mass parameters in formula (2.2) should not be more than 0(1 TeV), then the soft breaking terms on the scale M_I of the $SU(3)_{HV}$ intermediate breaking may be neglected, and it is possible to go on working in the approximation of conserved SUSY. The SUSY preserving stationary vacuum solutions are degenerate in the models with global SUSY. In the construction of the stationary solutions, only the following contributions of the scalar potential are taken into account:

$$V = \sum_i |F_i|^2 + \sum_a |D^a|^2 = V_F + V_D \geq 0 \quad (2.5)$$

$$\text{where} \quad V_F = \sum \left|\frac{\partial P_F}{\partial F_i}\right|^2 = \left|\frac{\partial P_F}{\partial F_{\Phi^a}}\right|^2 + \left|\frac{\partial P_F}{\partial F_{\xi_i}}\right|^2 + \left|\frac{\partial P_F}{\partial F_{\eta_i}}\right|^2 \quad (2.6)$$

The case $< V >= 0$ of supersymmetric vacuum can be realized within different gauge scenarios (2.4). By switching on the SUGRA, the vanishing scalar potential is no more required to conserve supersymmetry with the necessity. Hence, different gauge breaking scenarios (2.4) do not result in obligatory vacuum degeneracy, as in the case of the global SUSY version. Let us write down each of the terms of formula (2.6):

$$\begin{aligned} P_F(\Phi, \xi, \eta) &= \lambda_0 \Big[\frac{i}{4\times 3} f^{abc}\Phi^a\Phi^b\Phi^c + \frac{1}{4\times 3} d^{abc}\Phi^a\Phi^b\Phi^c + \frac{1}{4} M_I \Phi^c\Phi^c \Big]_F + \\ &+ \lambda_1 \Big[\eta_i \, (T^c)^i_j \, \xi^j \Phi^c + M' \eta_i \xi^i \Big]_F + \qquad (2.7) \\ &+ \lambda_2 \Big[\frac{i}{4} f^{abc} \, h^a_i \Phi^b H^c_j \epsilon^{ij} + \frac{d^{abc}}{4} \, h^a_i \Phi^b H^c_j \epsilon^{ij} \Big]_F + h.c. \end{aligned}$$

The contribution of D-terms into the scalar potential will be :

$$\begin{aligned} V_D &= g_H^2 |\eta^+ T^a \eta - \xi^+ T^a \xi + i/2 \; f^{abc}\Phi^b\Phi^{c+} + i/2 \; f^{abc} h^b h^{c+} + i/2 \; f^{abc} H^b H^{c+}|^2 \\ &+ g_2^2 |h^+ \tau^i/2 \; h + \; H^+ \tau^i/2 \; H|^2 + (g')^2 |1/2 \; h^+ h - \; 1/2 \; H^+ H|^2 \qquad (2.8) \end{aligned}$$

The SUSY-preserving condition for scalar potential (2.5) is determined by the flat F_i- and D^a directions: $< F_i >_0 = < D^a >_0 = 0$. It is possible to remove the degeneracy of the supersymmetric vacuum solutions taking into account the interaction with supergravity, which was endeavored in SUSY GUT's, e.g. in the $SU(5)$ one[12] ($SU(5) \to SU(5), SU(4)\times U(1), SU(3)\times SU(2)\times U(1)$).

The horizontal symmetry spontaneous breaking to the intermediate subgroups in the first three cases of (2.4) can be realized, using the scalar components of the chiral complex superfields Φ, which are singlet under the standard gauge group. The Φ-superfield transforms as the adjoint representation of $SU(3)_H$. The intermediate scale M_I can be sufficiently large: $M_I > 10^5 - 10^6$GeV.

The complete breaking of the remnant symmetry group V_H on the scale M_H will be occurred due to the nonvanishing VEV's of the scalars from the chiral superfields $\eta(3_H)$ and $\xi(\bar{3}_H)$. Again, the V_{min} corresponds to the flat directions: $< F_{\eta,\xi} >_0 = 0$. The version (iv) corresponds to the minimum of the scalar potential in the case when $< \Phi >_0 = 0$.

As for the electroweak breaking, it is due to the VEV's of the fields h and H, providing masses for quarks and leptons. Note that VEV's of the fields h and H must be of the order of M_W as they determine the quark and lepton mass matrices. On the other hand, the masses of physical Higgs fields h and H, which mix the generations, must be some orders higher than M_W, so as not to contradict the experimental restrictions on FCNC. As a careful search for the Higgs potential shows, this is the picture that can be attained.

3. The intermediate horizontal symmetry breaking

As was noted in the previous Section, the spontaneous horizontal gauge symmetry breaking takes place when the fields ϕ, η and ξ get nonvanishing VEVs. We are interested in the possibility of realizing the structure, when some of the horizontal gauge bosons (and the corresponding gauginos) may have relatively small masses ($M_H \sim 1-10$TeV) [13]. Our consideration of the family symmetry breaking will be done in two steps. To this end, we look for the SUSY stationary vacuum solutions, such that $< \Phi >_0 \gg < \eta >_0, < \xi >_0$. So, the degeneracy of the corresponding H-gauge bosons is assumed near one or two scales. The complete breaking of the $SU(3)$ group corresponds to the "condensation" of all eight bosons near the M_H scale. For intermediate $SU(3)_H$ breakings, some of the gauge massive superfields will have the masses around the scale M_I, while the other superfields from the remnant symmetry group will be condensed on the scale $M_H (M_H << M_I)$. We will analyze several subgroups of $SU(3)_H$ and check if the low scale M_H is consistent with the experimental data for these models. Such an analysis will allow us to get a deeper insight into the dynamics of horizontal forces and investigate the effects of their compensation, especially in pure leptonic and pure quark processes. At first stage, due to the nonvanishing VEV of Φ, the horizontal symmetry group breaks down to some subgroup V satisfying $[V, < \Phi >_0] = 0$. At the second stage, the remnant group V will be broken completely, as fields η and ξ will acquire nonzero VEVs. Let us consider several cases of this breaking.

Case (i): $V = SU(2) \times U(1)$. As has already been mentioned, in the gauge model with the global SUSY stationary supersymmetry conserving vacuum solutions are degenerate: $V_{min} = 0$. Let us recall that the superinvariance condition for the model on the scale M_I requires the existence of flat D^a- and F_Φ^a- directions: $< D^a >_0 = < F_\Phi^a >_0 = 0 \quad (a = 1,2,3,8)$. Equations (2.6-2.8) give the following form of these constraints:

$$\begin{aligned} 1/2\ d^{abc}(\Phi_1^a\Phi_1^b - \Phi_2^a\Phi_2^b) + M_I\Phi_1^c = 0 \\ d^{abc}\Phi_1^a\Phi_2^b + M_I\Phi_2^c = 0 \end{aligned} \qquad (< F_\Phi >_0 = 0) \qquad (3.1)$$

$$i f^{abc}\Phi^b\Phi^{c+} = 0 \qquad (< D^a >_0 = 0), \qquad (3.2)$$

where $\Phi^a = \Phi_1^a + i\Phi_2^a$, d^{abc} and f^{abc} are the $SU(3)$ structure constants. From equations (3.1) and (3.2) it is easy to verify that the SUSY $SU(3)_H$ group can be broken down to the SUSY $SU(2)_H \times U(1)_H$ if, for example, the 8-th component of the field Φ acquires a nonvanishing VEV:

$$< \Phi^8 >_0 = \frac{\sqrt{3}a_8}{2} = \frac{\sqrt{3}M_I}{2} \qquad (3.3)$$

In this case of the gauge symmetry breaking the supersymmetry conservation allows to describe the mass spectrum of new massive $N = 1$ supermultiplets in a rather simply way. We start with eight vector massless superfields $V_H^a(1,\ 1/2)$ $(4 \times 8^a = 32$ degrees of freedom) and eight chiral massless superfields $\Phi^a(1/2\ ;\ 0,\ 0)$ $(4 \times 8^a = 32$ degrees of freedom). As a result of the super-Higgs effect, we get four massive vector supermultiplets $(1\ ,\ \frac{1}{2}) + (\frac{1}{2}\ ,\ 0 + 0) = (1\ ,\ \frac{1}{2} + \frac{1}{2}\ ,\ 0)_{massive}$ with $8 \times 4^a = 32$ degrees of freedom and with the same universal mass. The formula for the gauge boson mass is

$$\begin{aligned} (M^2)_{ab} &= 1/2\ g_H^2 f^{8ac} f^{8bc} a_8^2 = 3/8\ g_H^2 a_8^2 \delta^{ab} \\ a,b &= 4,5,6,7 \quad or \\ M^2_{4,5,6,7} &= 3/8\ g_H^2 M_I^2\ ,\ \ M^2_{1,2,3,8} = 0 \end{aligned} \qquad (3.4)$$

The mass term of λ-gauginos is expressed as follows:

$$\begin{aligned} \mathcal{L}_M &= 1/\sqrt{2}\ g_H f^{8bc}\psi_\Phi^b\lambda^c a_8 = \frac{\sqrt{3}\ g_H}{2\ \sqrt{2}}\ M_I[\psi_\Phi^4\lambda^5 - \psi_\Phi^5\lambda^4 + \psi_\Phi^6\lambda^7 - \psi_\Phi^7\lambda^6] \\ &- M_I\ 3/4\ (\psi_\Phi^1\psi_\Phi^1 + \psi_\Phi^2\psi_\Phi^2 + \psi_\Phi^3\psi_\Phi^3 -\ 1/3\ \psi_\Phi^8\psi_\Phi^8) + h.c. \end{aligned} \qquad (3.5)$$

So the gauginos $\lambda^4, \lambda^5, \lambda^6, \lambda^7$ combining with fermions $\psi_\Phi^4, \psi_\Phi^5; \psi_\Phi^6, \psi_\Phi^7$ give the Dirac gauginos with the masses $M = \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} g_H M_I$. Four real scalar states from the supermultiplets $\Phi^{4,5,6,7}$ transform into the longitudinal components of four corresponding massive vector bosons, while the remaining four scalar states contribute to four massive $N = 1$ supermultipets $(1\ ,\ \frac{1}{2} + \frac{1}{2}\ ,\ 0)_{massive}$. There are also four massless vector superfields $V_H^{1,2,3,8}$ (16 degrees of freedom) and four massive chiral superfields (ψ_Φ^a, Φ^a) $(a = 1, 2, 3, 8)$ at this stage of breaking. So, due to the super-Higgs mechanism of SUSY breaking four massless vector superfields have absorbed four massless chiral superfields and formed four massive vector superfields. The chiral superfields $\Phi^{4,5,6,7}$ play the role of Higgs superfields and they all have been absorbed completely.

At the second stage of the $SU(2)_H \times U(1)_H$ gauge symmetry breaking with a simultaneous supersymmetry conservation, one can use the chiral superfields η_α, ξ_α with equal VEV's: $< \eta_{\alpha i} >_0 = < \xi^i_\alpha >_0 = \delta^i_\alpha \gamma$ $(i, \alpha = 1, 2, 3,)$. As a result of this breaking, four massive vector supermultiplets $V_H^{1,2,3,8}(1\ ,\ \frac{1}{2} + \frac{1}{2}\ ,\ 0)$ acquire the universal mass $M_H^2 = 2g_H^2\gamma^2$. A detailed analysis shows that the Majorana higgsinos from the supermultiplets η_α, ξ_α participate in the formation of four Dirac gauginos, whose upper components are $\lambda^1, \lambda^2, \lambda^3$ and λ^8. The degenerate mass of these Dirac gauginos will be $\sqrt{2} g_H \gamma$:

$$\begin{aligned} \mathcal{L}_M &= i/\sqrt{2}\ \ g_H\gamma\Big\{\lambda^3[(\eta_{11} - \xi^1_1) - (\eta_{22} - \xi^2_2)] + \ 1/\sqrt{3}\ \ \lambda^8[(\eta_{11} - \xi^1_1) + \\ &+ \ (\eta_{22} - \xi^2_2) - 2(\eta_{33} - \xi^3_3)] + \lambda^1[(\eta_{21} - \xi^2_1) + (\eta_{12} - \xi^1_2)] \\ &- \ i\lambda^2[(\eta_{12} + \xi^1_2) + (\eta_{21} + \xi^2_1)]\Big\} + h.c. \end{aligned} \tag{3.6}$$

It is easy now to rewrite the Lagrangian of the interactions in terms of physical states (remembering that for the matter fields $\psi_{mi} = U_{ij}\psi_{oj}$ and $A_{mi} = \tilde{U}_{ij}A_{oj}$, where A denotes the scalar partner of ψ–fermions). The gauge boson interactions with matter fields have the form:

$$\begin{aligned} \mathcal{L} = g_H H^\mu_a\ \ \Big\{ & \ \bar{\psi}_u\gamma_\mu U_L T^a U_L^+ \ \frac{1+\gamma_5}{2}\ \psi_u + \bar{\psi}_u\gamma_\mu U_R T^a U_R^+ \ \frac{1-\gamma_5}{2}\ \psi_u + \\ & + \ (u \to d, l, \nu)\Big\} \end{aligned} \tag{3.7}$$

Let us consider now the gaugino interactions. The initial Lagrangian has the form:

$$\mathcal{L} = ig_H\sqrt{2}(A_i^+ T^a_{ij}\lambda^a\psi_j - h.c.) \quad a = 1, 2, 3, 8 \tag{3.8}$$

Consider only the interaction between left "up" quarks, left "up" squarks and gauginos. The generalization of this Lagrangian to all leptons and quarks will be obtained by simply adding similar terms to u_R, d, d_R, ν, l and l_R. Expression (33.8) for "up" quarks looks like:

$$\mathcal{L} = ig_H\Big(A_1^*,\ A_2^*,\ A_3^*\Big)_L \begin{pmatrix} \lambda^3+\frac{1}{\sqrt{3}}\lambda^8 & \lambda^+ & 0 \\ \lambda^- & -\lambda^3+\frac{1}{\sqrt{3}}\lambda^8 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}\lambda^8 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} + h.c. =$$

$$= -g_H\Big[A_{iL}^*\tilde{U}_{ik}(\Lambda')_{bk}U_{jb}^*\ \frac{1+\gamma_5}{2}\ \psi_j\Big] + h.c. \qquad (3.9)$$

where $A_{iL} = \tilde{U}_{ij}A_{0jL}$, $i, j, ..., b = 1,\ 2,\ 3$.

Case (ii). To realize this version of the intermediate $SU(3)_H$ symmetry breaking, one has to use the pair of the chiral superfields Φ, $\bar{\Phi}$ with different $U(1)_R$ quantum numbers. Then one easily verifies that the stationary supersymmetric vacuum solutions will be realized in accordance with equations (2.6-2.8). These solutions will look like

$$< \Phi_1^3 >_0 = < \Phi_1^8 >_0 = \ -\sqrt{3}M\ , \quad < \bar{\Phi}_1^8 >_0 = \ 2\sqrt{3}M \qquad (3.10)$$

When these fields are applied simultaneously with the above VEVs (3.10), the following gauge boson mass spectrum is obtained: $M_{H_4} = M_{H_5} = 3\sqrt{2}M_I$, $M_{H_1} = M_{H_2} = M_{H_6} = M_{H_7} = 3M_I$, $M_{H_3} = M_{H_8} = 0$. The remnant group in this case is the $SUSY\ U(1)_{T_3} \times U(1)_{T_8}$ - group. As one would expect, the rank of the group did not change, whereas the remnant group was broken by the chiral superfields η, ξ on the scale M_H. Here it makes no difficulty to get the mass degeneracy of the superfields V_3 and V_8 conserving SUSY while doing this. Again, the super-Higgs mechanism is applied leading to the formation of the massive superfields with the universal mass M_H. In this connection, a rather simple way be proposed to estimate the bound on M_H from the comparison with the data on rare processes.

And, finally, consider case (iii) when $V = U(1)_{T_8}$. We confine ourselves to the case when the scalar components of the complex chiral superfield $\bar{\Phi}$ have the nonzero VEVs: $< \Phi_1^1 >_0 \neq 0$, $< \Phi_1^2 >_0 \neq 0$, $< \Phi_1^3 >_0 \neq 0$, $< \Phi_1^8 >_0 \neq 0$.

Although this choice of VEV's fulfils the equations for the flat F_Φ directions with the solutions $< \Phi_1^8 >_0 = -\sqrt{3}M_I$, $< \Phi_1^1 >_0^2 + < \Phi_1^2 >_0^2 - < \Phi_2^3 >_0^2 = 9M_I^2$, this solution does not determine the vacuum of the theory as might be expected. The corresponding solutions for $D^{1,2}$ are incompatible with the F_Φ-flat solutions. As in the previous case (ii), in order to overcome this difficulty one has to introduce a new Higgs superfield to compensate for the nonvanishing contributions of D-terms to the scalar potential of the theory. This compensation requires a specific choice of the vacuum expectations for the second Higgs superfield $\tilde{\Phi}$. In this case, only one vector supermultiplet V_H^8 is left on the intermediate scale.

The above examples are enough to research further into the regularity of the behavior of the violation scale M_H by comparing model predictions with experiment. Here we just note that the SUSY stationary solutions with CP violation in the horizontal sector are available both for the scales M_I and M_H. Indeed, for instance, in case (iii) the CP violation occurs on the scale M_I. Another supersymmetric vacuum, $< \Phi_1^8 >_0 = -\sqrt{3}M_I$, $< \Phi_1^1 >_0^2 + < \Phi_1^2 >_0^2 + < \Phi_1^3 >_0^2 = 9M_I^2$, corresponding to the intermediate symmetry group $SU(2)_H \times U(1)_H$, results in the CP violation in the neutral K-meson decays due to only horizontally acting forces on the scale M_H. In the last case, in the electroweak sector of CP violation one may have $\delta_{KM} = 0$. That is the very case outlined in our introduction.

4. The role of horizontal interactions with intermediate symmetry breaking scale in rare processes

Let us analyze the contribution of horizontal interactions to rare processes. We will consider first the oscillations of K^0, B_d^0 and B_s^0 mesons. The experimental data on $K^0 \longleftrightarrow \bar{K}^0$ oscillations are as follows:

$$\left(\frac{\Delta m_K}{m_K}\right)_H < \left(\frac{\Delta m_K}{m_K}\right)_{exp} < 7 \times 10^{-15} \qquad (4.1)$$

The theoretical expression for the $m(K_L) - m(K_S) = \Delta m_K$ mass difference is given by the equation:

$$\begin{aligned}\Delta m_K &= \frac{g_H^2}{M_H^2} Re(C_K^0) f_K^2 m_K \Big[\frac{7}{6}\, R_{1P}^V + \frac{1}{6}\, R_{1P}^A + \frac{1}{6} + \frac{1}{3}\, R_{1P}^P + \\ &+ \frac{1}{3}\frac{m_K^2}{(m_s - m_d)^2} - \frac{1}{3}\, R_{1P}^S\Big], \end{aligned} \qquad (4.2)$$

where $C_K^0 = \Sigma_a'(DT^aD^+)_{21}(DT^aD^+)_{21}$, C_K^0 being a unitary coefficient showing the contributions of the Feynman diagrams with the exchange of the horizontal bosons from the considered gauge groups. The symbol "$\prime$" denotes that the sum is over the definite set of indexes "a", but it should be noted that the sum over the complete set (a=1,2,...8) is equal to zero. "D" is the orthogonal matrix diagonalizing the Fritzsch-like mass matrix for "down" quarks, R_{1P}^V and R_{1P}^A are one-particle contributions to the vector and axial currents calculated in Ref. [17]; R_{1P}^S and R_{1P}^P are one-particle contributions to the scalar and pseudoscalar currents, their calculation is analogous.

From formula (4.2), using the experimental data and the values for $\alpha_H = g_H^2/4\pi \approx 1.9 \cdot 10^{-2}$ on the scale M_H [11],[13], one can obtain the lower limits on the light H-boson masses . These values are given in Table 2 together with the C_K^0 values.

Analogous expressions can be obtained for the B_d and B_s mesons. The mixing elements for B_d and B_s mesons are as follows: $C_{B_d}^0 = \Sigma_a'(DT^aD^+)_{31}(DT^aD^+)_{31}$ and $C_{B_s}^0 = \Sigma_a'(DT^aD^+)_{32}(DT^aD^+)_{32}$. Their values are given in Table 2. Using the H-boson mass limits from Table 2 and the value $f_{B_d} \approx 150$ MeV, one can calculate the B_d meson mass difference (see Table 2). The one-particle contribution (R_1) to the B_d (and B_s) meson amplitudes is unknown. But, assuming that it is not much greater than the vacuum contribution, one can see that the $\Delta m_{B_d}/m_{B_d}$ values given in Table 2 are very close to those obtained from ARGUS [18] (except for case (ii)):

$$\left(\frac{\Delta m_{B_d}}{m_{B_d}}\right)_H < \left(\frac{\Delta m_{B_d}}{m_{B_d}}\right)_{exp} = (0.73 \pm 0.14) \times 10^{-13} \qquad (4.3)$$

Table 2. The $M_H, C_K^0, C_{B_d}^0, C_{B_s}^0$, $\Delta m_{B_d}/m_{B_d}$ and $\Delta m_{B_s}/\Delta m_{B_d}$ Values for Different Models.

Models	C_K^0	M_H (TeV)	$C_{B_d}^0$	$\frac{\Delta m(B_d)}{m(B_d)}$	$\frac{\Delta m(B_s)}{\Delta m(B_d)}$
$SU(2)_H \times U(1)_H$	3.8×10^{-5}	$8 \div 9$	1.5×10^{-3}	$(1.1 \div 0.8) \times 10^{-13}k$	15.7
$U(1)_{3H} \times U(1)_{8H}$	1.6×10^{-2}	$170 \div 195$	1.5×10^{-3}	$(2.3 \div 1.8) \times 10^{-16}k$	16.2
$U(1)_{8H}$	2.8×10^{-5}	$7 \div 8$	1.1×10^{-3}	$(1 \div 0.8) \times 10^{-13}k$	15.7
$SU(3)_H$	$< (10^{-5} \div 10^{-6})$	$O(1\,TeV)$	–	–	–

$k = (R_{1P} + 1/2)$, $g_H \approx g_{EW}$.

Let us calculate now the relative mass difference of mesons B_d and B_s. Assuming that $f_{B_s} \approx f_{B_d}$, one obtains the values given in Table 2. So the oscillations of B_s mesons in such models must be stronger than those of B_d mesons. Let us consider now the decay $\mu \to 3e$. The branching ratio of this process will be

$$B(\mu \to 3e) = 12 \frac{g_H^4}{g_W^4} \frac{m_W^4}{M_H^4} |C^0_{(\mu)}|^2 \qquad (4.4)$$

where $C^0_{(\mu)} = \Sigma'_a (LT^aL^+)_{21}(LT^aL^+)_{11} = \Sigma'_a L^a_{21}L^a_{11}$; L- is the orthogonal matrix diagonalizing the real Fritzsch- like mass matrix for "down" leptons. Using the experimental value [19] $B(\mu \to 3e) < 10^{-12}$, it is easy to obtain the limits for the horizontal boson masses:

$$\begin{array}{llll} (i) & M_H > 7.5\, TeV & (ii) & M_H > 28.0\, TeV \\ (iii) & M_H > 4.8\, TeV & (iv) & M_H > O(1\, TeV) \end{array} \qquad (4.5)$$

Let us turn next to the process of the muon-to-electron conversion in the presence of a nucleus. The branching ratio of this process for the nucleus with equal numbers of protons and neutrons and large Z is [10]:

$$\frac{\Gamma(\mu N \to eN)}{\Gamma(\mu N \to \nu N)} = 432 \frac{|\Sigma'_a L^a_{21}[U^{a\,*}_{11} + D^{a\,*}_{11}]|^2}{1/4\,(1+3g_A^2)} \frac{m_W^4}{M_H^4} \frac{g_H^4}{g_W^4} \qquad (4.6)$$

In eq.(4.6), L^a_{21}, U^a_{11} and D^a_{11} are the mixing elements for leptons, up- and down-quarks. Using the recent experimental value for the μ to e conversion : $\Gamma(\mu N \to eN) < \Gamma(\mu N \to \nu N) \times 5 \cdot 10^{-12}$ [20], from eq. (4.6) one can obtain the limits for the horizontal gauge bosons masses:

$$\begin{array}{llll} (i) & M_H > 60.4 \div 97.4 TeV & (ii) & M_H > 59.9 \div 64.2\, TeV \\ (iii) & M_H > 10.5 \div 11.3 TeV & (iv) & M_H > 62.8 \div 124.3 TeV \end{array} \qquad (4.7)$$

In these estimates we have used that L is an orthogonal matrix diagonalizing the real Fritzsch-like leptons mass matrix, and D,U are unitary matrices diagonalizing the complex Fritzsch-like quark mass matrices. So, the limits for the horizontal gauge boson masses depend on the conditions for the complex phases. We chose for the phases to be: $\alpha_A - \tilde{\alpha}_A = (\alpha_d - \beta_d) - (\alpha_u - \beta_u) \simeq \frac{\pi}{2}, \alpha_B - \tilde{\alpha}_B = (\alpha_d - \gamma_d) - (\alpha_u - \gamma_u) \simeq 0$ (see Appendix 7.2); $\alpha_u - \beta_u = 0,\ \frac{\pi}{4},\ -\frac{\pi}{4}$; $\alpha_u - \gamma_u = 0,\ \frac{\pi}{4}$; $\alpha_l,\ \beta_l,\ \gamma_l = 0$.

If we consider another choice for the forms of the quark and lepton mass matrices, for instance, the "improved" Fritzsch ansatz like Matumoto [7c] (see Section 6), the corresponding estimates will do not change much (except for $U(1)_8$)

$$\begin{array}{llll} (i) & M_H > 59.2TeV & (ii) & M_H\ 65.2TeV \\ (iii) & M_H > 2.7TeV & (iv) & M_H > 59.2TeV \end{array} \tag{4.8}$$

This fact can easily be explained by the coincidence of the values of the mass matrix element $(M_d)_{12}$ in these two ansatzes and its dominant role in the definition of the V_{us}-CKM matrix element.

From another very important quark-lepton rare decay $K^+ \longrightarrow \pi^+\mu^+e^-$, whose partial width is now experimentally estimated as

$$Br(K^+ \longrightarrow \pi^+\mu^+e^-) < 2.1 \times 10^{-10}, BNL - E777, \tag{4.9}$$

the constraints on M_{H_0} are also rather large (except in (iii)):

$$M_{H_0} > g_H/g_W \times 35TeV, \tag{4.10}$$

Let us compare it with the bounds on the pure quark or lepton rare processes. For the $U(1)_8$ group, the corresponding bound on the scale M_{H_0} is approximately some TeVs.

Finally, let us consider the decay $\mu \to e\gamma$. The one-loop contribution with the H-boson exchange is suppressed against the $\mu \to 3e$ decay: $\Gamma(\mu \to e\gamma) \ll \Gamma(\mu \to 3e)$. So, the major contribution to the $\mu \to e\gamma$ decay width will come from the one-loop diagram with the exchange of horizontal gauginos and scalar charged leptons. The branching ratio of this decay is :

$$B(\mu \to e\gamma) = \frac{48\pi^2}{G_F^2 m_\mu^2} F^2 \tag{4.11}$$

where F^2 is given in ref.[21]. Using the experimental value [22]: $B(\mu \to e\gamma) < 4.9 \cdot 10^{-11}$, one can easily obtain the bounds on the gaugino masses:

$$\begin{array}{llll} (i) & \tilde{M}_H\ 0.58\ TeV & (ii) & \tilde{M}_H > 0.25\ TeV \\ (iii) & \tilde{M}_H > 0.31\ TeV & (iv) & \tilde{M}_H > O(100\ GeV) \end{array} \tag{4.12}$$

where the scalar lepton mass is 80 GeV.

To conclude, let us note that the analysis of the supersymmetric horizontal model shows that in several schemes of H-symmetry breaking (cases (i), (iii) and (iv)) the limits for the lower bounds of some H-bosons from the experimental results on the amplitudes of pure quark and pure leptonic rare processes ($|\Delta H| \neq 0$) can be relatively low ($\leq 10 TeV$). In this case the contribution of H-interaction to B_d^0 meson oscillations may turn out to be the major contribution and explain, in principle, the experimental value of the $B_{d1}^0 - B_{d2}^0$ mass difference. However, similar bounds on M_H, derived from some quark-lepton rare reactions ($|\Delta H| = 0$), may turn out to be much more than the above estimates, except for case (iii).

Really, we should look closer at this situation : in particular, we should clear up whether our understanding of the origin of quark and lepton generations is correct in that here we see one and the same quark and lepton mixing mechanism in operation. Indeed, by now no reliable evidences to the lepton mixing have been obtained. It's natural to think that the problem of the nature of mixings ascends to the main question of the SM, i.e. the origin of quark and lepton masses. So we state that we know nothing about how the mixings of quark and lepton families are correlated and can only hypothesize it.

Before considering a particular model of such a nontrivial correlation and discussing some of its consequences for the breaking scale of the horizontal gauge interaction M_H, we have to investigate the breaking of the $SU(3)_{HV}$-model without the intermediate scale M_{H_I}, trying to connect the splitting of its global breaking scale M_{H_0} with known heavy quark masses. This will enable us to get more information about the 8-gauge boson masses and make our estimates more predictive.

5. The SUSY $SU(3)_{HV}$ gauge model with (3+1) generations.

5.1. The $SU(3)_{HV}$-singlet 4th generation.

The requirements for the model or, more exactly, its superpotential, should match the "appropriate" form of the CKM mixing matrix for charged electroweak currents following from experimental observations. To have a more detailed and accurate information on the amplitudes of rare processes due to horizontal interactions, we are to penetrate into the mechanism of constructing the mixing matrices both for "up"-quarks (U-matrix), and for "down"-quarks (D-matrix), since their elements must be observable quantities. For definiteness we choose the Fritzsch scheme. So far, the "direct" measurements of the matrix elements V_{ud} , V_{us} , V_{cd} , V_{cs} , V_{ub} , V_{cb} belonging to the CKM matrix,- all found directly from the hadron decays with the b-, c-, s-, d-quarks already observed,- have not contradicted explicitly the Fritzsch scheme, even if the conventional ratios of the "running" quark masses

$$
\begin{aligned}
m_d/m_s &= 0.051 \pm 0.004 \ , \ m_u/m_c = 0.0038 \pm 0.0012 \ , \\
m_s/m_b &= 0.033 \pm 0.011 \ , \ m_c(\mu = 1GeV) = 1.35 \pm 0.05 \\
& \qquad and \quad m_t^{phys} \sim 0.6 m_t(M = 1GeV)
\end{aligned}
\tag{5.1}
$$

were used. Note, in the standard scheme with three generations the experimental limitation $V_{cb} = 0,046 \pm 0,05$ yields the prediction for the t-quark mass whose value must lie near the lower experimental bound $m_t > 89GeV$, at the 95% confidence level. Thus, for $m_s/m_b \geq 0,022$ and $|V_{cb}| < 0,051$ we obtain that $m_t^{phys} \leq 90$ GeV.

$$
\sqrt{\frac{m_c}{m_t}} > \sqrt{\frac{m_s}{m_b}} - |V_{cb}| \tag{5.2}
$$

Following Ref.[14], in order to realize the Fritzsch scheme, we introduce another 4th generation, singlet under the $SU(3)_H$-symmetry. It is easy to show that such an extension makes it possible to lower the upper bound of the t-quark mass. So, to conclude finally on the validity of the Fritzsch approach, further "direct" and more accurate measurements of the matrix elements of the CKM matrix, as well as the search for t-quark itself, are required. Note, limitations on the CKM mixing matrix elements derived from the indirect measurements may depend on the contributions from new hypothetical interactions, which is the prime goal of our investigation. These limitations are obtained from the amplitudes of rare processes going on at the one-loop level (and higher) and involving the exchange of the virtual t-quark, whose amplitudes are dependent on the elements V_{td} , V_{ts} of the CKM matrix. There apparently may exist other schemes explaining the observed hierarchies of quark masses, e.g., the models with additional massive quarks and leptons $(U_i \ , \ D_i)$ $(E_i^- \ , \ E_i^0)$ in each generation.

To understand clearly whether the dynamics of rare K-meson decays, as well as CP-violation in K^0- meson decays and the mass difference $\Delta m(B_d)$ saturation observed experimentally are completely described by the Standard Model, it might be helpful to find the matrix element $|V_{td}|$ of the CKM matrix from "direct" experiments. The first thing to be done is to discover the t-quark and study its decays. Then, proceeding from the solution of the "unitary" triangle $(1 \cdot V_{ub}^* - S_{12}V_{cb} + V_{td} \cdot 1 \approx 0)$ in the Standard Model with 3 generations, formed in the complex plane by the matrix elements V_{ub}^* , S_{12} , V_{cb}^* , V_{td} , one could determine the phase δ_{13}. This, in turn, would allow one to understand whether the vertex resulting from the intersection of the sides V_{ub}^* and V_{td}^* makes the domain satisfying the limitations for the CP-violating parameter ε_K and the $B_d^0 \leftrightarrow \bar{B}_d^0$ mixing. Otherwise (if the vertex is beyond this domain) this would mean that new physics is in operation. beyond the SM scale.

Since the direct measurements of the value of V_{td} are a matter of future, it might be interesting to discuss how it may be defined in a model dependent way. To make our consideration consistent, we will define this element in the Fritzsch scheme introducing an additional 4th generation. The comparison of direct experimental measurements of the elements V_{ud} , V_{us} , V_{ub} , V_{cd} , V_{cs} , V_{cb} with their description within the Fritzsch scheme demonstrated the possibility of an unambiguous choice of the phase parameters $\alpha_A - \tilde{\alpha}_A \approx \pi/2$, $cos(\alpha_A - \tilde{\alpha}_A) \sim 0.07 \div 0.17$, $sin(\alpha_B - \tilde{\alpha}_B) \leq 0.2$, $sin(\alpha_C - \tilde{\alpha}_C) \leq 0.2$. Using the set of phases, one can obtain some limitations on the t , t'-, b'-quark masses from the experimental estimate of V_{cb} :

$$|V_{cb}| \approx \left| \sqrt{\frac{m_s}{m_b}} - \sqrt{\frac{m_c}{m_t}}(1 - m_t/m_{t'}) - \frac{\sqrt{\frac{m_c m_t m_b}{m_t m_{t'} m_{b'}}}}{(1 - m_t/m_{t'})} \right| \tag{5.3}$$

Relationship (5.3) makes the upper possible bound on the t-quark mass higher, provided the b'-quark is not very heavy: $m_{b'}^{phys} \leq 60 \div 80\ GeV$, and the masses of t'- and t-quarks do not differ much: e.g., if $m_s/m_b = 0.022$, $V_{cb} \cong 0.045$, then $m_t^{phys} = 100\ GeV$, $m_{t'}^{phys} = 120\ GeV$. Accordingly, with the given set of phases the following useful expressions are written for V_{ub} and V_{td} :

$$|V_{ub}| \cong \left| \sqrt{\frac{m_u m_s}{m_c m_b}} - \sqrt{\frac{m_u m_c}{m_c m_t}}(1 - m_t/m_{t'}) \right.$$

$$- \frac{\sqrt{\frac{m_u m_c m_t m_b}{m_c m_1 m_{t'} m_{b'}}}}{(1 - m_t/m_{t'})} \Bigg| \tag{5.4}$$

$$|V_{td}| \cong \Bigg| \sqrt{\frac{\frac{m_u m_d}{m_c m_s}}{(1 - m_t^2/m_{t'}^2)}} - \sqrt{\frac{\frac{m_d m_s}{m_s m_b}}{(1 + m_t/m_{t'})}}$$

$$- \sqrt{\frac{\frac{m_d m_s m_t m_b}{m_s m_b m_{t'} m_{b'}}}{(1 + m_t/m_{t'})}} \Bigg| \tag{5.5}$$

As a result, the value of $|V_{ub}|$ tends to its lower limit, i.e., $|V_{ub}| \leq 4 \cdot 10^{-3}$. Similarly, very small values are possible for V_{td}, e.g., for the considered range of the t , t'-, b'-quark masses $|V_{td}| \leq 3 \cdot 10^{-3}$. If this value is used in the Fritzsch scheme, it may well be that $|\varepsilon_K|_{Frit.} << |\varepsilon_K|_{exp.}$.

To construct the u- and d-quark mixing matrices and exclude from the super-potential some undesirable couplings , we use a new U(1)-symmetry for the scheme with (3+1) generations (see Table 3). Table 3 presents the quark and lepton quantum numbers for the following 2 cases:

Table 3a

	H	C	L	Y	Y_H		H	C	L	Y	Y
Q'	1	3	2	1/6	1	Y_1	$\bar{3}$	1	2	1/2	−1
t'^c	1	$\bar{3}$	1	−2/3	0	Y_2	$\bar{3}$	1	2	−1/2	1
b'^c	1	$\bar{3}$	1	1/3	-2	X_1	3	1	2	1/2	−1
L'	1	1	2	−1/2	1	X_2	3	1	2	−1/2	1
e^{iC}	1	1	1	1	0	κ_1	1	1	2	1/2	−1
ν^{iC}	1	1	1	0	−2	κ_2	1	1	2	−1/2	1

Table 3b

	H	C	L	Y	Y_H
t'	1	3	1	2/3	1
t'^c	1	$\bar{3}$	1	−2/3	0
b'^c	1	$\bar{3}$	1	1/3	−2
b'	1	3	1	−1/3	1

	H	C	L	Y	Y_H
Y_1	$\bar{3}$	1	2	1/2	−1
Y_2	$\bar{3}$	1	2	−1/2	1
X_1	3	1	1	0	1
X_2	3	1	1	0	−1
κ_1	1	1	1	0	−1
κ_2	1	1	1	0	1
X'_1	3	1	1	0	1
X'_2	3	1	1	0	−1

A. The quarks of the 4th generation are transformed according to the doublet representation of the $SU(2)_L$-group.

B. These quarks are $SU(2)_L$−singlet. As follows from the experiments on e^+e^-−annihilation carried out at LEP, the experimental number of the generations is $Ng \approx 3$. In case A, this number sets the lower bound on quark and lepton masses, e.g., ν_4 $m_\nu > 41 GeV$, and in case B there is still a possibility of the existence of light neutrino of the 4-th generation. Remember that the bound on the number of generations obtained from cosmology is $N_g < 3.8$. The additional terms occurring in superpotential (2.2) have the following form:

$$\begin{aligned} P_A &= \lambda_7 Q Y_1 t'^c + \lambda_8 Q' X_1 u^c + \lambda_9 Q Y_2 b'^c + \\ &+ \lambda_{11} Q' \kappa_1 t'^c + \lambda_{12} Q' \kappa_2 b'^c + \lambda_{10} Q' X_2 d^c \\ P_B &= \lambda_7 Q Y_1 t'^c + \lambda_9 Q Y_2 b'^c + \lambda_8 t' X_1 u^c + \lambda_{10} b' X_2 d^c + \\ &+ \lambda_{11} t' \kappa_1 t'^c + \lambda_{12} b' \kappa_2 b'^c + \lambda'_8 b' X'_1 u^c + \lambda'_{10} t' X'_2 d^c \end{aligned} \tag{5.6}$$

5.2. Local H-symmetry breaking and gauge boson masses.

We assume that when the $SU(3)_H$−gauge symmetry of quark- lepton generations is violated, all the 8 gauge bosons acquire in the eigenspectrun of horizontal interactions the same mass equal to M_{H_0}. The such breaking is not difficult to get by, say, introducing the Higgs fields transforming in accordance with the triplet representation of the $SU(3)_H$ group. These fields are singlet under the Standard Model symmetries : ($z \in (3,1,1,0)$ and $\bar{z} \in (\bar{3},1,1,0)$, $< \bar{z}^{i\alpha} >_0 = \delta^{i\alpha} V$, $< z_i^{\alpha} >_0 = \delta_i^{\alpha} V$, $i, \alpha = 1,2,3$, where $V = M_{H_0}$). The degeneracy in the masses of 8 gauge horizontal vector bosons is eliminated by using the VEV's of the Higgs fields violating the electroweak symmetry and determining the mass matrix of u- and d- quarks (leptons). Thus, in case A the set of the Higgs fields from Table 3, with $H(8,\ 2,\ -1/2,\ -1)$, $Y_2(\bar{3},\ 2,\ -1/2,\ 1)$, $X_2(3,\ 2,\ -1/2,\ -1)$, $\kappa_2(1,\ 2,\ -1/2,\ 1)$, violates the $SU(2) \times U(1)$ symmetry and determines the mass matrix of d-quarks. Similarly, another set: h, Y_1 , X_1 , κ_1 , also violates the electroweak symmetry and determines the mass matrix for u-quarks. On the other hand, in order to calculate the splitting between the masses of horizontal gauge bosons, one has to take into account the VEV's of these two sets of the Higgs fields. The introduction into the model of a great number of the Higgs fields (H, h (8,2) and Y, X (3,2)), transforming nontrivially under to the horizontal $SU(3)_H$ and electroweak $SU(2)_L$ symmetries, may cause two difficulties . The first is a possible mixing between the horizontal Higgs H_μ^a and neutral Z-vector bosons. To avoid it, write down the covariant derivatives of the Higgs fields H(8,2) and $Y_2(3,2)$ (the brackets contain the $SU(3)_H - \ and\ SU(2)_L$ quantum numbers):

$$Tr \,| < D_\mu \hat{H} > |^2 \;=\; \sum_{a=1}^{8} [\frac{\bar{g}^2}{2} Z_\mu Z_\mu < H^a >^2 + \frac{g_H^2}{2} f^{abc} f^{ab'c'} H_\mu^b H_\mu^{b'} < H^c >< H^{c'} >] \quad (5.7)$$

$$| < D_\mu Y_2 > |^2 \;=\; \frac{1}{2} | < Y_2 > |^2 \, [g_H^2 (H_{4\mu}^2 + H_{5\mu}^2 + H_{6\mu}^2 + H_{7\mu}^2 \;+\; 4/3\ H_{8\mu}^2) + \bar{g}^2 Z_\mu^2 + 4 Y_H^2 \tilde{g}_H^2 C_\mu^2 + 2 \bar{g} g_H (-2/\sqrt{3}\,) H_{8\mu} Z_\mu + 4 \bar{g} \tilde{g}_H (Z_\mu C_\mu) Y \;-\; 4/3\ Y_H g_H \tilde{g}_H H_{8\mu} C_\mu] \quad (5.8)$$

In formula (5.8), C_μ corresponds to the $U(1)_H$–gauge degree of freedom. As has already been mentioned, the $U(1)_H$ group may be used to prohibit in superpotential (5.6) the constraints not needed for the construction of the mass matrix. This group can be chosen to be local and have a large violation scale, ($M_{U(1)_H} >> M_{SU(3)_H}$). It would also be interesting to consider the intrinsic $U(1)_R$–symmetry of the N=1 SUSY model as $U(1)_H$, in which case it becomes a global group. By analogy with Eqs.(5.7) and (5.8), the contributions to the mass matrix H_μ^a , Z_μ are written for all the Higgs fields listed in Tables 3a and 3b. As follows from expression (5.7), the vacuum expectations of the Higgs fields, transforming in the regular representation, do not lead to any mixings between the horizontal and Z_μ–gauge fields. After the summation of the contributions from the vacuum expectations of the Higgs fields, the equality $| < Y_i(3,2) > |^2 = | < X_i(3,2) > |^2$ may lead to cancellation of the terms $2\bar{g}g_H(-2/\sqrt{3})H_{8\mu}Z_\mu$. As a result there will be no mixing between the horizontal and Z_μ–gauge fields, either. Another possible difficulty is that, on the one hand, the vacuum expectations of two sets of the Higgs fields, transforming in representations (8,2) and (3,2) of the $SU(3)_H$ and $SU(2)$ groups, saturate the masses of t- and t'- , b'- quarks but, on the other hand, they contribute to the masses of $Z_\mu(W_\mu^\pm)$–bosons. Putting together the contributions from the nonzero vacuum expectations of the above sets of the Higgs fields, we obtain some limitations on the masses of heavy quarks versus the values of the Yukawa constants. Thus, for cases A and B we have :

$$M_Z^2 = \frac{\bar{g}^2}{2}\Big[\frac{A_d^2+B_d^2}{\lambda_3^2} + \frac{A_u^2+B_u^2}{\lambda_5^2} + \frac{C_d^2}{\lambda_9^2} + \frac{C_u^2}{\lambda_7^2} + \\ + \frac{C_d^2}{\lambda_{10}^2} + \frac{C_u^2}{\lambda_8^2} + \frac{D_u^2}{\lambda_{11}^2} + \frac{D_d^2}{\lambda_{12}^2}\Big] \qquad (5.9)$$

$$M_Z^2 = \frac{\bar{g}^2}{2}\Big[\frac{A_d^2+B_d^2}{\lambda_3^2} + \frac{A_u^2+B_u^2}{\lambda_5^2} + \frac{C_d^2}{\lambda_9^2} + \frac{C_u^2}{\lambda_7^2}\Big] \qquad (5.10)$$

The parameters $A_{d,u}$, $B_{d,u}$, $C_{d,u}$, $D_{d,u}$ were already given in Appendix 7.2 and $\bar{g}^2 = g_2^2 + g_1^2$. As the analysis of these relations show the possible noncontradictory mass ranges for t-, t'-, b'-quarks are: $m_{t,t'} \in (90 \div 150\ GeV)$ and $m_{b'} \in (60 \div 100\ GeV)$. Note that relationship (5.10) suggests that the choice of the 4th generation with SU(2)-singlet quarks allows greater values for t , t' , b'-quark masses . This set of t-, t'-, b'-quark masses will be used later on in the analysis of the mass differences between long- and short-living meson states Δm_K , Δm_{B_d} , Δm_{B_s}, and in the estimations of the CP-violation parameters.

Now we can come to constructing the horizontal gauge boson mass matrix M_{ab}^2 (a,b=1,2,...,8):

$$(M_H^2)_{ab} = M_{H_0}^2\delta_{ab} + (\Delta M_d^2)_{ab} + (\Delta M_u^2)_{ab} \qquad (5.11)$$

Here $(\Delta M_d^2)_{ab}$ and $(\Delta M_u^2)_{ab}$ allow for the contributions from the vacuum expectations of only those Higgs bosons that were used to construct the

Fritzsch mass matrix for d- (u-) quarks. As it turned out in our case, each of the (8×8)–dimensional mass matrices $(\Delta M_d^2)_{ab}$ and $(\Delta M_u^2)_{ab}$ are broken into two (4×4)–dimensional matrices:

$$\mathbf{\Delta M^2_{dp,q=a,b=1,2,6,7}} = \frac{g_H^2}{4}\Delta^2 \times diag(0,0,1,1) \;+\; \frac{g_H^2}{4}\times$$

$$\begin{pmatrix} \varphi_2^2+\frac{1}{4}(\varphi_6^2+\varphi_7^2) & -\varphi_1\varphi_2 & -\frac{1}{4}(\varphi_1\varphi_6+3\varphi_2\varphi_7) & -\frac{1}{4}(\varphi_1\varphi_7-3\varphi_2\varphi_6) \\ -\varphi_1\varphi_2 & \varphi_1^2+\frac{1}{4}(\varphi_6^2+\varphi_7^2) & -\frac{1}{4}(\varphi_2\varphi_6-3\varphi_1\varphi_7) & -\frac{1}{4}(\varphi_2\varphi_7+3\varphi_1\varphi_6) \\ -\frac{1}{4}(\varphi_1\varphi_6+3\varphi_2\varphi_7) & -\frac{1}{4}(\varphi_2\varphi_6-3\varphi_1\varphi_7) & \frac{1}{4}(\varphi_1^2+\varphi_2^2)+\varphi_7^2 & -\varphi_6\varphi_7 \\ -\frac{1}{4}(\varphi_1\varphi_7-3\varphi_2\varphi_6) & -\frac{1}{4}(\varphi_2\varphi_7+3\varphi_1\varphi_6) & -\varphi_6\varphi_7 & \frac{1}{4}(\varphi_1^2+\varphi_2^2)+\varphi_6^2 \end{pmatrix}$$

$$\mathbf{\Delta M^2_{dr,s=a,b=4,5,3,8}} = \frac{g_H^2}{4}\Delta^2 \times diag(1,1,0,\tfrac{4}{3}) \;+\; \frac{g_H^2}{4}\times$$

$$\begin{pmatrix} \frac{1}{4}(\varphi_1^2+\varphi_2^2+\varphi_6^2+\varphi_7^2) & 0 & \frac{3}{4}(\varphi_1\varphi_6-\varphi_2\varphi_7) & \frac{\sqrt{3}}{4}(\varphi_2\varphi_7-\varphi_1\varphi_6) \\ 0 & \frac{1}{4}(\varphi_1^2+\varphi_2^2+\varphi_6^2+\varphi_7^2) & \frac{3}{4}(\varphi_1\varphi_7+\varphi_2\varphi_6) & -\frac{\sqrt{3}}{4}(\varphi_1\varphi_7+\varphi_2\varphi_6) \\ \frac{3}{4}(\varphi_1\varphi_6-\varphi_2\varphi_7) & \frac{3}{4}(\varphi_1\varphi_7+\varphi_2\varphi_6) & \varphi_1^2+\varphi_2^2+\frac{1}{4}(\varphi_6^2+\varphi_7^2) & -\frac{\sqrt{3}}{4}(\varphi_6^2+\varphi_7^2) \\ \frac{\sqrt{3}}{4}(\varphi_2\varphi_7-\varphi_1\varphi_6) & -\frac{\sqrt{3}}{4}(\varphi_1\varphi_7+\varphi_2\varphi_6) & -\frac{\sqrt{3}}{4}(\varphi_6^2+\varphi_7^2) & \frac{3}{4}(\varphi_6^2+\varphi_7^2) \end{pmatrix}$$

Here $\lambda_3\,(\varphi_1 - i\,\varphi_2) = A_d\,e^{-i\alpha_A}$, $\lambda_3\,(\varphi_6 - i\,\varphi_7) = B_d\,e^{-i\alpha_B}$, $\lambda_{10}\,(\Delta_1 - i\,\Delta_2) = C_d\,e^{-i\alpha_C}$. After substituting $\lambda_5\,(\tilde\varphi_1 - i\,\tilde\varphi_2) = A_u\,e^{-i\tilde\alpha_A}$, $\lambda_5\,(\tilde\varphi_6 - i\,\tilde\varphi_7) = B_u\,e^{-i\tilde\alpha_B}$, $\lambda_8\,(\tilde\Delta_1 - i\,\tilde\Delta_2) = C_u\,e^{-i\tilde\alpha_C}$ the matrix $(\Delta M_u^2)_{ab}$ takes a similar form.

Under the condition that $\varphi_1/\varphi_2 = \tilde\varphi_1/\tilde\varphi_2$ and $\varphi_6/\varphi_7 = \tilde\varphi_6/\tilde\varphi_7$ the matrices $(M_d^2)_{pq}$ and $(M_u^2)_{pq}$ (p,q = a,b = 1,2,6,7) are diagonalized via the same orthogonal transformations[14]. Note, that the largest value of the mass splitting between horizontal gauge bosons is proportional to $\sim \frac{g_H^2}{\lambda^2} m_t' m_t$ and this splitting is not complete. So, write down some rough equalities between the masses of horizontal gauge bosons:

$$M_{H_1}^2 \approx M_{H_2}^2 \approx M_{H_3}^2 \approx M_{H_0}^2 + \frac{g_H^2}{4}\left[\frac{1}{\lambda_5^2}\frac{m_c m_t m_{t'}}{1-\frac{m_t}{m_{t'}}} + \frac{1}{\lambda_3^2} m_s m_b\right] + ...$$

$$M_{H_4}^2 \approx M_{H_5}^2 \approx M_{H_6}^2 \approx M_{H_7}^2 \approx M_{H_0}^2 + \frac{g_H^2}{4}\left[\frac{1}{\lambda_{7,8}^2} m_t m_{t'} + \frac{1}{\lambda_{9,10}^2} m_b m_{b'}\right] + ...$$

$$M_{H_8}^2 \approx M_{H_0}^2 + \frac{g_H^2}{3}\left[\frac{1}{\lambda_{7,8}^2} m_t m_{t'} + \frac{1}{\lambda_{9,10}^2} m_b m_{b'}\right] + ...$$

Probably, to achieve a more accurate definition of the gauge boson mass splitting within this approach, one should also take into account radiative mass corrections due to scalar quark, or scalar lepton exchange in the one-loop diagrams. We hope to do so in the our next publications. The exact knowledge of this mass spectrum would be very important to do more accurate estimates of the universal scale M_{H_0} of the horizontal gauge symmetry breaking, derived from the amplitudes of pure lepton, or pure quark rare processes. Note, when the quantum number of $(Q-L)$- generation is not changed, the amplitudes of rare processes do not depend on the splitting value.

5.3. The analysis of phenomenological parameters and estimation of M_{H_0}.

Within the SUSY $SU(3)_H$ version of horizontal interactions, it is interesting to study the following rare processes :

a) pure quark processes: $K^0 \leftrightarrow \bar{K}^0$, $B_d^0 \leftrightarrow \bar{B}_d^0$, $B_s^0 \leftrightarrow \bar{B}_s^0$ oscillations and CP-violating effects in K-, B-, D-meson decays;

b) quark-lepton processes: $K^\pm \to \pi^\pm \mu^\pm e^\pm$, $(\mu - e)-$conversion on nuclei, $K^0 \to \pi^0 \mu e$, $K^\pm \to \pi^\pm \nu_i \nu_j$, etc.;

c) purely lepton processes: $\mu \to e\gamma$, $\mu \to 3e$, $\tau \to \mu\gamma$, etc.

The mixing of the neutral $Z_\mu-$gauge boson and horizontal H_μ^a-bosons will lead to rare $Z_\mu-$boson decays followed by the violation of the quantum number of quark-leptonic generations, e.g., $(Z \to \mu e\ ,\ \tau\mu\ ,\ \nu_i\nu_j\ ,\ ...)$. In this subsection we will investigate the most pronounced processes among those mentioned above and specify the lower bound on the scale of hypothetical horizontal interactions under study. Let us begin with studying of the processes of type (a). First, consider the contribution to the value of the $K_L^0 - K_S^0$ meson mass difference $\Delta m_K = m(K_L^0) - m(K_S^0)$, which is due to the tree Feynman diagrams with the H_μ^a horizontal gauge boson exchanges. This interaction is described by the relevant part of the SUSY $SU(3)_H-$Lagrangian and has the form

$$\mathcal{L}_H = g_H \bar{\psi}_d \gamma_\mu \ (\ D P_d \frac{\Lambda^a}{2} P_d^* D^T\ )\ \psi_d O_{ab} H_\mu^b\ , \qquad (5.12)$$

where

$$\mathbf{\Lambda^a} = \begin{pmatrix} \lambda^a & \vdots \\ \dots & 1 \end{pmatrix}.$$

Here we have (a,b=1,2,...,8). The matrix O_{ab} determines the relationship between the bare, $H^b_{0\mu}$, and physical, H^b_μ, gauge fields and is calculated for the mass matrix $(M^2_H)_{ab}$ diagonalized; $\psi_d = (\psi_d\ ,\ \psi_s\ ,\ \psi_b\ ,\ \psi_{b'})$; g_H is the gauge coupling of the $SU(3)_H$ group.

After rather simple though cumbersome calculations in the leading approximation, the expression for the $K^0_L - K^0_S$ meson mass difference takes the form:

$$\left[\frac{\Delta m_K}{m_K}\right]_H = \frac{g^4_H}{2M^4_{H_0}}\left[\frac{4}{\lambda^2_5}\frac{m_d}{m_b}\frac{m_c m_t}{(1-m_t/m'_t)} + \frac{m_d m_s}{\lambda^2_3} + \frac{m_u m_c}{\lambda^2_5}\right] f^2_K\ R_K \quad (5.13)$$

Using the experimental bound for $\Delta m_K/m_K$ (4.1) and putting the values of the quark masses: $m_d = 8 \times 10^{-3}\ GeV$, $m_c = 1.35\ GeV$, $m_t^{phys} = 100\ GeV$, $m_b = 5\ GeV$, $m_{t'}^{phys} = 130\ GeV$ into formula (5.13), we obtain the following estimate: $M_{H_0} \approx g_H \cdot 2.9 TeV$. Here we use the following value for the Yukawa constraints: $\lambda_5 \sim O(1)$. Using the condition $\alpha_H = \alpha_{e.w.} = g^2_2/4\pi \approx 1.9 \times 10^{-2}$, we obtain the following scale for the $SU(3)_H$ violation: $M_{H_0} > 1.4\ TeV$ ($M_{H_0} > 4.2 TeV$, if $\lambda_5 \sim O(0.1)$). Constructing the model for the H-gauge boson mass splitting and comparing with expression (4.2), we get convinced that the value of the unitary compensation coefficient is very small: $C_K \approx (2g^2_H/\lambda^2_5)\{m_c m_t/[M^2_{H_0}(1 - m_t/m'_t)]\}(m_d/m_b) \sim (10^{-5} \div 10^{-6})$ (see Tab.2). Let us note, that the similar compensation coefficient for $(K^0 - \bar{K}^0)$- oscillation in SM is also very small: $(C_K)_{GIM} \sim (m^2_{q_i}/M^2_W)[V_{1i}V_{2j}V^*_{2i}V^*_{1j}]$.

To make our reasoning complete, we should also bring in the expressions for Δm_{B_d} and Δm_{B_s}. In the leading approximation we get:

$$\left[\frac{\Delta m_{B_d}}{m_{B_d}}\right]_H \cong \frac{g^4_H}{2M^4_{H_0}}\left[\frac{1}{\lambda^2_5}\frac{m_d}{m_s}\frac{m_c m_t}{(1-m_t/m'_t)} + \frac{m_b m_d}{\lambda^2_3}\right] f^2_{B_d} R_{B_d} \quad (5.14)$$

$$\left[\frac{\Delta m_{B_s}}{m_{B_s}}\right]_H \cong \frac{g^4_H}{2M^4_{H_0}}\left[\frac{1}{\lambda^2_5}\frac{m_c m_t}{(1-m_t/m'_t)} + \frac{m_b m_s}{\lambda^2_3}\right] f^2_{B_s} R_{B_s} \quad (5.15)$$

$$R_{B_d} = \left[\frac{7}{6}R^V_{1P} + \frac{1}{6}R^A_{1P} + \frac{1}{3}R^P_{1P} - \frac{1}{3}R^S_{1P} + \frac{1}{6} + \frac{1}{3}\frac{m^2_{B_d}}{(m_b - m_d)^2}\right] \quad (5.16)$$

Comparing formulas (5.13) and (5.14), we arrive at the conclusion that $(\Delta m_{B_d})_H \sim 100(\Delta m_K)_H$, suggesting that the contribution from the horizontal interactions could saturate the experimentally observed amplitude of the mixing $B_d^0 \leftrightarrow \bar{B}_d^0$ ($x_{B_d} = \Delta M/\Gamma = 0.67 \pm 0.10$, $\Delta m_{B_d} = (3.5 \pm 0.8) \times 10^{-13} GeV$). Now consider the case of u- and d-quark states when these states do not coincide: $P_u \neq P_d$. In the leading approximation, the imaginary part of the transition amplitude $M_{12}(K^0 \to \bar{K}^0)$ takes the form

$$\frac{ImM_{12}(K^0 \to \bar{K}^0)}{m_K} \simeq \frac{g_H^4}{4M_{H_0}^4}\frac{1}{\lambda_5^2}\Big\{m_u m_c \sin 2(\alpha_A - \tilde{\alpha}_A)$$
$$- 2\sqrt{\frac{m_d}{m_s}\frac{m_u m_c^2 m_t}{(1-\frac{m_t}{m_{t'}})}}[\sin(\alpha_A - \tilde{\alpha}_A + \alpha_B - \tilde{\alpha}_B) + \sin(\alpha_C - \tilde{\alpha}_C)]\Big\} f_K^2 R_K \tag{5.17}$$

Putting into this expression, the values of the running quark masses choosing λ_5 as $\lambda_5 \sim 0(1)$, remembering that $\alpha_C - \tilde{\alpha}_C \sim 0.2$, $\alpha_B - \tilde{\alpha}_B \sim 0.2$, $\alpha_A - \tilde{\alpha}_A \sim \pi/2$, and taking the experimental estimate to be $ImM_{12}(K^0 \to \bar{K}^0) \approx 2{\cdot}10^{-17}$ GeV , we obtain the limitation on the SU(3) horizontal symmetry violation scale: $M_{H_0} > 1,9$ TeV. The choice of the angles: $\alpha_C - \tilde{\alpha}_C \leq 0.2$, $\alpha_B - \tilde{\alpha}_B \leq 0.2$, $\alpha_A - \tilde{\alpha}_A \sim \pi/2$ fits well the experimental "direct" measurements of the CKM matrix elements V_{ud} , V_{us} , V_{ub} , V_{cd} , V_{cs} , V_{cb} of the Fritzsch scheme.

Again, to make the reasoning complete, let us represent the expression for the mass difference $\Delta m_K = m(K_L^0) - m(K_S^0)$ in the case when $P_u \neq P_d$:

$$\Big[\frac{\Delta m_K}{m_K}\Big]_H \approx \frac{g_H^4}{2M_{H_0}^4}\Big[\frac{4}{\lambda_5^2}\frac{m_d}{m_b}\frac{m_c m_t}{(1-m_t/m_t')}cos^2(\alpha_B - \tilde{\alpha}_B) +$$
$$+ \frac{m_d m_s}{\lambda_3^2} + \frac{m_u m_c}{\lambda_5^2}cos2(\alpha_A - \tilde{\alpha}_A)\Big] f_K^2 R_K \tag{5.18}$$

Now let us calculate the contribution from the horizontal gauge interactions to the CP-violation parameter ε_K

$$|\varepsilon_K|_H = \frac{|ImM_{12}|_H}{\sqrt{2}\Delta m_K}$$

With the help of expression (5.17) one can easily obtain that

$$|\varepsilon_K|_H = \frac{g_H^4}{4M_{H_0}^4}\Big| - \frac{2}{\lambda_5^2}\sqrt{\frac{m_d}{m_b}}\sqrt{\frac{m_u m_c^2 m_t}{(1-m_t/m_t')}}\Big[\sin(\alpha_A - \tilde{\alpha}_A + \alpha_B - \tilde{\alpha}_B)$$

$$+ \sin(\alpha_C - \tilde{\alpha}_C)]\Big] + \frac{m_u m_c}{\lambda_5^2} \sin 2(\alpha_A - \tilde{\alpha}_A)\Big| \frac{f_K^2 R_K}{\sqrt{2}\,\Delta m_K} \tag{5.19}$$

Now, using the values of the previously found phases and taking into account that $m_{H_0} > 1.9\,TeV$, $m_t^{phys} = 100\,GeV$, $m_{t'}^{phys} = 130\,GeV$, one can easily see that the CP-violation parameter $|\varepsilon_K|_H$ may lie near its experimental value: $|\varepsilon_K|_H \cong |\varepsilon_K|_{exp} = 2\cdot 10^{-3}$. Note that, as is clear from expression (5.19), in the case of $P_u = P_d$ there is no CP-violation in the horizontal sector. Accordingly, in the Fritzsch scheme the CKM matrix ($O_{CKM} = UP_u P_d^* D^+$) becomes real and there is no CP-violation in the electroweak sector either. This is explained by the CP-violation mechanism which is determined by the vacuum expectations of the Higgs fields with nonzero quantum numbers in both the electroweak and horizontal groups. The contributions from these two interactions to the CP-violation effects may differ in their magnitude, and this difference may be estimated in the Fritzsch scheme if we use the whole set of the experimental data available.

For the completeness of our reasoning, let us present the CP-violation parameters of the ($B_d^0 \leftrightarrow \bar{B}_d^0$) and ($B_s^0 \leftrightarrow \bar{B}_s^0$) oscillations:

$$\begin{aligned} \frac{ImM_{12}(B_d^0 \leftrightarrow \bar{B}_d^0)}{m_{B_d}} &\simeq \frac{g_H^4}{4M_{H_0}^4}\frac{1}{\lambda_5^2}\Big\{ \frac{m_d}{m_s}\frac{m_c m_t}{(1 - m_t/m_{t'})} \sin 2(\alpha_B - \tilde{\alpha}_B) \\ &+ 2\sqrt{\frac{m_d}{m_b}}\sqrt{\frac{m_u m_c^2 m_t}{(1 - m_t/m_{t'})}} \sin(\alpha_C - \tilde{\alpha}_C) \Big\} f_{B_d}^2 R_{B_d}, \end{aligned} \tag{5.20}$$

$$\begin{aligned} \frac{\Delta m(B_d)}{m_{B_d}} &\simeq \frac{g_H^4}{2M_{H_0}^4}\Big\{ \frac{m_d m_b}{\lambda_3^2} + \\ &+ \frac{1}{\lambda_5^2}\frac{m_d}{m_s}\frac{m_c m_t}{(1 - m_t/m_{t'})} cos2(\alpha_B - \tilde{\alpha}_B) \Big\} f_{B_d}^2 R_{B_d}, \end{aligned} \tag{5.21}$$

$$ImM_{12}(B_s^0 \leftrightarrow \bar{B}_s^0) = \frac{m_s}{m_d} ImM_{12}(B_d^0 \leftrightarrow \bar{B}_d^0) \tag{5.22}$$

$$\Delta m(B_s) = \frac{m_s}{m_d} \Delta m(B_d). \tag{5.23}$$

Provided

$$\frac{p}{q} = \frac{\sqrt{M_{12} - \frac{\Gamma_{12}}{2}}}{\sqrt{M_{12}^* - \frac{\Gamma_{12}^*}{2}}} = \frac{(1+\epsilon_B)}{(1-\epsilon_B)}, \tag{5.24}$$

and neglecting the values of $|Re\Gamma_{12}|, |Im\Gamma_{12}|$, for the CP-violating parameters $\frac{1}{2}Im(p/q)_H$, we obtain that

$$\frac{1}{2} \times Im(\frac{p}{q})_{B_{d,s}} \approx \frac{ImM_{12}(B_{d,s})_H}{ReM_{12}(B_{d,s})_H} \approx tg2(\alpha_B - \tilde{\alpha}_B) \leq 0.3 \div 0.4, \tag{5.25}$$

In this approach, we might also have two CP- violating contributions from electroweak and horizontal interactions to the amplitudes of B-meson decays. The last fact might lead to a very interesting CP -violation asymmetry $A_f(t)$ for the decays of neutral B_d^0 - and $\bar{B}_d^0$ - mesons to final hadron CP -eigenstates ,for example, to $f = (J/\Psi K_S^0)$ or $(\pi\pi)$

$$A_f(t) \approx \sin(\Delta m_{B_d} t) Im(\frac{p}{q} \times \rho_f), \qquad \rho_f = \frac{A(\bar{B} \to f)}{A(B \to f)}.$$

In the standard model with the Kobayashi- Maskawa mechanism of CP-violation, the time integrated asymmetry of B_d^0- and $\bar{B}_d^0$- meson decays to the $J/\Psi K_S^0$- final state is:

$$A(J/\Psi K_S^0) \approx \frac{x_d}{1+x_d^2} \times \sin 2\phi_3, \tag{5.26}$$

where $\phi_3 = argV_{td}$ is one of the angles: $(\phi_i, i = 1,2,3)$ of the unitary triangle (Fig. 1). Let us compare this asymmetry with an analogous asymmetry of the B^0 and $\bar{B}^0$-decays to the (π^+, π^-) final state, the latter being known to depend on the magnitude of V_{ub}. Then:

$$A(\pi^+\pi^-) \approx -\frac{x_d}{1+x_d^2} \times \sin 2\phi_2, \tag{5.27}$$

where $\phi_2 = \pi - \phi_1 - \phi_3$ and $\phi_1 = argV_{ub}^* = \delta_{13}$ ($\delta^{KM} = \phi_1 + \phi_3$).

For pure horizontal CP- violation mechanism these effects will coincide for both B^0- decays. If, as in our example, there are two sources of CP-violation, then it becomes still more problematic to distinguish between the two contributions, and a more thorough analysis is needed.

As is clear from the analysis, in the given approach there is a mass region, $O(1\ TeV) \leq m_{H_0} \leq O(10\ TeV)$, of horizontal gauge bosons, where the contributions of new forces might be noticeable and cause intensive CP-violation in $B_{d,s}$ - meson decays.

For the ratio $\frac{x_s}{x_d}\Big|_H$ we obtain:

$$\frac{x_s}{x_d}\Big|_H = \frac{(\Delta m_{B_s})_H}{(\Delta m_{B_d})_H} \approx \frac{m_s}{m_d} \approx 15 \div 19 \qquad (5.28)$$

In the SM, for the Wolfenstein parameters $\rho = (|V_{ub}|/|V_{bc}V_{us}|)\cos\delta_{13}$ and $\eta = (|V_{ub}|/|V_{bc}V_{us}|)\sin\delta_{13}$ chosen so that $\rho \geq 0$ and $\sqrt{\rho^2+\eta^2} \simeq \frac{1}{2}$ $(|V_{ub}|/|V_{bc}| \approx 0.11)$, this ratio becomes:

$$\frac{x_s}{x_d}\Big|_{SM} = \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{1}{\lambda^2[(1-\rho)^2+\eta^2]} \simeq 20 \div 100. \qquad (5.29)$$

Now let us consider in the given approach a purely leptonic rare process , for example, $\mu \Longrightarrow 3\,e$-decay:

$$Br(\mu \Longrightarrow 3\,e)_H = 12\frac{g_H^4}{g_W^4}\frac{M_W^4}{M_{H_0}^4} \times C_{(\mu)}^2, \qquad (5.30)$$

where

$$C_{(\mu)} = \sum_{a,b=1}^{8} \frac{\Delta M_{ab}^2}{M_{H_0}^2}(LT^aL^+)_{21}(LT^bL^+)_{11} \qquad (5.31)$$

In formula (5.31), ΔM_{ab}^2 are the elements of the horizontal gauge boson mass matrix , and L is the matrix diagonalizing the charge lepton mass matrix.

In the *Fritzsch ansatz* the main contribution to expression (5.31) has the following form:

$$C_{(\mu)} \approx -\frac{1}{3}\frac{g_H^2}{\lambda_{10}^2}\frac{m_t m_{t'}}{M_{H_0}^2}\frac{\sqrt{m_\mu m_e}}{m_\tau} \qquad (5.32)$$

So, one has

$$Br(\mu \Longrightarrow 3e) = \frac{4}{3}(\frac{g_H^4}{g_W^4})(\frac{g_H^4}{\lambda_{10}^4})(\frac{M_W^4}{M_{H_0}^4})(\frac{m_t^2 m_{t'}^2}{M_{H_0}^4})\frac{m_\mu m_e}{m_\tau^2} \qquad (5.33)$$

At $m_t^{phys} = 100\,GeV$ and $m_{t'}^{phys} = 130\,GeV$ one can easily obtain the low bound on the gauge symmetry breaking:

$$M_{H_0} > \frac{1}{\sqrt[3]{\lambda_{10}}} 310\,GeV \qquad (5.34)$$

6. The $SUSY\ SU(3)_{HV}$-gauge model with 3 generations.

For the case of $N_g = 3$ generations the large mass value expected for the t- quark inspired the search for forms of quark mass matrix different from Fritzsch ansatz.

In this Section we will confine ourselves to the consideration of two types of Hermitian mass matrices and, going on with the previous analysis, estimate the local H- symmetry breaking scale. As the 1st approach, we consider a somewhat modified Fritzsch ansatz for 3 generations, with nonzero antidiagonal mass matrix elements, and the upper bound on the t-quark from the experimental data on the matrix element V_{cb} will be much larger. We will not study now the most general form of hermitian quark mass matrices of the three families. It will be enough, and more useful, for us to consider the specific forms of quark ansatzes which would fit well the CKM- mixing matrix elements with the experimental accuracy attainable today.

Another (democratic) ansatz is noteworthy for the possibility to single out the t-quark mass value. And besides, within this approach the mass matrices of the above form can, at least, be more correctly interpreted in physical terms - e.g., via the compositeness of quarks.

6.1. The *improved* Fritzsch ansatz

Let us consider some of the "calculable" ansatzes for 3×3 "*up*" and "*down*" quark mixing matrices [7c] consistent with the modern values of the CKM-matrix for charged currents.

$$\mathbf{M^u} = \begin{pmatrix} 0 & \sqrt{|m_u| m_c} & \sqrt{|m_u| m_c} \\ \sqrt{|m_u| m_c} & m_c & -m_c \\ \sqrt{|m_u| m_c} & -m_c & m_t \end{pmatrix} \tag{6.1}$$

and

$$\mathbf{M^d} = \begin{pmatrix} 0 & i\sqrt{|m_d| m_s} & \sqrt{|m_d| m_s} \\ -i\sqrt{|m_d| m_s} & m_s & m_s \\ \sqrt{|m_d| m_s} & m_s & m_b \end{pmatrix}. \tag{6.2}$$

In the leading approximation the unitary matrices U and D ($O_{CKM} = U\,D^+$), diagonalizing these mass states, have the following form:

$$\mathbf{U} \approx \begin{pmatrix} 1 & -\sqrt{\frac{m_u}{m_c}} & -2\sqrt{\frac{m_u}{m_c}\frac{m_c}{m_t}} \\ \sqrt{\frac{m_u}{m_c}} & 1 & \frac{m_c}{m_t} \\ \sqrt{\frac{m_u}{m_c}\frac{m_c}{m_t}} & -\frac{m_c}{m_t} & 1 \end{pmatrix} \tag{6.3}$$

and

$$\mathbf{D} \approx \begin{pmatrix} e^{-i\frac{\pi}{4}} & \sqrt{\frac{m_d}{m_s}}e^{-i\frac{3\pi}{4}} & \frac{\sqrt{2m_d m_s}}{m_b}e^{i\frac{\pi}{2}} \\ \sqrt{\frac{m_d}{m_s}} & e^{i\frac{\pi}{2}} & \frac{m_s}{m_b}e^{-i\frac{\pi}{2}} \\ \frac{\sqrt{m_d m_s}}{m_b}e^{i\frac{\pi}{2}} & \frac{m_s}{m_b}e^{i\frac{\pi}{2}} & e^{i\frac{\pi}{2}} \end{pmatrix}. \tag{6.4}$$

Using the above ansatz for the $b \to c$ transition, one can get a higher restriction for the upper bound on the t- quark mass ($V_{cb} \approx \frac{m_s}{m_b} + \frac{m_u}{m_t}$.

The experimental precision of the measurements of V_{cb} could indicate, and only to a certain extent, the magnitude of the d_{23}. Unfortunately, our knowledge of the d_{13} -element is still not enough because of big experimental uncertainties in q or $\sqrt{\rho^2 + \eta^2}$ values ($q \leq 0.2$). In scheme [7c] $q = |V_{ub}|/|V_{cb}| \approx 0.1$ and the Wolfenstein parameters are $\rho \approx -0.15$ and $\eta \approx 0.49$ ($\sqrt{\rho^2 + \eta^2} \approx \frac{1}{2}$), so that the ($V_{ub} - V_{td}$)- intersection of the unitary triangle lies in the second (left) quadrant of the ρ, η- complex plane (see Fig. 1).

To obtain the form of these mass matrices, it's necessary to consider, besides H - and h -Higgs superfields (see (2.3)), the additional superfields $H_0\,(1_H,\,2_L,\,-\frac{1}{2})$ and $h_0\,(1_H,\,2_L,\,\frac{1}{2})$, which are $SU(3)_H$ -singlets. Then the corresponding addenda $k_1\,Q\,h_0\,u^c$ and $k_2\,Q\,H_0\,d^c$ will appear in the superpotential (2.3).

The splitting between the horizontal gauge boson masses will be determined only by the nonvanishing VEV's of the $H(8_H, 2_L, -\frac{1}{2})$ and $h(8_H, 2_L, +\frac{1}{2})$ Higgs fields: $<\hat{H}>_0 = \varphi^a(\frac{\lambda^a}{2})$ and $<\hat{h}>_0 = \tilde{\varphi}^a(\frac{\lambda^a}{2})$, where

$$\begin{aligned} \varphi_2 &= -2\sqrt{m_d m_s}/\lambda_3;\ \varphi_4 = 2\sqrt{m_d m_s}/\lambda_e;\ \varphi_6 = 2m_s/\lambda_3; \\ \varphi_3 &= -2m_s/\lambda_3;\ \ \varphi_8 = -\frac{2}{\sqrt{3}}(m_b - \frac{m_s}{2})/\lambda_3. \end{aligned} \tag{6.5}$$

and similarly for the nonvanishing VEV's of the h-Higgs fields:

$$\begin{aligned} \tilde{\varphi}_1 &= \tilde{\varphi}_4 = 2\sqrt{m_u m_c}/\lambda_5;\ \tilde{\varphi}_6 = -2m_c/\lambda_5; \\ \tilde{\varphi}_3 &= -m_c/\lambda_5;\ \ \tilde{\varphi}_8 = -\frac{2}{\sqrt{3}}(m_t - \frac{m_c}{2})/\lambda_5. \end{aligned} \tag{6.6}$$

Now we have the possibility to calculate the contributions of the horizontal interactions to the amplitudes of rare processes. After the calculations like in (5.18) and (5.19), the expression for the $K_L^0 - K_S^0$-meson mass difference takes the following general form:

$$\Big[\frac{(M_{12})_{12}^K}{m_K}\Big]_H = \frac{1}{2}\frac{g_H^4}{M_{H_0}^4}\sum_{a=1}^{8}\Big\{\Big[\tilde{\varphi}_a(D\frac{\lambda^a}{2}D^+)_{12}\Big]^2 + \Big[\varphi_a(D\frac{\lambda^a}{2}D^+)_{12}\Big]^2\Big\} f_K^2 R_K.$$

or

$$\begin{aligned} \Big[\frac{(M_{12})_{12}^K}{m_K}\Big]_H &= \frac{1}{8}\frac{g_H^4}{M_{H_0}^4} \\ &\Big\{\Big[- i\tilde{\varphi}_1(d_{21}d_{12}^* + d_{22}d_{11}^*) + \tilde{\varphi}_2(d_{22}d_{11}^* - d_{21}d_{12}^*) \\ &- i\tilde{\varphi}_4(d_{21}d_{13}^* + d_{23}d_{11}^*) + \tilde{\varphi}_5(d_{23}d_{11}^* - d_{21}d_{13}^*) \\ &- i\tilde{\varphi}_6(d_{22}d_{13}^* + d_{23}d_{12}^*) + \tilde{\varphi}_7(d_{23}d_{12}^* - d_{22}d_{13}^*) \\ &- i\tilde{\varphi}_3(d_{21}d_{11}^* - d_{22}d_{12}^*) + i\sqrt{3}\tilde{\varphi}_8(d_{23}d_{13}^*)\Big]^2 \\ &+ \Big[\tilde{\varphi} \to \varphi\Big]^2\Big\} f_K^2 R_K. \end{aligned} \tag{6.7}$$

Putting into formula (6.7) the expressions for φ, $\tilde{\varphi}$ and the elements d_{ij} of the D- mixing matrix, we can obtain the lower limit for the value M_{H_0}. So, we analyze the ratios:

$$\Big[\frac{\Delta m_K}{m_K}\Big]_H = \frac{g_H^2}{M_{H_0}^2} Re[C_K] f_K^2 R_K < 7.10^{-15} \tag{6.8}$$

and

$$\left[\frac{ImM_{12}}{m_K}\right]_H = \frac{1}{2}\frac{g_H^2}{M_{H_0}^2} Im[C_K] f_K^2 R_K < 2.10^{-17}. \qquad (6.9)$$

In formulas (6.8) and (6.9) the expression for C_K is as follows :

$$C_K = -\frac{g_H^2}{2\lambda_5^2}\frac{m_t^2}{M_{H_0}^2}\left[\frac{m_c}{m_t}\left(\sqrt{\frac{m_u}{m_c}} + \sqrt{\frac{m_d}{m_s}\frac{m_s}{m_b}}\right) + i\left(\sqrt{\frac{m_u}{m_c}\frac{m_c}{m_t}} + 3\sqrt{\frac{m_d}{m_s}\frac{m_c}{m_t}\frac{m_s}{m_b}}\right.\right.$$
$$\left.\left. + 2\sqrt{\frac{m_d}{m_s}\frac{m_s^2}{m_b^2}}\right)\right]^2 + \frac{m_s^2}{2\lambda_3^2}\frac{m_d}{m_s}\left[\left(1+\frac{m_d}{m_s}\right) - i\left(1+\frac{m_d}{m_s} - 2\frac{m_s}{m_b}\right)\right]^2, \qquad (6.10)$$

In the leading approximation, for the value of $R_K = \frac{1}{6} + \frac{1}{3}\frac{m_K^2}{(m_s - m_d)^2}$,$f_K = 0.163\,GeV$, $m_t = 150GeV$ and $g_H \simeq 0.488$ we obtain from formulas (6.7) and (6.8) that:

$$\begin{aligned} M_{H_0} &> 180\,GeV \ \ (\lambda_5 \simeq \lambda_3 \simeq O(1)), \\ M_{H_0} &> 550\,GeV \ \ (\lambda_5 \simeq \lambda_3 \simeq O(0.1)). \end{aligned} \qquad (6.11)$$

Similar estimates from formulas (6.7) and (6.9) yield:

$$\begin{aligned} M_{H_0} &> 510\,GeV \ \ (\lambda_5 \simeq \lambda_3 \simeq O(1)), \\ M_{H_0} &> 1.6\,TeV \ \ (\lambda_5 \simeq \lambda_3 \simeq O(0.1)). \end{aligned} \qquad (6.12)$$

In quite an analogous way, we should also write the expression for $M_{12}(B_d)_H$ $(M_{12}(B_d)_H)$:

$$\left[\frac{M_{12}(B_d)}{m_{B_d}}\right]_H = \frac{1}{2}\frac{g_H^2}{M_{H_0}^2} C_{B_d} f_{B_d}^2 R_{B_d}. \qquad (6.13)$$

The unitary suppression coefficient will take the following form:

$$C_{B_d} \approx \frac{g_H^2}{2\lambda_5^2}\frac{m_t^2}{M_{H_0}^2}\left[e^{-i\frac{3\pi}{4}}\sqrt{\frac{m_u}{m_c}\frac{m_c}{m_t}} - e^{-i\frac{5\pi}{4}}\sqrt{\frac{m_d}{m_s}\frac{m_c}{m_t}} + \sqrt{\frac{2m_d}{m_s}\frac{m_s}{m_b}}\right]^2,$$

From this formula and assuming that $\Delta m(B_d)/m(B_d) < (0.73 \pm 0.14)10^{-13}$, $R_{B_d} \simeq \frac{1}{6} + \frac{1}{3}\frac{m_{B_d}^2}{(m_b - m_d)^2}$, $f_{B_d} \simeq 0.14GeV$, we obtain :

$$\begin{aligned} M_{H_0} &> 380\,GeV \ \ (\lambda_5 \simeq \lambda_3 \simeq O(1)), \\ M_{H_0} &> 1.2\,TeV \ \ (\lambda_5 \simeq \lambda_3 \simeq O(0.1)). \end{aligned} \qquad (6.14)$$

Note that, if we take the value $f_{B_D} = 0.2\,GeV$, the lower bounds on the horizontal symmetry breaking scales (6.12) and (6.14) are approximately equal.

In this ansatz for the CP- violation parameter $\frac{1}{2} \times Im(\frac{p}{q})_H$ we have a well-defined magnitude:

$$\frac{1}{2} \times Im(\frac{p}{q})_{B_d} \approx \frac{ImM_{12}(B_d)_H}{ReM_{12}(B_d)_H} \approx 0.3. \qquad (6.15)$$

Finally, let us give a very useful estimate:

$$\left[\frac{\Delta m_{B_s}}{\Delta m_{B_d}}\right]_H \approx \left[\frac{d_{23}}{d_{13}}\right]^2 \approx \frac{1}{2}\frac{m_s}{m_d} \approx 7 \div 10 \qquad (6.16)$$

and compare it with a similar ratio from the above ansatz of the SM

$$\left[\frac{\Delta m_{B_s}}{\Delta m_{B_d}}\right]_{EW} \approx \frac{1}{2}\frac{m_s}{m_d} \times (1 + \frac{m_c}{m_t}\frac{m_b}{m_s})^2 \approx 17 \div 20. \qquad (6.17)$$

We have assumed that $(f_{B_s} \simeq f_{B_d})$ and $m_t^{phys} \approx 100 - 120GeV$. Taking into account only the electroweak interactions and the values of the ρ, η-parameters in this scheme, we'll see that this ratio will be equal to $\sim 17 - 20$. This result could be interpreted, for instance, in the following way. Let us assume that the experimental ratio x_s/x_d is considerably less than 17, and the electroweak interactions will saturate the $B_s^0 - \bar{B}_s^0$- mixing. Then the electroweak contributions to a similar mixing between the B_d^0- and $\bar{B}_d^0$- meson states will be suppressed by the factor of $\sim$ 17. But in this case the contributions from horizontal interactions to the $B_d^0 - \bar{B}_d^0$- mixing might prove to be dominant [23,11]. Then the predictions for the CP- violation effects of $B_d^0, \bar{B}_d^0$- meson decays might be relevant for the horizontal interactions (6.15). This situation seems to be rather amazing because of the two "sources" of CP- violation in this approach. Really, CP- violation is evoked by the complex VEV's of the Higgs field $H^a(8_H, 2_L, -1/2), a = 2$, and manifests itself via the contribution from both EW interactions (δ^{KM}) and horizontal ones. Therefore, in the case of the dominant contribution of horizontal interactions the CP- violation asymmetry will be equal in both processes: $B \to J/\Psi K_S$ and $B \to \pi^-\pi^+$.

After all, in this section we calculate the branching ratio for the $\mu \to 3e$-decay. For this we have proposed the form of the charged lepton mass matrix

- the one that was used for down- quarks. Using this ansatz for a charged lepton mixing, we find out that:

$$\begin{aligned} Br(\mu\to 3e) & \simeq 12\frac{g_H^4}{g_W^4}\frac{M_W^4}{M_{H_0}^4}\Big\{f^{abc}\Big(L\frac{\lambda^a}{2}L^+\Big)_{21}(g_H\phi^b)f^{a'b'c}\Big(L\frac{\lambda^{a'}}{2}L^+\Big)_{11}(g_H\phi^{b'})\Big\}^2 \\ & \simeq \frac{3}{2}\frac{g_H^4}{g_W^4}\frac{M_W^4}{M_{H_0}^4}\frac{g_H^4}{\lambda_5^4}\frac{m_t^4}{M_{H_0}^4}\Big[-\frac{1}{\sqrt{2}}\frac{m_c^2}{m_t^2}\sqrt{\frac{m_u}{m_c}}+\frac{m_c}{m_t}\Big(\sqrt{\frac{m_u}{m_c}}-\sqrt{\frac{2m_e}{m_\mu}}\Big)\frac{m_\mu}{m_\tau}\Big]^2, \end{aligned} \quad (6.18)$$

where($\phi = \tilde{\varphi}^a, \varphi^a$) are the nonvanishing VEV's of the h^a- and H^a-Higgs fields, respectively.

The comparison of expression (6.18) with the experimental limit for this branching ratio gives us a very small value for the horizontal symmetry breaking scale:

$$\begin{aligned} M_{H_0} &> 0.49\,TeV\,, \quad if\ \lambda_5\sim 1 \\ M_{H_0} &> 1.56\,TeV\,, \quad if\ \lambda_5\sim 0.1. \end{aligned} \quad (6.19)$$

6.2. The $SUSY SU(3)_{HV}$-gauge model with "Democratic" ansatz for quark and lepton families

In the electroweak $SU(2)\times U(1)$- model, it's impossible to define separately the "mixings" in up- and down-quark mass matrices ("absolute mixing"). The SM still provides a certain freedom in choosing the primary mass matrices for quarks in such a way as to get large mixings both for up-, U, and down-quarks D, provided $UD^+ = V_{CKM}$. To get information on this "absolute mixing", one should investigate rare processes in the framework of the horizontal gauge model. Therefore, it would be very interesting to consider a scheme, in which these mixings are large, and to study possible constraints on the horizontal symmetry breaking scale M_{H_0}. So, we consider the "up"- and "down"- fermion mass matrices of the following ("democratic") form, which has been used for explaining the special role of the t- quark mass ($m_t \sim \Lambda_{EW} \gg m_c, m_u$) [7a,24]:

$$\mathbf{M_f^0} = \frac{1}{3}m_f\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (6.20)$$

The diagonalization of these matrices yields a mass gap, i.e. the masses of t- or b- quarks are split far apart from all other degenerate masses of $c-$, u- or $s-$, d-quarks. The complete mass matrices are $M_f = M_f^0 + \Delta M_f$. As a result of the diagonalization, they yield the physical fermion mass matrices M^D for "up" and "down" quarks: $M_f^D = V_f^+ M_f V_f$, where $V_f = V_{f0} V_{f1}$ and $V_{di} = D_i$, $V_{ui} = U_i$, $i = (0,1)$. Here $V_{f0}(\ f = d\,or\,u)$ are unitary matrices constructed from the eigenvectors of the M_f^0-matrix and equal to :

$$(\mathbf{U}^0)^{\mathrm{T}} = (\mathbf{D}^0)^{\mathrm{T}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{6.21}$$

In the first approximation, there is a conservation of the isotopic symmetry of the mixing mechanism in the up" and "down" quark mass matrices. The matrices U_1 and D_1 ($U_1 \neq D_1$) are small corrections to produce the correct form of the V_{CKM}-matrix. Using the U_1- and D_1- correction matrices, we can construct the mass matrices M_f differing from M_f^0 by a small correction factor.

As in the previous subsection, let us construct the corresponding splitting of the horizontal gauge boson mass matrix. In this case, each of the 8×8-dimensional mass matrices: $\left[\Delta M_H^2\right]_d^{ab}$ and $\left[\Delta M_H^2\right]_u^{ab}$, a, $b = 1, 2...8$, is broken into 3×3- and 5×5-dimensional matrices. The additional contributions to the mass spectra of the $H_{2,5,7}^{\mu}$- and $H_{1,4,6,3,8}^{\mu}$- horizontal gauge bosons take, correspondingly, the following forms:

$$\left[\mathbf{\Delta M_H^2}\right]_{\mathbf{f}}^{\mathbf{a,b=2,5,7}} = \frac{g_H^2 m_f^2}{12\lambda_f^2} \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}, \tag{6.22}$$

$$\left[\Delta \mathbf{M}_{\mathbf{H}}^{2}\right]_{\mathbf{f}}^{\mathbf{a,b=1,4,6,3,8}} = \frac{g_H^2 m_f^2}{36\lambda_f^2}\begin{pmatrix} 2 & -1 & -1 & 0 & 2\sqrt{3} \\ -1 & 2 & -1 & 3 & -\sqrt{3} \\ -1 & -1 & 2 & -3 & -\sqrt{3} \\ 0 & 3 & -3 & 6 & 0 \\ 2\sqrt{3} & -\sqrt{3} & -\sqrt{3} & 0 & 6 \end{pmatrix}. \qquad (6.23)$$

The diagonalization of these mass matrices can easily be realized by the orthogonal matrices $O^{(-)}$ and $O^{(+)}$:

$$O^{(-)}\left[\Delta M_H^2\right]_f^{a,b=2,5,7} O^{(-)T} = \left[\Delta M_D^2\right]_f^{(-)}, \qquad (6.24)$$

so that $Z_a^{\mu(-)} = O_{ab}^{(-)} H_b^{\mu(-)}$ ($a,b = 2,5,7$) and

$$O^{(+)}\left[\Delta M_H^2\right]_f^{a,b=1,4,6,3,8} O^{(+)T} = \left[\Delta M_D^2\right]_f^{(+)}, \qquad (6.25)$$

where $Z_a^{\mu(+)} = O_{ab}^{(+)} H_b^{\mu(+)}$ $(a,b = 1,4,6,3,8)$. In accordance with expressions (6.22),(6.24) and (6.23),(6.25), let us write down the forms of the $O_{ab}^{(-)}$- and $O_{ab}^{(+)}$- diagonalizing matrices:

$$\mathbf{O}_{\mathbf{ab}}^{(-)\mathbf{T}} = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \qquad (6.26)$$

$$\mathbf{O}_{\mathbf{ab}}^{(+)\mathbf{T}} = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & -\frac{2}{3} & 0 & \frac{\sqrt{2}}{3} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{3} & \frac{1}{\sqrt{6}} & -\frac{1}{3\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{3} & -\frac{1}{\sqrt{6}} & -\frac{1}{3\sqrt{2}} \\ 0 & \frac{1}{\sqrt{3}} & 0 & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & \frac{\sqrt{3}}{3} & 0 & \sqrt{\frac{2}{3}} \end{pmatrix}. \qquad (6.27)$$

Note that the signs $(-)$ and $(+)$ indicate the opposite CP- transformation properties of the $J^{\mu}_{H_{2,5,7}}$- and $J^{\mu}_{H_{1,4,6,3,8}}$- gauge horizontal currents for each value of the index μ ($\mu = 0 or 1, 2, 3$). Until there is no mixing between these currents, there may be no CP- violation in the gauge sector of the horizontal interactions [9].

As a result of this approach, we get a very simple splitting between 8-gauge Z^{μ}_{a}- bosons:

$$\begin{aligned} M^2_{Z_1} &= M^2_{Z_4} = M^2_{Z_6} = M^2_{Z_7} = M^2_{H_0} \\ M^2_{Z_3} &= M^2_{Z_8} = M^2_{Z_2} = M^2_{Z_5} = M^2_{H_0} + \frac{g^2_H}{4}\sum_f \frac{m^2_f}{\lambda^2_f}. \end{aligned} \tag{6.28}$$

The mass spectra of $Z^{\mu}_{1,4,6,7}$- gauge bosons correspond to the global $SU(2)_H \times U(1)_H$- symmetry in the gauge sector, which was considered in sections 3 and 4.

If we use the family mixing (6.1), the Lagrangian for the quark- gauge boson interactions will be

$$\begin{aligned} \mathcal{L}_Q &= \frac{g_H}{2}\bar{Q}\gamma_\mu\Big(\Big[-\lambda^8\Big]Z^{\mu}_1 \\ &+ \Big[\frac{\sqrt{3}}{2}\lambda^3 - \frac{1}{2}\lambda^1\Big]Z^{\mu}_4 + \Big[-\frac{1}{2}\lambda^3 - \frac{\sqrt{3}}{2}\lambda^1\Big]Z^{\mu}_6 \\ &+ \Big[\frac{1}{2}\lambda^4 - \frac{\sqrt{3}}{2}\lambda^6\Big]Z^{\mu}_3 + \Big[\frac{1}{2}\lambda^6 + \frac{\sqrt{3}}{2}\lambda^4\Big]Z^{\mu}_8 \\ &+ \Big[\lambda^5\Big]Z^{\mu}_2 - \Big[\lambda^7\Big]Z^{\mu}_5 - \Big[\lambda^2\Big]Z^{\mu}_7\Big)Q, \end{aligned} \tag{6.29}$$

where $Q = Q_d = (d, s, b)$ or $Q = Q_u = (u, c, t)$.

When the "democracy" is broken, we have to take into account a certain small quark mixing and, maybe, the Z^{μ}_{a}- gauge boson mixing. Then the previous expression changes to be:

$$\begin{aligned} \mathcal{L}_Q &= \frac{g_H}{2}\bar{Q}\gamma_\mu\Big(\Big[-D_1\lambda^8 D_1^+\Big]\tilde{Z}^{\mu}_1 \\ &+ \Big[\frac{\sqrt{3}}{2}D_1\lambda^3 D_1^+ - \frac{1}{2}D_1\lambda^1 D_1^+\Big]\tilde{Z}^{\mu}_4 - \Big[\frac{1}{2}D_1\lambda^3 D_1^+ + \frac{\sqrt{3}}{2}D_1\lambda^1 D_1^+\Big]\tilde{Z}^{\mu}_6 \end{aligned}$$

$$+ \left[\frac{1}{2}D_1\lambda^4 D_1^+ - \frac{\sqrt{3}}{2}D_1\lambda^6 D_1^+\right]\tilde{Z}_3^\mu + \left[\frac{1}{2}D_1\lambda^6 D_1^+ + \frac{\sqrt{3}}{2}D_1\lambda^4 D_1^+\right]\tilde{Z}_8^\mu$$
$$+ \left[D_1\lambda^5 D_1^+\right]\tilde{Z}_2^\mu - \left[D_1\lambda^7 D_1^+\right]\tilde{Z}_5^\mu - \left[D_1\lambda^2 D_1^+\right]\tilde{Z}_7^\mu\Big)Q\,, \qquad (6.30)$$

where again one has $Q = Q_d = (d, s, b)$, or $Q = Q_u = (u, c, t)$.

At this stage, we will not consider the additional gauge boson mixing and just assume that $M_{Z_i} \approx M_{\tilde{Z}_i}$, i=1,2,3..8. For our purpose, it will suffice to take into account only new small correction to the quark family mixing -D_1 and U_1 matrices, leading to the correct form of the CKM- matrix for charged EW currents.

Now, for the further estimates of the $SU(3)_H$- symmetry breaking scale M_{H_0} we use the results from section 4. To this end, we need the following useful relation

$$\sum_a (DT^a D^+)_{ik}(DT^a D^+)_{mn} = \frac{1}{2}\Big(\delta_{in}\delta_{km} - \frac{1}{3}\delta_{ik}\delta_{mn}\Big)\,. \qquad (6.31)$$

Then, the expression for the K_L^0- K_S^0 meson mass difference (4.2), derived from formulas (6.31), (6.28) and (6.30), will change only due to the new suppression coefficient $\sim \frac{m_f^2}{M_{H_0}^2}$, e.g.

$$\left\{\frac{\Delta m_K}{m_K}\right\}_H = \frac{g_H^2}{M_{H_0}^2}\left\{\frac{g_H^2}{4M_{H_0}^2}\sum_f \frac{m_f^2}{\lambda_f^2}\right\}Re(C_K^0)f_K^2 R_K\,. \qquad (6.32)$$

where $C_K^0 = \sum_a'(D_1T^aD_1^+)_{21}(D_1T^aD_1^+)_{21}$, and the index ($\prime$) indicates that summation is only over the diagrams with the exchanges of $Z_1^\mu, Z_4^\mu, Z_6^\mu, Z_7^\mu$- gauge horizontal bosons. In this approximation, the lower bound on the local horizontal symmetry breaking scale may be smaller than in case (i) from section 4 (the $SU(2)_H \times U(1)_H$ symmetry). For instance, if we take that $m_f = m_t$ and make our usual assumption for the relation $\frac{g_H}{\lambda_f}$, we can get:

$$M_{H_0} \approx \sqrt{\frac{g_H}{2\lambda_f}}\sqrt{\frac{m_t}{M_H}} \times M_H > O(0.8)TeV\,. \qquad (6.33)$$

In the last inequality we use the estimate for $M_H > 8 - 9TeV$ taken from Table 2. Note, that the consequences for the $B_{d,s}^0 \longleftrightarrow \bar{B}_{d,s}^0$ oscillations remain as in Table 2- e.g., this value for the gauge symmetry breaking scale (6.33) corresponds to the present quantity of the $B_{d_1}^0$- $B_{d_2}^0$- meson mass difference.

Really, one could expect a very low horizontal local symmetry breaking scale- M_{H_0} in pure quark (or pure lepton) rare processes due to the changes of the quantum numbers of generations therein: $|\Delta H| \neq 1$. From the experimental limits on the amplitudes of quark- lepton rare processes, where $\Delta H = 0$, we obtain considerably larger values for this scale (4.7;4.8),(4.10). What are the consequences of the studies of the lower bound on the horizontal local symmetry breaking scale ? Here are some of them:

1.The most pronounced processes promoting the discovery of a new hypothetic interaction are quark- lepton rare processes like $K \longrightarrow \pi + \mu + e$-, or the μ/e- conversion on nuclei. Within this class, the decay $K \longrightarrow \pi + \nu_i + \nu_j$ may also turn out to be very important. In this case, the "traditional" construction for the quark- lepton families has been assumed: $C_1 = (Q_1[u,d]; L_1[\nu_e, e]), C_2 = (Q_2[c,s]; L_2[\nu_\mu, \mu]), C_3 = (Q_3[t,b]; L_3[\nu_\tau, \tau])$ and here the mixings between Q_i- and L_j- families are not very large. Then we should think that the local horizontal symmetry breaking scale is very large, as follows from the limits (4.7;4.8) - e.g., it may be more than 60 Tev. For this large enough scale, the contributions to the pure quark rare reactions (meson mixings), or pure lepton rare decays ($\mu \rightarrow 3e$ etc.), from these forces will be very small. Especially so, if the splitting between the masses of 8- gauge horizontal bosons is as in our previous examples: $|(\Delta M_H^2)_a| \ll M_H^2$. For the case of the large splitting $|(\Delta M_H^2)_a| \gg minM_{H^a}^2$, we may use , for practical purposes, the results of the $U(1)_{T_3} \times U(1)_{T_8}$-gauge group (section 4), where it was established that the lower limits on the M_H-scale are: $M_H > 170 - 195TeV$ (Δm_K ,Table 2); $M_H > 60-100TeV$ (from the μ/e- conversion on nuclei); and $M_H > 35TeV$ (from $K \longrightarrow \pi + \mu + e$ if $B \leq 10^{-10}$ (4.10)), or $M_H > 100TeV$ (if the limit 10^{-12} is reached in the nearest future in BNL- experiment). The lower bound on M_H'obtained from the modern experimental limit on a pure lepton rare decay, like $\mu \rightarrow 3e$, is compatible with the bounds resulting from the $K \longrightarrow \pi + \mu + e$ - experiment. So, we have $M_H > 28TeV$ (4.5).

2. An alternative scenario we have to consider is connected with searching for another possible mechanism of the (q-l) - mixing to diminish the scale M_H to the values approaching the region of (1-10) Tev. For the purpose, we may also use an indefinite correlation both between the Q_i- and L_j- family mixings, and, within L_j- lepton families, - between charged lepton and neutrino mixings, so far as the experimental situation allows us to do so. These explicit differences in the origin of quark and lepton mass spectra make one also suppose that leptonic families might mix by quite a different mechanism, different from the above example of quark mixing. It should be remembered

that in the SM it's impossible, in principle, to establish a correlation between the Q_i-quark and L_j- lepton mixings. Due to electroweak interactions, we can only get information on the correlations between up- and down- quark mixing. But now there is still a certain freedom in the choice of the mixing models for charged leptons or neutrino, especially in the case of very small neutrino masses.

From the analysis of pure quark (lepton) rare processes in the gauge horizontal model we may get complete information about separate "absolute" mixings of up- quarks (neutrinos) and down- quarks (charged leptons). And from quark- lepton rare reactions in the frames of gauge horizontal interactions we may define correlations between $[d, s, b]$ ($[u, c, t]$) quark- and $[e, \mu, \tau]$ or $[\nu_e, \nu_\mu, \nu_\tau]$- lepton bases. In the above examples (see section 4), the supposition of the absence of correlation between down- quark and charged lepton mixings resulted in rather high limits for the M_H-scale, obtained from quark-lepton processes, compared to those from pure "q", or pure "l" -processes. Now let us consider the scheme when the magnitude of correlation between quark- and charged lepton mixings is large.

$$\begin{aligned}
\mathcal{L}_l &= \frac{g_H}{2}\bar{\Psi}_l\gamma_\mu\Big(\Big[\frac{\sqrt{3}}{2}L_1\lambda^3L_1^+ + \frac{1}{2}L_1\lambda^8L_1^+\Big]Z_1^\mu \\
&+ \Big[\frac{\sqrt{3}}{4}L_1\lambda^3L_1^+ - \frac{3}{4}L_1\lambda^8L_1^+ - \frac{1}{2}L_1\lambda^6L_1^+\Big]Z_4^\mu \\
&- \Big[\frac{1}{4}L_1\lambda^3L_1^+ - \frac{\sqrt{3}}{4}L_1\lambda^8L_1^+ + \frac{\sqrt{3}}{2}L_1\lambda^6L_1^+\Big]Z_6^\mu \\
&+ \Big[-\frac{\sqrt{3}}{2}L_1\lambda^1L_1^+ + \frac{1}{2}L_1\lambda^4L_1^+\Big]Z_3^\mu + \Big[\frac{1}{2}L_1\lambda^1L_1^+ + \frac{\sqrt{3}}{2}L_1\lambda^4L_1^+\Big]Z_8^\mu \\
&- \Big[L_1\lambda^5L_1^+\Big]Z_2^\mu + \Big[L_1\lambda^2L_1^+\Big]Z_5^\mu - \Big[L_1\lambda^7L_1^+\Big]Z_7^\mu\Big)\Psi_l , \qquad (6.34)
\end{aligned}$$

where $\Psi_l = (e, \mu, \tau)$.

It is obvious from this scheme that the lower bound on M_{H_0} can also be very small ($\sim O(1)TeV$) if pure charge lepton rare processes are considered (e.g., the modern high experimental limit for the $\mu^+ \to e^+ + e^+ + e^-$ - decay). Again, in this approach there appears a similar additional suppression factor $\sim \frac{m_f^2}{M_H^2}$ for the partial width of this process. After repeating the calculations we did for the bound (6.33) of this process, here we get the value compatible with the limit for the horizontal breaking scale M_{H_0} in case (ii) $(U(1)_{3H} \times U(1)_{8H}$-symmetry) of section 4, with $M_H > 28TeV$. Now, assuming that $m_f \approx m_t$

and $(L_1T^3L_1^T)_{21}(L_1T^3L_1^T)_{11} = L^3 \approx \sqrt{\frac{m_e}{m_\mu}}$, as was accepted in formula (4.5) (Fritzsch ansatz for lepton mixing), we have: $M_{H_0} > \sqrt{\frac{g_H}{2\lambda_f}}\sqrt{m_f M_H} > 1.5TeV$.

Besides, in this model we could obtain lower limits for the horizontal symmetry breaking scale by analyzing quark- lepton rare processes like $K \longrightarrow \pi + \mu + e$-, or the μ/e- conversion. For example, for process of the first type the estimate (4.10) comes to be

$$M_{H_0} \approx \sqrt{2|d_{13}|}\, M_H \;\; (K^+ \longrightarrow \pi^+ + \mu^+ + e^-)\,, \tag{6.35}$$

$$M_{H_0} \approx \sqrt{2|d_{12}d_{23}|}\, M_H (K^+ \longrightarrow \pi^+ + \mu^- + e^+)\,. \tag{6.36}$$

If we take the values for $(D_1)_{ij} = d_{12}, d_{23}, d_{13}$ from all the three ansatzes, we may verify using formulas (6.35) and (4.10) that the scale M_{H_0} will be rather low: $M_{H_0} > O(1TeV)$ for $M_H > 35TeV$. The last estimate is conditioned by the model-dependent value of d_{13}, the latter being not very precisely defined from the comparison with the V_{cb}- matrix element. Here we assume that $|d_{13}| \ll |V_{cb}|$, which does not contradict the experiment. Note, in this scheme we make a very interesting prediction for the heavy quark-, or heavy τ- lepton rare decay. For example, for this horizontal symmetry breaking scale ($M_{H_0} \sim 1TeV$) the partial width for the τ- lepton decay - $\tau \rightarrow \mu K^{0*}$ may be rather large $\sim 10^{-5}$.

By analogy, we find that the rate for μ to e- conversion (4.6) is reduced by the factor $2|d_{12}d_{13}|$. Combining this factor and the expression (4.6), we come to the following limit for M_{H_0}:

$$M_{H_0} \approx \sqrt{2|d_{12}d_{13}|}\, M_H > O(1-2)\,TeV\,, \tag{6.37}$$

if the limits we obtain for M_H are as in formula (4.7).

In this scheme the bounds (6.35) and (6.37) differ by the factor $\sqrt{|d_{12}|}$. Note, that our earlier limits for M_H (4.10) and (4.7) from for these two very important processes also differ by the same factor $\approx 2 - 2.5$.

Finally, we have to consider the decay $K \longrightarrow \pi + \nu + \nu$. Now the experimental lower limit for the partial width of this process is :

$$Br\Big(K \longrightarrow \pi + \nu_i + \nu_j\Big) < 5 \times 10^{-9}.BNL. \tag{6.38}$$

According to (6.38), the immediate estimate of this decay in our approach gives us the following constraint: $M_{H_0} > 10TeV$. To lower this limit, it's necessary to elucidate the origin of the neutrino mass spectrum. Clearly, to achieve this one has to consider an extension of the fermion matter spectrum of new particles- first of all, new neutral neutrino- like particles ($T_{SU(2)} = \frac{1}{2}, SU(3)_H$-singlet). This could result in an efficient decrease in the value of the coupling constant gH in the neutrino horizontal interaction. The studies of the regularities in the observed mass spectrum of ordinary particles might indicate possible existence of new partcles like those occurring in GUT's (E(6)) or in other earlier extensiions of the SM (Left- Right models with mirror particles or with the fermion spectrum doubled). Composite models could also promote the understanding of the mass spectrum origin (except, probably, the neutrino mass spectrum).

So, as long as we do not have a clear understanding of the mass spectrum origin, we cannot claim that our predictions for the gauge horizontal symmetry breaking scale ought to coincide for different types of experiments (pure quark-, pure lepton-, quark-charged lepton-, quark- neutrino- rare processes). That is why it is important to go on with the searches for the local horizontal symmetry parallel to the above-mentioned experiments:

1a) Quark- charged lepton rare processes: $K \longrightarrow \pi + \mu + e$-, μ/e- conversion, $\tau \longrightarrow (\mu/\nu) + meson/mesons$, $B_{d,s} \longrightarrow L_i^+ + L_j^- + meson/mesons; \cdots$.

1b) Quark- neutrino rare processes: $K \longrightarrow \pi + \nu + \nu \cdots$.

2) Pure quark rare processes: K-,B-,D,- rare decays and the CP- violation effects .

3a) Pure charged lepton rare processes: $\mu \longrightarrow e^+ + e^- + e^+$, $\tau \longrightarrow L_i + L_j + \bar{L}_j; \cdots$.

3b) The searches for neutrino oscillations and Dirac/Majorana neutrino masses.

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7. Appendix

7.1

To estimate $g_H(\mu)$, one could use the renorm-group equations for the running coupling constants:

$$\begin{aligned} \frac{d\alpha_i}{dln\mu} &= \frac{b_i}{2\pi}{\alpha_i}^2, \\ i &\longrightarrow SU(3^c),\, SU(2)_L,\, U(1)_Y,\, SU(3)_H, \end{aligned} \qquad (7.1)$$

where b_i for the group SU(N) in the supersymmetric case is given by the expression:

$$b_i = -3N + \sum n_R C(R) \quad (N = 0 for\ group U(1)). \qquad (7.2)$$

Here n_R means the number of R- dimensional representations, while C(R) is used for the corresponding Casimir operator. To make a qualitative estimation, one may neglect in the RG equations the contributions from possible heavy matter particles, e.g., those from mirror fermions. In the given approximation, for b_2, b_3 and b_1 the following standard values can be used:

$$\begin{aligned} b_3 &= -9 + 2N_g = -3; \\ b_2 &= -6 + 2N_g + \frac{n_2}{2} = \frac{n_2}{2}; \\ b_1 &= 2N_g + \frac{3}{10}n_2. \end{aligned} \qquad (7.3)$$

with n_2 the number of the Higgs EW- doublets of the theory exploited in the electroweak sector. However, to calculate the value of the function b_{3H}, one should, first of all, allow (in addition to ordinary quark and lepton superfields) for the superfields participating in horizontal interactions. The result is:

$$b_{3H} = -3N + \frac{1}{2} \times 16 + \frac{1}{2} \times n_3 + N \times n_8 \qquad (7.4)$$

with n_3, n_8 meaning the number of the Higgs superfields, triplet and octet under the $SU(3)_H$, which we consider in the framework of the given theory. Proceeding from the equality of the constants on the scale gauge M_X - $g_H^2(M_X) = g_3^2(M_X) = g_2^2(M_X) = \frac{5}{3}g_1^2(M_X)$ - the quantity g_H may be compared with g_2 on the scale $\mu \sim M_H$. The difference between these two quantities is mainly due to the particular choice of the Higgs fields leading to the breaking of the electroweak and horizontal symmetries, respectively:

$$\begin{aligned} \frac{1}{\alpha_{3H}(\mu)} &= \frac{1}{\alpha_2(\mu)} + \frac{b_{3H} - b_2}{2\pi} \ln \frac{M_X}{\mu} \\ &= \frac{1}{\alpha_2(\mu)} + \frac{-1 + 1/2\, n_3 + 3\, n_8 - 1/2\, n_2}{2\pi} \ln \frac{M_X}{\mu}. \end{aligned} \tag{7.5}$$

So, for instance, when one takes into account the fields $\Phi(8_H, 1_L)$, $H(8_H, 2_L)$, $h(8_H, 2_L)$ the difference between $\alpha_{3H}(\mu)$ and $\alpha_2(\mu)$ is expressed by the formula:

$$\frac{1}{\alpha_{3H}(\mu)} - \frac{1}{\alpha_2(\mu)} = \frac{1}{2\pi} \ln \frac{M_X}{\mu} \tag{7.6}$$

whence $\alpha_{3H}(\mu) \leq \alpha_2(\mu)$. It is interesting to estimate this difference for the matter supermultiplet 27 from (3-generations) the E_6- group. As a result on the scale $\mu \sim 1TeV$ ($M_X \sim 10^{16} GeV$) we get $\frac{1}{\alpha_{3H}(\mu)} - \frac{1}{\alpha_2(\mu)} = \frac{7}{4\pi}$. Finally note, that the inclusion of both matter and Higgs superfields, singlet under the $SU(2)_L$, makes the quantity α_{3H} still smaller compared to α_2.

7.2

The Fritzsch mass matrix for the d- quarks is defined in terms of the following vacuum expectations of the Higgs fields:

$$\lambda_3 < \mathbf{H_{ij}} > = \begin{pmatrix} 0 & A_d e^{-i\alpha_A} & 0 \\ A_d e^{i\alpha_A} & 0 & B_d e^{-i\alpha_B} \\ 0 & B_d e^{i\alpha_B} & 0 \end{pmatrix} \qquad \lambda_9 < \mathbf{Y_2^0} > = \begin{pmatrix} 0 \\ 0 \\ C_d e^{i\alpha_C} \end{pmatrix}$$

$$\lambda_{10} < \mathbf{X_2^0} > = \begin{pmatrix} 0 \\ 0 \\ C_d e^{-i\alpha_C} \end{pmatrix} \qquad \lambda_{12} < \kappa_2^0 > = \begin{pmatrix} D_d \end{pmatrix}$$

Similarly, the Fritzsch mass matrix for the u-quarks is derived from superpotential (2.5) when nonzero vacuum expectations of the Higgs fields h^2_{ij} , Y^0_1 , X^0_1 , κ^0_1 , arise, which have the form similar to down-quark mass matrix with the notations changed as $A_d \rightarrow A_u$, $B_d \rightarrow B_u$, $C_d \rightarrow C_u$, $D_d \rightarrow D_u$, $\alpha_A \rightarrow \tilde{\alpha_A}$; $\alpha_B \rightarrow \tilde{\alpha_B}$; $\alpha_C \rightarrow \tilde{\alpha_C}$, whereas $\lambda_3 \rightarrow \lambda_5$, $\lambda_9 \rightarrow \lambda_7$, $\lambda_{10} \rightarrow \lambda_8$, $\lambda_{12} \rightarrow \lambda_{11}$,. As a result, the mass matrix acquires the standard form:

$$\bar{\Psi}^{\rm d}_{\rm 0L}{\rm M}^{\rm d}\Psi^{\rm d}_{\rm 0R} + {\rm h.c.} = \bar{\Psi}^{\rm d}_{\rm 0L}\begin{pmatrix} 0 & A_d e^{-i\alpha_A} & 0 & 0 \\ A_d e^{i\alpha_A} & 0 & B_d e^{-i\alpha_B} & 0 \\ 0 & B_d e^{i\alpha_B} & 0 & C_d e^{-i\alpha_C} \\ 0 & 0 & C_d e^{i\alpha_C} & D_d \end{pmatrix}\Psi^{\rm d}_{\rm 0R} + h.c.$$

The signs of the eigenvalues are chosen in the standard way: $m_1 = -m_d$, $m_2 = m_s$, $m_3 = -m_b$, $m_4 = m_{b'}$ with $0 < m_d << m_s << m_b << m_{b'}$.

Assuming the above hierarchy of the masses, one can write the expression for $|A|$, $|B|$, $|C|$, $|D|$ in terms of the quark masses:

$$\begin{aligned} A_d &= \sqrt{m_d m_s};\ B_d = \sqrt{\frac{m_s m_b}{1 - \frac{m_b}{m_{b1}}}};\ C_d = \sqrt{m_b m_{b1}}; \\ D_d &= m_{b1} - m_b + m_s - m_d. \end{aligned} \quad (7.7)$$

For the case of up-quarks we have $m_1 = -m_u$, $m_2 = m_c$, $m_3 = -m_t$, $m_4 = m_{t1}$ with $0 < m_u << m_c << m_t < m_{t1}$. In this case

$$\begin{aligned} A_u &= \sqrt{m_u m_c};\ B_u = \sqrt{\frac{m_c m_t}{1 - \frac{m_t}{m_{t1}}}};\ C_u = \sqrt{m_t m_{t1}}; \\ D_u &= m_{t1} - m_t + m_c - m_u. \end{aligned} \quad (7.8)$$

Now, redefine the fermionic fields as where

$$\bar{\Psi}^d_{0L} M^d \Psi^d_{0R} + h.c. = \bar{\Psi}^d_{0L} D^+ D M^d D^+ D \Psi^d_{0R} + h.c. \quad (7.9)$$

where

$$D = d_{ij} diag(e^{i\alpha}, e^{i\beta}, e^{i\gamma}, e^{i\delta}) \quad (7.10)$$

Notice, that for the Fritzsch scheme $D_L = D_R = D$; $P_L = P_R = P$, and with the phase parameters of the matrix P chosen as $\alpha_A = \alpha - \beta$, $\alpha_B = \beta - \gamma$ $\alpha_C = \gamma - \delta$, the Fritzsch matrix becomes real and takes the form

$$\mathbf{M_d} = \begin{pmatrix} 0 & A_d & 0 & 0 \\ A_d & 0 & B_d & 0 \\ 0 & B_d & 0 & C_d \\ 0 & 0 & C_d & D_d \end{pmatrix}$$

and

$$M_u = M_d\,(A_d \to A_u;\ B_d \to B_u;\ C_d \to C_u;\ D_d \to D_u). \tag{7.11}$$

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