ISR-RF/JB/ps *16th March 1979*

TECHNICAL NOTE

FAST FOURIER TRANSFORM SPECTRUM ANALYSIS EQUIPMENT

FOR MEASUREMENT OF SCHOTTKY NOISE SPECTRA AND BEAM TRANSFER FUNCTION

TECHNICAL EXHIBIT 1979

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1. BEAM SIGNALS DUE TO THE "NATURAL" STATISTICAL VARIATION AROUND THE MEAN OF A LARGE BUT FINITE NUMBER OF REVOLVING PROTONS IN A STORAGE RING

1.1 Spectra of the signal produced by the revolving particles or so-called longitudinal Schottky scan

1.1.1 Principle

The beam current in a storage ring is the summation of a large but finite number of protons, each with a single positive charge, revolving at about the speed of light. It is therefore subject to statistical fluctuations, or noise, about its mean value corresponding to the dc current. This situation is equivalent to the free electron motion forming the current in a conductor or in vacuum, as described by Schottky in his relation:

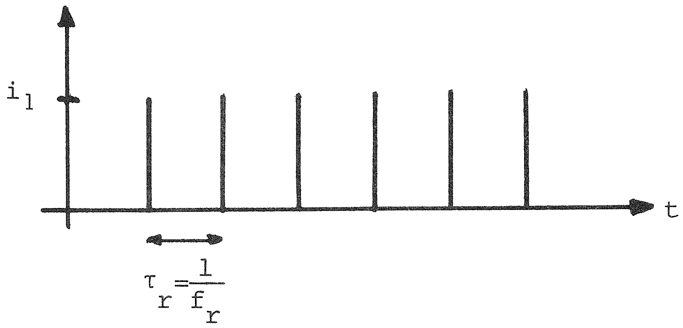
$$I^2_{noise} = 2e\ I_{dc}\ \Delta f$$

where the shot noise current or ac part is proportional to the square root of the dc or mean current part. Such noise current has a constant spectral density (white noise).

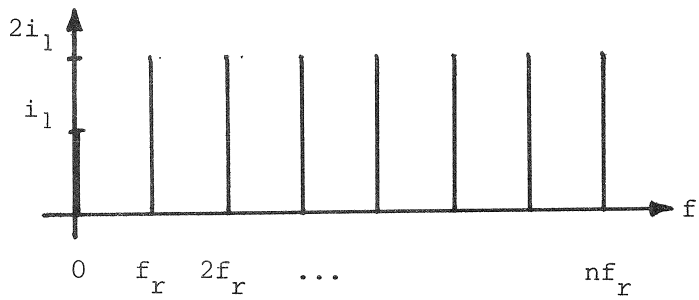
But due to the periodic nature of the particles' azimuthal motion, the spectrum exists only at harmonics of the revolution frequencies. If a single proton circulates in a storage ring, its current can be expressed as

$$i_1 = ef_r$$

where f_r is the **revolution** frequency and e the electron charge. Such a current is a train of very narrow pulses or Dirac functions.



The spectrum of such a signal is by definition a series of harmonics with constant amplitude and Dirac pulse shape:



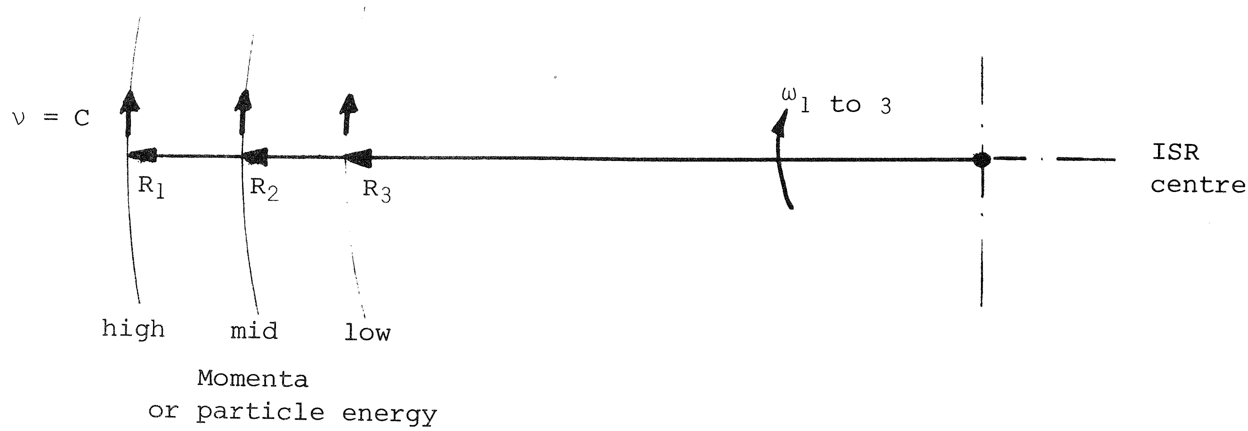
Now if the current in the storage ring is made of N protons randomly distributed around the circumference the total current is then

$$I = \sum_{j=1}^{N} i_j \partial (t-t_j)$$

over one period.

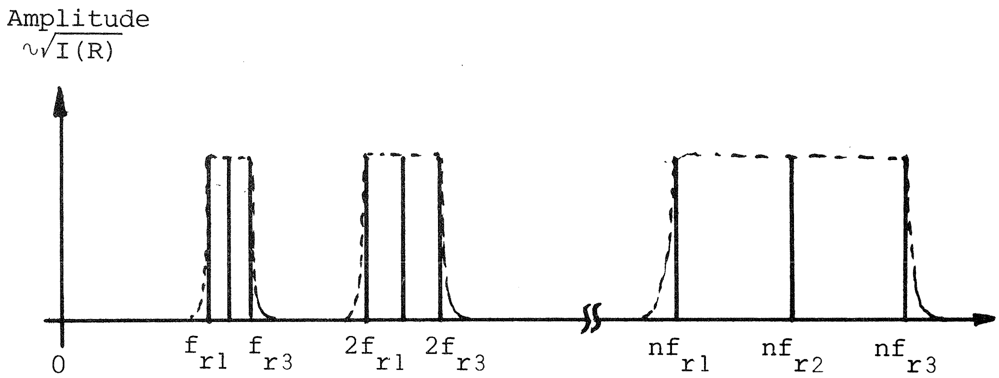
Since each of the N protons produces an identical spectrum the resulting total spectrum is similar to that of a single particle, but with an rms amplitude proportional to $\sqrt{N}$.

One stage further is to consider particles with different energies. Since for the ISR energy range, particle velocity is close to the speed of light, tangential velocities of the protons are almost identical. But particles with higher momentum or energy are less deflected by the machine magnetic bending field and circulate on orbits with larger radii.



$$f_{r1} < f_{r2} < f_{r3} \quad \text{for} \quad R_1 > R_2 > R_3$$

If one looks now at the spectrum of the current produced by particles circulating on different orbits, one obtains the following spectra:

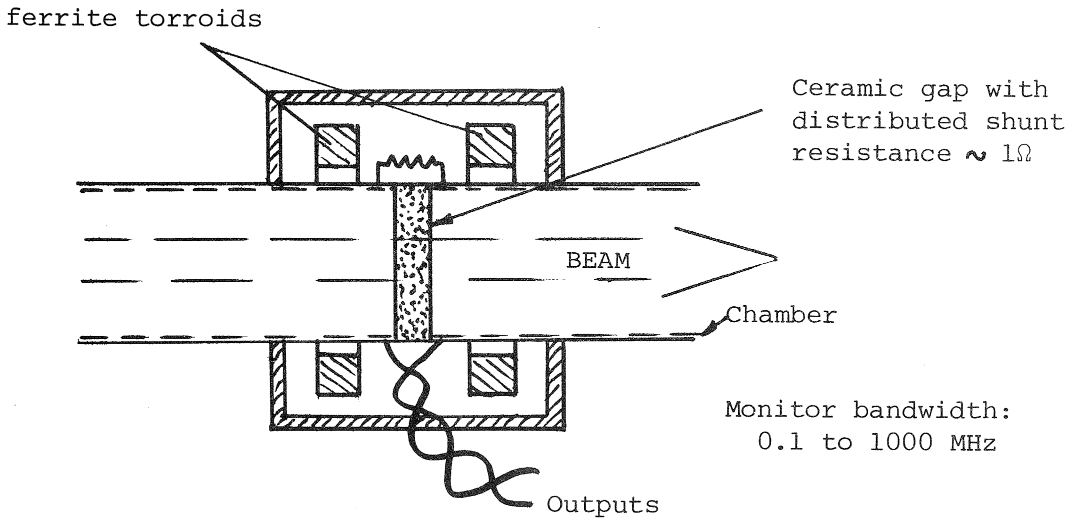


Since the distribution of particle energies is practically continuous (this is the case of the stack or total current in a storage ring), the spectrum is also continuous and is the mirror image of the square root of the current density as a function of the radial position or momentum. It is called the longitudinal Schottky scan.

1.1.2 Monitors

The proton current circulates in a metallic chamber and its accompanying electromagnetic field induces an image current in the chamber wall.

The monitor or PU used to detect the beam current is simply a gap made of ceramic interrupting the chamber. The image current induces a voltage proportional to the instantaneous beam current across the monitor shunt resistance. Torroids of magnetic material have been added to increase the coupling at low frequencies and such a monitor can be described as a coaxial single turn, very broad band, low-impedance transformer.

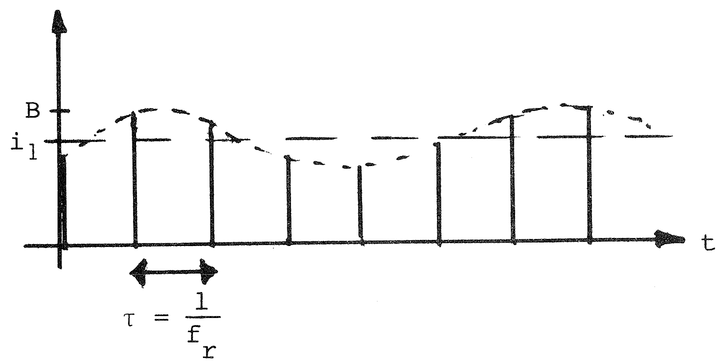


The signal is amplified by a broad-band low-noise amplifier and processed by a spectrum analyser (described in paragraph 1.3).

1.2 Spectra of the signal due to revolving protons having in addition vertical or radial betatronic oscillations

1.2.1 Principle

Again we consider a single revolving proton. If this particle also oscillates transversely (the so-called betatronic oscillations) and is observed with a device sensitive to the transverse position, the time domain signal observed is an amplitude-modulated Dirac pulse train:



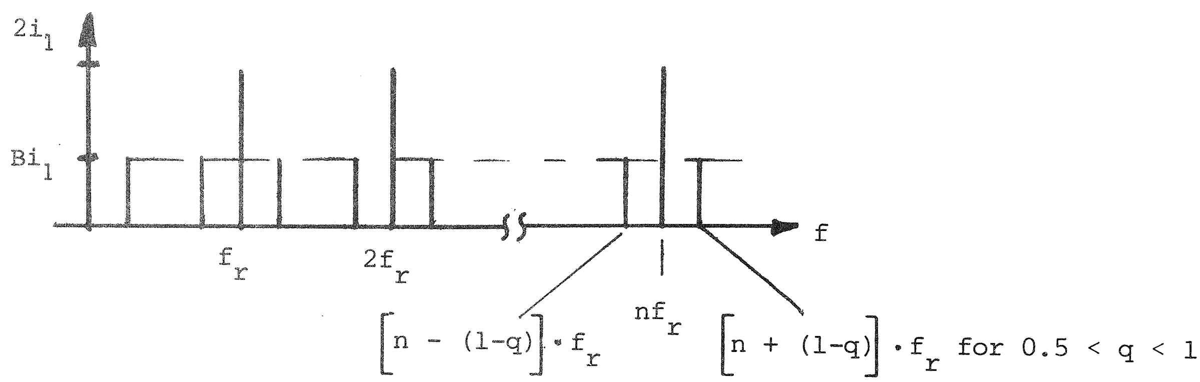
This signal is simply the longitudinal one as seen in paragraph 1.1.1, modulated in amplitude, and may be written as

$$i_1(1 + B\cos Q\,\omega_r t)\,\partial\,(t) \qquad t = K\,\tau_r \quad \text{for} \quad K = 1 \to \infty$$

where Q = number of betatron oscillations per revolution,

and B the peak betatronic amplitude.

The spectrum of such a signal is also a series of harmonics but with sidebands at $\pm q\, f_r$, where q is the non-integer part of Q.



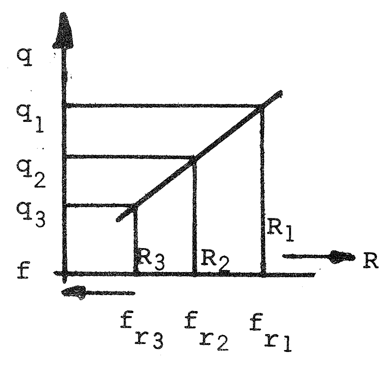
The n^{th} harmonic of this signal can be written as:

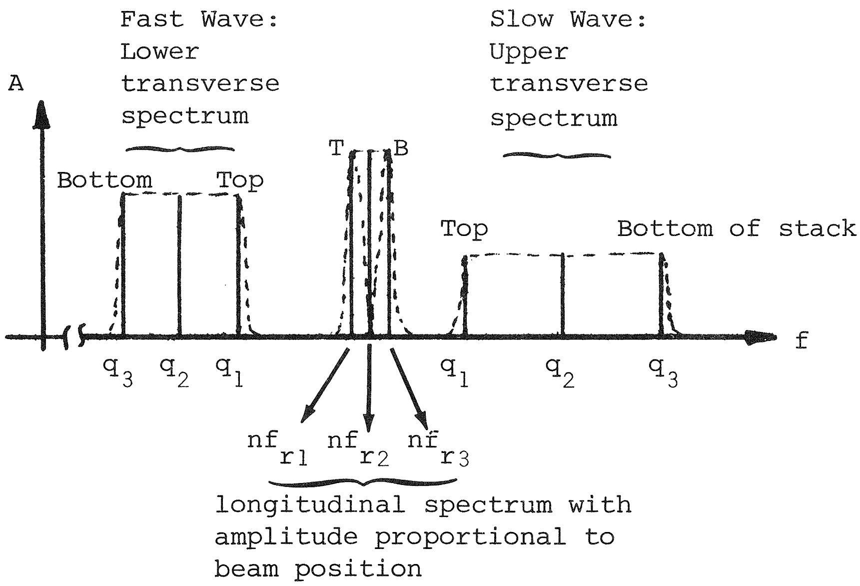
$$2\,i_1\cos\omega_r t + Bi_1\left(\cos\left[n - (1-q)\right]\omega_r t + \cos\left[n + (1-q)\right]\omega_r t\right)$$

For N protons circulating at the same revolution frequency and the same momentum, and whose centre of gravity fluctuates with amplitude B, the spectrum is multiplied by $\sqrt{N}$ and therefore the side bands are proportional to $\sqrt{N}\cdot B_{rms}$.

This argument may again be extended, as for the longitudinal spectra, to a continuous momentum distribution of all particles of a stack. In this case the resulting spectra are:

with $Q = 8+q$
and working line:

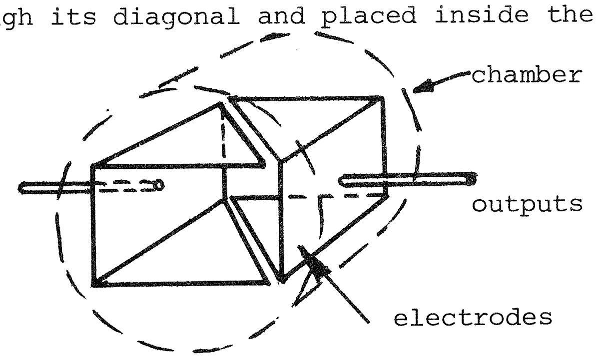




These are called the transverse Schottky scans or spectra.

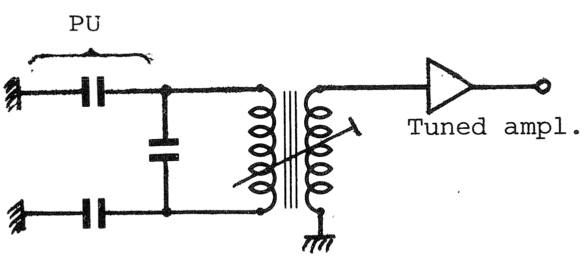
1.2.2 Monitors

The transverse motion of the particles is detected by electrostatic monitors consisting of a pair of electrodes forming a rectangular tube cut through its diagonal and placed inside the vacuum chamber.

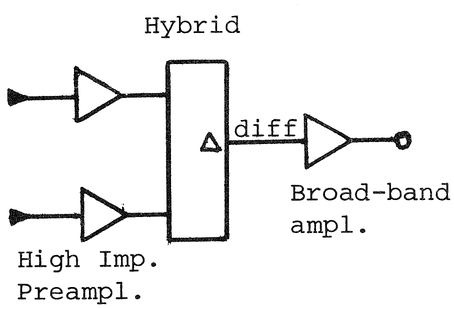


Aperture:	160×60 mm
Length:	∿250 mm
Capacity to ground:	∿80 pF per electrode

The electrodes are connected to two preamplifying circuits:



Tuned transformer



Broad-band circuit

The tuned amplifier is centred on 10.8 MHz and has a bandwidth of 300 kHz. It is mainly used for the "natural" transverse Schottky spectra observation.

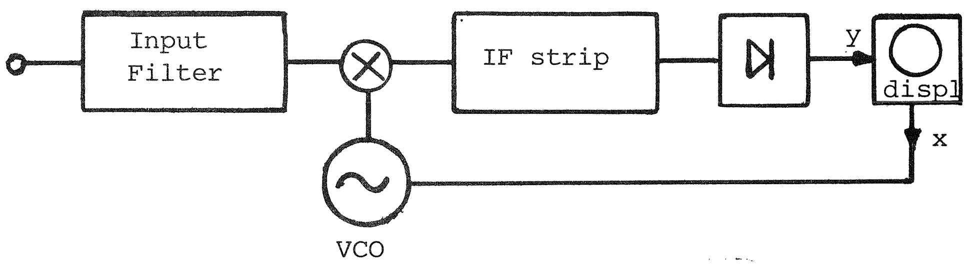
The broad-band system using very low noise amplifiers covers a frequency range from 1 to 100 MHz.

Both systems transmit the difference signals in order to attenuate the common mode, or longitudinal, signals. This is necessary because the transverse sidebands have a magnitude about 100 times smaller, since they are the result of particle transverse oscillations of magnitude ≤ 1 mm corresponding to an amplitude modulation factor ≤ 1%.

The signals are then processed by a spectrum analyser (see 1.3).

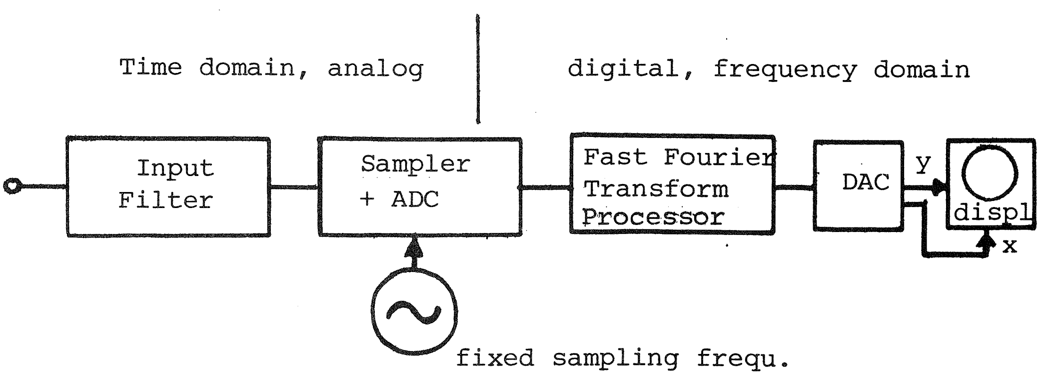
1.3 Spectrum analyser and Fast Fourier Transform

The commonly-known spectrum analysers are based on the following scheme:



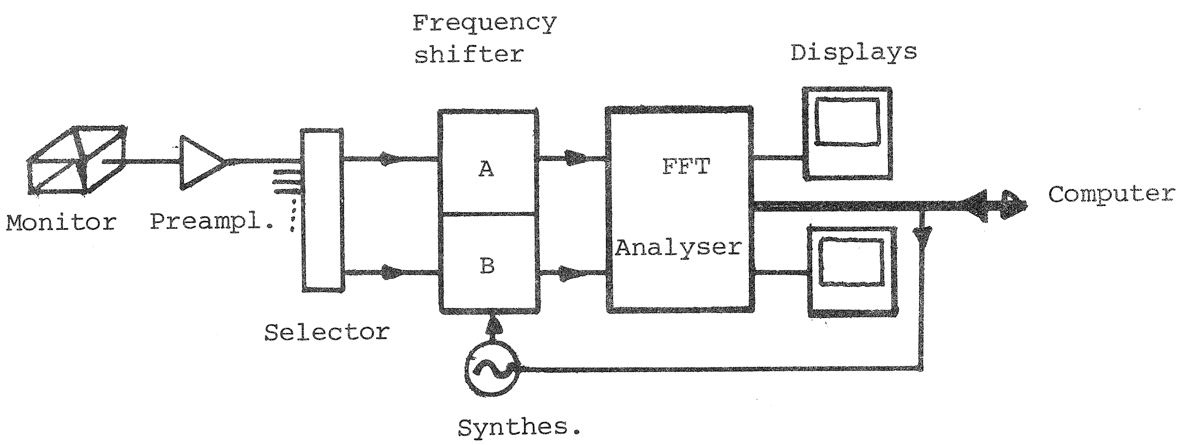
The disadvantage of the swept analyser is that the scanning speed is the reciprocal of the IF bandwidth or resolution. This means that for sharp frequency resolution or narrow IF bandwidth, the scanning speed is slow and in addition, if the signal evolves quickly in time, its variation cannot be tracked.

Another approach possible with the fast processing speed of present digital circuitry is represented in the following scheme:



Such a Fast Fourier Transform spectrum analyser has the advantage of using all available information if the processor is fast enough, and one can speak of real-time analysis. For the same resolution in the frequency domain such an FFT analyser is equivalent to N fixed analysers operating in parallel, where N is the number of channels of the FFT. The FFT analyser is therefore N times faster than the swept spectrum analyser. In the case of noise spectra analysis, i.e. Schottky spectra, averaging is necessary in order to obtain the mean power density or rms value of such a signal. The mean is obtained usually from the average of 250 to 1000 instant spectra and hence the speed of spectra processing becomes an important factor.

However the frequency range of an FFT analyser is from 0 to f_{max} (f_{max} = 100 kHz for this equipment). A dual-channel frequency shifter has been built to displace the FFT central frequency into the observation range of 0 Hz to 100 MHz. The FFT analyser is then used as IF strip and detector. The equipment block diagram is shown on the following figure:

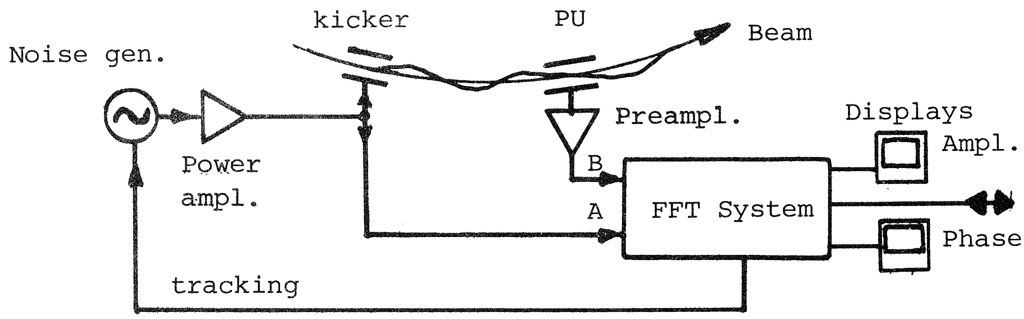


The dual-channel FFT analyser not only allows the processing of two sources of data in parallel but also that of cross functions such as the system transfer function.

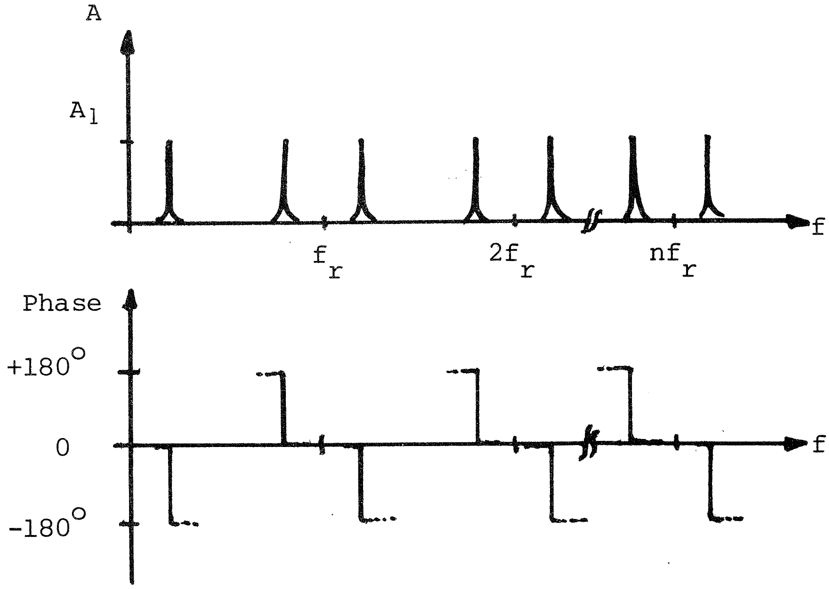
2. BEAM TRANSFER FUNCTION AS OBTAINED FROM THE BROAD-BAND TRANSVERSE EXCITATION AND WITH A DUAL-CHANNEL CROSS-FUNCTION ANALYSER USING FFT

2.1 Principle

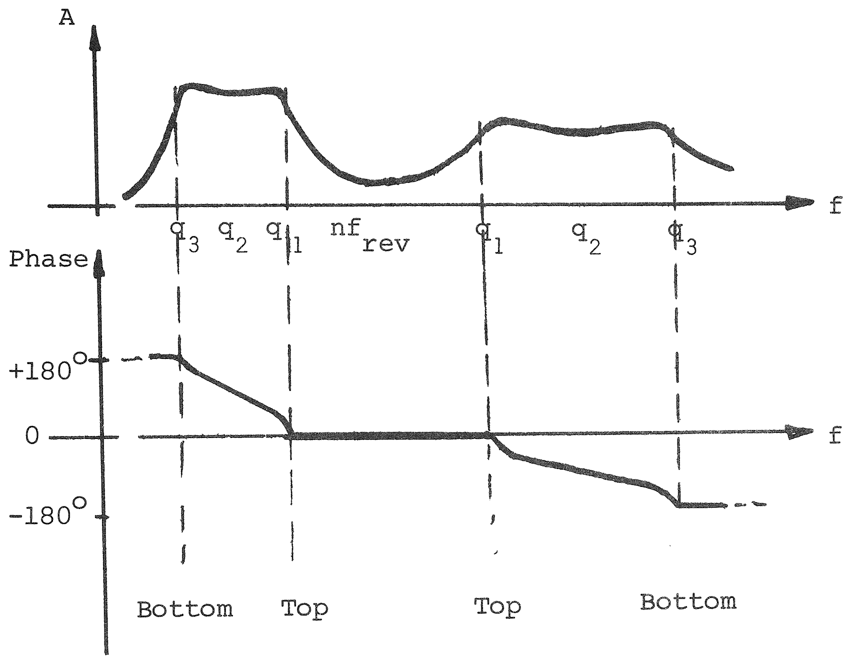
A kicker and an electrostatic monitor (PU) are installed in the storage ring close to each other. They are identical in construction but the kicker is fed with a stochastic or noise signal of medium power, about 1 to 10 W, which has a constant power density spectrum in the analysis bandwidth. The following figure shows the set-up.



A single particle of given momentum and circulating in the storage ring has a unique betatronic frequency defined by its Q value due to the machine working line characteristic. The response of such a particle to transverse excitation is similar to that of a resonant circuit of very high quality factor. It will respond at all harmonics of the revolution frequency $\pm(1-q)f_{revolution}$. Such a transfer function is represented here.



For N particles of the same momentum the magnitude A_1 is multiplied by N. This argument can also be extended to a stack with continuous momentum distribution. The transfer function of such a stack excited transversely appears as follows (for a harmonic number n):



This means that the transfer function is that of a system with continuous distribution of resonators and is the global response. For frequencies below that of all resonators the relative phase is +90°. For frequencies above, the phase is -90°, resulting in a smooth phase change of 180° across the sideband. In addition the total phase change across the lower and upper sidebands is 360°. The magnitude of the transfer function is proportional to the proton density and to the inverse of the frequency spread across a sideband.

Now considering the total stack, the centre of the beam is deflected transversally. This coherent modulation is transmitted by the beam and picked up by the monitor. The processor calculates the transfer function with the kicker signal as reference. The magnitude of the transfer function is directly proportional to N (number of protons) and not to $\sqrt{N}$ as for the "natural" Schottky scans. This method also has the advantage of eliminating uncorrelated signals such as longitudinal spectra and transmitting very little energy to the beam. No blowing-up effect on the beam is produced thanks to the sensitivity of this technique.

Fig. 1 shows a typical example of such a transfer function, a vertical one, measured on a physics stack of 28 A at 31 GeV/c. The interest of this function is that it is the inverse of what is better known as the transverse stability limit diagram. This contains globally the contribution of the effective working line or Q as function of momentum, the influence of the vacuum chamber wall, the transverse feedback or damping systems and non-linear resonances including beam-beam influences. The magnitude of the transfer function is directly related to the actual stability of the beam and it can be investigated at all revolution frequency harmonics up to 100 MHz. The real part of the transfer function is identical to the "natural" Schottky transverse power spectrum if the rms betatronic amplitude is constant across the stack.

This technique is also applicable to longitudinal excitation and its transfer function.

2.2 Instrumentation

The principal difficulty of the equipment described in Fig. 2 was the design of the dual-channel frequency shifter with symmetric transfer function across the full frequency range. The frequency shifter automatically sets the central frequency of the chosen range in the middle of the FFT analyser range.

This equipment is connected to the ISR computer for further processing of the transfer function.

3. DISCUSSION

This FFT instrumentation is an important step in the ISR beam monitoring facilities and it started with the observation of longitudinal and transverse Schottky spectra in 1972. A first set-up using swept spectrum analysers for "natural Schottky scans was operational around 1974 and the new transfer function equipment with FFT has been ready in its manual mode since 1978.

The following persons contributed to this work:

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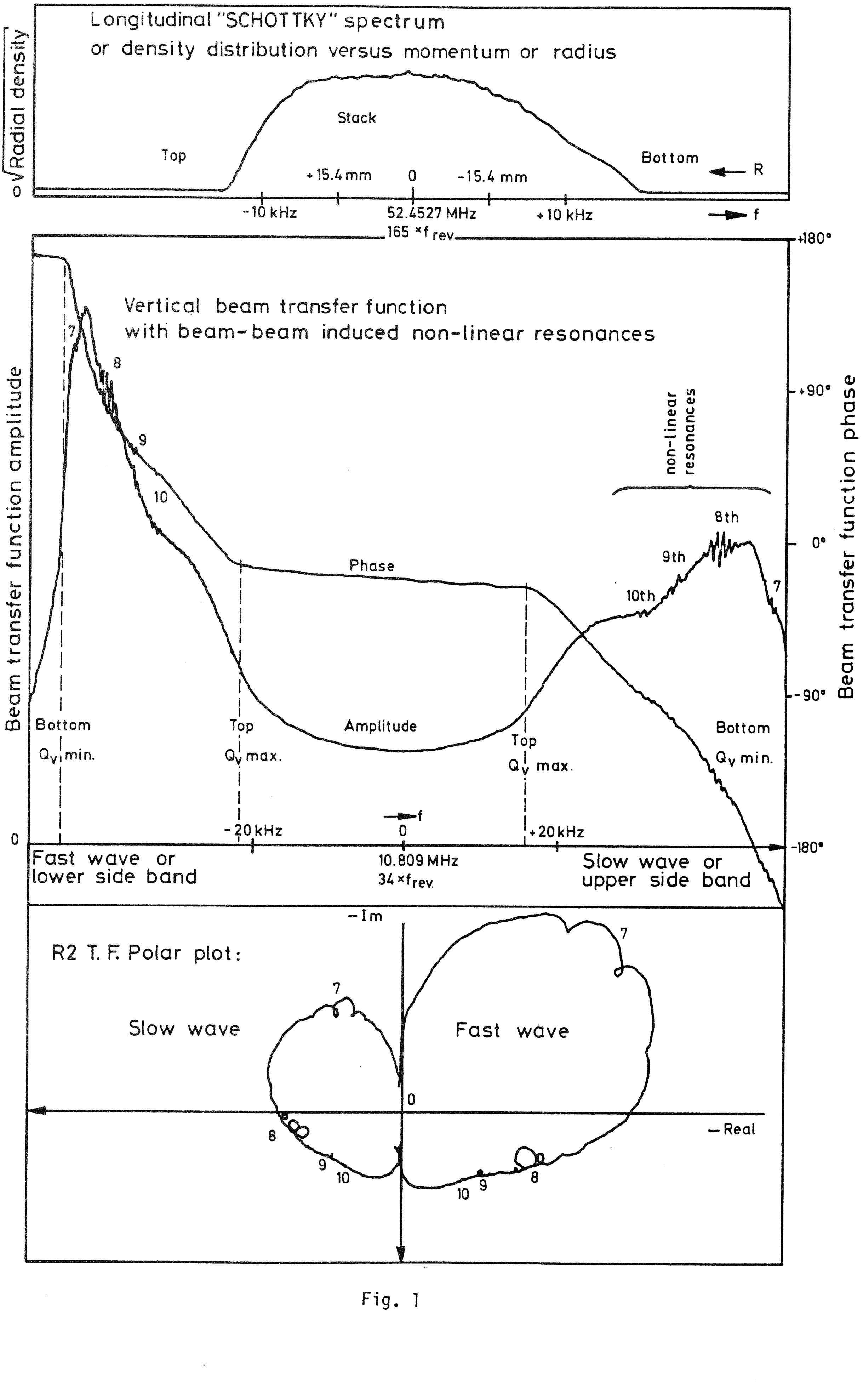
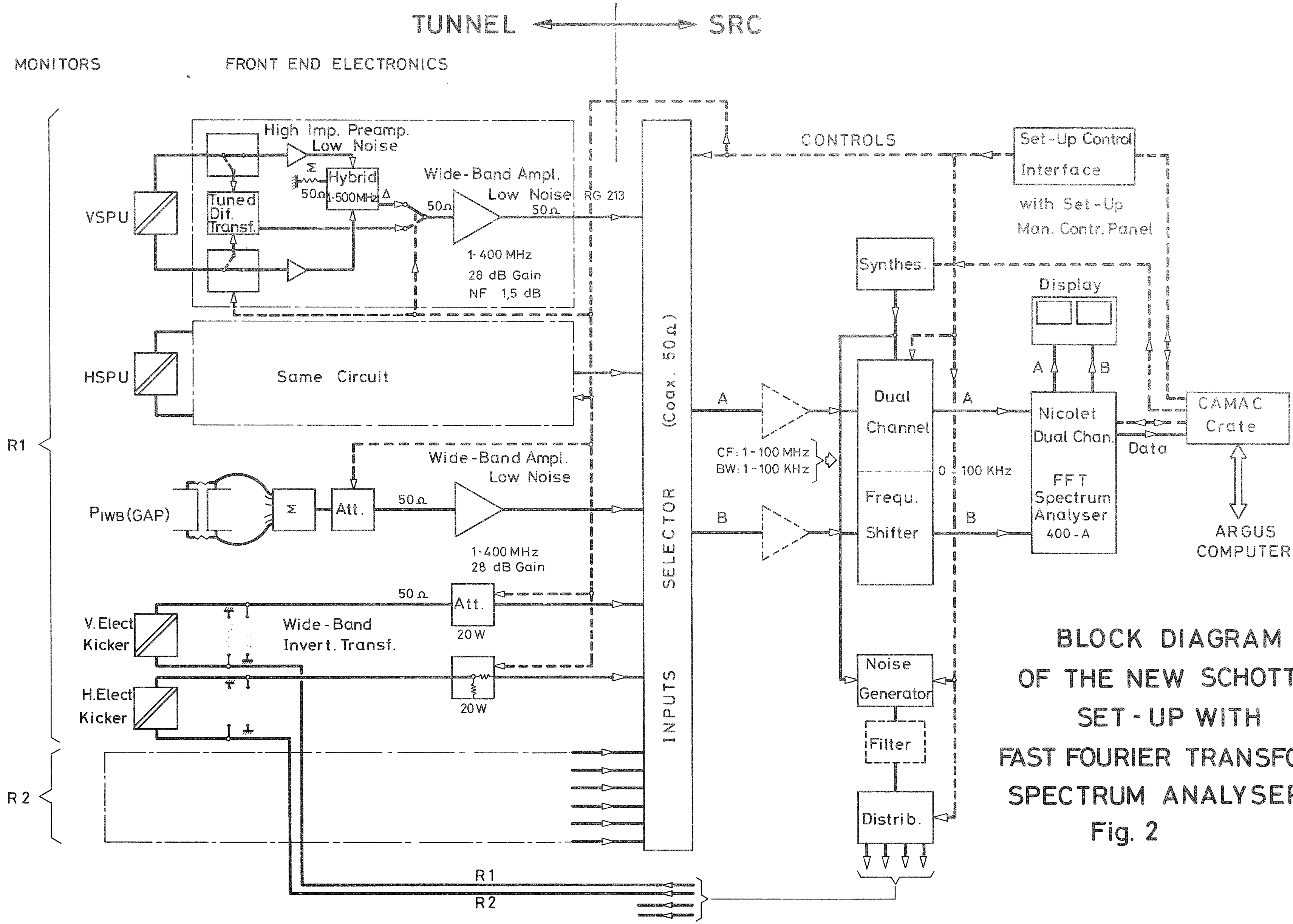
J-C. Juillard
D. Kemp
S. Myers
E. Peschardt
F. Sacherer
W. Schnell
L. Thorndahl
H. Verelst
B. Zotter

Reference: CERN-ISR-RF-TH-BOM/79-20: Information from beam response to longitudinal andtransverse excitation, by J. Borer et al.



Longitudinal "SCHOTTKY" spectrum
or density distribution versus momentum or radius
0 √Radial density
Stack
Top
Bottom
R
+15.4 mm
0
-15.4 mm
-10 kHz
52.4527 MHz
+10 kHz
f
165 ×f rev.
Vertical beam transfer function
with beam-beam induced non-linear resonances
Beam transfer function amplitude
Beam transfer function phase
+180°
+90°
0°
-90°
-180°
non-linear resonances
8th
9th
10th
7
8
9
10
Phase
Amplitude
Bottom
Qv min.
Top
Qv max.
-20 kHz
0
+20 kHz
Fast wave or
lower side band
10.809 MHz
34×frev.
Slow wave or
upper side band
-Im
R2 T. F. Polar plot:
Slow wave
Fast wave
-Real

Fig. 1



BLOCK DIAGRAM OF THE NEW SCHOTTKY SET-UP WITH FAST FOURIER TRANSFORM SPECTRUM ANALYSER

Fig. 2