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High energy physics and cosmic rays

Department of Theoretical Nuclear Physics

Preprint N 93

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ELECTROMAGNETIC FORM FACTORS OF HADRONS IN THE TIME-LIKE REGION

Moscow 1991

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Hadron electromagnetic form factor in the time-like region at the boundary of the physical region is considered. The energy behaviour of the form factor is shown to be dominantely determined by the strong hadron–antihadron interaction. The experiments to extract this interaction by using of hadron form factor properties are proposed.

1. Introduction

The main goal to study electromagnetic form factor of hadrons is an obtaining of information about structure of these particles. The most complete data on the behaviour of the form factor as a function of four-momentum transferred is obtained for a pion and a nucleon. To investigate form factor of hadron (h) two reactions are used: the reaction of elastic scattering of electrons by hadron $e^-h \to e^-h$ (so-called space-like region of the four-momentum transferred); the reaction of hadron–antihadron pair production in the electron–positron annihilation $e^+e^- \to \bar{h}h$ or inverse reaction (time-like region).

Recently high precision data on the electromagnetic form factor of proton in the time-like region became available due to good quality experiment PS-170 performed at Low Energy Antiproton Ring at CERN [1]. The data were obtained from the reaction $\bar{p}p \to e^-e^+$ in the energy region from the threshold of $\bar{p}p$ (square of energy in center-of-mass is equal to 3.52 GeV^2) up to 4.2 GeV^2. These data with previous one are presented in Fig. 1 and reveal a feature which principally differs behaviour of the proton form factor in the time-like region from that of pion. The form factor drops enormously quickly just near threshold (from about 0.56 at the threshold to 0.36 at 3.6 GeV^2).

To describe the behaviour of the form factor different vector dominance models (VDM) are used. These models successfully reproduce the behaviour of the pion form factor both in space-like and time-like regions [2] and the behaviour of the nucleon (proton and neutron) form factors in the space-like region [3,4], but they fail in description of the proton form factor in the time-like region (see for instance dashed curve in Fig. 1).

Another approach was proposed [5,6] to understand physics of the behaviour of the nucleon electromagnetic form factor in the time-like region. This approach takes into account final state interaction (interaction in the nucleon–antinucleon system) as a dominant physical reason giving energy form factor behaviour and its value near $\bar{N}N$ threshold. In this model, the form factor is separated into two parts according different physical processes. Form factor is presented as a product of a factor corresponding to singularities of transition amplitude lying far from $\bar{N}N$ threshold and a factor reflecting strong final state interaction. The energy dependence of the form factor is given by the latter factor. Moreover, the behaviour of the form factor appears to be directly connected to other observables in the $\bar{N}N$ system. For instance, it is possible extract a value of imaginary part of nucleon-antinucleon scattering length by using of the momentum dependence of the form factor just near $\bar{N}N$ threshold.

This approach predicts pecular behaviour of the proton form factor in the time-like region and is able to reproduce recent

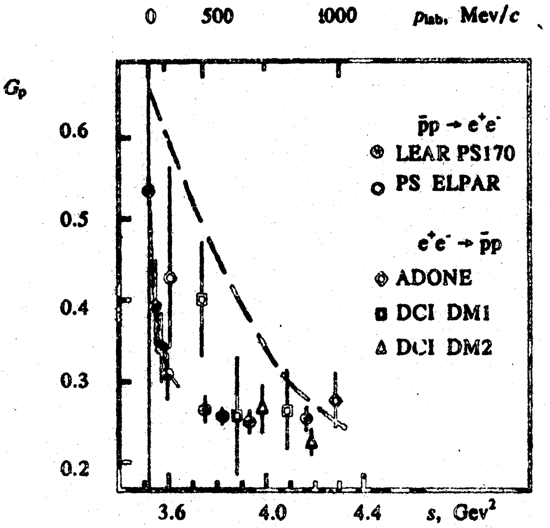

Fig. 1. Proton electromagnetic form factor in the time-like region. Experimental data are taken from [1]. Dashed line is the VDM calculations [3]. Solid lines crossing first four experimental points is calculation of this work.

experimental data. These results are presented in this article. Moreover it turns out to be possible predict behaviour of the neutron form factor. Our predictions differ sufficiently from the predictions of standart VDM. The neutron form factor can not exceed the proton one, whereas all vector dominance models obtain the neutron form factor five–ten times more than the proton one.

Established connection of the behaviour of the hadron form factor with the interaction in the hadron–antihadron systems gives us unique possibility to investigate these systems. The main advantage of electron–positron production of baryon–antibaryon pairs is that there is no initial state interaction and the transition mechanism is well-determined. When we use baryon–antibaryon beams both factors are unknown. However, even existing facilities allows us obtain such an information from the "form factor" experiments with e^+e^--beams (for instance for systems with hidden strangeness like $\bar{Y}Y$, or with hidden charm and beauty like $\bar{D}D$, $\bar{B}_cB_c$ etc).

There is an additional group of experiments with e^+e^- beams, which can give more information about hadron–antihadron interaction in final state. For a case of nucleon–antinucleon we can call experiments on investigation of electron–positron annihilation into multipion systems which are dominant modes of nucleon-antinucleon annihilation. In these reactions, system nucleon-antinucleon appears as an intermediate state. Hence quantum numbers of the $\bar{N}N$ system are definitely fixed, because nucleon and antinucleon are created from electron–positron pair through a photon what gives photon quantum numbers to the nucleon–antinucleon system. Even or odd amount of pions in the final state fixes isospin of a system. So this kind of experiments allows us prepare the nucleon–antinucleon system in a state with definite quantum numbers, that is practically difficult in any other experiment. This fact makes investigations proposed here very fruitfull. For example, the broad deep–bump structure observed in the electron–positron annihilation into six pions near nucleon–antinucleon threshold was interpretated as an expected manifestation of Green function zero for interacting $\bar{N}$ and N in the intermediate state [6]. From these data the existence of a new narrow vector $\bar{N}N$ state was predicted [7].

The main advantage of the approach presented in [5-7] and here is its ability to describe simultaneously the properties of nucleon-antinucleon interaction, form factor of the nucleon in the time-like region, and multipion electron-positron annihilation near $\bar{N}N$ threshold. These features could be found in other hadron-antihadron systems considered here.

The article is organized by the following way. Section 2 is devoted to the general properties of the form factor in the time-like region. The manifestation of these properties on the example of the nucleon form factor is considered in Section 3. In Section 4 we discuss form factor properties of other hadrons. Conclusion and proposals for experiments are presented in Section 5.

2. General properties of the form factor

In this Section we investigate general properties of the hadron form factor in the time-like region near the boundary of the physical region. For different hadrons we can have different number of form factors, because it depends on spin of hadron. Hence as a concrete example we will write formulas for a nucleon (proton and neutron). But all physical results can be trivially generalized for other hadrons what will be done in Section 4.

The form factor of the nucleon (N) in the time-like region is determined from the reaction of e^+e^--annihilation into nucleon-antinucleon pair $e^+e^- \to \bar{N}N$ or vice versa. This is so-called *s*-channel in contrast to *t*-channel corresponding to $e^-N \to e^-N$ scattering or to the nucleon form factor in the space-like region.

The differential cross section $d\sigma/d\Omega$ of the reaction $\bar{p}p \to e^+e^-$ is connected to the form factor of the proton in the vicinity of $\bar{p}p$ threshold by the formula:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{32kE}\{|G_M|^2(1+\cos^2\theta) + \frac{4M_P^2}{s}|G_E|^2\sin^2\theta\},$$

here k and E is center-of-mass momentum and energy in the pp system, θ is angle in c.m.s, α is the fine structure constant, M_P is the proton mass. G_E and G_M are electric and magnetic form factors of the proton, correspondingly. They are connected to Pauli form factors F_1 and F_2:

$$G_E = F_1 + F_2,$$

$$G_M = F_1 - \frac{q^2}{4M_N^2}F_2,$$

here q^2 is four-momentum transferred, which is equal to t in the center-of-mass system of the reaction $e^-N \to e^-N$ and to s in the reaction $e^+e^- \to \bar{N}N$. Threshold of the latter reaction corresponds to $q^2 = -4M_N^2$ (M_N is the nucleon mass).

At the pp threshold G_E and G_M are equal and for symplicity hereafter they are taken to be equal in the kinetic energy region of few tens MeV near the threshold.

Before doing of any calculation we can make some conclusions about the nucleon form factor near threshold. Let's consider a diagram corresponding to the process $e^+e^- \to \bar{N}N$ (Fig. 2).

Dashed block in this diagram presents the final state interaction in the system $\bar{N}N$. We know that this interaction is very strong and even can produce bound states in the $\bar{N}N$ system. Left part of the diagram corresponds to the transition amplitude from e^+e^- pair into $\bar{N}N$ pair. Black circle in this transition amplitude denotes a connection between a photon and $\bar{N}N$ pair, which can be realized for example by vector mesons (ρ or ω).

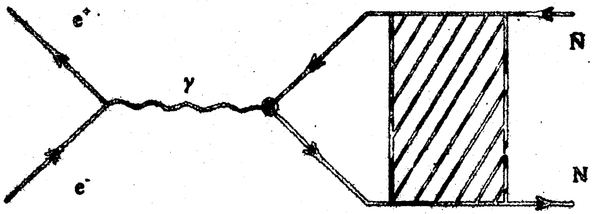

Fig. 2. The diagram corresponding to electron–positron annihilation into the nucleon–antinucleon system

This transition amplitude without final state interaction corresponds to the s-channel diagrams and therefore has no t-singularities. So this amplitude in r-representation is proportional to δ-function of r. In the first Born approximation on the fine structure constant (Fig. 2) this fact allows us rewrite immediately the expression for the transition amplitude $e^+e^- \to \bar{N}N$ and therefore for the form factor G in the following way [5]

$$G = G_0 \,|\psi(0)|. \qquad (1)$$

where $\psi(0)$ is the $\bar{N}N$ wave function in the origin.

Roughly speaking, the form factor is separated into two parts G_0 and $\psi(0)$. The former corresponds to singularities far from $\bar{N}N$ threshold (for instance, to a connection of the photon to the nucleon through ρ- or ω-exchanges, as it can be written in usual vector dominance models). The latter reflects strong final state interaction. Main dependence on energy of the form factor in the nearthreshold region will be given by the second factor in the formula (1). We are interested in the kinetic energy region of few tens MeV in the $\bar{N}N$ center of mass. Here the first factor G_0 is practically constant, so to determine energy dependence of the form factor it is necessary to investigate $\psi(0)$.

The wave function of the $\bar{N}N$ system in the origin is directly connected to the Jost function $f(k)$ of the $\bar{N}N$ system [5]:

$$\psi(0) = \frac{1}{f(-k)},$$

which can be rewritten in the form:

$$f(k) = \frac{e^{i\delta(k)}}{\tau(k^2)}.$$

Here $\delta(k)$ is the phase shift of the $\bar{N}N$ scattering and $\tau(k^2)$ is an even function of k (k is the momentum in c.m.s.).

When we are working in the region close to the $\bar{N}N$ threshold we can neglect the k^2-dependence of $\tau(k^2)$ and use scattering length approximation for the phase shift

$$\delta(k) = a\,k,$$

where a is the $\bar{N}N$ triplet scattering length.

So in the nearthreshold region the behaviour of the form factor is determined by the formula [6]

$$G = C\, e^{-\mathrm{Im}\, a k} \qquad (2)$$

Exponential factor here provides us very sharp peak in the form factor behaviour near threshold. Moreover we can directly investigate the properties of the final system and determine a value of the scattering length. Just near threshold we can expand the exponent:

$$G = C\,(1 - \mathrm{Im}\, a\, k).$$

Therefore in high precision experiment we can see linear k-term in the behaviour of the form factor. Let us emphasize the principal difference with VDM predictions for analytical behaviour of the form factor. We see linear dependence on k, whereas in any VDM approach we are dealing with q^2 (energy or k^2). This linear k-dependence is a direct manifestation of the threshold (a cut, not a pole) and final state interaction.

In the following Section we consider how these properties of the form factor manifest themselves in the behaviour of the proton form factor.

3. Investigation of the nucleon form factor

Recently new very precise experimental data on the proton form factor just near threshold appeared [1]. These data with previous one are presented in Fig. 1. We see very sharp peak at small relative momenta with decreasing of the form factor in two times. VDM [3] denoted by dashed line can not reproduce this energy behaviour.

Note that in the approach presented here we can easily reproduce this energy dependence of the form factor. We can extract value of imaginary part of scattering length from the data on the protonium (pp Coulomb atom) using the experimental values of shifts and widths of atomic S-levels [9]. (In fact, we need scattering length correspondig to triplet state because final pp system in diagram in Fig. 2 is produced only in state with quantum numbers of a photon). This imaginary part of the scattering length from protonium data is equal to $\mathrm{Im}\, a\,(^3S_1) = 0.8$ fm. The calculations of the form factor by using formula (2) with this value of $\mathrm{Im}\, a$ and $C = 0.52$ are presented in Fig. 1 by solid line (this line crosses four first LEAR experimental points).

By using of this experimental data on the proton form factor we can predict a value of the neutron form factor just near threshold. For this, let's write more accurately the definition of the nucleon form factor in terms of the isoscalar (isospin is equal to 0) $G(0)$ and isovector $G(1)$ (isospin 1) form factors:

$$G_p = |G(0) + G(1)|,$$

$$G_n = |G(0) - G(1)|$$

We can use decomposition (1) for both isospins and rewrite this formulas in terms of input form factors and wave functions of final state in pure isospin states:

$$G_p = |G_0(0)\psi(0) + G_0(1)\psi(0)|,$$

$$G_n = |G_0(0)\psi(0) - G_0(1)\psi(0)|.$$

Depending on the relative values of $G_0(0)$ and $G_0(1)$ there are two possible situations.

First, $G_0(0) \approx G_0(1)$, i.e. there is now sufficient difference between isoscalar and isovector input form factors. In this case in the previous formulas we can take out of brackets the common factor G_0. We see immediately that neutron form factor G_n is less than proton one. Note that in this case behaviour of the neutron form factor can be different from the proton one. Just near threshold it can be practically constant or even increasing function of the energy (see predictions for the neutron form factor in ref. [6]).

Second, one of the input form factors is dominant. For instance, $G_0(1) \gg G_0(0)$. This situation seems to be probable if we believe that these form factors are determined by ρ- and ω-mesons correspondingly. In this case we have dominance of $G_0(1)$ because ρ-meson has a product of coupling constants with nucleon and photon larger than that for ω-meson. So we can neglect $G_0(0)$ contributions to the nucleon form factor and we obtain that proton and neutron form factors are approximately equal.

Therefore, we see that neutron form factor does not exceed proton one in any case. This conclusion directly contradicts to predictions of the VDMs which gives neutron form factor five or even ten times more than proton one.

4. Form factor of other hadrons

It is clearly seen that consideration presented above can be directly applied to investigation of the form factor of any hadron in time-like region.

Trivial generalization can be done for other baryons, first of all for baryons with strangeness. Closest threshold to the $\bar{N}N$ one is a threshold of $\bar{\Lambda}\Lambda$ production in e^+e^--annihilation. This reaction can be measured even with facilities existing now [10].

We can estimate value of the lambda form factor near the $\bar{\Lambda}\Lambda$ threshold. If we consider the same ideology of e^+e^--transition through a vector meson into $\bar{\Lambda}\Lambda$ pair with consequent final state interaction, we obtain the following. A contribution of φ-meson seems to be dominant. The value of G_0 for lamda is proportional to the coupling constants of φ-meson with photon and φ-meson with lambda. The former is known from the experiment. The latter can be estimated from the SU(3)-relations. Both are of the same order as for a nucleon.

Final state interaction in the $\bar{\Lambda}\Lambda$ system according to existing approaches is approximately the same as in case of $\bar{N}N$. Hence there are no reasons for the lambda form factor to be strongly suppressed as compared to nucleon one and we expected that they have the same order of magnitude. Moreover the final state interaction in $\bar{\Lambda}\Lambda$ system in some sence is even more clear that in the case of nucleon–antinucleon interaction, because here we are dealing with pure isospin $(I=0)$ state.

Therefore this experiment can give direct information on $\bar{\Lambda}\Lambda$ interaction. This information is not available in any other present experiment.

Due to the same reason an investigation of the reactions of e^+e^--annihilation near other $\bar{Y}Y$ thresholds will give unique information about these systems.

Absolutely analogous ideology can be applied to study meson–antimeson and other baryon–antibaryon systems (including systems with hidden charm and beauty). It will be very interesting to measure form factors of K-, D-, F-mesons, B_c-, B_b-baryons etc.

Note that very sharp behaviour can not be seen in the pion form factor, because $\pi^+\pi^-$ system has only elastic scattering and has no absorption at the threshold (moreover, it is difficult imagine final state interaction in this system at all).

5. Conclusion

The analysis of the experimental data shows that behaviour of the electromagnetic form factor of the hadron is mostly determined by the interaction of the hadron-antihadron in the final state. Therefore the measurements of the form factor properties can serve as a very fruitful source information about hadron-antihadron interaction, especially in situations when direct investigation of this interaction is impossible.

To obtain more ellaborate information about hadron–antihadron interaction the following experiments with electron–positron annihilation are desirable:

1. Precise measurement of the proton and neutron form factors in the time-like region just near threshold of the reaction $e^+e^- \to \bar{N}N$ give us opportunity of high quality determination of $\bar{N}N$ scattering parameters.

2. Investigation of the strange and charm particles form factors, first of all for lambda hyperon, because only these experiments can provide direct information about $\bar{\Lambda}\Lambda$ interaction.

3. There is a possibility to discover a $^3S_1$ bound state in the $\bar{\Lambda}\Lambda$ system, which can manifest itself as a vector meson. To do it, the experiment to measure proton form factor near $\bar{\Lambda}\Lambda$ threshold is desirable, because this state will manifest itself as a bump in the form factor behaviour $(e^+e^- \to \bar{\Lambda}\Lambda \to \bar{p}p)$.

4. Bound states with photon quantum numbers in baryon–antibaryon systems will manifest themselves also as a broad deep–bump structures in the electon–positron transition into main annihilation channels of these systems. It will be very interesting to search phenomena connected with such a meson in the $\bar{\Lambda}\Lambda$ system near threshold in the reaction $e^+e^- \to K\bar{K}4\pi$ by analogy with 6π annihilation channel near $\bar{N}N$ threshold.

5. To determine interaction in the systems with hidden new quantum numbers, the experiments on precise measurement of the cross sections $e^+e^- \to \bar{K}K$, $\bar{D}D$, $\bar{F}F$, $\bar{B}_cB_c$, $\bar{B}_bB_b$ etc near corresponding thresholds can be very informative.

Roughly speaking, just near each hadron-antihadron threshold it will be very intresting to measure both cross section of electron-positron transition into hadron–antihadron pair and cross sections of electron–positron transition into the systems corresponding to main modes of hadron–antihadron annihilation.

The authors would like to thank Prof. R. Baldini-Ferroli for providing us of new experimental data.

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Our address is: USSR, 117924, Moscow B-333, Leninsky prospect, 53

Подписано в печать 4 июля 1991 года
Заказ № 401. Тираж 100 экз. П.л.0,8
Отпечатано в Отделе научно-технической
информации ФИАН СССР
Москва, В-333, Ленинский проспект, 53