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ON THE PROBLEM OF SPIN OF A CHARGE–MONOPOLE SYSTEM: TRANSPOSITION OPERATOR ON TWO DYONS FIBRE BUNDLE, ZEEMAN EFFECT IN A CHARGE–MONOPOLE SYSTEM

Moscow – 1990

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High energy physics and cosmic rays

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A B S T R A C T

Transposition operator of two dyons on the nontrivial two dyons fibre bundle is constructed. So we can correctly define its action on local sections. It is shown that symmetric wave functions defined on this bundle can not be transformed into antisymmetric ones by gauge transformation, apart from the well known statement, incorrect as it will be seen, firstly pointed out in connection with dyon spin problem. It is shown also, that Zeeman energy levels splitting for dyons differs from the splitting for ordinary fermions.

1. I N T R O D U C T I O N

For a long time the system consisted of monopole and charge (dyon) has been an example of how spin is generated by two particles interaction, bosons make up a fermion. In particular, the statistics problem of dyon including Dirac monopole was examined in well known Goldhaber's work [1], cited in many reviews and popular lectures (see, for example, [2,3]).

As known, the monopole - charge system has dynamical integral of motion $\vec{J}$, whose components satisfy to commutation relations of su(2)-algebra. The popular is assertion that through the monopole - charge interaction a spin $n/2 = e\mu$ (lowest j) is generated. In particular, when n is odd, two bosons (spinless particle and spinless Dirac monopole) make up a fermion.

Assuming that n/2 is adequate to spin, we should solve a puzzle connected with spin and statistics (when n is odd). Goldhaber [1] gives the solution of this problem in which he studies two dyons wave function at their transposition. The puzzle is smoothed out by the following result. There are two gauge equivalent descriptions of two dyons system. In the first case with symmetric wave function ψ and certain Hamiltonian H, the dyons are considered as bosons. In the second one with antisymmetric wave function ψ' and Hamiltonian H' the dyons are considered (when n is odd) as fermions. These descriptions are connected by a gauge transformation.

However, an approach proposed in this work is not correct. It does not take into account that the system including Dirac monopole does not have global wave function. Instead, the sections of complex line bundle are to be examined. Charge in the Dirac monopole field is a well known case [4,5]. Within the correct approach we consider the two dyons bundle and construct an operator of dyons transposition, defined on this this bundle. Trivial bundle being a case, there is no problem to construct it. But when we have nontrivial bundle, the problem arise - how to lift the action of operator on base to the action on whole bundle? Hopf fibering, as an example, displays the lift not possibly existing. In that case, there is no lift for reflection operator on base. This fact may be proved through using a degree of mapping and Lefschetz number.

2. TRANSPOSITION OPERATOR IN NONTRIVIAL TWO DYONS FIBRE BUNDLE

System of one Dirac monopole and a charge have two Hamiltonians (see, for example [4]) caused by two choises of monopole potential

A^{+} $(A^{+}_{r} = A^{+}_{\theta} = 0,\ A^{+}_{\varphi} = \frac{\mu}{r}\tan\frac{\theta}{2})$ and A^{-} $(A^{-}_{r} = A^{-}_{\theta} = 0,\ A^{-}_{\varphi} = -\frac{\mu}{r}\cot\text{an}\frac{\theta}{2})$ and corresponding to them domains $U_{+} = R^{3}\setminus\{z\in[0,+\infty)\}$ and $U_{-} = R^{3}\setminus\{z\in(-\infty,0]\}$. Transition from wave function ψ_{+} on U_{+} to wave function ψ_{-} on U_{-} is determined by transition function $T_{+-} = \exp(2ie\mu\varphi(\vec{r}))$, where $\varphi(\vec{r})$ is azimuthal angle of vector $\vec{r}$ from monopole to charge. Thus, state of the system is described by the pair of functions (each has its own domain) which are connected on the overlap $U_{+}\cap U_{-}$ by the transition function T_{+-}.

In two dyons case, by a natural way, one obtains 16 respective domains and Hamiltonians

$$H = \frac{1}{2m}(\vec{p}_{e1} - eA(\vec{r}_{1}) - eA(\vec{r}_{12}))^{2} + \frac{1}{2m}(\vec{p}_{e2} - eA(\vec{r}_{2}) - eA(\vec{r}_{21}))^{2} +$$

$$+ \frac{1}{2M} \vec{p}^{\,2}_{\mu 1} + \frac{1}{2M} \vec{p}^{\,2}_{\mu 2} + V(\vec{r}_1 + \vec{R}_1 - \vec{r}_2 - \vec{R}_2),$$

corresponding to choises A^+ and A^- for each of four variables

$$\vec{r}_1, \quad \vec{r}_2, \quad \vec{r}_{12} = \vec{r}_1 + \vec{R}_1 - \vec{R}_2, \quad \vec{r}_{21} = \vec{r}_2 - \vec{R}_2 + \vec{R}_1$$

(see Fig. 1).

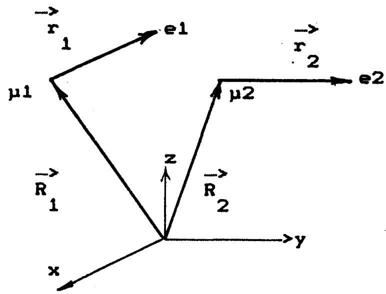

Fig. 1

Transition between wave functions defined on different domains (local sections) is combined of those $\exp(2ie\mu\varphi)$.

For instant,

$$\psi_{+++-} = \psi_{+--+} \exp(2ie\mu[\varphi(\vec{r}_{12}) + \varphi(\vec{r}_2) - \varphi(\vec{r}_{21})]) .$$

Thus, two dyons bundle has base

$$M2 = \{(\vec{r}_1, \vec{R}_1, \vec{r}_2, \vec{R}_2) \mid \vec{r}_1 \neq 0, \ \vec{r}_{12} \neq 0, \ \vec{r}_2 \neq 0, \ \vec{r}_{21} \neq 0\},$$

covering of this base by 16 regions

$$\{ U_\alpha \mid \alpha = (\pm\pm\pm\pm)\}, \quad U_\alpha = U_\pm \times U_\pm \times U_\pm \times U_\pm$$

with transition functions $T_{\alpha\beta}$.

Equivalently, instead of sections of afore-mentioned complex line bundle (let us denote it as LD2) one makes use functions determined on total space of principal U(1) - fibre bundle D2 associated with it. States of system are described either by 16 functions, every pair of them being connected by transition function $T_{\alpha\beta}$ on the overlap of their domains, or by single function defined on the total space D2 (and satisfying to condition (4)).

The problem of lifting to D2 of an arbitrary element of the group Diff(M2) of diffeomorphisms on the base M2 is connected with exact sequence

$$1 \longrightarrow Aut_V(D2) \xrightarrow{i_*} Aut(D2) \xrightarrow{p_*} Diff(M2) , \qquad (1)$$

where

$$Aut(D2) = \{ f \mid f(ug) = f(u)g , u \in D2 , g \in U(1)\}, \qquad (2)$$

$Aut_V(D2)$ - gauge transformations group, i.e. subgroup of the group Aut(D2) consisted of elements f inducing identical transformations on the base: $p_* f = 1$ (homomorphism p_* is induced by projection $p : D2 \longrightarrow M2$).

Dyon transposition operator is automorphism $\hat{\tau}$ of fibre bundle, satisfying $\hat{\tau}^2 = 1$ and being a lift of mapping on base

$$\tau : (\vec{r}_1, \vec{R}_1, \vec{r}_2, \vec{R}_2) \longrightarrow (\vec{r}_2, \vec{R}_2, \vec{r}_1, \vec{R}_1) . \qquad (3)$$

If lift $\hat{\tau}$ for $\tau \in Diff(M2)$ exists, then $\hat{\tau} \in Aut(D2)$. Since we are interested in how $\hat{\tau}$ acts on local sections ψ of line bundle LD2 (or, equivalenly, on functions $\Psi(u)$ on principal bundle D2 with equivariant condition

$$\Psi(ug) = g^{-1}\Psi(u) , \qquad (4)$$

$g \in U(1)$, $u \in D2$; $\psi_\alpha(X) = (6^*_\alpha\Psi)(X) = \Psi(6_\alpha(X))$ for local sections 6_α in D2), then important is how it acts on regions U_α covering the base. It is evident $\tau : U_\alpha \longrightarrow U_{\alpha^c}$, where $\alpha^c := (klij)$ if $\alpha =$ $= (ijkl)$. So it is convinient to define automorphism $\hat{\tau}$ in the form

$(g, X)_\alpha \longrightarrow (h_{\alpha^C}(X^C)g, X^C)_{\alpha^C}, (X^C := (\vec{r}_2, \vec{R}_2, \vec{r}_1, \vec{R}_1),$
if $X = (\vec{r}_1, \vec{R}_1, \vec{r}_2, \vec{R}_2))$.

One can show, that correspondance

$$(g, X)_\alpha \longrightarrow (g, X^C)_{\alpha^C}$$

correctly defines automorphism $\hat{\tau} \in \mathrm{Aut}(D2)$, $\hat{\tau}^2 = 1$ on two dyons fibre bundle D2.

Then

$$\hat{\tau} : (g, X)_\alpha \longrightarrow (T_{\alpha\alpha^C}(X^C)g, X^C)_\alpha \qquad (5)$$

for $X \in U_\alpha \cap U_{\alpha^C}$.

Therefore, on the sections

$$\hat{\tau}\,\psi_\alpha(X) := \psi_\alpha(X^C)\cdot T_{\alpha^C\alpha}(X^C).$$

For example, when $\alpha = (++++)$ $\alpha^C = \alpha$, $T_{\alpha^C\alpha} \equiv 1$, one has

$$\hat{\tau}\,\psi_{++++}(X) := \psi_{++++}(X^C),$$

but when $\alpha = (+-++)$, one obtains

$$\hat{\tau}\,\psi_{+-++}(X) := \exp(2ie\mu[\varphi(\vec{r}_{12}) - \varphi(\vec{r}_{21})])\cdot\psi_{+-++}(X^C).$$

Symmetry condition $\hat{\tau}\,\Psi = \Psi$ of global function on these regions has the form

$$\psi_\alpha(X^C) = \psi_\alpha(X), \quad \alpha = (++++),$$

$$\psi_\beta(X^C) = \exp(2ie\mu[\varphi(\vec{r}_{12}) - \varphi(\vec{r}_{21})])\cdot\psi_\beta(X), \quad \beta = (+-++) .$$

When $|\vec{R}_1 - \vec{R}_2| \longrightarrow \infty$, the right hand side of last equality tends to $\exp(2ie\mu\pi)\psi_\beta(X) = -\psi_\beta(X)$ for $2e\mu$ being odd.

Transition from ψ_α satisfying $\psi_\alpha(X^C) = \psi_\alpha(X)$ to ψ_β satisfying $\psi_\beta(X^C) = -\psi_\beta(X)$ was interpreted in [1] as the dyons are fermions ($2e\mu$ being odd). We have different forms of single symmetry condition $\hat{\tau}\,\Psi = \Psi$ for different local sections. In general, on nontrivial fibre bundle the global section can be formed by symmetric and antisymmetric functions on respective regions. As an instance, the Moebius band is simplest case (see Fig.2).

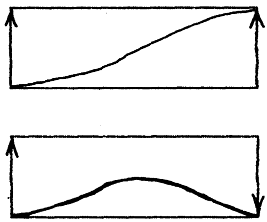

Fig.2

Note, that on fiber bundle the notion of symmetricity is correct in regard to automorphism $\hat{\tau}$. The property $\hat{\tau}\,\Psi = \Psi$ is definition of symmetric Ψ, $\hat{\tau}\,\Psi = -\,\Psi$ - of antisymmetric one. Hence, the symmetric global function (when we pass to language of local sections) can take antisymmetric form on certain regions. At first glance, this contradicts to Pauli principle. Since the antisymmetric function must be equal to zero at $\vec{r}_2 = \vec{r}_1$, $\vec{R}_2 = \vec{R}_1$, in contradistinction to symmetric one. But on two dyons fibre bundle it is not the problem, since symmetric global function takes antisymmetric local form only when $|\vec{R}_1 - \vec{R}_2| \longrightarrow \infty$.

Return now to assertion [1] that symmetric wave function of two identical dyons can be converted by gauge transformation to antisymmetrical form when $n = 2e\mu$ is odd. Exactness of sequence (1) results that if there exists some lift on D2 of τ defined on the base, then exact sequence

$$1 \longrightarrow Aut_V(D2) \xrightarrow{i_*} E \underset{s}{\overset{p_*}{\rightleftarrows}} \{1, \tau\} \longrightarrow 1, \qquad (6)$$

takes place, here E denotes the subgroup of Aut(D2), being extension of the group $T = \{1, \tau\}$ with the aid of $Aut_V(D2)$. In addition, to satisfy condition $\hat{\tau}^2 = 1$ the homomorphism p_* must have the right inverse s, $p_* s = 1$ (i.e. exact sequence (6) have to split). Let us prove that this condition is satisfied.

Evident is that any lift $\tilde{\tau}$, $\tilde{\tau} = \mathrm{lift}(\tau)$ satisfies $\tilde{\tau}^2 = \alpha \in \in Aut_V(D2)$, where α may be considered as a function on the base, $\alpha = \alpha(pu) = \alpha(X)$ as it results from U(1) is Abelian. The base of two dyons fibre bundle D2 can be represented as subset

$$M2 = \bigcup_{(a,b) \in R^3 \times R^3} (a,b) \times (R^3 \setminus \{a,b\}) \times (R^3 \setminus \{a,b\})$$

of $R^3 \times R^3 \times R^3 \times R^3$. One can show that set of homotopy classes of mappings α: M2 ——> U(1) is trivial. It follows, that $\gamma = \arg(\alpha)$ is function γ : M2 ——> R for arbitrary $\alpha(X)$. This fact is sufficient for the sequence (6) to split. Actually, let $\tilde{\tau}^2 = \alpha(X) = = \exp(i\gamma(X))$, then using the associativity in E, one can obtain $\tilde{\tau}\cdot\exp(i\gamma(X)) = \exp(i\gamma(X))\cdot\tilde{\tau}$ (i.e. $\gamma(X^c) = \gamma(X)$). Let us define $\hat{\tau} = \tilde{\tau}\cdot\exp(-i\gamma(X)/2)$. Then $\hat{\tau}^2 = 1$.

Having the operator $\hat{\tau}$ ($\hat{\tau}^2 = 1$), one can extract symmetric and antisymmetric functions on the total space D2, $Y_\pm = \{\Psi |\ \hat{\tau}\Psi = \pm \Psi\}$, satisfying the equivariance condition (4). It is evident there is no gauge transformation ($\alpha \in Aut_V(D2)$) from any $\Psi_+ \in Y_+$ to $\Psi_- \in Y_-$, since in opposite case it should mean vanishing of $\alpha^{-1}(X)$ on whole set of stationary points

$$M_0 = \bigcup_{(a,a) \in \mathrm{diag}(R^3 \times R^3)} (a,a) \times \mathrm{diag}((R^3 \setminus \{a\}) \times (R^3 \setminus \{a\})) \subset M2$$

of involution τ on the base M2.

Summarizing all mentioned above, we can assert that "prons" to gauge equivalence [1] of two descriptions of dyon as boson and as fermion are not right in principle (and so there is no contradiction to the connection spin and statistics). The structure of non-trivial fibre bundle is crucial here, it affects the definition of two dyons transposition operator.

Another contrargument to "the dyon is fermion" results from Zeeman effect in a weak field for the system formed by spinless charge and spinless monopole [6]. The splitting for such dyon differs significantly from the splitting of levels for ordinary fermion.

3. ZEEMAN EFFECT IN A CHARGE - DIRAC MONOPOLE SYSTEM

The dynamical integral of motion $\hat{\vec{J}}$ of spinless charged particle in the Dirac monopole field is most simple in the total space of of fiber bundle $P = C^2\backslash\{0\}$ (with the base $R^3\backslash\{0\}$ and fibre U(1)) on to which one can transport the dynamics of particle [7,8,9]. The corresponding to this integral are generators of $SU(2) \subset \mathrm{Aut}P$

$$\hat{J}_i = \frac{1}{2}(\bar{Z}\sigma_i\bar{\partial} - \partial\sigma_i Z)\ , \quad i = 1,2,3\ . \qquad (7)$$

In the formula (7) $(Z^1,Z^2) \in C^2\backslash\{0\}$, $\bar{Z}\sigma_i Z = X_i$ - the projection p on the base $R^3\backslash\{0\}$,

$$[\hat{J}_i,\hat{J}_j] = i\varepsilon_{ijk}\hat{J}_k\ , \quad [\hat{J}_i,H_0] = 0\ ,$$

where

$$H_0 = -\frac{1}{2M\bar{Z}Z}\left(\partial\bar{\partial} + \frac{n^2}{4\bar{Z}Z}\right)$$

is the Hamiltonian [7] of a spinless charged particle in the Dirac monopole field with $n = 2e\mu$. With the aid of integral of motion (7) one can separate angular variables (determined on three-dimentional sphere S^3) from the radial variable $r = \bar{Z}Z$ [10]. Angular part of wave function on the total space P is element of rotation matrix

$$D^{j}_{-n/2\ m}(\varsigma),\ \varsigma \quad S^3;\ m = -j,\ -j+1,\ \dots,\ j\ ;$$

$j = n/2,\ n/2+1,\ n/2+2, \dots$ (suppose $n > 0$).

The generators $\hat{J}_i$ (7) can be written in other form

$$\hat{J}_i = \varepsilon_{ijk} \bar{Z}\sigma_j Z \frac{1}{i} h_k - \frac{\bar{Z}\sigma_i Z}{\bar{Z}Z} \frac{V}{2}, \qquad (8)$$

where operators h_i and V are defined (since monopole connection is $\omega = (\bar{Z}dZ - Zd\bar{Z})/2\bar{Z}Z)$ by conditions $ph_i = \partial_i = \partial/\partial X_i$, $\omega(h_i) = 0$, $\omega(iV) = i$:

$$h_i = \frac{1}{2\bar{Z}Z}(\bar{Z}\sigma_i\bar{\partial} + \partial\sigma_i Z),\ V = Z\partial - \bar{Z}\bar{\partial}, \qquad (9)$$

$\partial_\alpha = \partial/\partial Z_\alpha$, $\bar{\partial}_\alpha = \partial/\partial\bar{Z}_\alpha$.

In local description on the base $R^3\backslash\{0\}$ the generators $\hat{J}_i$ are represented by locally defined operators J_i acting on local sections $\psi(X)$ corresponding to functions $\Psi_n(Z,\bar{Z})$ on the total space P:

$$J_i = \frac{1}{i}\varepsilon_{ijk} X_j(\partial_k - inA_k(X)) - \frac{X_i}{X}\frac{n}{2}, \qquad (10)$$

$[J_i, J_j] = i\varepsilon_{ijk} J_k$, $A_i(X)$ - local potential of Dirac monopole. Operator J_i (10) is well known in Dirac monopole theory. Relation between (8) and (10) has been clarified by Solov'ev [7]. The last term in (10) is often interpreted as internal angular momentum of the system. One concludes from that interpretation that spin n/2 is generated in the system of charge and monopole.

Note, that extra term $-\frac{\bar{Z}\sigma_i Z}{\bar{Z}Z}\frac{V}{2}$ in (8) arise not by chance, it has an algebraic origin caused by topology of fibre bundle P.

When there is some Lie algebra of differential operators of the order 1 on the base $R^3\backslash\{0\}$, for example, spatial symmetries generators algebra, and we want to lift this algebra to total space of bundle, then operators $\partial_i = \partial/\partial X_i$ change by horyzontal operators h_i ($ph_i = \partial_i$) acting on functions $\Psi(Z,\bar{Z})$ of total space P.

Commutation relations $[h_i, h_j] = i\Omega_{ij}V$ ($\Omega = d\omega$ – the curvature of connection ω), in terms of homological algebra, determine extension [11, 12] of differential operators of order 1 Lie algebra Σ on base with the aid of commutative algebra Λ consisting of operators of the form $a(Z,\bar{Z})V$ (here $a(Z,\bar{Z})$ are the functions constant along every fiber: $Va(Z,\bar{Z}) = 0$, and, consequently, they may be considered as functions on the base). Extension is solution of the problem: to find an algera Ξ, which can be included in exact sequence

$$0 \longrightarrow \Lambda \xrightarrow{i} \Xi \xrightarrow{p_*} \Sigma \longrightarrow 0 ,$$

where homomorphism i is inclusion and p_* indused by projection p on to base. If the lift ε maps operator $\xi = \xi_i(X)\partial_i$ to operator $\varepsilon(\xi) = \xi_i(\bar{Z}\sigma_j Z)h_i$, then

$$[\varepsilon(\xi), \varepsilon(\eta)] = \varepsilon([\xi,\eta]) + i\Omega(\xi,\eta)V = \varepsilon([\xi,\eta]) + \alpha^2(\xi,\eta),$$

$\Omega(\xi,\eta) = \Omega_{ij}\xi_i\eta_j$, where $\alpha^2(\xi,\eta) = i\Omega(\xi,\eta)V \in H^2(\Sigma,\Lambda)$ realizes 2-cocycle on Lie algebra Σ with values in Λ. Its closeness $\delta\alpha^2 = 0$ is equivalent to closenesss $d\Omega = 0$ of the curvature form Ω. This extension is nontrivial, since $\alpha^2(\xi,\eta) = i\Omega(\xi,\eta)V \neq (\delta\alpha^1)(\xi,\eta)$, i.e. curvature form Ω of connection ω defined nontrivial element in de Rahm cohomology group $H^2(R^3\backslash\{0\}) = R$. As the result Ξ consists of elements of the form $\xi_i(\bar{Z}\sigma_j Z)h_i + ia(\bar{Z},Z)V$. If we take Lie algebra of Lie group SO(3), given by generators $L_i = -i\varepsilon_{ijk}X_j\partial_k$ in Σ, then by the extension we can map operators L_i to $\hat{J}_i$ (8), which make up finite dimensional subalgebra in Ξ.

For scalar particle, taking into account only linear on field $\vec{H}$ terms, the Hamiltonian H in total space of fibre bundle has the form

$$H = H_0 + \Delta H = -\frac{1}{2M}\left(\frac{1}{\bar{Z}Z}\partial\bar{\partial} + \frac{n^2}{4(\bar{Z}Z)^2}\right) + W - \frac{eH_i}{2M}\left(\frac{\bar{Z}\sigma_i Z}{\bar{Z}Z}\frac{n}{2} + \hat{J}_i\right), \qquad (11)$$

where W is a potential localizing the charged particle. The monopole position is assumed to be fixed. As an instance, for a point dyon at centre $W = -\alpha/\bar{Z}Z$, and discrete spectrum energy levels of nonperturbated hamiltonian $H_0 + W$ are not degenerated on j.

Hence, levels splitting is given by matrix elements

$$\Delta E = -\frac{eH}{2M}\langle njm|\hat{J}_3 + \frac{\bar{Z}\sigma_i Z}{\bar{Z}Z}\frac{n}{2}|njm\rangle, \qquad (12)$$

where

$$|njm\rangle = R_j^{(n)}(\bar{Z}Z)\sqrt{\frac{2j+1}{4\pi}}\, D^j_{-n/2\ m}(\varsigma), \quad \vec{H} = H\vec{e}_3 .$$

Integrating in (12) is over total space having volume element $dV = (\bar{Z}Z/\pi)dZ^1\wedge dZ^2\wedge d\bar{Z}^1\wedge d\bar{Z}^2$. After integrating the product of three D-functions $D^{*j}_{-n/2\ m}\, D^j_{-n/2\ m}\, D^1_{00}$ (here $D^1_{00} = \bar{Z}\sigma_3 Z/\bar{Z}Z$) over 3-dimentional sphere, one obtains

$$\Delta E = -\frac{eHm}{2M}\left(1 - \frac{n^2}{4j(j+1)}\right). \qquad (13)$$

Consider the lowest orbital state with $j = n/2$. Have we true spin, the formula for magnetic sublevels would be

$$\Delta E = -\frac{e}{2M}2mH, \quad -n/2 \leq m \leq n/2. \qquad (14)$$

In our case, as it is seen from (13), we have

$$\Delta E = -\frac{e}{2M}mH\left(\frac{n}{2}+1\right)^{-1}. \qquad (15)$$

The difference between (14) and (15) is quite evident, taking into account that operators of dynamical symmetry $\hat{J}_i$ being carried on to base $R^3\setminus(0)$ (i.e. after local trivialization) do not concern the generators for rotations of coordinate system. It show itself, in particular, in the evolution of wave function on account of its being in magnetic field. During the period corresponding to Larmor precession of spin with frequency $\vec{\mu}\vec{H}$ the wave function gets phase factor

$$\exp(\pi i n/(2n + 2)) ,$$

while for true spin n/2 this factor should be

$$(-1)^n .$$

(Remember, that the latter was used in the experimental check of neutron spin 1/2).

Thus, it is clear that dynamical symmetry group generators (7) can not be interpreted as spin operators. The spin operators may be defined by $\Psi'(Z') = S(\hat{g})\Psi(Z)$ at automorphism $\hat{g} \in SU(2) \subset AutP$, $\hat{g}: Z \longrightarrow Z'$(with generators $\hat{J}_i$), of bundle P covering a rotation $g: X \longrightarrow X'$ by the projection p, in our case s = 0.

4. ZEEMAN EFFECT FOR T'HOOFT-POLAKOV MONOPOLE INTERACTING WITH SCALAR ISODOUBLET

The work [1] was stimulated by papers of Jackiw and Rebby [13] and of Hassenfratz and t'Hooft [14], which are devoted to spin generation problem for t'Hooft-Polyakov monopole interacting with scalar isotopical doublet. In [13,14] and many papers after one asserts that though starting fields are bosonic, the half-integer spin arises in this system through existence of integral

$$\vec{J} = \vec{L} + \vec{T}, \tag{16}$$

where $\vec{L} = [\vec{r} \times \vec{p}]$ and $\vec{T} = \vec{\sigma}/2$ - isospin operator.

That is why interesting is (on our point) to examine how this system behaves itself in a weak homogenious magnetic field. It is shown in this section, that the splitting of levels differs from that for the particle having true spin.

It is convenient to study this system in ordinary space, since relevant fibre bundle is trivial.

In a weak homogenious magnetic field the Hamiltonian

$$H = \frac{1}{2M}[\vec{p} - e\vec{A}^{a} T_{a}] + W,$$

where $eA^{a}_{j} = \varepsilon_{ajk}[(1 - h(r))/r^{2}]X_{k}$ – t'Hooft-Polyakov monopole potential gets extra terms

$$\Delta H = -\frac{eH}{2M}((T_{3} + \frac{1}{2})L_{3} + \frac{1}{2}(1 - h(r))\cdot$$

$$\cdot[(T_{3} + \frac{1}{2})\sin^{2}\theta/2 - \frac{1}{2}\cos\theta(T_{+}(n_{1} - in_{2}) + T_{-}(n_{1} + in_{2}))]\}, \quad (17)$$

$\vec{n} = \vec{r}/r$, $T_{\pm} = T_{1} \pm T_{2}$,

which leads to the splitting level

$$\Delta E = -\frac{eH}{2M}\,\frac{\ell + m + 1/2}{2\ell + 1}\,(m - \frac{1}{2} + \frac{1 - \overline{h(r)}}{4(\ell - 1/2)(\ell + 3/2)}\cdot$$

$$\cdot[(\ell - \frac{1}{2})(\ell + \frac{3}{2}) + m^{2} + m(\ell + m - \frac{1}{2})]), \quad (18)$$

$j = \ell \pm 1/2.$

Here $\overline{h(r)}$ – mean value with radialfunction $R_{j\ell}(r)$ arises. When extra potential $W(r)$ localizes the particle at distance $a >> 1/\mu$ of monopole size ($h(r) \sim Ar\exp(-\mu r)$, when $r \longrightarrow \infty$), then we may ignore $\overline{h(r)}$ and splitting takes the form

$$\Delta E = -\frac{eH}{2M}\,\frac{\ell + m + 1/2}{2\ell + 1}\,(m - \frac{1}{4} + \frac{1}{4}\,\frac{m^{2} + m(\ell + m - 1/2)}{(\ell - 1/2)(\ell + 3/2)}), \quad (19)$$

$j = \ell \pm 1/2.$

Splitting ΔE is nonlinear on number m. Thus, Zeeman effect is quite anomalous in arbitrarily weak field.

It is worth to ephasize that splitting of energy levels in a weak field for the dyons considered above differs significantly from the the splitting of levels for ordinary fermion. We may conclude that spin is not generated in the system of charge and monopole.

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