Coherent dipole oscillations and orbit effects induced by long range beam-beam interactions in the LHC

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Abstract

The large number of bunches foreseen for the Large Hadron Collider (LHC) requires a non-zero crossing angle to avoid multiple collisions outside the interaction point. The two beams however experience long range beam-beam interactions during the time they are in one common vacuum chamber. These long range interactions can excite coherent beam-beam modes and due to the finite separation they are in general non-linear leading to possible coherent resonances of higher order. A computer simulation based on a strong-strong model has been developped and was used to investigate this coherent beam-beam effect. In particular the excitation of coherent dipole oscillations of the bunches around the closed orbit was studied. Another consequence of the dipolar nature of long range interactions is a distortion of the closed orbit which can possibly lead to a loss of luminosity. Possible constraints on the LHC parameters are investigated and discussed.

1 Introduction

The LHC is designed to reach very high luminosities above $10^{34}cm^{-2}s^{-1}$ [1] and to achieve this a very large number of closely spaced bunches (more than 4000) is foreseen. Normally, the bunches of the two beams are separated into two beam pipes except in the experimental interaction regions where they share a vacuum chamber. For such a large number there will always be a certain number of bunches simultaneously in the common vacuum chamber near the interaction points. To avoid that all those bunches collide, a finite crossing angle in the order of 200 μrad if foreseen. However, long range beam-beam effects cannot be avoided between the incoming and outgoing bunches and it has been shown [2] that these long range forces constrain the choice of parameters for the crossing angle and the β^* functions at the interaction point. Other effects due to the crossing angle, such as the excitation of synchro betatron resonances have been studied earlier [3, 4] and have shown no detrimental effect on the particle. In a former study of the long range effects, the incoherent tune shifts and induced spreads have been investigated and in this paper the coherent effect will be examined. The excitation of coherent bunch oscillations induced by the beam-beam effect is well known and studied extensively [5, 6] for the case of head on collisions where the coherent motion is excited by a small transverse displacement of the colliding bunches. For the future high energy hadron colliders such as the SSC and the LHC a geometry with a finite crossing angle if foreseen and the coherent oscillations can be excited by long range collisions [7].

The strongest effect expected is the excitation of coherent rigid dipole oscillations, i.e. small deviations from the nominal orbit, and their stability is investigated with a simulation program. In the second part of the report the effect of long range interactions on the closed orbit is discussed.

2 LHC parameters

2.1 Parasitic long range collisions in interaction region

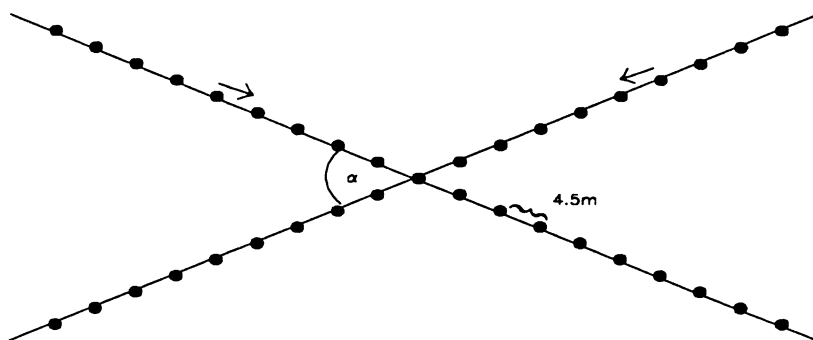The schematic view of the interaction region is shown in Fig.1.

Figure 1: Schematic view of LHC collision region

The smallest possible bunch spacing in the LHC is 15 ns corresponding to a bunch distance of 4.5m. Since the two beams are simultaneously in the same vacuum chamber for about 43 m on both sides of the interaction point, the total number of parasitic long range collisions is about 19 on each side of the collision point. Assuming a zero phase at the collision point,

the phase advance in the insertion changes fast from -90^0 before to $+90^0$ after the crossing. Most long range collisions therefore occur with a relative phase of $\pi/2$ with respect to the collision point. Since the phase difference between the long range collisions left and right of the interaction point is close to π their effect is additive in the horizontal plane. Therefore, although the actual kicks received from opposing bunches are relatively small their effect is cumulated over the entire interaction region and the total kick can become very significant [2]. The fact that a given bunch "collides" with many other bunches in every interaction region leads eventually to a coupling of all bunches and a large number of coherent modes can be expected. In order to ensure a stable system, the motion of all these multi-bunch modes with different frequencies must be stable independently.

2.2 Beam separation in interaction region

The normalised emittance of the LHC beams is $\epsilon = 3.75$ μm ($\epsilon = \sigma^2 \gamma / \beta$) and with a β^* value of 0.5 m at the interaction point this results in a beam size of about 15 μm. The separation between the two orbits increases linearly with the distance from the interaction point like $d(s) = \alpha s$ and it can be easily demonstrated that the normalised separation $d_{sep} = d(s)/\sigma(s)$ is constant between the interaction region and the first quadrupole and can be written as $d_{sep} = \alpha \beta^* / \sigma^*$ with α the full crossing angle and β^* and σ^*, the betatron function and beam size at the interaction point (see e.g. [2]). For the nominal LHC parameters this gives a separation of about 6.5 σ for all long range collisions. This approximation is only valid until the first quadrupole of the focusing triplet is reached. Since in this triplet the two beams still share the vacuum chamber, beyond the first quadrupole the separation has to be calculated for every parasitic collision individually [2]. The separation in the triplet is slightly larger (up to 10 σ) than in the driftspace between the collision point and the triplet.

3 Simulation model

3.1 Phase space variables

The variables used for the simulation are the transverse coordinate and angle of the centre-of-mass of a bunch with respect to the design orbit (x,x'). The evolution of these variables is followed for N bunches in each beam. The bunches are considered as rigid objects in this simulation since only the coherent dipole oscillation is considered. A complete simulation with more than 4000 bunches is unrealistic since the computer time required for the simulation would be too large. The maximum number of bunches in the simulation was 256 per beam and in order to make the model realistic several parasitic collisions have been accumulated into a single kick for the test bunch. The betatron phase advance between the long range collisions on one side of the interaction region is very small (see 2.1) and such a cumulation is justified. This overestimates the effect slightly because the bunches oscillate with different phases with respect to each other. The effect of this is discussed in a later chapter. The main features of the coherent motion can be investigated and conclusions can be extrapolated to the actual number of bunches. For the simulation a linear, uncoupled machine is used and the simulation of a single turn can be separated into two different parts, transport from one interaction point to the next and the beam-beam interaction. These two ingredients will be described in the next sections.

3.2 Particle transport

The phase space coordinates (x,x') are transformed from one interaction point to the next with the transport matrix A:

$$A = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}cos(2\pi Q_x) & \sqrt{\beta_2\beta_1}sin(2\pi Q_x) \\ -\sqrt{\frac{1}{\beta_2\beta_1}}sin(2\pi Q_x) & \sqrt{\frac{\beta_1}{\beta_2}}cos(2\pi Q_x) \end{pmatrix}$$

where Q_x is the phase advance between the interaction points and β_1 and β_2 are the betatron functions at these points. A similar expression holds for the vertical plane. The number of potential interaction points is determined by the number of bunches and if the particles are equally spaced around the ring the total number of interaction points is 2N, when N is the number of bunches. The tracking is performed by stepping from one interaction point to the next, i.e. 2N steps constitute a single turn in the machine. However, only a limited number of interactions can occur, dependent on the lattice configuration. These interactions take place in the regions around the experimental interaction points before the beams are separated into their respective beam pipes. All interaction regions are considered identical in the simulation, i.e. the same length and the same β functions at the interaction point. To determine where at any of the 2N potential collision points a long range collision occurs, a vector is used inside the program which can be specified as a parameter to determine the collision scheme.

3.3 Beam-beam interaction

A bunch entering the interaction region is kicked by all opposing bunches from the other beam during the time it travels in the common beam pipe. The distance of the opposing bunches from their respective orbits is denoted by x_1 and x_2 (Fig.2). The distance of the two design orbits is d(s).
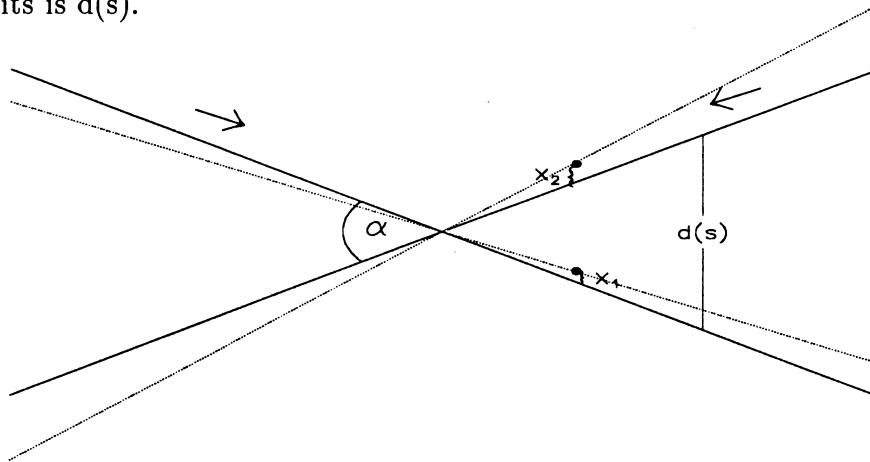

Figure 2: Geometry of the LHC interaction region. The solid line is the closed orbit and the dotted line is the bunch trajectory

The distance between the two bunches is then given as $\delta x = x_2 - x_1 + d$ where d is the distance of the design orbits at the position of the parasitic collision. The beam-beam interaction between the two bunches is approximated by kicks which have the form:

$$f(r) = \frac{8\pi\xi\sigma^2}{r\beta}(1 - e^{\frac{r^2}{2\sigma^2}})$$

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where: $r = \sqrt{x^2 + z^2}$, σ is the transverse beam size and ξ is the linear beam-beam tune shift. The angle x' is changed by δx' computed as:

$$\delta x' = f(x_2 - x_1 + d(s)) - f(d(s))$$

The opposing bunch receives a kick with the opposite sign.

Contrary to normal head-on collisions, long range interactions have a dipole term which causes an orbit distortion. The term f(d(s)) has to be subtracted since it is the orbit kick caused by the dipole term and has to be compensated by other means such as tuning dipoles near the interaction points. In this part of the study we are only interested in oscillations of bunches around the orbit. The effect on the orbit is investigated in chapter 5.

The separation for the nominal LHC parameters is about 6.5 σ and experiments have shown that a linearised beam-beam force as it is usually used to study coherent beam-beam effects from head-on collisions is not sufficient to describe the behaviour of the bunches. A linearised force would suppress all resonances of orders higher than two and results in too optimistic limits for the stability. It can be shown that a separation of at least 10 - 12 σ is necessary to suppress non-linear resonances.

3.4   Simulation strategy

The program has been written for two dimensions, but the stability was only studied in the horizontal plane where the two beams cross at an angle. After the number of bunches is chosen, one has to assign initial displacements to the individual bunches of the two beams. These are taken as random numbers in the range of $\pm 0.1\sigma$. The results have been shown to be not very sensitive to the initial displacement as long as no particular pattern is chosen.

The program follows all bunches of both beams and keeps track of the phase space variables. The displacement of some bunches is stored for a Fourier analysis. An unstable situation is assumed whenever a bunch displacement exceeds a certain limit. In most cases a limit of twice the initial deviation from the design orbit is enough and the detection of instability is relatively insensitive to the actual choice of this value. For this detection, the number of turns tracked in each case is in the order of 250 - 1000, but in some cases up to 16000 turns have been tracked to study the possible dependence of the number of turns on the onset of unstable motion. Should the motion remain stable, i.e. bound, then the linear beam-beam tune shift ξ is increased in small steps to find the maximum value where the motion remains stable. To find stability diagrams in the (Q,ξ)-plane, the initial Q-values of the machine are scanned. It has been found, that both, the ξ and Q step have to be chosen fine enough in order not to miss resonances, especially in the case of multi bunch resonances and clustered interaction points where the resonance width is very small (see 4.2.2).

4   Results

4.1   Equally spaced interaction regions

Although equally spaced interaction regions with a limited number of bunches are not a good model for the LHC, such a scheme is treated for completeness and to compare with the results obtained for clustered interaction regions and for many bunches. It also allows to understand the dynamics of the process and to draw conclusions about the applicability of the model and the validity of the extrapolation. The dependence of the tune shift on separation, number of long range interactions and collision scheme is also studied with simple configurations.

4.1.1 EQUALLY SPACED INTERACTION REGIONS WITH 4 BUNCHES

A simulation has been performed to investigate the stability of a scheme with only four bunches and equally spaced interaction regions, in this particular example with 4 collision points, i.e. every second collision is omitted. The Fig.3 shows a Fourier spectrum of a bunch where the beam-beam tune shift used was ξ = -0.0034, i.e. the nominal LHC tune shift, the separation was 6.5 σ and 19 long range collisions were cumulated on each side in every interaction region.
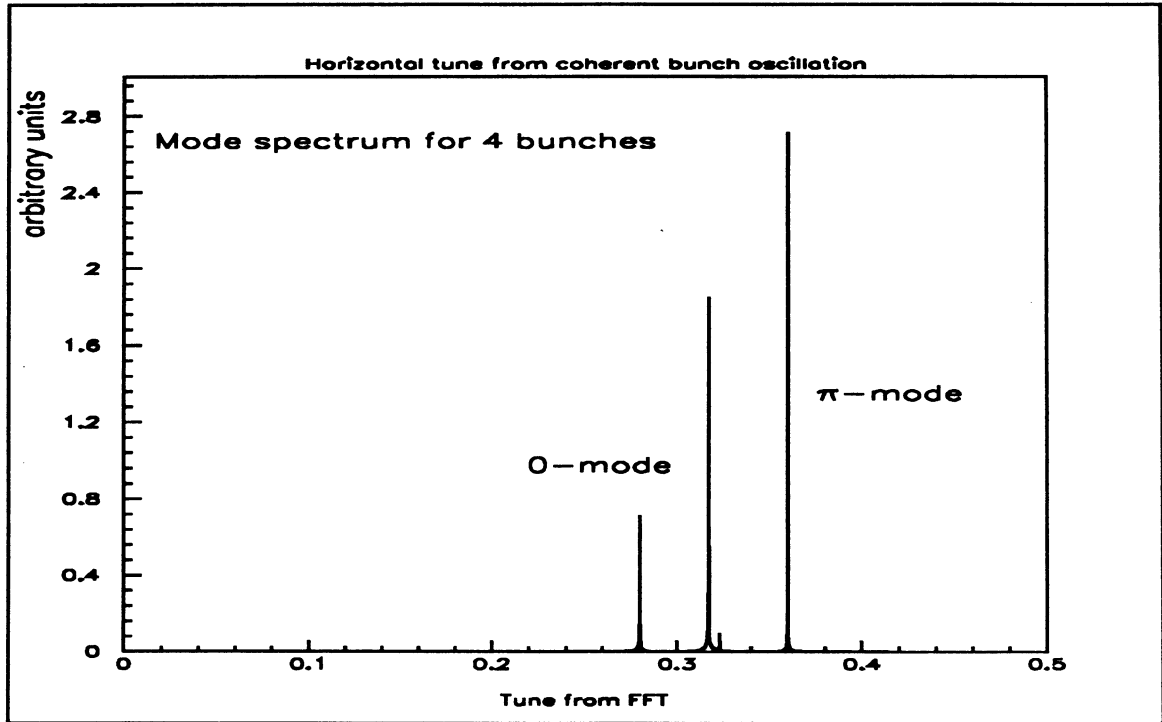

Figure 3: Mode spectrum for four bunches and equally spaced interaction regions

4.1.2 MODE SPECTRUM

The horizontal tune was set to Q_h = 71.28. Three peaks in the spectrum can be observed: the peak at the lowest frequency corresponds to the so-called 0-mode and experiences no frequency shift, i.e. it is found at a fractional tune of 0.28. For this mode the bunches oscillate in phase and it is always stable since the beam-beam force has no coherent effect when the bunches move together. The mode where the bunches move maximally out of phase is the so-called π-mode and it experiences the maximum frequency shift. This mode can become unstable. Near the central peak a very small peak can be observed which belongs to the incoherent tune. Between the 0-mode and the π-mode are the multi-bunch modes and in the very simple case of equally spaced and equal bunches these modes are degenerated into a single mode. The symmetry of the system is the reason for this degeneration and breaking the symmetry by using two non equidistant interaction points or different phase advances between the interaction points the other modes become visible [8]. Such a mode can become unstable when the bunch distribution characteristic for a certain mode is contained in the initial bunch distribution and if the frequency of the mode is close enough to a low order resonance.

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The parameters which determine the mode spectrum are the number of bunches, the collision scheme and asymmetries such as different phase advances between the collision points.

4.1.3 STABILITY DIAGRAM

The stability of coherent motion is usually presented in the form of a (Q,ξ)-diagram where for every value of the tune Q the tune shift ξ is plotted where instability occurs, i.e. at which values of ξ the π-mode is shifted onto a resonance which can be driven by the force. For the usual coherent motion induced by a small offset these are the resonances of order one and two.

Increasing ξ further would again result in stable motion as long as none of the multi bunch modes is shifted onto a resonance. The stability diagram in the (Q,ξ)-plane is shown in Fig.4.
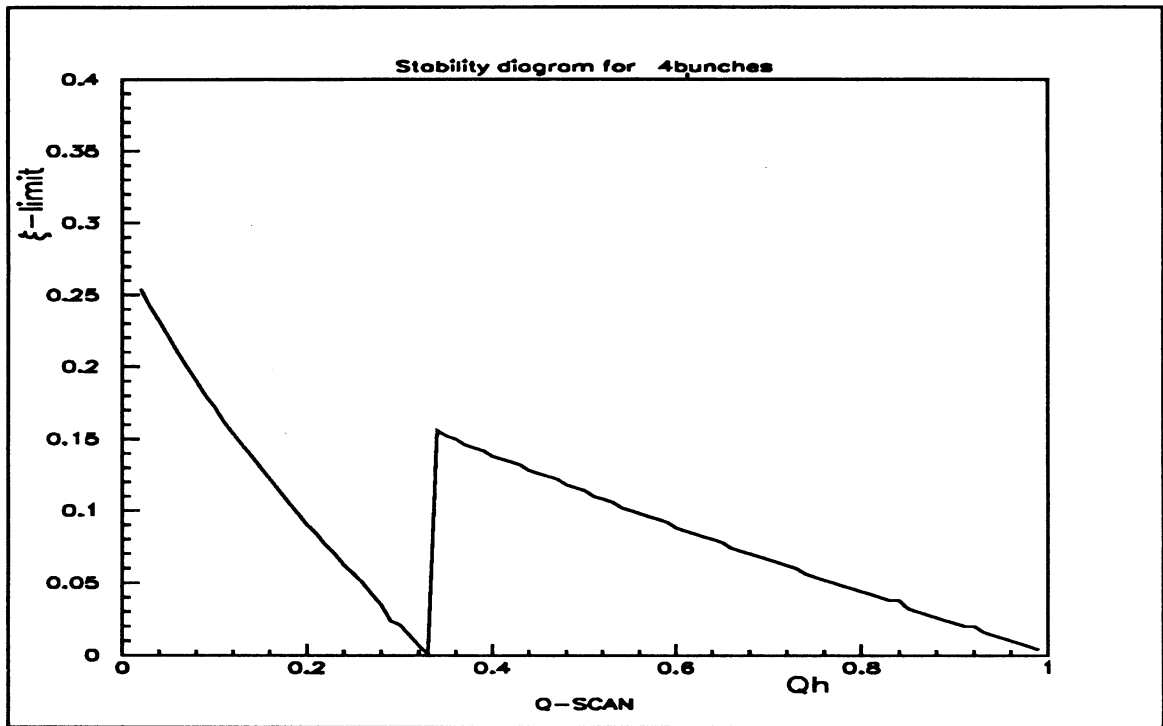

Figure 4: Stability diagram for four bunches and equally spaced interaction regions

Two regions are visible where the bunch motion becomes unstable for small values of the linear tune shift indicating the type of resonance excited: the regions for tune values just below the integer and around the third order resonance. The resonances at 1/2 and 2/3 are suppressed due to the symmetry of the collision scheme. The same type of plot for a single interaction region would show these resonances. For a linearised beam-beam force this third order resonance disappears. Non-linear resonances of orders higher than three have however not been seen in the simulations.

4.1.4 FREQUENCY SHIFT

An important quantity is the frequency shift of the π-mode because it determines the frequency span of the coherent beam-beam modes and therefore, at a given tune Q, the limit of

stability ξ_{limit}. From simulations with simple configurations it can be shown that this shift depends only on the total number of long range collisions, the linear beam-beam tune shift ξ, the number of interaction points and the separation of the two orbits. This frequency shift is independent of the total number of bunches as long as the above parameters are kept constant, i.e. for smaller number of bunches the long range kicks can be accumulated into a single kick in the simulation to get the realistic tune shifts. This allows a simple extrapolation of the frequency span to the nominal LHC parameters. For the case of the four equally spaced collision points one estimates a shift of approximately 3.6 | ξ | per interaction point assuming a separation of 6.5 σ and 19 long range interactions on each side of the collision point.

For small values of the linear beam-beam tune shift ξ the frequency span is usually small enough to be placed between low order resonances, i.e. orders lower than 3.

For a separation of more than $\approx 1.5\sigma$ the frequency shift has a sign opposite to that expected for head-on collisions with a small offset. The frequency shift from such head-on collisions has been calculated and is $\delta Q_{head-on} = 1.34\xi$ [9, 10].

4.2   Clustered interaction regions

In the SSC and the LHC the interaction points are clustered (SSC) or not equally distributed around the ring (LHC), i.e. with different phase advance. In addition, the long range collisions are clustered around the interaction regions since normally the two beams are separated.

4.2.1   FREQUENCY SHIFT

Assuming three identical interaction regions with a phase advance of Q/2, Q/4 and Q/4 between the collisons, a separation of 6.5 σ and 19 long range collisons on both sides of the collision point, a frequency shift of the π-mode of about 11 | ξ | is found, i.e. with the design tune values of Q_x=71.28 and Q_z=70.31 this results in a shift to 71.317 of the horizontal π-mode. To get the total freqency shift, the shift from the head-on collision has to be added. The coherent tune shift from this head-on collision at the interaction point however has an opposite sign and partially compensates the long range tune shift. It can therefore be extrapolated that a linear tune shift of more than 0.008 is necessary to get unstable motion on the 3rd order resonance, i.e. when the tune of the π mode approaches $3Q_x = 214$. For a linearised beam-beam force a linear beam-beam tune shift as large as ξ=-0.02 would be needed to make the motion unstable since only first and second order resonances would be excited.

4.2.2   CLUSTERED LONG RANGE INTERACTIONS WITH MANY BUNCHES

To investigate the effect of many long range interactions clustered around the collision points, a larger number of bunches has been simulated (up to 256 per beam) and the long range collisons have been localised around the interaction points. Because a given bunch collides with many other bunches, all bunches eventually couple together and a very large number of multi-bunch beam-beam modes can be observed.

In Fig.5 the spectrum is shown for 256 bunches and one interaction point with eight clustered long range collisons. A large number of beam-beam modes can be seen filling the space between the 0-mode and the π-mode. For the limit of a very large number of bunches
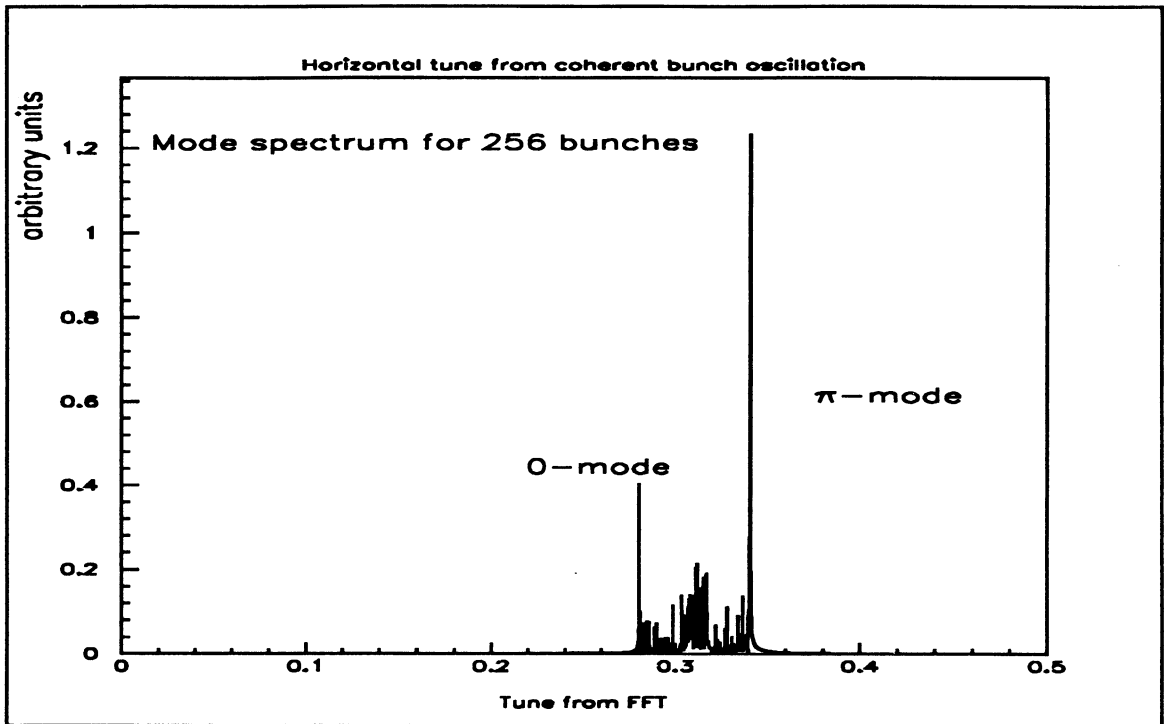
Figure 5: Mode spectrum for clustered long range collisions

the spectrum will become a continuum of modes and the entire tune region up to the π-mode is potentially unstable if a low order resonance is within this region.

The stability diagram for clustered long range collisions around one interaction point is shown in Fig.6.

The resonances of orders one, two and three are clearly visible and since the symmetry of the system is broken no resonances are suppressed.

4.2.3 RESONANCE WIDTH

The stability limit ξ_{limit} only depends on the frequency shift and not on the number of long range collisions as long as the frequency shift is kept constant. However, the widths of the individual resonances decrease very fast when the number of parasitic collisions is increased: the long range collisions all occur at different phases between the bunches and a decoherence effect is observed. For larger number of collisions (≥ 6) the linear beam-beam tune shift in the program input has to be adjusted with a precision of $\approx 10^{-5}$ to get unstable motion. The limit of instability in Fig.6 had to be carefully searched since with a too coarse scan of ξ the instability could be missed and only the first and second order resonance would be seen. A scan with the required granularity in ξ would not be feasible since the computer time required would be enormous. A method was therefore implemented in the simulation program to find the stability limit for the required resonance by adjusting the π-mode on to the required frequency. Higher order resonances were also studied using this method, but have not been identified, at least not with the number of turns studied (up to 16000 turns).
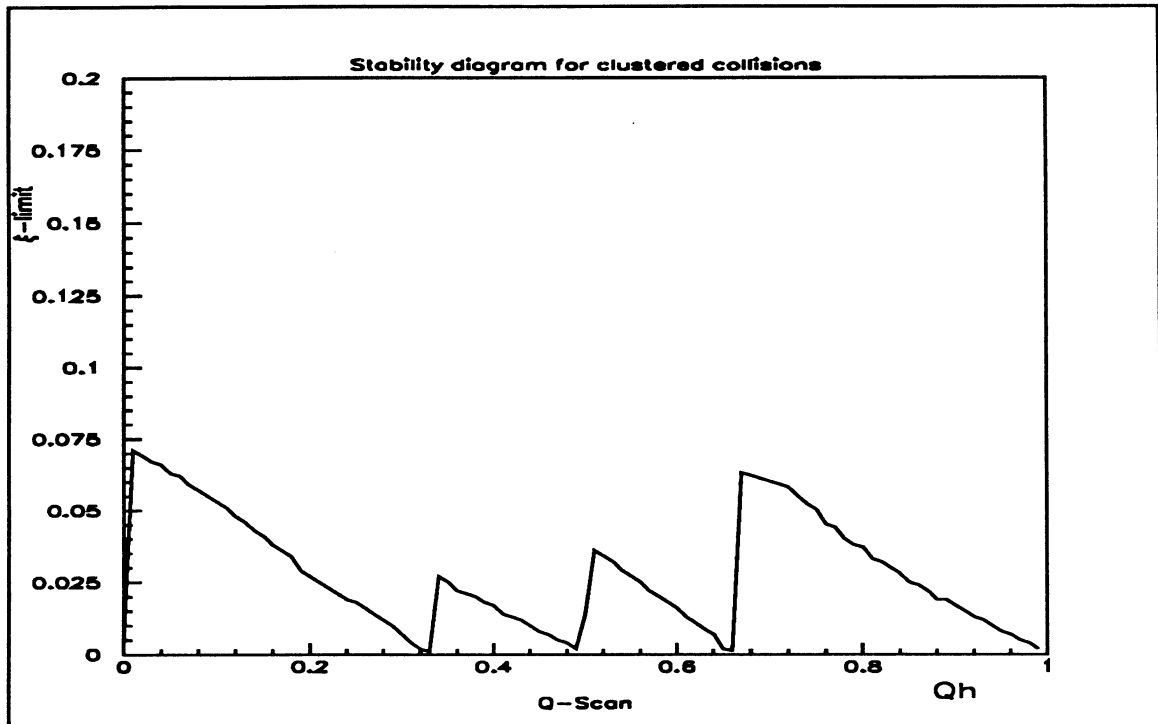
Figure 6: Stability diagram for clustered long range collisions

4.2.4 DEPENDENCE ON SEPARATION

The separation determines not only the total frequency shift of the π-mode and therefore the frequency span of the beam-beam modes, but also the non-linearity of the beam-beam potential.

In Fig.7 the stability limit for a working point of $Q_h = 71.28$ has been plotted as a function of the separation in terms of beam size σ. The value of ξ necessary for unstable motion increases with the separation as one would expect and for the nominal separation of around 6-7 σ a value of $\xi \geq 0.008$ is necessary, i.e. about twice the nominal linear beam-beam tune shift. This nominal beam-beam tune shift is indicated as a dashed line in the figure. From Fig.7 it is also clear that a separation below 5σ would be potentially dangerous.

For very large separation, i.e. above 10 - 12 σ the beam-beam force is sufficiently linear to avoid non-linear resonances and the stability limit is strongly increased.

4.2.5 DEPENDENCE ON INTEGER PART OF TUNE

The integer part of the tune is of great importance for the excitation of so-called systematic resonances and to study the dependence on this value the stability has been studied for different integer values. This has shown that the current integer values of 71 or 70 respectively are well chosen, while an integer value of 69, corresponding to a 3rd order systematic resonance, would result in a strong broadening of the resonances.

The integer part of the tune is also important for the study of closed orbit effects caused by long range interactions (see chapter 5).
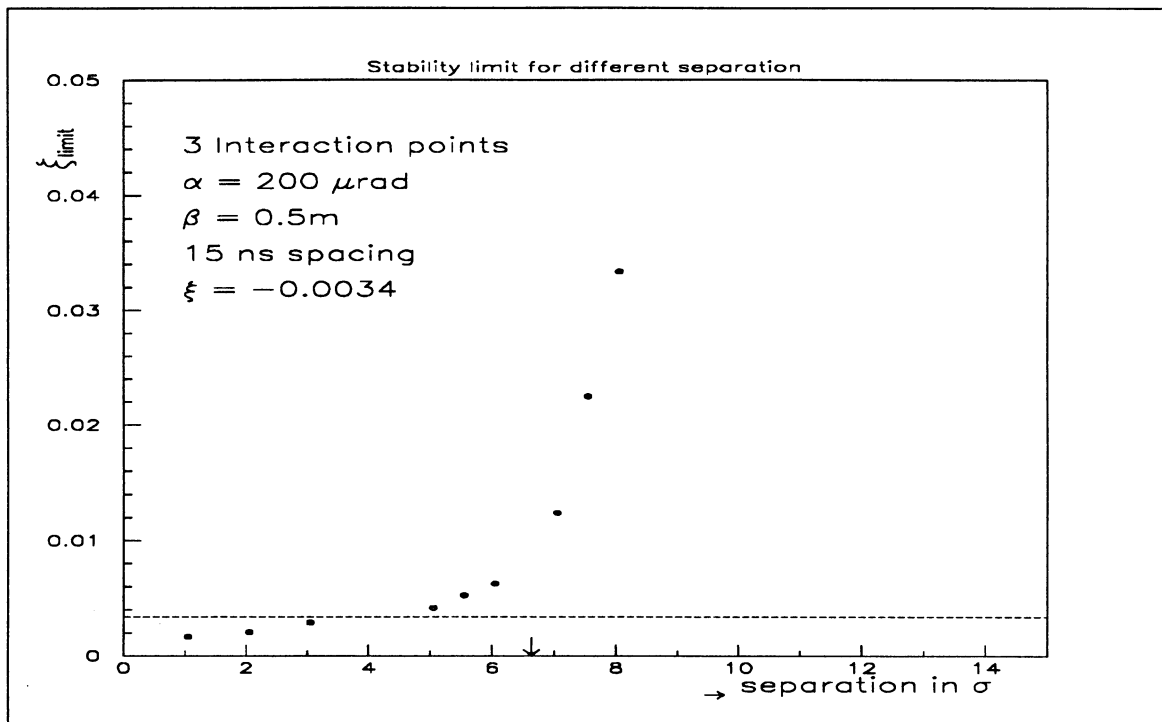
Figure 7: Stability diagram for clustered long range collisions

4.3 Very high luminosity interaction region

The nominal luminosity for the LHC experiments is $1.65 \ 10^{34} cm^{-2} s^{-1}$ but for special use in a single experiment configuration, higher luminosities up to $6.7 \ 10^{34} cm^{-2} s{-}1$ can be reached [1]. To achieve this luminosity the betatron function at the interaction point can be decreased to $\beta^* = 0.3$ m with an increased crossing angle of $\alpha = 230$ μrad to avoid large tune spread due to long range beam-beam interactions [2]. Since the beams collide only in one interaction region the bunch intensity is increased by a factor of 3 while the bunch spacing is increased from 15 ns to 50 ns. This configuration results in a linear beam-beam tune shift of $\xi = -0.01$.

The increased bunch spacing results in a much smaller number of long range collisions and it can be calculated that a coherent instability on the 3rd order resonance occurs at beam-beam tune shifts as large as $\xi \geq 0.035$, i.e. more than three times larger than the beam-beam parameters foreseen for this configuration.

5 Orbit effects

As mentioned in a previous chapter, long range collisions also give a constant dipole kick to the bunch which results in a small closed orbit distortion. For the analysis of incoherent tune shifts or coherent oscillations this orbit kick has to be subtracted since it would change the reference frame of the simulation. It can however be important to study this orbit effect since it might cause the bunches to collide with an offset in the interaction regions. A large displacement would lead to a substantial loss of luminosity and/or to excitation of coherent oscillations. In addition, a collision at an offset could excite odd order non-linear resonances and is therefore not desirable. However, due to the crossing angle geometry of the interaction

region, odd order resonances cannot be avoided and have to be considered in any case.

5.1 Single interaction region

For a single interaction region (for the geometry of the interaction see Fig.2) the orbit kick can easily be evaluated from the formulae in chapter 3.3. This kick only depends on the separation of the two closed orbits, the number of long range collisions and the linear beam-beam tune shift ξ. Assuming 19 long range interactions on each side and a separation of 6.5 σ one calculates an orbit displacement of 0.2 σ at the interaction point where the orbit kick occurs. The reduction of luminosity for two separated bunches (assuming Gaussian distribution and equal bunch sizes in both beams) can be expressed as:

$$\frac{L(d)}{L(0)} = e^{\frac{-d^2}{4\sigma^2}}$$

where d is the separation of the two closed orbits and σ the beam size.

From this expression a luminosity reduction of less than 4 percent can be expected but a fine tuning of the collision point is foreseen in the design and the orbit distortions can therefore be compensated to a large extend. A detrimental effect on coherent bunch oscillations is not expected since they are dominated by the non-linear fields due to long range collisions before and after the direct collision.

5.2 Leading and trailing bunches

The bunch train in the LHC is complicated by the existence of beam gaps which are necessary for the rise-times of the injection kickers of the LHC and its injectors [1]. A few bunches at the beginning and the end of the bunch batches experience less long range interactions at each interaction point and are displaced in the tune diagram [2]. As a consequence, the closed orbit correction which is tuned for the "normal" bunches would not properly correct those bunches which only get an orbit kick on one side of the interaction region and therefore they need special attention.

5.3 Effect on other interaction regions

Since for the leading and trailing bunches the orbit effects cannot be fully compensated, they will propagate around the ring and may have an effect on the following interaction points. In the worst case they could accumulate and would largely increase the separation at the interaction point. The displacement x(s) of the closed orbit at a position s caused by a kick δk is given as:

$$x(s) = \frac{\delta k \sqrt{\beta(s)\beta_k}}{2 sin(\pi Q)} cos(\pi Q - \mid \mu(s) - \mu_k \mid)$$

where Q is the horizontal tune, $\beta(s)$ and β_k the betatron functions at a position s and at the position of the closed orbit kick and $\mu(s)$ and μ_k are the respective phases.

It is clear that the precise values are very sensitive to the actual choice of the tunes and the phase advances between the collision points. Two scenarios have been investigated [11]: a horizontal tune of 70.28 and collisions in points 1 and 2 and a horizontal tune of 71.28 with collisions in 1 and 5, i.e. at opposite locations in the ring. In both cases the effect of the distortion onto the other collision point is very small ($\leq$ 0.05 of the distortion in

the interaction region where the kick takes place). With a different choice of the tune and different collision schemes these effects could accumulate and reach a separation up to 0.5 σ.

From experience with the CERN SPS as a proton-antiproton collider we believe that a separation of a few tenth of σ can be tolerated. The resulting loss of luminosity would be very small.

6 Conclusion

The coherent dipole oscillations induced by long range beam-beam interactions for the LHC have been investigated and the results can be summarised as follows:

- Coherent dipole oscillations can be excited by long range collision.

- The separation of several sigma causes the excitation of non-linear coherent resonances of low (3rd) order.

- Clustered interaction regions increase the number of beam-beam modes which can potentially become unstable.
  The frequency space between the 0-mode and the π-mode should be free from low order resonances.

- For the LHC parameters a linear tune shift of $\xi = 0.0080$ per interaction is necessary for unstable motion on the 3rd order resonance.

- For the special very high luminosity configuration a coherent unstable motion is very unlikely with the parameters foreseen.

- Strong decoherence from long range collisions and other damping mechanisms will make it unlikely that the coherent beam-beam effect is a limitation for the LHC with the current parameters.

- The effect of the dipole term of the long range interaction on the closed orbit is not negligible for the PACMAN (leading and trailing) bunches since it cannot be compensated by simple means. It is however unlikely that this will cause problems for the operation of the machine or substantially affect the luminosity.

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