SI/Note MAE/70-3
12.2.1970

DESIGN OF THE PSB CORRECTING LENSES

(MULTIPOLES)

by

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1. Introduction

A number of correcting lenses, normal and skew quadrupoles, sextupoles and octupoles are foreseen for combating various instabilities of the PSB beams. Location and performance specifications for these lenses are given in ref. (1). A somewhat different design than used for the pultipole model (2) is necessary, because the location of the pick-up electrodes (3) required a considerably bigger multipole aperture.

2. Lay-out

The multipoles are of the shielded air-core type. Octupole, sextupole and skew-quadrupole or normal quadrupole are superimposed on each other.

The elements given in Table IV of ref. (1) are grouped into 3 constructional units :

1. 12 units per ring with octupole, sextupole, skew quadrupole
2. 4 " " " " " " , quadrupole
3. 4 " " " " quadrupole only

Sextupole and octupole are designed such that they can be operated continuously with the maximum required integral gradient strength. For the sake of standardization, the skew quadrupoles are of the same type as the normal (stopband) quadrupoles having a duty cycle corresponding to the main magnet cycle.

The derivation of formula used for the calculations are given in Appendix I. Coils having a uniform current density as suggested in (4) and (5) are used. The parameters are listed in Appendix II, a cross-sectional view of the multipole with its main dimensions is given in fig. 1.

3. Construction

The coils for octupoles and sextupoles are made from hollow water-cooled copper conductors, resulting in a compact single layer design. If necessary further increase of magnetic strength would be possible with special cooling arrangements (higher pressure). The current densities of quadrupoles and skew-quadrupoles are low and solid conductors are used.

The conductors will be insulated with a suitable tape (glass-mica or glass fiber) of 0.2 to 0.3 mm thickness before being bent. After mounting and connecting of the coils, the free space between coils will be filled with a radiation resistant epoxy resin.

The outer iron shield will be used for positioning and mounting of the four multipoles into one unit.

The following tolerances and errors can be expected :

I. Radial position of individual coils (copper) with respect to supporting tubes $< \pm 0.3$ mm

The resulting error in field distribution if for

the quadrupole	$\pm$ 0.3 %
the sextupole	$\pm$ 0.5 %
the octupole	$\pm$ 1.0 %

II. Precision of alignment :

i) Centering of coils with respect to the outer surface of the iron shield $\pm$ 0.2 mm

ii) Positioning of the multipoles
with respect to the alignment target $\pm$ 0.2 mm

iii) Precision of alignment $\pm$ 0.1 mm

These errors are statistical errors; they are independent of each other.

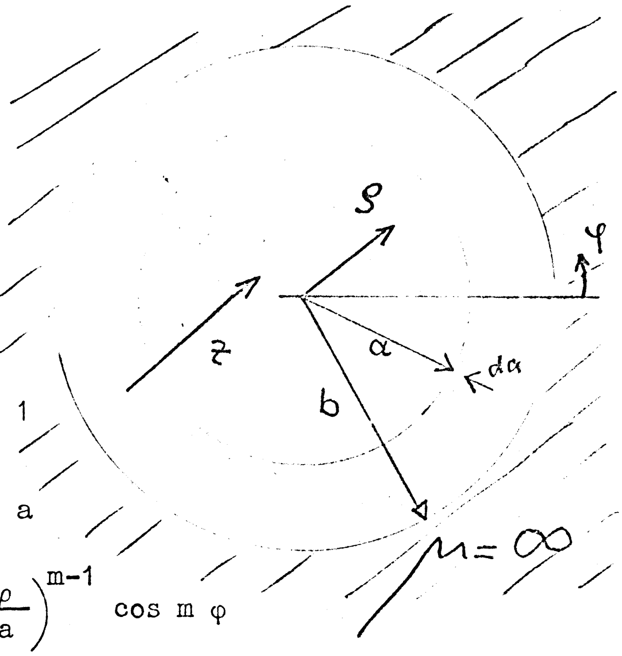
We have

$$\Delta R_{rms} = \frac{1}{2} \sqrt{\Sigma \Delta R^2} \approx 0.15 \text{ mm} \quad (\text{deviation } 2\sigma)$$

The azimuthal coil position is not critical and an error of $\pm$ 0.5^{o} could be tolerated.

APPENDIX I

Multipole fields and gradients



The field within a cylinder of $\mu = \infty$ material produced by a current at radius a in a shell of thickness da with the current density

$$i_z = i_o \cos m\varphi \qquad m = \text{integer} \geq 1$$

can be shown to be [6] for $\rho < a$

$$dB_\varphi = \mu_o\, i_o\, da\, \frac{a^{2m} + b^{2m}}{2b^{2m}} \left(\frac{\rho}{a}\right)^{m-1} \cos m\varphi$$

$$dB_\rho = \mu_o\, i_o\, da\, \frac{a^{2m} + b^{2m}}{2b^{2m}} \left(\frac{\rho}{a}\right)^{m-1} \sin m\varphi \qquad (1)$$

for $\rho > a$

$$dB_\varphi = \mu_o\, i_o\, da\, \frac{1}{2}\left[\left(\frac{a}{b}\right)^{m+1}\left(\frac{\rho}{b}\right)^{m-1} + \left(\frac{a}{\rho}\right)^{m+1}\right] \cos m\varphi$$

$$dB_\rho = \mu_o\, i_o\, da\, \frac{1}{2}\left[\left(\frac{a}{b}\right)^{m+1}\left(\frac{\rho}{b}\right)^{m-1} + \left(\frac{a}{\rho}\right)^{m+1}\right] \sin m\varphi \qquad (2)$$

For a current distribution between radii a_1 and a_2 one obtains by integration for $\rho < a_1$, $i_z = i_o \cos m\varphi$, $m = 2$:

$$B_\varphi = \frac{\mu_o\, i_o}{2} \left\{ \ell n \frac{a_2}{a_1} + \frac{1}{4}\left[\left(\frac{a_2}{b}\right)^4 - \left(\frac{a_1}{b}\right)^4 \right] \right\} \rho \cos 2\varphi \qquad (3)$$

for $m = 3,4 \ldots$

$$B_\varphi = \frac{\mu_o\, i_o}{2} \left[\frac{1}{m-2} \left(\frac{1}{a_1^{\,m-2}} - \frac{1}{a_2^{\,m-2}} \right) + \frac{a_2^{\,m+2} - a_1^{\,m+2}}{(m+2) b^{2m}} \right] \rho^{m-1} \cos m\varphi \qquad (4)$$

and for the multipole gradients from

$$g^{(m-2)} = \left(\frac{\vartheta^{m-1} B_\varphi}{\vartheta \rho^{m-1}} \right) \varphi = \pi/m \qquad (5)$$

for $m = 2$

$$g = \frac{\mu_o\, i_o}{2} \left(\ell n \frac{a_2}{a_1} + \frac{a_2^{\,4} - a_1^{\,4}}{4b^4} \right) \qquad (6)$$

for $m = 3,4 \ldots$

$$g^{(m-2)} = \frac{\mu_o\, i_o}{2} (m-1)! \left[\frac{1}{m-2} \left(\frac{1}{a_1^{\,m-2}} - \frac{1}{a_2^{\,m-2}} \right) + \frac{a_2^{\,m+2} - a_1^{\,m+2}}{(m+2)\, b^{2m}} \right] \qquad (7)$$

With a uniform current distribution j between 0 and $\varphi_o = \pi/3\,m$ degrees instead of $i_z = i_o \cos m\,\varphi$, one finds from harmonic analysis the current densities for the first and higher harmonics of each multipole :

$$i_{m,n} = \frac{4m}{\pi} \int_0^{\pi/3m} j \cos nx \, dx \qquad (8)$$

$$i_{m,n} = \frac{4m}{\pi n} \sin \frac{n\pi}{3m} \; j$$

With

$m = 2$,	$n = 2$	quadrupole
$m = 2$,	$n = 6,10 \ldots$	harmonics produced by quadr. coil
$m = 3$,	$n = 3$	sextupole
$m = 3$,	$n = 9,15 \ldots$	harmonics produced by sext. coil
$m = 4$,	$n = 4$	octupole
$m = 4$,	$n = 12,20 \ldots$	harmonics produced by oct. coil

one has

$$i_{2,2} = i_{3,3} = i_{4,4} = \frac{4}{\pi} \sin\frac{\pi}{3} \; j$$

$$i_{2,6} = i_{3,9} = i_{4,12} = 0 \qquad (9)$$

$$i_{2,10} = i_{3,15} = i_{4,20} = \frac{4}{5\pi} \sin\frac{\pi}{3} \; j$$

Introducing eq. (9) into eqs. (6) and (7), one obtains with F as the coil filling factor, a_1, a_2, b in [m] and j in [A/mm^2] for :

Quadrupole

$$g = 0.694 \; j \; F \left(\ell n \frac{a_2}{a_1} + \frac{a_2^{\;4} - a_1^{\;4}}{4b^4} \right) \quad [T/m] \tag{10}$$

Sextupole

$$g' = 1.39 \; j \; F \left\{ \frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{5b} \left[\left(\frac{a_2}{b} \right)^5 - \left(\frac{a_1}{b} \right)^5 \right] \right\} \quad [T/m^2] \tag{11}$$

Octupole

$$g'' = 2.08 \; j \; F \left\{ \frac{1}{a_1^{\;2}} - \frac{1}{a_2^{\;2}} + \frac{1}{3b^2} \left[\left(\frac{a_2}{b} \right)^6 - \left(\frac{a_1}{b} \right)^6 \right] \right\} \quad [T/m^3] \tag{12}$$

The higher harmonics produced by each coil can as well be found from eq. (7) and (9), but may be safely neglected for the lay-out of the Booster correction multipoles; their influence on the field distribution within the aperture is smaller than 1 °/o.

APPENDIX II

List of Parameters

	Octupole	Sextupole	(Skew-) Quadr.
Performance specification :			
Magnetic length [m]	0.4	0.36	0.31
Max. gradient strength	2/3 T/m^3	6.7 T/m^2	0.17 T/m
Continuous " "	"	"	0.11 T/m
Mechanical dimensions of coils (location of copper)			
Inner radius a_1 [mm]	90	102.5	122
Outer radius a_2 [mm]	98.5	111.0	134
Inner radius of iron shield b [mm]		135	
Conductor :			
Height x width [mm^2]	8.5 x 5.4		4 x 12
Insulated [mm^2]	9.1 x 6		4.6 x 12.6
	2.7 ∅ cooling hole		
Area [mm^2]	38		46
Number of turns	4	6	14
Filling factor	0,73	0.73	0.8
Electrical parameters :			
Current density max. [A/mm^2]	7.1		1.85
Max. current [A]	270		85
rms current [A]			57
Resistance R 40° C [mΩ]	14	15.4	20
Inductance [mH]	≤ 0.7	≤ 0.4	0.7
Power [kW]	1.05	1.15	0.065

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	Octupole	Sextupole	(Skew-) Quad.
Cooling :			
Hydraulic connection per element	series	series	
Pressure drop [atm]	4	4	
Flow/element [ℓ/min]	0.36	0.4	
Total [ℓ/min]	9		
Max. temperature increase [° C]	42	42	

References :

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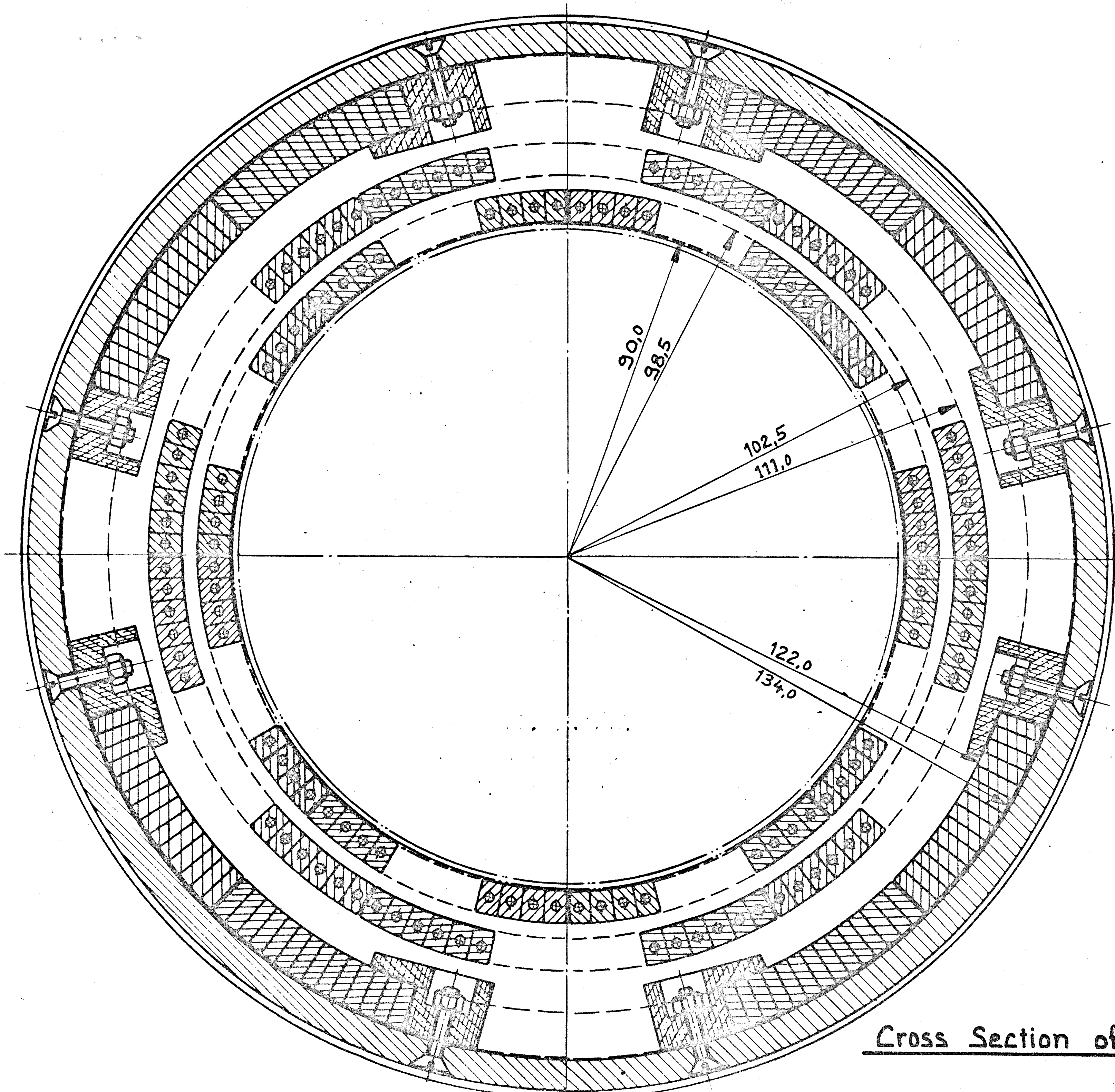

Fig.1

Cross Section of Multipole