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NOTE ON BEAM-LOADING EFFECT ON PSB PHASE-LOCK

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INTRODUCTION

The beam intensity influences the block-diagram and the response of a phase-control system.

The problem is to study the transfer function of the accelerating cavity to the phase and amplitude modulations of the currents of the generator and the beam.

Presently, we study only the "quasi-stationary" solution and we will consider the transient response later.

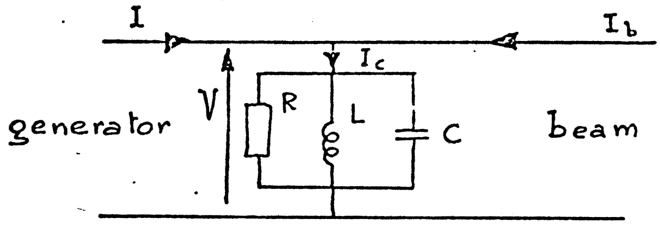
A transfer function is given, following ref. 1, and also the results of some tests on a practical model.

1. Calculation of transfer function

1.1 Hypothesis:

a) We consider only the fundamental of the beam current,

b) As ref. 1-3, we schematize the system composed of the cavity, the RF generator, and the beam as in the following figure



with the following definitions:

R	shunt impedance of the cavity in parallel with power tube resistance
I_o	amplitude of generator current
I_b	" " fundamental of the beam current
I_c	$= I + I_b$
V_o	nominal value of voltage
i, i_b, v	amplitude modulation of the corresponding value
ϕ_V	phase of V_o, <u>reference for phase measurement</u>
ϕ	phase of generator current
ϕ_s	stable phase
ϕ_b	phase of beam current (in general $\neq \phi_s$)[3] (defined from $\phi_V - \frac{\pi}{2}$)
$\Delta_\phi, \delta\phi_\beta, \delta\phi_V$	phase modulation terms

The sign of ϕ, $\delta\phi$, $\delta\phi_b$, $\delta\phi_V$ is negative for clock-wise rotation.
The sign of ϕ_b and ϕ_s is positive for clock-wise rotation

$\phi_s > 0$ accelerating condition

$\phi_s < 0$ decelerating condition.

c) We consider only a quasi-stationary solution (after transient)

d) The phase modulation index is small, compared with one radian

e) The beam and the RF have the same frequency.

1.2 Transfer function

For hypothesis (d) the transfer function from (I) to (V), is well approximated[1] by the low-pass analogue, defined in eq. 1

$$Z = \frac{R}{1 + j^2 \frac{Q}{\omega_o}\Delta\omega *} = \frac{Ry}{y + j\Delta\omega *} \qquad (1)$$

with $j\Delta\omega^{*} = j\omega + s + j\omega_o = s + j\Delta\omega$

$\omega_o = (LC)^{-1/2}$

ω = angular frequency of RF

$s = j\omega_m$ (modulation) complex operator

$y = \omega_o/2Q$ half bandwidth of the resonant circuit.

We have

$$I_c = (I_o + i)\, e^{j(\omega t + \varphi + \delta\varphi)} + (I_b + i_b)e^{j(\omega t - \frac{\pi}{2} - \varphi_b + \delta}$$

We have in phasors terms

$$I_c = (I_o + i)e^{j(\omega t + \varphi + \delta\varphi)} + (I_b + i_b)e^{j(\omega t - \frac{\pi}{2} - \varphi_b + \delta\varphi_b)} \qquad (2)$$

$$V = (V_o + v)e^{j(\omega t + \delta\varphi_V)} \qquad (3)$$

and $$I_c\, Z = V\ .$$

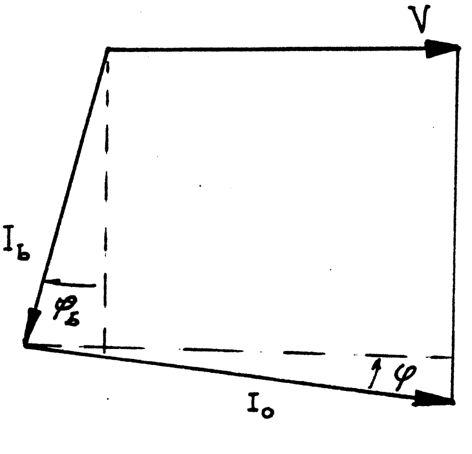
Since from eqs. 1,2,3 , neglecting second order terms, we find:

$$\Big\{ \left[I_o \cos\varphi + i\cos\varphi - I_o \sin\varphi\delta\varphi\right] + j\left[I_o \sin\varphi + i\sin\varphi + I_o\cos\varphi\delta\varphi\right]$$

$$- \left[I_b \sin\varphi_b + i_b \sin\varphi_b - I_b\cos\varphi_b\delta\varphi_b\right] - j\left[I_b\cos\varphi_b + i_b\cos\varphi_b + I_b\sin\varphi_s\delta\varphi_b\right]\Big\}$$

$$= \left[(V_o + v)(y + s) - \Delta\omega\, V_o\, \delta\varphi_V\right] + j\left[V_o\, \delta\varphi_V(y + s) + \Delta\omega(V_o + v)\right]. \qquad (4)$$

We can split the $(\Delta\omega)$ in three parts, and calculate some terms from the figure below



$$\frac{\Delta\omega}{y} = \left(\frac{\Delta\omega}{y}\right)_o + \frac{\delta\omega_o}{y} + \frac{\delta\omega}{y} = - \frac{RI_b \cos\varphi_b - RI_o \sin\varphi}{RI_o \cos\varphi - RI_b \sin\varphi_b} + \frac{\delta\omega}{y} \tag{5}$$

$$\left(\frac{\Delta\omega}{y}\right)_o = - \frac{RI_b \cos\varphi_b}{RI_o - RI_b \sin\varphi_s} \quad \text{detuning to have } \varphi = 0 \tag{6}$$

$\left(\frac{\delta\omega_o}{y}\right)$ = detuning due to $\varphi \neq 0$

$\frac{\delta\omega}{y}$ = modulation term due to tuning system

From eq. 4 considering the constant part of the real component we have

$$V_o = RI_o \cos\varphi - RI_b \sin\varphi . \tag{7}$$

Then from eqs. 4, 5, 6 separating real and imaginary components

$$\frac{v}{V_o}\left(1 + \frac{s}{y}\right) = - \frac{RI_o}{V_o}\delta\varphi + \frac{RI_b}{V_o}\cos\varphi_b\,\delta\varphi_b + \frac{R}{V_o}\cos\varphi\, i - \frac{R}{V_o}\sin\varphi_b\, i_b$$
$$+ \left[\left(\frac{\Delta\omega}{y}\right)_o + \frac{\delta\omega_o}{y}\right]\delta\varphi_V + \frac{\delta\omega}{y}\,\delta\varphi_V \tag{8}$$

$$\delta\varphi_V\left(1 + \frac{s}{y}\right) = \frac{RI_o \cos\varphi}{V_o}\delta\varphi - \frac{RI_b \sin\varphi_b}{V_o}\delta\varphi_b + \frac{R \sin\varphi}{V_o}\, i$$
$$- \frac{R}{V_o}\cos\varphi_b\, i_b - \left[\left(\frac{\Delta\omega}{y}\right)_o + \frac{\delta\omega_o}{y}\right]\frac{v}{V_o} - \frac{\delta\omega}{y} \tag{9}$$

To test these formulae we choose some particular conditions:

$$i = 0 \qquad \delta\varphi_b = 0$$
$$i_b = 0 \qquad \delta\omega = 0 \tag{10}$$

Then eliminating the $\left(\frac{v}{V_o}\right)$ term from eqs. 8 and 9 with conditions of eq. 10, we find:

$$\left.\begin{aligned}
&\delta\varphi_V \left[(1 + \tfrac{s}{y})^2 + (\tfrac{\Delta\omega}{y})^2\right] = \delta\varphi \left[\frac{RI_o \cos\varphi}{V_o}(1 + \tfrac{s}{y}) + \frac{\Delta\omega}{y}\frac{RI_o \sin\varphi}{V_o}\right] \\
&V_o = RI_o \cos\varphi - RI_b \sin\varphi_b \\
&\frac{\Delta\omega}{y} = -\frac{RI_b \cos\varphi_b - RI_o \sin\varphi}{RI_o \cos\varphi - RI_b \sin\varphi_b}
\end{aligned}\right\} \quad (11)$$

$\boxed{\text{Case A}}$ $I_b = 0$

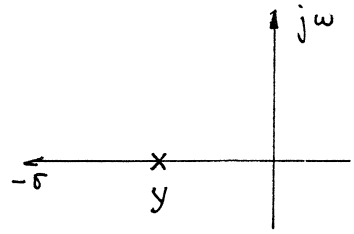
$$\delta\varphi_V \left[(1 + \tfrac{s}{y})^2 + (\tfrac{\Delta\omega}{y})^2\right] = \delta\varphi \left[\frac{RI_o \cos\varphi}{V_o}(1 + \tfrac{s}{y}) + \frac{\Delta\omega}{y}\frac{RI_o \sin\varphi}{V_o}\right]$$

$$V_o = RI_o \cos\varphi$$

$$\frac{\Delta\omega}{y} = \operatorname{tg}\varphi$$

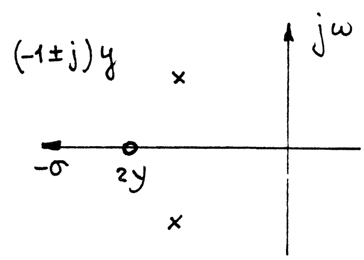
A1 $I_b = 0 \qquad \varphi = 0$

$$\delta\varphi_V(1 + \tfrac{s}{y}) = \delta\varphi$$



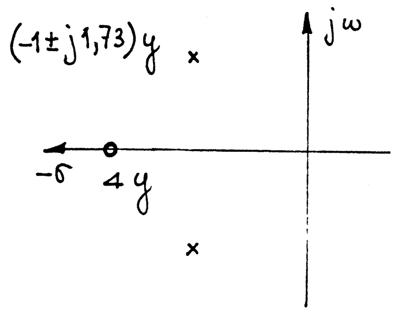
A2 $I_b = 0 \qquad \varphi = 45^\circ$

$$\delta\varphi_V\left[s^2 + 2sy + 2y^2\right] = y\delta\varphi(s + 2y)$$



A3 $I_b = 0 \qquad \varphi = 60^\circ$

$$\delta\varphi_V\left[s^2 + 2sy + 4y^2\right] = y\delta\varphi(s + 4y)$$



See Fig. 1 for log and phase diagram.

$\boxed{\text{Case B}}$ $\quad \varphi = 0 \qquad \varphi_b = 0 \qquad I_b \neq 0$

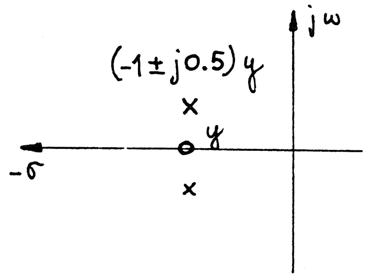
$$\delta\varphi_V \left\{ s^2 + 2sy + y^2 \left[1 + \left(\frac{\Delta\omega}{y}\right)^2 \right] \right\} = \delta\varphi y \, \frac{RI_o}{V_o} \, (s + y)$$

$$V_o = RI_o$$

$$\frac{\Delta\omega}{y} = - \frac{I_b}{I_o}$$

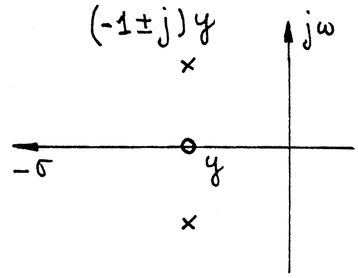
(B1) $\qquad I_b = \frac{1}{2} I_o$

$$\delta\varphi_V \left[s^2 + 2sy + 1.25y^2 \right] = y\delta\varphi(s + y)$$



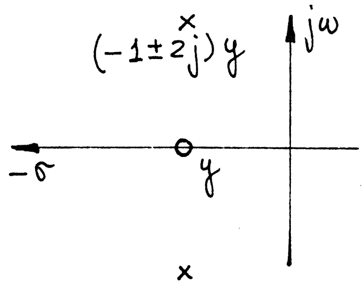
(B2) $\qquad I_b = I_o$

$$\delta\varphi_V \left[s^2 + 2sy + 2y^2 \right] = \delta\varphi y(s + y)$$



(B3) $\qquad I_b = 2I_o$

$$\delta\varphi_V \left[s^2 + 2sy + 5y^2 \right] = \delta\varphi y(s + y)$$



See Fig. 2 for log and phase diagram..

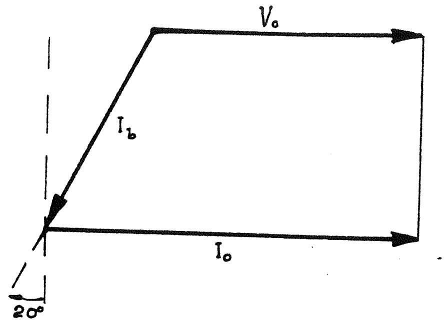
$\boxed{\text{Case C}}$ $\quad \varphi = 0 \qquad \varphi_b \neq 0 \qquad I_b \neq 0$

$$\delta\varphi_V \left\{ s^2 + 2sy + y^2 \left[1 + \left(\frac{\Delta\omega}{y}\right)^2 \right] \right\} = \delta\varphi y \, \frac{RI_o}{V_o} (s + y)$$

$$V_o = RI_o - RI_b \sin \varphi_b$$

$$\frac{\Delta\omega}{y} = - \frac{RI_b \cos \varphi_b}{RI_o - RI_b \sin \varphi_b}$$

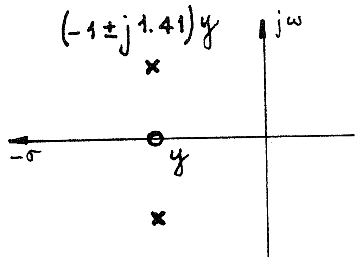
(C1) $\varphi = 0 \qquad \varphi_b = +20^\circ \qquad I_b = I_o$



$V_o = 0.66\ RI_o$

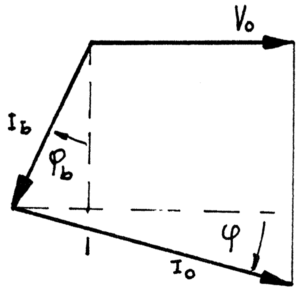
$\frac{\Delta\omega}{y} = -1.42$

$\delta\varphi_V[s^2 + 2sy + 3] = 1.52y(s + y)\delta\varphi$



See Fig. 3.

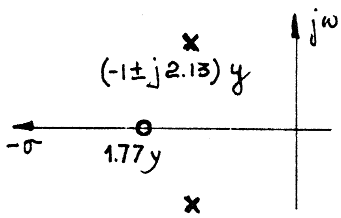
$\boxed{\text{Case D}}$ $\varphi \neq 0 \qquad \varphi_b \neq 0 \qquad I_b \neq 0$



(D1) $\varphi = -20^\circ \qquad \varphi_b = +20^\circ \qquad I_b = I_o$

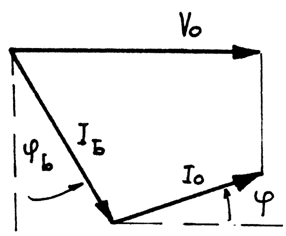
$V_o = 0.6\ RI_o$

$\frac{\Delta\omega}{y} = -2.13$



$\delta\varphi_V[s^2 + 2sy + 5.55y^2] = 1.56y\delta\varphi[s + 1.77y]$

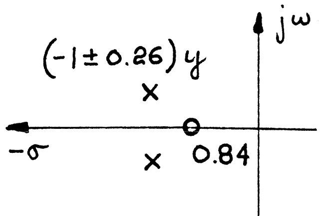
(D2) $\varphi = +30 \qquad \varphi_b = -30 \qquad I_b = I_o$



$V_o = 1.36\ RI_o$

$\frac{\Delta\omega}{y} = -0.27$

$\delta\varphi_V[s^2 + 2sy + 1.07y^2] = 0.63\ y\delta\varphi[s + 0.84]$



See Fig. 3.

1.3 Test of transfer function

To test the validity of the evaluated formulae, we measure the transfer function in the conditions A÷D, on a model of the accelerating cavity.

This model and the measuring system are shown in Fig. 4:
- a fixed generator simulates the beam current;
- a voltage controlled oscillator simulates the generator; its average phase is controlled to a fixed value ($\varphi_b+\varphi$) by a phase-lock, which has a bandwidth and a gain chosen to permit a phase modulation ($\delta\varphi$) at high frequency;
- input and output phase are measured with the linear phase discriminator used for PSB beam-control[2].

Test conditions are:
- input phase modulation is $\delta\varphi = \pm\ 11°$ and $\delta\varphi = \pm\ 22°$
- carrier frequency is $\omega_o = 6$ MHz
- circuit characteristics $Q \cong 40$

Figs. 5, 6, 7 give the results of measurements.

If we compare these curves with the corresponding ones evaluated in 1.2 we see the agreement is good.

As the working conditions of the PSB beam control meet the hypothesis of par. 1.1, we can use formulae 8-9 to study the influence of beam-loading.

2. Beam control diagram with beam-loading

Ref.[2] gives the scheme of the phase and radial loops proposed for the PSB.

At present we consider a more complete block diagram including:

- beam loading effect; defined by formulae 8-9;
- tuning circuit; defined by

  gain K_T

  transfer function $F_T(s)$

- AVC circuit; defined by

  gain K_V

  transfer function $F_V(s)$

We obtain the diagram of Fig. 8, note that

$$\varphi_b \equiv \varphi_i \ , \ \varphi_V \equiv \varphi_o$$

where φ_i and φ_o are defined in Ref.[2] as the phase error between the beam (or voltage) and an ideal frequency program.

2.1 Order of magnitude

We test the importance of the different feedback paths under a few possible operating conditions.

$I_b = I_o$ for $\varphi_s = 0$, $\varphi = 0$

V_o = constant

$\varphi_b \cong \varphi_s = \pm 20°$, corresponding to a transient in radial displacement

$\varphi = \pm 30°$, detuning corresponding to about 25 kHz, i.e. 5 synchrotron harmonics evaluated at injection with a Q = 50, and 15 harmonics at ejection with Q = 80.

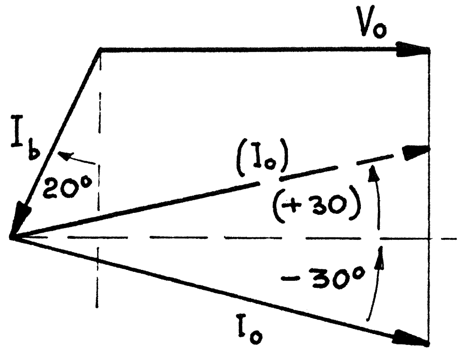
$K_\varphi = 6.2\ 10^7 \left[\frac{\text{rad/sec}}{\text{rad}}\right] = 170$ [kHz/o], gain of phase loop[2].

bandwidth of the AVC loop larger than 50 kHz [4].

Referring to Fig. 8 we calculate the value of the feedback gains.

Case A	$\varphi_s = + 20°$	$\varphi = - 30°$ (+ 30°)

To have V_o = const., we find (in brackets corresponds to $\varphi = + 30°$)



$$\frac{I_o}{I_b} = 1.55$$

then $V_o = 1$

$$\frac{\Delta\omega}{Y} = - 1.71 \quad (- 0.17)$$

A = 1.34 E = 0.94

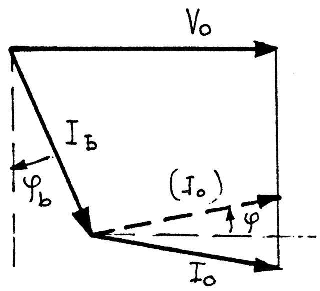
C = - 0.34 F = + 0.77 (- 0.77) H = + 1.71 (+ 0.17)

D = - 1.71 (- 0.17) G = 1.34

L = - 1.45 M = - 0.53

Case B	$\varphi_s = - 20°$	$\varphi = - 30$ (+ 30°)

To have V_o = const., we find



$$\frac{I_o}{I_b} = 0.76$$

then $V_o = 1$

$$\frac{\Delta\omega}{y} = -1.32 \quad (-0.56)$$

$A = 0.65$	$D = -1.32 \quad (-0.56)$	$G = 0.66$
	$E = 0.94$	$H = 1.32 \quad (0.56)$
$C = +0.34$	$F = 0.38 \quad (-0.38)$	$L = -0.71$
		$M = 0.26$

2.2 Discussion

Referring to Fig. 8 and to the previous values we find:

a) Tuning Circuit

Within its bandwidth the tuning partially compensates the terms:

$A \equiv$ variation of gain of phase feedback
$C \equiv$ interaction between $\delta\varphi_b$ and $\delta\varphi_v$
$L \equiv$ influence of i_b (amplitude modulation of the beam fundamental) on $\delta\varphi_v$
$H \equiv$ influence of v (voltage modulation) on $\delta\varphi_v$

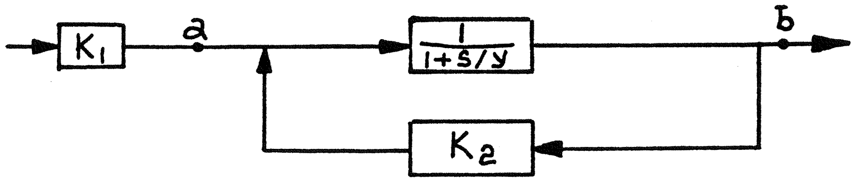
as we will see later, it can also help the AVC loop to compensate i_b modulations.

Nevertheless in the following discussion we make a worst case assumption and ignore the tuning loop.

b) AVC Circuit

The gain of the loop is proportional to the term G, in the previous conditions it varies within the range $0.66 \div 1.34$ with $\varphi_s = -20 \div +20°$.

Four paths, in parallel with normal phase-loop feedback, pass through the AVC loop (E, D, F, M). The partial gains, as previously calculated, vary between 0 ÷ 2 (absolute value). We can treat the diagram as follows:



where $K_1 \equiv$ sum of the feedback terms

$$\frac{K_2}{1+\frac{s}{y}} \equiv \text{open loop transfer functions of AVC}$$

Roughly we can say that the transfer function between points a-b is:

- equal to $1/k_2$ in the bandwidth of the AVC
- equal to the direct path outside the bandwidth of the AVC.

As the gain K_2 is large and the bandwith of AVC is larger than (y), the feedback path a → b is negligeable.

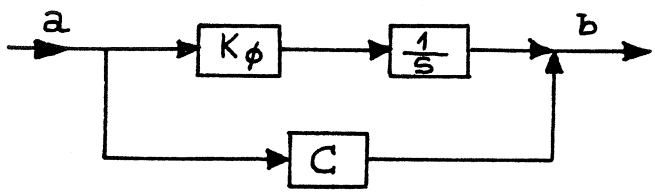
The beam current modulation i_b/I_o acts on $\frac{v}{V_o}$ in two manners.

- Directly by term (M), which is in general < 1 ($\propto \sin \varphi_s$).
- Indirectly via the tuning system, by the $\delta\varphi_V$ modulation ($\propto \cos \varphi_s$). As we need less than 1°/o voltage modulation at 10 kHz (2 x $f_{synchr.}$), the "help" of the tuning system to the AVC loop is an important element.

c) Phase loop

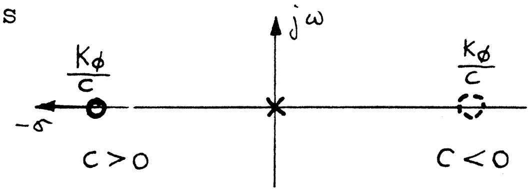
We neglect the tuning system (see a)), and the feedbacks through the AVC loop (see b)). Then:

- the gain of the phase-lock is modulated by the beam-loading effect, the term A varies between 0.65 $\div$ 1.34 with $\varphi_s = -20 \div +20^\circ$. In general $\varphi_s > 0$ (accelerating conditions) which means $A > 1$.
- a feedback (c) is in parallel to the phase loop. Its influence can be schematised as follows



where we study the phase loop only with the $\frac{1}{s}$ term. The a-b transfer function is

$$= c \, \frac{s + K_\varphi / c}{s}$$



then we add a zero to the transfer function. This is positive or negative depending on the sign of c.
As $|c| < 0.5$, we find $f_r = \frac{K_\varphi}{c}$ 350 kHz; the phase error is $< \pm 9^\circ$ at $f \cong 60$ kHz, unity gain point of phase lock system[2].

- the pole $(s + y)$ introduced by the cavity, $y = \frac{\omega_o}{2Q}$ must be compensated with a filter. This is in general at a fixed value (in the CPS, $\frac{\omega_o}{Q} \cong$ const. in the accelerating cycle) [5].

The phase modulation induced by (i_b/I_o) must be splitted in two terms:

- the first is the change of the "quasi-static" value I_b during bunching period: as the frequency is within the phase-loop bandwidth ($\leqslant$ 10 kHz) it can be easily compensated.

- the second is a real i_b modulation due to $\delta\varphi_b$ variation. The transfer function $[i_b/\delta\varphi_b]$ can be studied as in ref. 6. As a first approximation we can neglect this term: as the PSB works with a small stable-phase, the longitudinal focusing forces are independent of phase modulation.

3. Q values admissible for the phase control system

The Q value measured on the PSB cavity with the 4L2 type ferrites[4)] with a total gap capacity c = 130 pf are about

at	3 MHz	V = 1 kV	Q = 50
	3 MHz	V = 12 kV	Q = 60
	8 MHz	V = 1 ÷ 12 kV	Q = 80

3.1 Value of the time constant (y)

We have:

$$y = \frac{\omega_o}{2Q}$$

then we find

at 3 MHz $\qquad$ y = 30 ÷ 25 kHz (depending on the V value)

at 8 MHz $\qquad$ y = 50 kHz

As:
- this value is always within the phase-loop bandwidth
- this value changes by a factor 2

we must compensate the response with a roughly programmed filter. The problem would be difficult with a larger range of Q values versus voltage.

3.2 Variation of the phase-loop gain

Figure 9 shows the:

- value of I_b (fundamental component) at injection during RF trapping 7)
- two extreme values of I_b at transfer, for a bunch length $\Delta\varphi = 145^o$ and $\Delta\varphi = 220^o$
- value of the current I_o evaluated at $\varphi = 0$ (tuned circuit) and $\varphi_s = 0$ (non accelerating conditions).

The value of I_b corresponds to $N = 2.5\ 10^{12}$ particles per ring. The value of I_o is evaluated with $I_o = V_o\ c\ \omega/Q$. As we can see in Fig. 9 the worst case, excluding the reduced voltage case at transfer, is $I_b = I_o$ as we have studied in par. 2. Then it follows that the term A, of the phase loop gain, can change by about 6 dB when φ (tuning phase) varies between $+ 30^o$ and -30^o, and φ_s (stable phase) varies in the range of $\pm 20^o$. This variation can still be compensated by:

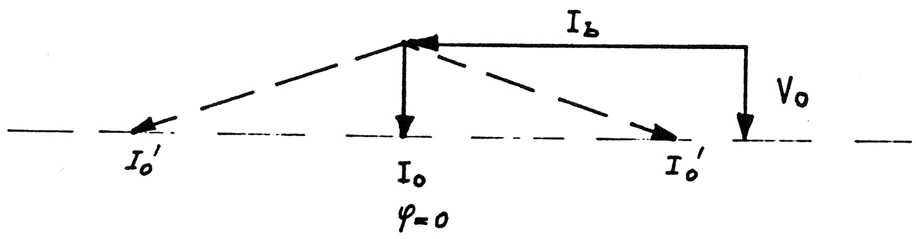
a) reducing by 6 dB the total gain, and the bandwidth; we should find $K_\varphi \cong 80$ kHz/^{o} and $f_\tau \cong 30$ kHz . Though these values are still convenient for the phase-loop, they are low from the point of view of the synchronisation system.

b) compensating the gain with an element controlled by the radial control input (φ_s) and by the tuning phase program (φ). This control is possible but it means a study of the simultaneous action of the phase tuning and AVC loops.

A particular care must be taken at two moments in the accelerating cycle:

- when the phase-control is switched on, about 50 µs after injection, $I_b \cong 0.5\ I_o$, but the phase φ_b can take any value and the speed of phase drift is relatively low ($\sim$ 50 µs/90^o for $\Delta f = 5$ kHz). Let us suppose that we have as worst case $\varphi_b = -90^o$, the gain A would be reduced by 6 dB.
- at transfer, with reduced voltage at 3 kV we found $I_b = 3.2\ I_o$, which is not an acceptable condition.

A possible solution to these cases, proposed by G. Nassibian, is to detune the cavity with a phase program



to increase the ratio I_o/I_b.

With a suitable study of the control system it is then possible to work in normal conditions with the Q values given in par. 3. Nevertheless the working conditions at transfer with reduced voltage and full intensity seem critical and would require an improvement in the feedback paths.

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SI Senior Staff

REFERENCES

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4. D. Zanaschi, Private communication.

5. W. Schnell, Proc. CERN Conference, 1959, p. 455-490.

6. H.G. Hereward, Proc. 1961 Int. Conf. HEA Brookhaven, p. 236-243.

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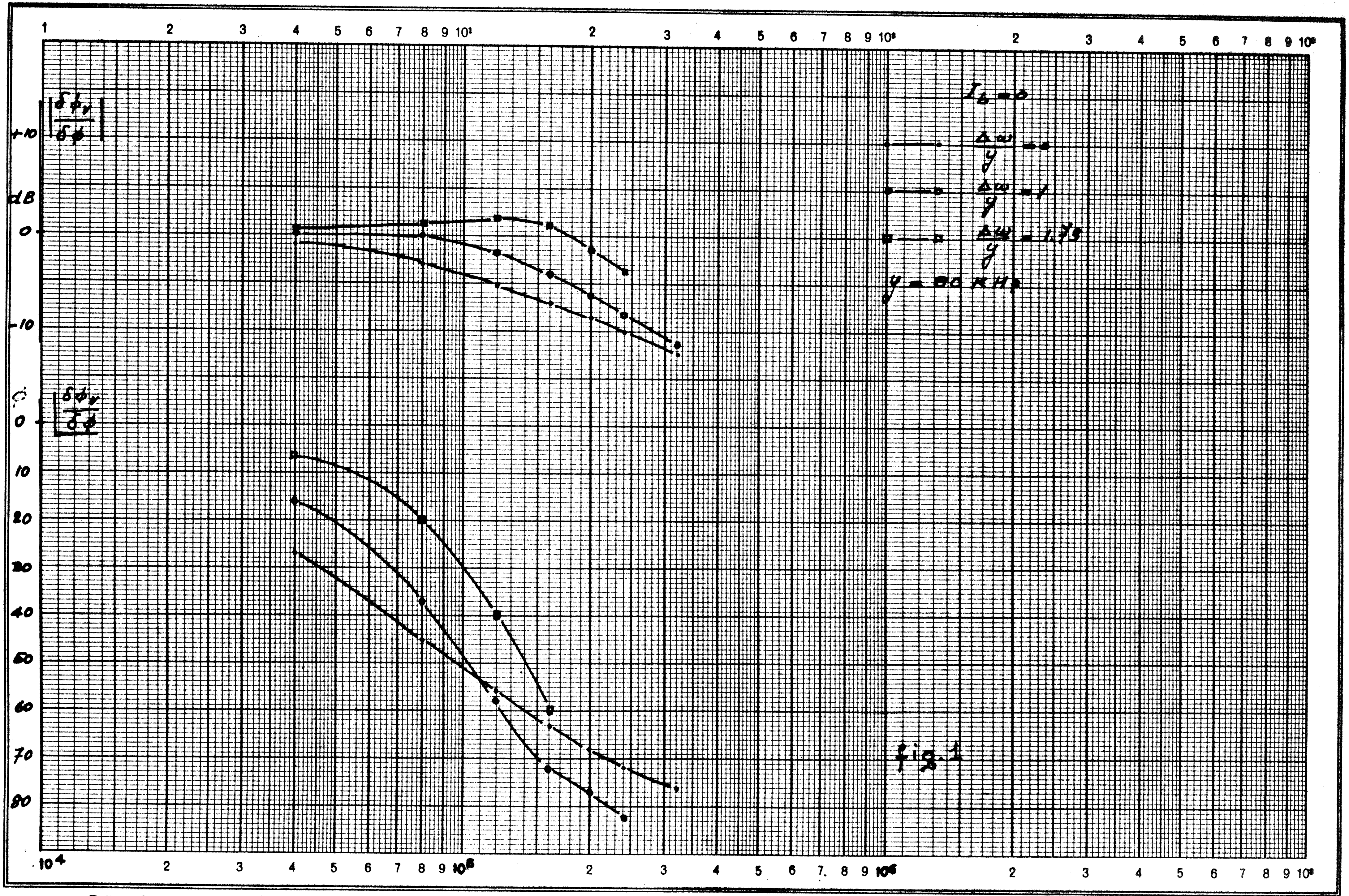
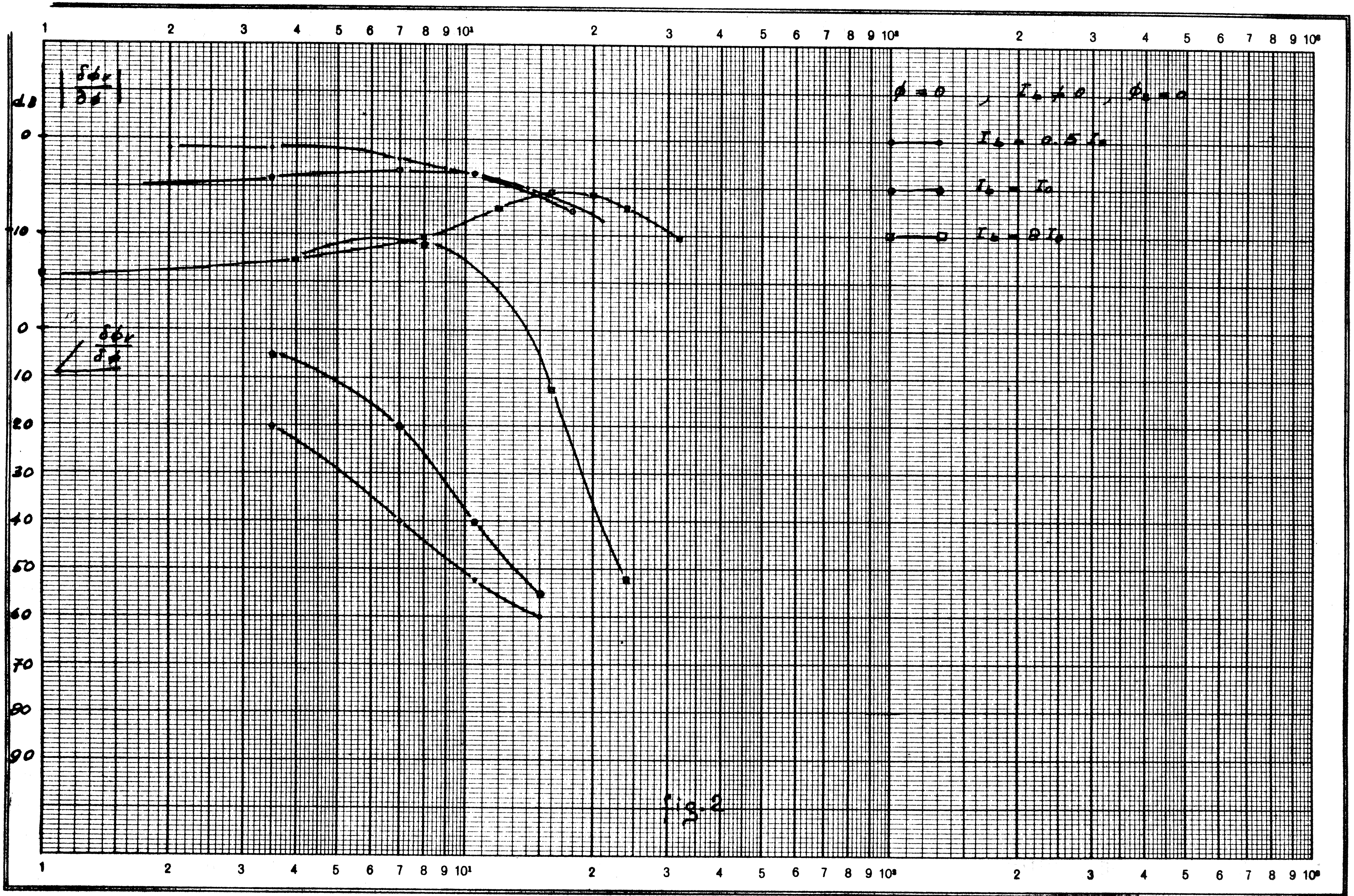
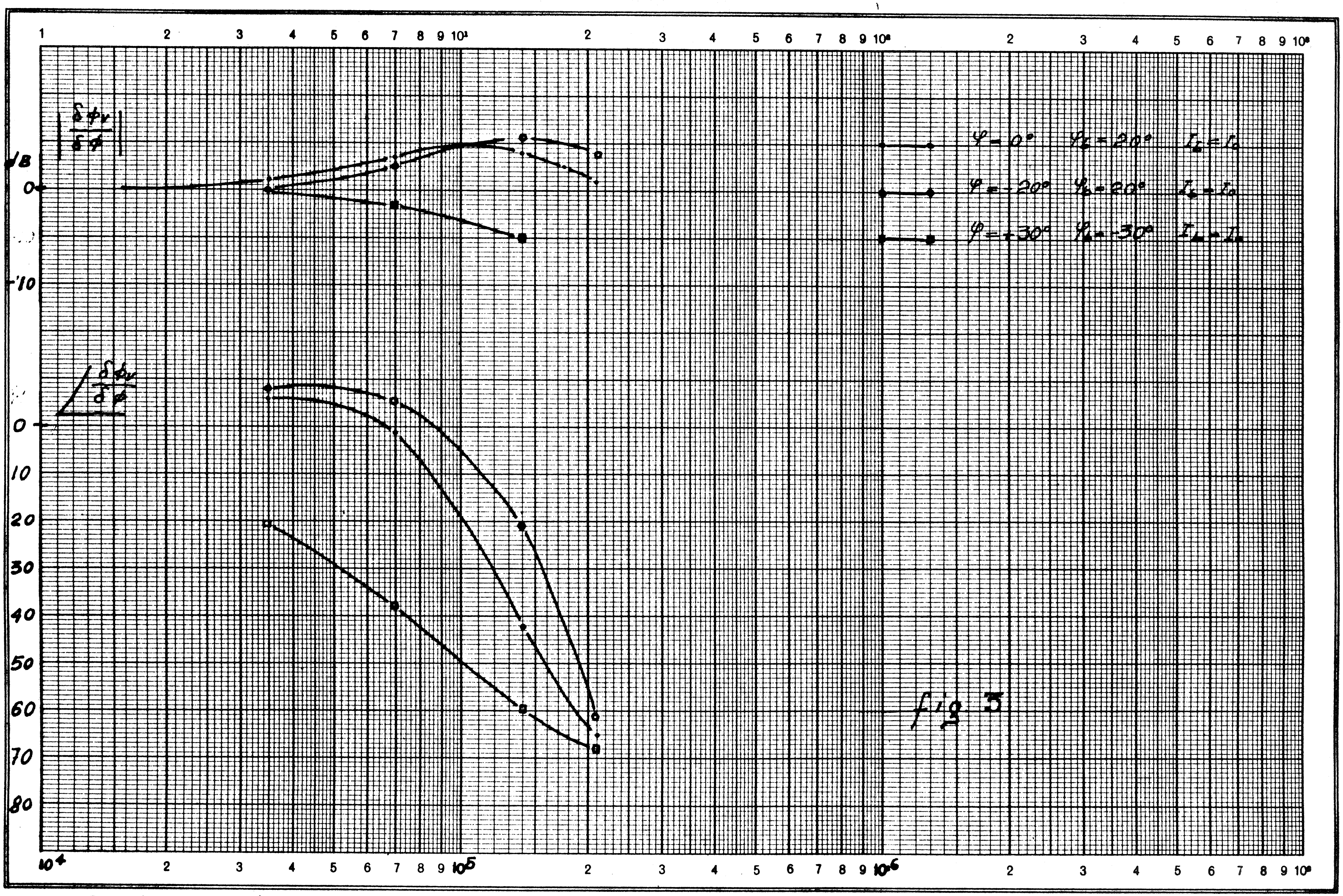

dB
+10
0
-10
0
10
20
30
40
50
60
70
80
10^4
10^5
10^6
$I_b = 0$
$\frac{\Delta\omega}{\gamma} = 0$
$\frac{\Delta\omega}{\gamma} = 1$
$\frac{\Delta\omega}{\gamma} = 1.73$
$\gamma = 80$ KHz

fig. 1



dB
$\left|\frac{\delta\phi_r}{\delta\phi}\right|$
$\angle\frac{\delta\phi_r}{\delta\phi}$
$\phi = 0$, $I_b \neq 0$, $\phi_s = 0$
$I_b = 0.5\, I_o$
$I_b = I_o$
$I_b = 2\, I_o$

fig. 2



|δφv / δφ|
dB
0
-10
∠ δφv / δφ
0
10
20
30
40
50
60
70
80
φ = 0°   φL = 20°   IL = I0
φ = -20°   φL = 20°   IL = I0
φ = +30°   φL = -30°   IL = I0
10^4
10^5
10^6
fig 3

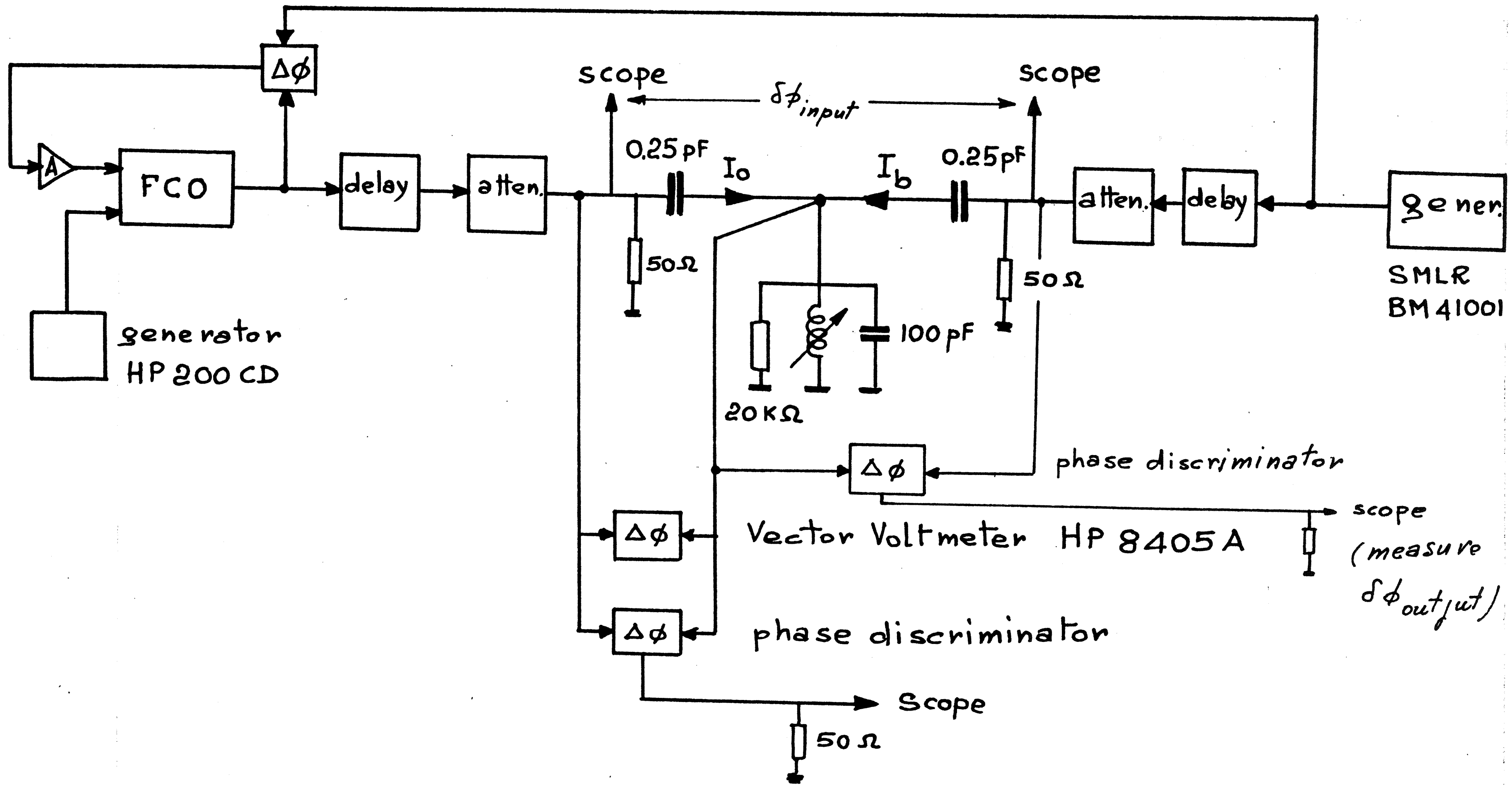


fig. 4 model for beam Loading test

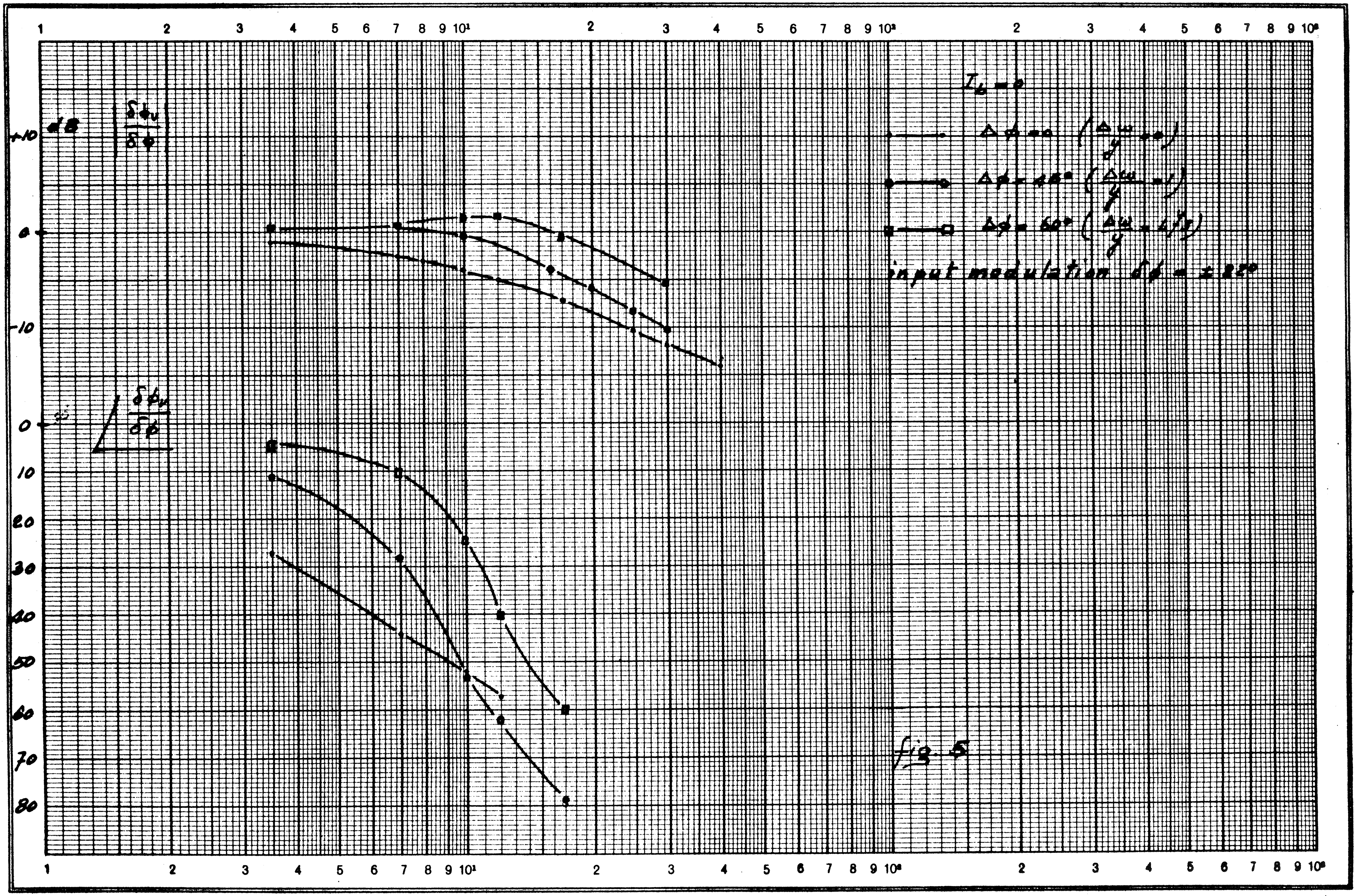
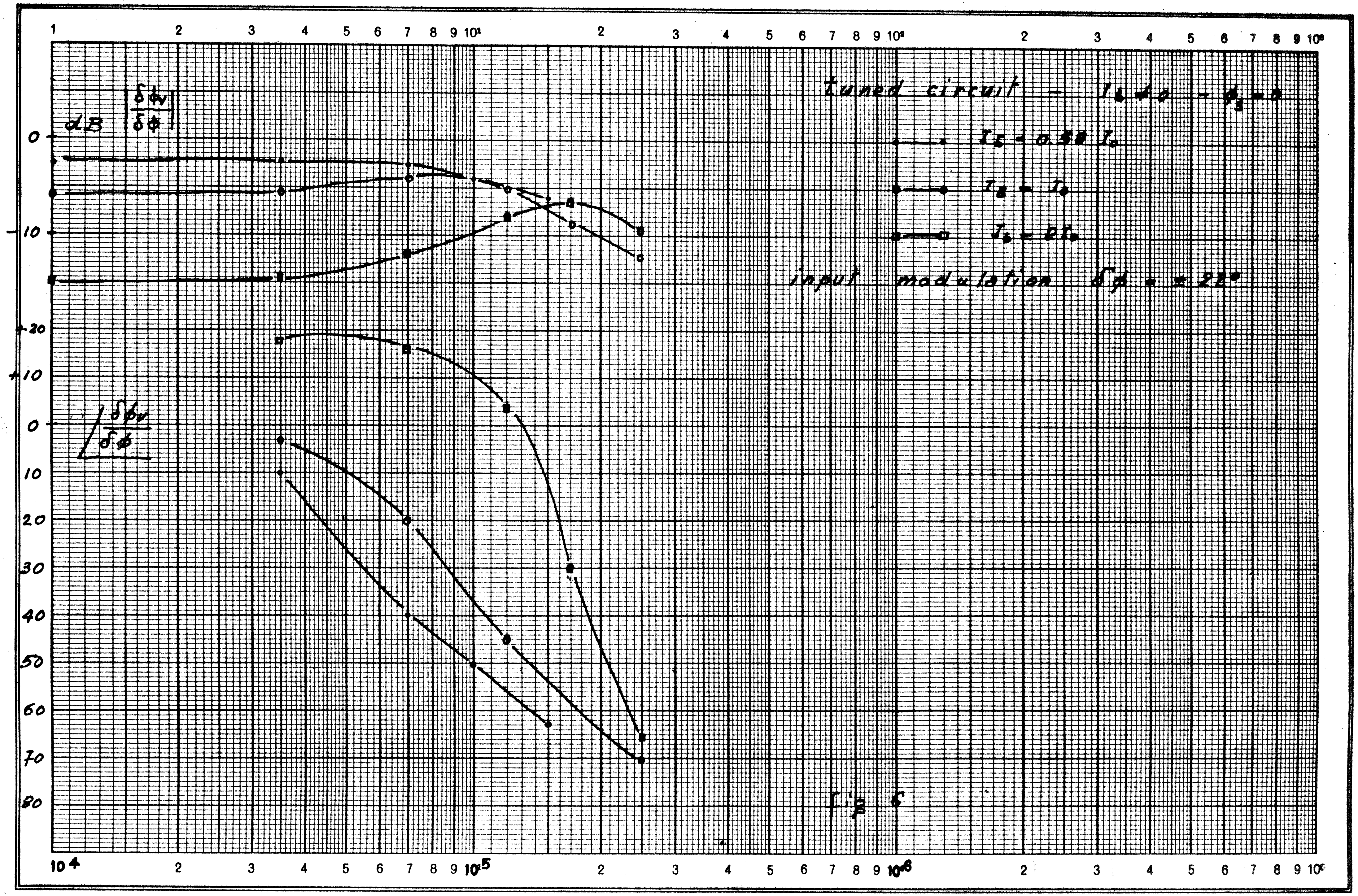
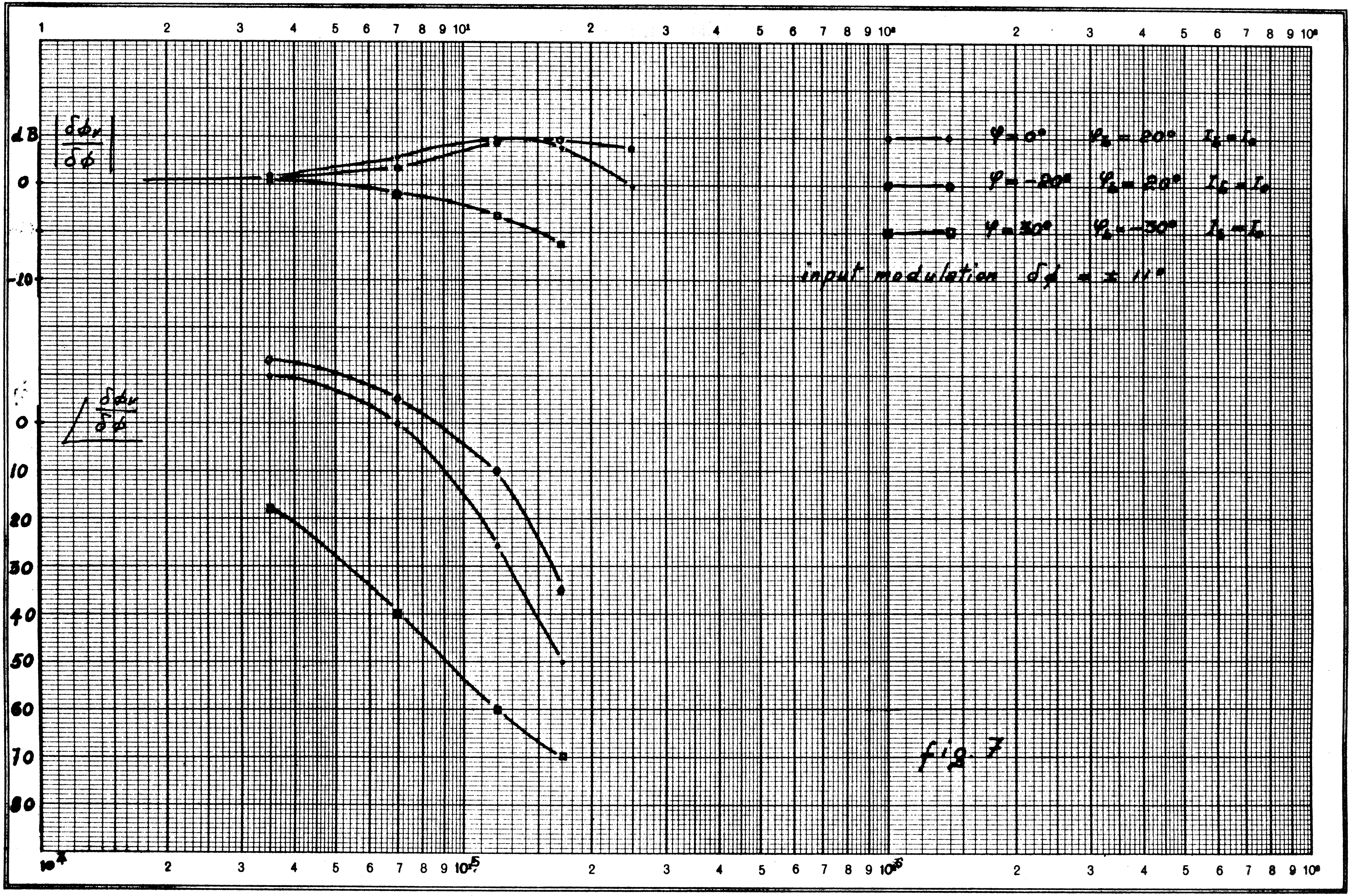


fig. 5



tuned circuit
input modulation
dB
0
10
+20
+10
0
10
20
30
40
50
60
70
80
10^4
10^5
10^6
Fig 6



dB
$\left|\frac{\delta\Phi_v}{\delta\Phi}\right|$
$\angle\frac{\delta\Phi_v}{\delta\Phi}$
$\psi = 0°$ $\psi_L = 20°$ $I_L = I_0$
$\psi = -20°$ $\psi_L = 20°$ $I_L = I_0$
$\psi = 30°$ $\psi_L = -30°$ $I_L = I_0$
input modulation $\delta\phi = \pm 11°$
0
-10
0
10
20
30
40
50
60
70
80
10^4
10^5
10^6

fig. 7

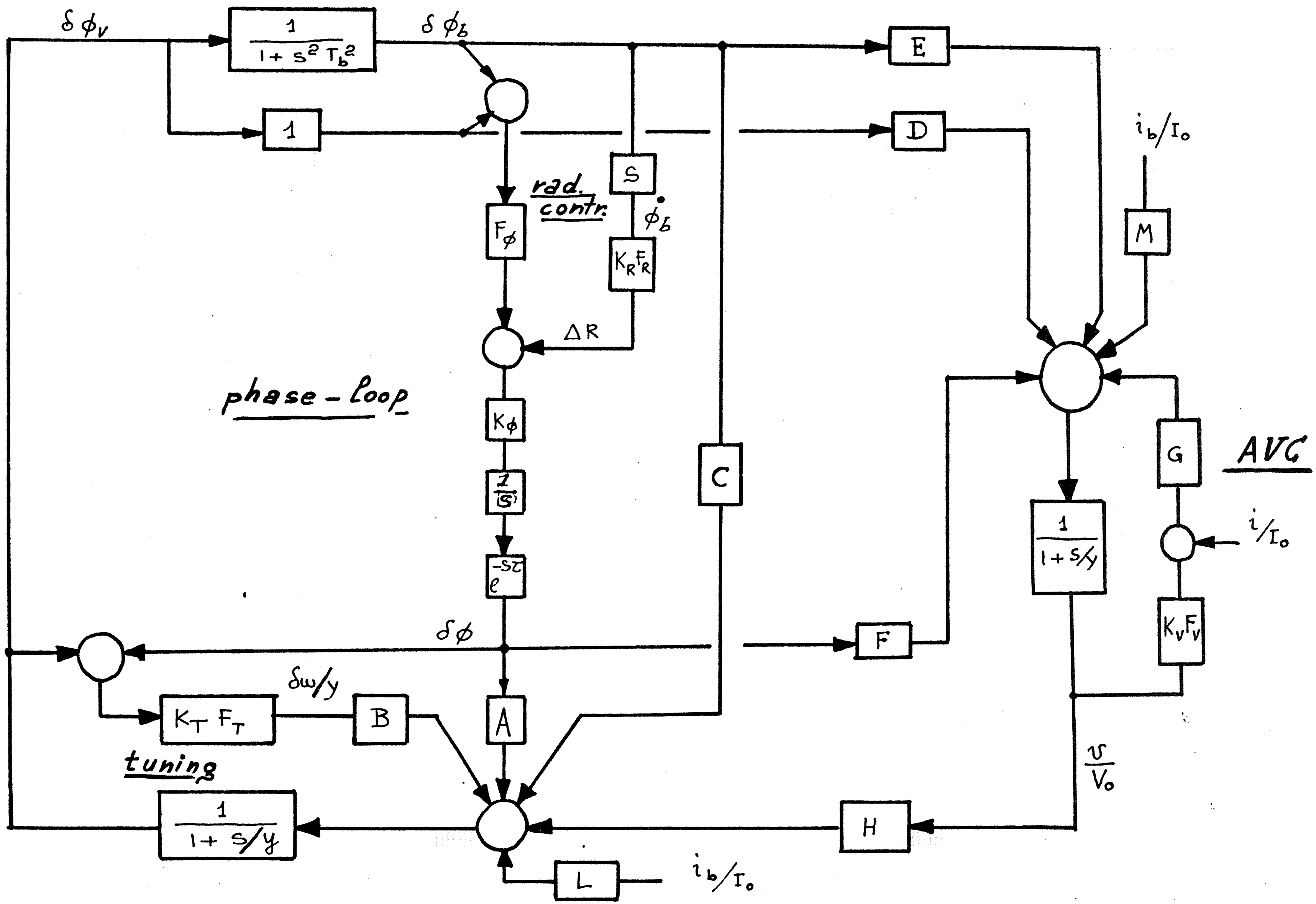


fig. 8

$$A = \frac{R I_0 \cos\varphi}{V_0}$$

$$B = -1$$

$$C = \frac{-R I_b \sin\varphi_b}{V_0}$$

$$D = \frac{\Delta\omega}{y}$$

$$E = \frac{R I_b \cos\varphi_b}{V_0}$$

$$F = -\frac{R I_0 \sin\varphi}{V_0}$$

$$G = \frac{R I_0 \cos\varphi}{V_0}$$

$$H = -\frac{\Delta\omega}{y}$$

$$L = -\frac{R I_0}{V_0}\cos\varphi_b$$

$$M = -\frac{R I_0}{V_0}\sin\varphi_b$$

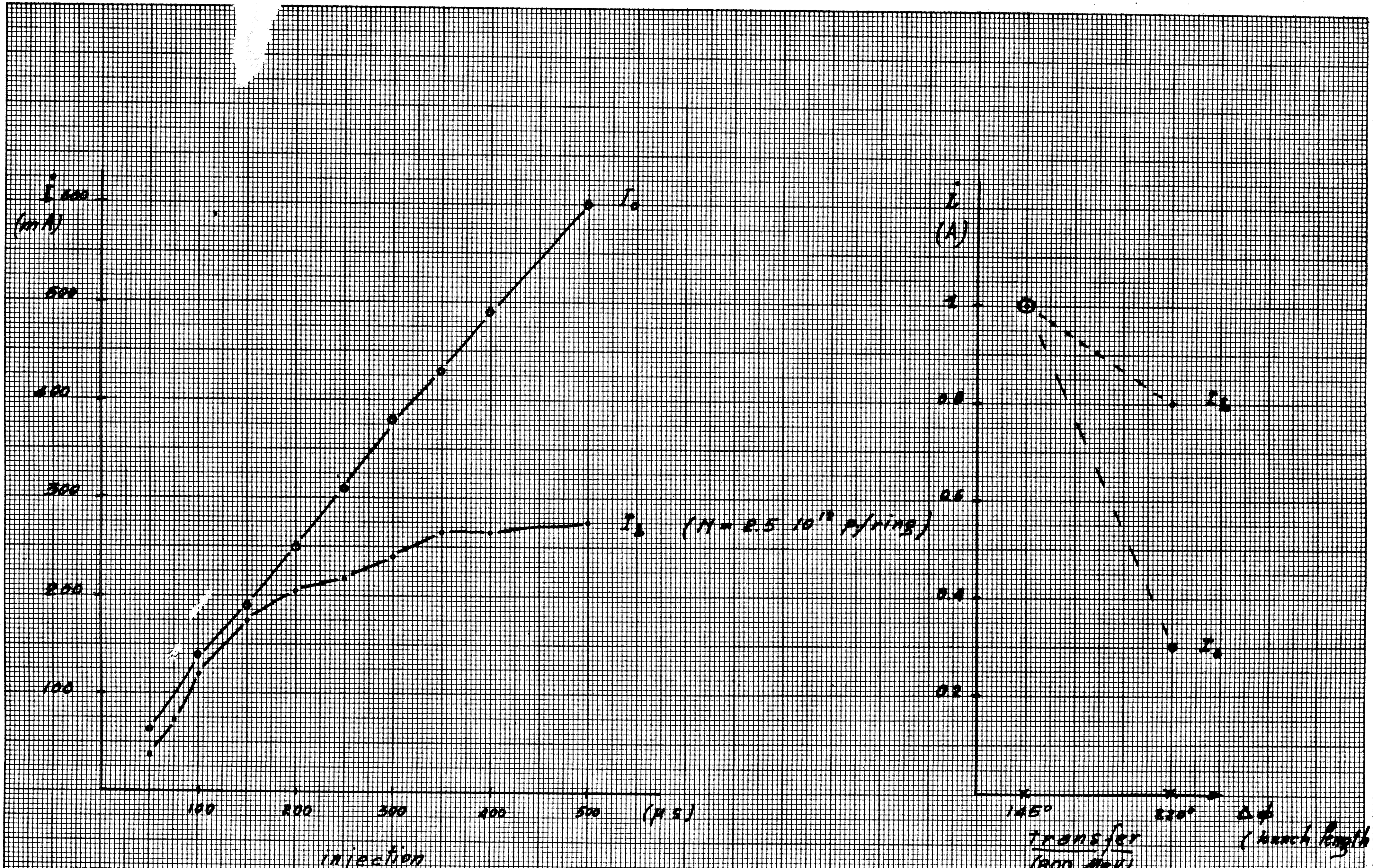

I (mA)
100
200
300
400
500
600
I₀
I_b
(N = 2.5 10¹² p/ring)
100
200
300
400
500
(μs)
injection
I (A)
1
0.8
0.6
0.4
0.2
I_b
I₀
145°
220°
Δφ
(bunch length)
transfer
(800 MeV)

fig. 9