

CONSIDERATIONS ON LONGITUDINAL STABILITY AND
BEAM-EQUIPMENT INTERACTIONS IN THE PSB

D.W. Lamotte, K.H. Reich

1. Introduction
2. Basic theory
3. Formulae used for evaluating beam-equipment interactions
 - 3.1 Admissible total coupling impedance
 - 3.2 Individual coupling impedances
4. Numerical results
 - 4.1 "Critical" total impedance
 - 4.2 Individual contributions
5. Growth times
6. Discussion and conclusions

Acknowledgements

Appendix

References

1. INTRODUCTION

Our knowledge of collective effects in circular accelerators has been reviewed successively by Teng¹⁾, Courant²⁾, Sessler³⁾, Neil⁴⁾, Kolomenskij and Lebedev⁵⁾, and Courant⁶⁾. We attempt here to extract from this knowledge what is relevant for the longitudinal motion in the PSB, and to apply it numerically as regards beam-equipment interactions. We shall begin by putting these particular interactions in their context.

Through a number of different mechanisms operative on the longitudinal motion, collective effects may produce the ultimate common result that the intensity and/or the density of the protons which can be accelerated are smaller than would be the case in the absence of these effects. While circulating in a synchrotron, protons experience the superposed action of the various perturbing fields and a complete theory should therefore treat all effects simultaneously. However, as each effect in itself is already so complicated that it can only be dealt with in an approximate fashion (due to, amongst other things, the non-linearity of space charge and RF-fields and the different time-variation of the self-fields), it is customary to deal with them only one or two at a time. A better justification for this procedure is the fact that in some machines the perturbing fields are often small with respect to the external accelerating fields and can thus be taken into account by using perturbation methods.

Coulomb repulsion between the protons circulating in a lossless (i.e. $\sigma = \infty$) vacuum chamber reduces the area of stable motion (RF "bucket") below transition energy (and increases it above transition; see Ref. 7-10 and papers quoted concerning the motion of individual particles). This effect can be compared to the incoherent space-charge detuning (Laslett tune shift) in the transverse phase plane. In both cases we deal with a stationary situation, i.e. inside the stable phase space area the charge distribution is supposed to be independent of time (except for adiabatic effects). As regards the PSB,

the potential "bucket" reduction was offset by raising the RF voltage from 10 to 12 kV. - Above transition energy the same Coulomb forces can lead to the negative mass instability^{10,11)} (which affects the whole beam, corresponding to the coherent space-charge limit in transverse phase space) but for once we obviously need not be concerned, the PSB working always below transition. In contrast, the related two-stream instability¹¹⁾ may also occur below transition, but its excitation requires two maxima in the equilibrium energy distribution, which we do not normally expect in the PSB.

Next we list the various aspects of beam-cavity interactions, i.e. effects due to the voltages induced in the RF cavity(ies) by the circulating proton current (beam loading). We separate somewhat arbitrarily the stationary effects^{12,13)} from the dynamic effects. By the former we refer to those due to the unmodulated voltage induced at RF frequency. While these stationary effects are in their result similar to those of the Coulomb repulsion (i.e. a change of effective RF voltage, and in the present case also of RF phase), we simply rely on the AVC and the phase lock systems of the PSB to correct them. The coherent bunch oscillations are a special class of dynamic beam-cavity interactions. If all bunches tend to oscillate simultaneously in phase, this tendency should of course be suppressed rapidly by the phase lock system. However, in a standard synchrotron usually no deliberate damping mechanism is provided for longitudinal oscillations of individual bunches with respect to other bunches (leaving the total centre of gravity unaffected). Such oscillations are essentially due to a modulation of the accelerating voltage per turn and have been observed notably at the AGS and the CPS^{14,15)} Gumowski has started a study in 1968 in view of the specifications of the quality factor and the impedance of the PSB RF cavity, and this work is now being finalised¹⁶⁾. For completeness sake we mention that some coherent bunch shape oscillations would be taken care of by the PSB phase lock system¹⁷⁾. Finally there are the coherent beam effects which we shall treat as part of the general beam-equipment interaction.

These general beam-equipment interactions being the substance of this report, we shall deal with them in the following more extended manner. First we present the physical mechanism in descriptive terms and we sketch briefly the mathematical method, and then we retrace the historical development of the subject. In Section 3 we present the necessary formulae and apply them to the PSB in Section 4.

2. BASIC THEORY

At the outset interactions with non-resonant and resonant equipment, respectively, were treated completely separately. However, as the unifying concept of the equipment impedance with respect to the beam perturbation current developed (based on a number of simplifying assumptions), it became possible to work in terms of a single formalism and simply to add suitably the different contributions from resistive walls, cavities, fast "kicker" magnets, etc. and even from the Coulomb repulsion.

Let us discuss the basic mechanism of the instabilities for the case of a coasting unbunched beam⁵⁾. The n^{th} azimuthal harmonic of a proton density fluctuation represents a travelling wave $\sim \exp j(\omega t - n\theta)$ where ω is the unknown radian frequency and $\theta = \omega_0 t$ (with ω_0 as the angular particle velocity) is the azimuthal position. Assuming $\omega = n\omega_0 \pm n \Delta \omega_0(n)$ (where $\Delta \omega_0 \ll \omega_0$), the speed of propagation of the perturbation wave along the beam in direction of increasing θ is

$$v_W = \beta_W c = \omega R_m / n = [\omega_0 \pm \Delta \omega_0(n)] R_m,$$

where R_m is the mean machine radius. In other words one has a fast wave with $\omega_0 + \Delta \omega_0$ and a slow wave with $\omega_0 - \Delta \omega_0$. As particles move up to the peak of the slow wave they are being braked by the tail fields, and hence the particle density increases.

The standard mathematical method^{3,18,19,20)} is based on the collisionless Boltzmann (Vlasov) equation. The more important assumptions and approximations are:

- i) Perturbation theory is applicable, i.e. the "equipment" creates only a "small" change of the basic situation
- ii) energy losses are small enough to permit the use of the standard Hamiltonian formalism
- iii) large use is made of average fields (in particular use of the average electric field on the beam axis, neglecting any radial dependence)
- iv) the beam is unbunched in most cases.

Assuming a distribution function, $\Psi(q, p, t)$, the problem consists in finding the complex frequency shifts of the perturbation waves from solving Vlassow's equation ($d\Psi/dt = 0$ because of ii))

$$\frac{\partial \Psi}{\partial q} \frac{dq}{dt} + \frac{\partial \Psi}{\partial p} \frac{dp}{dt} + \frac{\partial \Psi}{\partial t} = 0 \quad (1)$$

where q and p are a set of generalised coordinates and momenta describing the dynamical behaviour of a particle. To obtain the derivatives $dq/dt (= \partial H / \partial p)$ and $dp/dt (= - \partial H / \partial q \propto \bar{E}_s \text{ perturb.})$, one has to introduce into the Hamiltonian the assumed distribution function (along with the perturbation fields obtained from Maxwell's equations). This "loop" method then automatically insures approximate self-consistency (dynamical equilibrium) of the perturbation in the sense that the resulting distribution will at the same time produce the perturbation fields and be the result of the forces of these particular fields.

In view of the difficulty in solving (1), Ψ is often assumed to be of the form (unbunched beam)

$$\Psi(q, p, t) = \Psi_0(p) + \Psi_1(q, p, t) \quad (2)$$

where Ψ_0 is known and Ψ_1 is a small perturbation term. Furthermore substituting (2) into (1) and keeping terms of first order only yields a linear equation defining Ψ_1 . Considering in this equation ω as the

unknown of interest leads to a dispersion relation, i.e. a relation connecting the unknown frequency ω with the wavelength ($\lambda = 2\pi R_m/n$) of the perturbation. It is often written as

$$1 = - (U - jV) \int \frac{d\psi_0}{dp} \frac{dp}{\omega - n\theta} \quad (3)$$

where $(U - jV)$ is proportional to the perturbing force with the "conservative" term U mainly determining the threshold and V the growth of the instability (for $V \ll |U|$).

After the basic work by Kolomenskij and Lebedev¹⁰⁾, and Nielson, Sessler and Symon¹¹⁾, Neil, Judd and Laslett²¹⁾, and Laslett, Neil and Sessler²²⁾ studied the case of coupling instabilities in a coasting beam due to resonant cavities^{*)}.

The details of the theory of longitudinal resistive instabilities were worked out by Neil and Sessler²³⁾. Briggs and Neil^{24,25)} discuss the stabilisation of beams by means of inductive walls, and Sessler and Vaccaro by means of a helical insert²⁶⁾. Vaccaro¹⁸⁾, and Sessler and Vaccaro¹⁹⁾ extended the theory to vacuum chambers with walls of arbitrary electrical properties, Zotter²⁷⁾ to laminated chambers (also useable for stabilisation), Courant²⁸⁾ considered the effect of using insulation between vacuum chamber sections, and Keil and Zotter²⁹⁾ worked out in detail the case of periodically changing chamber cross-section ("bellows"). Continuing the work of Pease³⁰⁾ (who, in addition to the Gaussian and Lorentz energy distribution used previously, introduced four further energy distributions:

rectangular	$F_0(\xi) = \frac{1}{2}$,
elliptic	$F_{1/2}(\xi) = \frac{2}{\pi} \sqrt{1 - \xi^2}$,
parabolic	$F_1(\xi) = \frac{3}{4} (1 - \xi^2)$,
and quartic	$F_2(\xi) = \frac{15}{16} (1 - \xi^2)^2$, approaching \cos^2 ,

Ruggiero and Vaccaro²⁰⁾ studied a total of nine distributions and gave

*) In this case, sufficiently small cavity impedance or a radial beam control system is required to ensure the validity of ii).

a stability diagram in terms of the reduced quantities

$$U' - jV' = 2(U - jV)\omega_0 / \pi n [(\Delta E)^2 d\omega_0 / dE] \quad (4)$$

where ΔE is the full energy spread measured at the half height of the distribution function. Keil and Schnell³¹⁾ linked these quantities to the coupling impedance (used previously by Lebedev and Zhilkov³²⁾ and Sessler and Vaccaro^{18,19)})

$$Z = R + jX = - \frac{2\pi R_m \overline{E}_s}{I} = j \frac{2\pi(U - jV) \beta}{\omega_0 N e^2 \beta_W} \quad (5)$$

where I = perturbed beam current, \overline{E}_s = average longitudinal electric field produced by I and β = reduced particle velocity), thereby establishing a physically more obvious relation with the electromagnetic properties of the arrangement formed by the beam and the equipment (such as vacuum chambers, electrodes, kicker magnets, etc.). For $\beta_W = \beta$ this relation can be expressed as³¹⁾

$$U' - jV' = -j \frac{N e^2 \omega_0^2 Z}{\pi^2 n d\omega_0 / dE (\Delta E)^2} \quad (6a)$$

or

$$U' - jV' = -j \frac{2 I_0 e \gamma}{\pi E_0 \eta} \frac{Z}{n (\Delta p / m_0 c)^2} \quad (6b)$$

where I_0 is the average³³⁾ proton current and e , γ , η , Δp , m_0 and c have their usual meanings, Δp being again the full width at half height.

These theories have been applied to the AGS^{21,34)} and its proposed booster³⁴⁾, the Bevatron^{21,23)}, the Cambridge Electron Accelerator¹²⁾, the Cosmotron³³⁾, the CPS^{21, 35,36)}, the ISR^{18,20,29,37)}, the MURA electron accelerator^{22,23,24,25)} and the 300 GeV machine^{38,39,40)}

As these instabilities may occur below transition energy, we have to look at them for the PSB (though hopefully we can be some-

what less concerned than the ISR people with their long beam life times or the CPS people with their stringent requirements for uniformly debunched beams). While the existing theory (for unbunched beams) is directly applicable at injection, it would be desirable to have explicit solutions of the theory for bunched beams^{41 to 45)} for the other parts of the PSB cycle. This corresponds to using $\psi_0(q,p)$ in (2) instead of $\psi_0(p)$, and including the RF field in the Hamiltonian (thereby increasing the difficulty of the problem).

If one takes into account the internal degrees of freedom of a bunch one comes to a situation corresponding to the "head-tail effect" in transverse phase space. Hereward has made first estimates of possible high-order bunch-shape instabilities from longitudinal short-memory wake field in the CPS⁴⁶⁾.

3. FORMULAE USED FOR EVALUATING BEAM-EQUIPMENT INTERACTIONS

(MKS units are used throughout unless specified otherwise)

3.1 Admissible total coupling impedance

The standard equation³¹⁾ is used, viz

$$\frac{|Z|}{n} = \frac{0.7\pi}{2} E_0 \frac{|\eta|}{[\text{eV}] \gamma I_0} \left(\frac{\Delta p}{m_0 c} \right)^2 \quad (7)$$

This equation applies to an unbunched beam and therefore to the PSB at injection. However, to get a "feel" for the situation, we have also put in numbers corresponding to a bunched beam. While aware of Ref. 33, we use in the latter case $I_0 = I_{\text{mean}}/B$ where B is the bunching factor (< 1).

3.2 Individual coupling impedances

a) RF accelerating cavity

The appropriate quantity is

$$Z_c = \frac{\Delta V_c}{\Delta i_b} \quad (8)$$

where ΔV_c is the change of cavity voltage produced by a change of beam current Δi_b with the PA switched on and the relevant servo-loops working. While awaiting measured values of Z_c for the production cavities, the following formula is used for arriving at approximate values

$$Z_c = \frac{1/R_g - j \omega C_c [1 - (\omega_c/\omega)^2]}{1/R_g^2 + \omega^2 C_c^2 [1 - (\omega_c/\omega)^2]^2} \quad (9)$$

where R_g = equivalent shunt resistance

C_c = equivalent resonator capacity

ω_c = resonant radian frequency

$\omega = n\omega_0$ = radian frequency of mode considered
(with ω_0 = angular particle frequency)

and the standard electrical engineering convention has been adopted for the sign, i.e. j for an inductive and $-j$ for a capacitive impedance.

$$\omega = \omega_c$$

This case is disregarded, assuming that the AVC is sufficiently strong to reduce Z_c to admissible values⁴⁷⁾, and that any remaining tendency for self-bunching would anyway not be harmful at this frequency.

$$\omega \gg \omega_c$$

Values are taken from the measured curves $Z_c(f)$.

b) Space charge (infinitely conducting wall)

$$Z_{sp.c}/n = -j g_0 Z_0 / (2\beta\gamma^2) \quad (10)$$

where $g_0 = 1 + 2 \ln \frac{a_{vc}}{a_{beam}}$ is the usual geometric term of the beam-chamber capacitance, and $Z_0 = (c\epsilon_0)^{-1} = 377 \Omega$ is the impedance of free space.

c) Insulated vacuum joints

These joints present a complex impedance, depending on the earthing of the vacuum chamber, etc. For the present purpose they are assumed to be purely capacitive, i.e.

$$Z_j/n = -j^2 / (n^2 \omega_0 C_j) \quad (11)$$

where C_j is the capacitance of the joint.

d) Ferrite kickers

While awaiting measurements of the production kickers we use the values computed by Brückner⁴⁸⁾.

e) Resistive wall³⁵⁾.

$$Z_w/n = (1 + j) R_{PSB} R_{surf} / (n a_{vc}) \quad (12)$$

where $R_{surf} = \sqrt{\frac{n \omega_0 \rho \mu}{2}}$ is the surface resistance with ρ the wall resistivity (in Ωm).

f) Bellows²⁰⁾

$$Z_b/n = j Z_0 \beta \tau \alpha / a_{vc} \quad (13)$$

where τ is the corrugation depth and α is the fraction of the circumference with bellows.

g) Electrodes¹⁹⁾

$$Z_e/n = 2j M \ell_e \beta Z_\ell / [2R_{PSB} (1 + j n \omega_o C Z_t)]$$

where M = number of electrodes of length ℓ_e
 Z_ℓ = characteristic impedance of electrode
 considered as infinitely long waveguide
 C = electrode capacitance
 Z_t = impedance of termination.

4. NUMERICAL RESULTS

4.1 "Critical" total impedance (Equ. 7)

Input data and results are as follows.

	Injection (before trapping)	Transfer
B	1.0	0.27
η	- 0.85	- 0.24
I_o [A]	0.3	2.4
$\frac{\Delta p}{m_o c}$	0.51×10^{-3}	1.84×10^{-3}
$\frac{ Z }{n}$ [Ω]	725	190

4.2 Individual contributions

a) RF cavity

Values of Z_c/n are shown below (using $C_c = 120$ pF and I_o in Equ. 9 and measured values⁴⁹⁾ where appropriate).

n	$Z_c/n[\Omega]$ at injection	$Z_c/n[\Omega]$ at transfer
1 - 3	$3 + j 130$	$j 35$
4	$17 + j 245$	$j 92$
6	$17 - j 200$	$- j 75$
7	$4 - j 90$	$- j 28$
8	$2 - j 55$	$- j 16$
18		35
48	10	

b) Space charge⁵⁰⁾

Using $g_o = 2.9$ at injection and $g_o = 4.3$ at transfer we obtain from Equ. 10 :

$$\frac{Z_{sp.ch}}{n} = - j 1560 \Omega \text{ and } - j 280 \Omega \text{ respectively.}$$

c) Vacuum joints (Equ. 11)

With $C_j = 20$ nF we have the maximum values ($n = 1$)

$$Z_j/n = - j 13.4 \Omega \text{ and } - j 5.1 \Omega \text{ respectively.}$$

d) Kickers

The following values of $\Sigma|Z_K|/n$ at injection are used⁴⁸⁾

	n = 5	13	67
$\Sigma Z_K /n$	9	10	7.5

e) Resistive wall (Equ. 12)

With $\rho = 1.2 \cdot 10^{-6} \Omega \text{m}$ and $a_{vc} = 0.05 \text{ m}$, the largest contribution ($n = 1$) becomes at injection

$$Z_w/n = (1 + j) 0.85 \Omega.$$

f) Bellows (Equ. 13)

Taking $\tau = 0.003 \text{ m}$, $a_{vc} = 0.043 \text{ m}$ and $\alpha = 1/3$, we obtain $Z_b/n = j 2.8 \Omega$ and $j 7.4 \Omega$, respectively.

g) Electrodes (Equ. 14)

$$\Sigma Z_e/n \approx j 1 \Omega \quad \text{for } n < 150.$$

5. GROWTH TIMES

Assuming a monoenergetic beam (conservative case) below transition, and $V \ll |U|$, the slow wave grows with an e-folding time τ_0 given by ²¹⁾

$$\tau_0 = \frac{2}{V} \left[\frac{2\pi |U|}{n \omega_0 (d\omega_0/dE)} \right]^{1/2}, \quad (15)$$

or, with Equ. (5), $\frac{d\omega_0}{dE} = -\frac{\omega_0 \eta}{E_0 \beta^2 \gamma}$, $\beta_w = \beta$ and $I_0 = Ne \omega_0 / 2\pi$

$$\tau_0 = \frac{2}{R\omega_0} \left(\frac{2\pi E_0 [eV] \beta^2 \gamma |X|}{n |\eta| I_0} \right)^{1/2}. \quad (16)$$

For injection into the PSB this becomes

$$\tau_0 [s] = 0.026 (|X|/n)^{1/2} / R. \quad (16 a)$$

Selected values of τ_0 are shown in the table below.

n	R [Ω]	X [Ω]	τ_0 [ms]
3	11.55	4290	86.6
4	71.4	5260	13.5
6	113.0	10570	9.7
7	33.95	11550	31.8
8	22.8	12920	46.6

6. DISCUSSION AND CONCLUSIONS

In most cases preliminary impedance values have been used, to be replaced by the final values once these have been measured. Of the "equipment" studied, only the contribution from the RF cavity and the "space charge" are really important, with the kicker magnets and the (shunted) vacuum joints still contributing noticeably.

The most uncomfortable situation appears at injection for $n = 6$ where we have

$$\Sigma Z/n \approx (Z_c + Z_{sp.c.})/n \approx -j 1800 \Omega$$

or more than twice the "critical" value. (At transfer, the corresponding figure is $-j 355 \Omega$, which is above the "critical" value given but would be below if $B = 1$ had been used; it is understood, of course, that anyway the theory is not strictly appropriate to a bunched beam.) With respect to the coupling impedances the situation at the CPS is comparable (see Appendix) but the e-folding times are an order of magnitude smaller. To our knowledge no longitudinal insta-

bilities have been identified (at injection), though there exist of course some unexplained beam losses.

If it should turn out that the overall beam coupling impedance in the PSB becomes troublesome, addition of inductive elements could be considered^{24,25)}. Using the same ferrite rings as for the wide band beam observation station ($\mu = 850$), a ferrite cylinder of $D_i = 160$ mm, $D_o = 240$ mm and $L = 0.7$ m mounted concentrically with the beam (in a long straight section) would lower the total $n = 6$ coupling impedance at injection to the "critical" value⁵¹⁾ ($6 \times 725 \Omega$).

A programmed decrease of this extra induction (of $47.5 \mu\text{H}$) could be necessary during the acceleration in order to match the decrease of the "critical" impedance.

ACKNOWLEDGEMENTS

This report benefited from discussions with several colleagues, notably U. Bigliani, I. Gumowski, G. Nassibian and D. Zanaschi.

Distribution (open)

List MPS-SI/1

K. Hübner
E. Keil
W. Schnell
B. Zotter

APPENDIX

Situation at the CPS at injection

1. With $I_0 = 100$ mA (2×10^{12} accelerated, 50% injection and trapping efficiency)

$$\eta = -0.873$$

we find for the critical coupling impedance (Equ. 7) for various energy spreads (due to linac and debuncher settings, and space charge action in the transport line)

$\Delta E[\text{keV}]$	150	200	250	300
$ Z /n[\text{k}\Omega]$	2.24	3.98	6.20	8.96

2. We compute the impedances for the most important components, i.e. the longitudinal space charge and the cavities

a) Space charge

Taking $g_0 = 2.8$ ($a_{vc} = \sqrt{72.5 \times 34} = 50$ mm, $a_b = 20$ mm, i.e. $\epsilon = 25 \times 10^{-6}$ rad m)

one has

$$\frac{Z_{\text{sp.ch.}}}{n} = -j 1530 \Omega$$

b) RF cavities

Assuming the cavities are an RLC parallel circuit for the beam with the values⁵²⁾

$$C = 120 \times 10^{-12} \text{ F}$$

and $R = 14.5 \times 10^3 \Omega$

we have for 14 cavities

$n^*)$	19	21
$\frac{Z_c}{n}(\text{k}\Omega)$	$0.875 + j 3.19$	$0.865 - j 3.02$

*) For $n = 20$ we have to take into account an AVC reduction factor of 6 to 15.

c) Thus , we have for the most dangerous case ($n = 21$) a total coupling impedance of

$$\frac{Z_{\text{tot}}}{n} = 0.865 - j 4.53 \text{ k}\Omega$$

3. The corresponding e-folding time (for $\Delta E = 0$) is (Equ. 16)

$$\tau_0 \approx 0.65 \text{ ms.}$$

REFERENCES

1. L.C. Teng, High intensity multi-GeV proton accelerators, Trans. IRE NS-11 No. 4, p.17-23 (1964).
2. E.D. Courant, Beam instabilities in circular accelerators (Proc. 1st US National Accelerator Conference), Trans. Nucl. Science Vol. 12, p. 550-555 (1965) and BNL internal report AADD-69 (17 references cited).
3. A.M. Sessler, Instabilities of relativistic particle beams, Proc. Frascati Conference, p. 319-330 (1965) and UCRL internal report 16440 (48 references cited).
4. V.K. Neil, Review of dynamic instabilities in circular accelerators (Proc. 1967 US National Accelerator Conference), Trans. Nucl. Science, Vol. 14, p. 522-528 (1967) and UCRL internal report 70205 (8 references cited).
5. A.A. Kolomenskij, A.N. Lebedev, Collective effects in circular accelerators, Proc. 1st USSR Accelerator Conference, Vol. 2, p. 261-273 (1968) and French translation by CERN (MPS/DL Note 70-20) and German translation by Kernforschungszentrum Karlsruhe (KFK-tr-302) (58 references cited).
6. E.D. Courant, Some high-current effects in accelerators, Proc. VII International Conference on High Energy Accelerators, Yerevan 1969.
7. M. Dory, Nonlinear azimuthal space charge effects in particle accelerators, Ph.D. Thesis, 1962, MURA-654.
8. W.S. Trzeciak, Longitudinal space charge effects in RF buckets, (Proc. 1967 US National Accelerator Conference), Trans. Nucl. Science, Vol. 14, p. 612-623 (1967).
9. I. Gumowski, K.H. Reich, Synchrotron motion in the presence of space charge, CERN/SI/Int. DL/70-6.
10. A.A. Kolomenskij, A.N. Lebedev, Certain beam-stacking effects in fixed-field magnetic systems, Proc. International Conference on High-Energy Accelerators and Instrumentation, CERN 1959, p.115-124.
11. C.E. Nielsen, A.M. Sessler, K.R. Symon, Longitudinal instabilities in intense relativistic beams, Proc. International Conference on High-Energy Accelerators and Instrumentation, CERN 1959, p.239-252.
12. V.K. Neil, A.M. Sessler, Interaction of a particle beam with an externally driven radio-frequency cavity, Review Scientific Instr., Vol. 32, No. 3, p.256-266 (1964).

13. H.H. Umstätter, On the distortion of the centre of accelerating buckets by beam loading effects, MPS/Int. RF 67-18.
14. O. Barbalat, Performance of the modified AGS RF system and its implications for the future CPS and PSB accelerating systems, MPS-SI/DL Note 70-5.
15. Y. Baconnier, D. Boussard, J. Gareyte, Some preliminary results on coherent longitudinal instabilities in the CPS, CERN/MPS/SR 70-6, and D. Möhl, Bunch to bunch frequency spread to stabilize coherent oscillations in the absence of active feedback.
16. I. Gumowski, Stability of coherent synchrotron motion, Part 1, CERN/SI/Int. DL/70-13.
17. U. Bigliani, Progress report on PSB beam control, SI/Int. EL/69-1.
18. V.G. Vaccaro, Longitudinal instability of a coasting beam above transition, due to the action of lumped discontinuities, ISR-RF/66-35.
19. A. Sessler, V. Vaccaro, Longitudinal instabilities of azimuthally uniform beams in circular vacuum chambers with walls of arbitrary electrical properties, CERN 67-2.
20. A.G. Ruggiero, V.G. Vaccaro, Solution of the dispersion relation for longitudinal stability of an intense coasting beam in a circular accelerator (application to the ISR), ISR-TH/68-33.
21. V.K. Neil, D.L. Judd, L.J. Laslett, Electromagnetic fields and resistive losses, Review Scientific Instr., Vol. 32, No. 3, p. 267-276 (1964).
22. L.J. Laslett, V.K. Neil, A.M. Sessler, Electromagnetic coupling instabilities in a coasting beam, Review Scientific Instr., Vol. 32, No. 3, p. 276-279 (1964).
23. V.K. Neil, A.M. Sessler, Longitudinal resistive instabilities of intense coasting beams in particle accelerators, Review Scientific Instr., Vol. 36, No. 4, p. 429-436 (1965).
24. R.J. Briggs, V.K. Neil, Stabilization of intense coasting beams in particle accelerators by means of inductive walls, Plasma Physics, Vol. 8, p. 255-269 (1966).
25. V.K. Neil, R.J. Briggs, Stabilization of non-relativistic beams by means of inductive walls, Plasma Physics, Vol. 9, p. 631-639 (1967).

26. A.M. Sessler, V.G. Vaccaro, Passive compensation of longitudinal space charge effects in circular accelerators; the helical insert, CERN 68-1.
27. B. Zotter, Longitudinal instability of relativistic particle beams in laminated vacuum chambers, CERN-ISR-TH/69-35.
28. E.D. Courant, Effect of insulation between vacuum chamber sections on resistive instabilities, BNL internal report AADD-136.
29. E. Keil, B. Zotter, The coupling impedance of corrugated vacuum chambers for cyclic particle accelerators, CERN-ISR-TH/70-30 and TH/70-33.
30. R.L. Pease, Longitudinal instability rise rates, BNL internal report AADD-101.
31. E. Keil, W. Schnell, Concerning longitudinal stability in the ISR, CERN-ISR-TH-RF/69-48.
32. A.N. Lebedev and E.A. Zhilkov, Steady-state acceleration of particles in the presence of space charge, Nucl. Instr. Meth. 45, 238 (1966)
33. PS-Machine development report: Debunching in the presence of high-frequency cavities, MPS/DL/Note 70-14.
34. R.L. Pease, Theory of longitudinal instabilities in synchrotrons, with applications, (Proc. 1967 US National Accelerator Conference), Trans. Nucl. Science, Vol. 14, p. 562-566 (1967) and BNL internal report AADD-128.
35. H.G. Hereward, Effects of cavities on debunching, rough estimates for the CPS, CERN/MPS/DL 69-7 and H.G. Hereward, A. Sørenssen, Debunching and the resistive wall, CERN/MPS/DL 69-13.
36. A. Sørenssen, What sort of coupling impedances are tolerable in the future CPS?, CERN/MPS/DL 70-1.
37. P. Strolin, V.G. Vaccaro, Beam equipment interaction in the ISR: stability considerations, CERN-ISR-TH/69-28.
38. H.G. Hereward, Effect of beam-induced RF on debunching in the 300 GeV machine, MPS/DL Note/69-20.
39. D. Möhl, W. Schnell, A. Sørenssen, C. Zettler, The feasibility of using the PS as an injector for "project B" RF problems, CERN/ISR-RF/70-31 and CERN/MPS-DL/70-6.(MC/31/Rev.).
40. W. Hardt, D. Möhl, Debunching in Project B, CERN internal report MC-34.

41. V.P. Grigorev, Longitudinal instability of a particle bunch in cyclic accelerators, Soviet Physics-Technical Physics, Vol. 11, No. 3, Sept. 1966.
42. E.A. Zhil'kov, A.N. Lebedev, Phase stability in accelerators with beam control, Atomnaya Energia 18, 22 (1965) and PTE No. 1, 17 (1965).
43. A.N. Lebedev, Coherent synchrotron oscillations in the presence of space charge, Atomnaya Energia, 25, 100 (1968).
44. Ia.S. Derbeniev, N.S. Didansky, Stability of a bunch interacting with a low Q resonator, Preprint IYAF-326, Novosibirsk 1969.
45. I. Gumowski, Internal report under preparation.
46. H.G. Hereward, Private communication.
47. U. Bigliani and D. Zanaschi, Private communication.
48. A. Brückner, Private communication.
49. D. Zanaschi, Private communication.
50. C. Bovet, I. Gumowski, K.H. Reich, Values of g_0 for the PSB, memorandum dated 12.10.1970.
51. G. Gelato, Private communication.
52. H.H. Umstätter, Measurements of frequency response, pulse response and beam interaction in the PS cavities, MPS/Int. RF/67-5.