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Λ_{μ} programmes at CERN

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Contents

Λ_μ PROGRAMMES AT CERN.

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1. Introduction.

A series of FORTRAN programmes (~ 40 subroutines) have been written at CERN in connection with the analysis of Λ_μ candidates in the T8 experiment (Interaction of 1.5 GeV/c K^- in the Ecole Polytechnique heavy liquid chamber).

A V^0 is noted as a Λ_μ candidate ($\Lambda \rightarrow P + \mu^- + \bar{\nu}$) if

a) the negative particle stops and has an electron at its end,
b) there is a possible origin of the V^0 in the chamber
c) There is no kink along the negative track greater than in advance established momentum dependent values (Six' criteria; e.g.(4^0 for P_μ = 207 MeV/c),(5^0, 160 MeV/c),(10^0, 79 MeV/c),(15^0, 56 MeV/c)).

The last condition eliminates about 87% of all normal lambda decays ($\Lambda \rightarrow P + \pi^-$) where the pion decays in flight. It leaves, however, those normal lambda decays, where the muon is emitted much forward or backward in the CM system of the pion. The total number of Λ_μ candidates, defined in the way above, is about 400 in all the collaboration. 75 % of the events have been analysed by the programmes described below and the remaining 25 % have been analysed by a slightly different method used in University College, London.

Most of the Λ_μ candidates are expected to be normal lambda decays.

In the first series of programmes we investigate if an observed Λ_μ candidate can be explained as a normal lambda decay (Section 2).

It is without any hesitation necessary to investigate how effective these testprogrammes are in recognizing normal lambda decays, where the "true" values of the measurable quantities have been subject to changes owing to different types of measurement errors, multiple scattering etc.

We have written a Monte Carlo programme, which generates normal lambda decays (Section 3). The momenta and angles of the decay particles are changed in such a way, that the new events are hoped to simulate "measured candidates". The efficiency of the testprogrammes is investigated by means of those generated events.

The artificial lambda decays are also useful when investigating some types of possible background events, i.e. normal lambda decays which by an accident happen to simulate Λ_μ events. In section 6 we estimate the background due to normal lambda decays where the pion is scattered very near the production point.

Some of the true Λ_μ events (if we have any) will of course also be eliminated by the test programmes because they can be explained as normal lambda decays. The detection efficiency of Λ_μ events has therefore to be determined. To be able to do this we have written a Monte Carlo programme, which generates true Λ_μ candidates (Section 4). These are sent through our test programmes.

2. Test programmes for Λ_μ candidates.

A Λ_μ candidate (measured or generated) is sent through two different systems of test programmes.

In the first one we assume that the lambda direction is known and in the second one we investigate the lambda without using the known direction. We can then compare the efficiency of the two methods.

We make two further assumptions in the analysis. First we assume that the pion decays after a range, which is greater than 5 mm. This means that the measured direction of the negative track is identical with the direction of the pion. Afterwards we assume that the pion decays immediately at the production point, which means that the negative track is due to the muon from the pion decay.

Before we start to describe the programmes we want to make clear two other things, namely the input quantities and the method to calculate errors in derived quantities.

2.1. Input quantities.

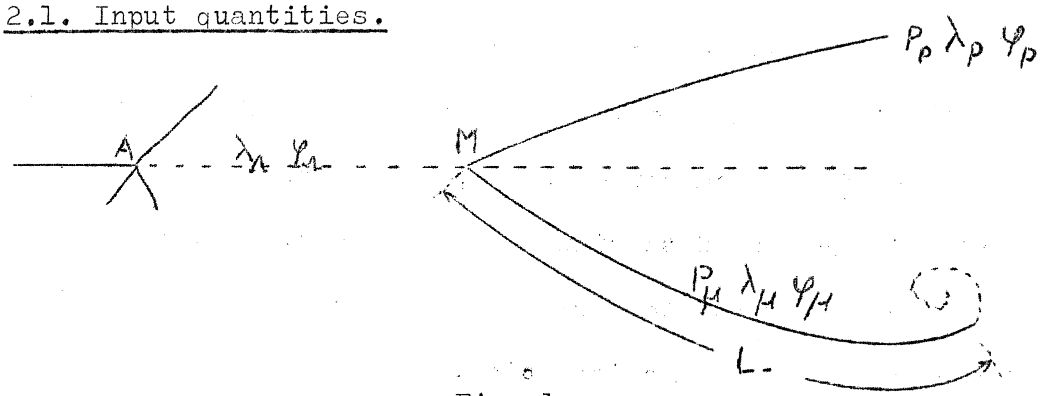

Fig. 1

The following 8 quantities (with errors) are assumed to be known for each event:

$\lambda_\Lambda,\ \varphi_\Lambda,\ P_p,\ \lambda_p,\ \varphi_p,\ P_\mu,\ \lambda_\mu,\ \varphi_\mu,$

where λ and φ are dip and azimuth angles respectively. P_μ is calculated from L_- (Fig. 1) assuming that the muon starts in M.

One programme (DIPFI) calculates λ_Λ and φ_Λ with errors in case the coordinates of A and M are given.

$$\operatorname{tg}\lambda_\Lambda = (z_M-z_A)/\sqrt{(x_M-x_A)^2+(y_M-y_A)^2}$$

$$\operatorname{tg}\varphi_\Lambda = (y_M-y_A)/(x_M-x_A) \qquad (1)$$

$$\sigma_{\lambda_\Lambda} = \frac{\cos^2\lambda_\Lambda\sqrt{\operatorname{tg}^2\lambda_\Lambda\left[(x_M-x_A)^2(\sigma^2_{x_M}+\sigma^2_{x_A})+(y_M-y_A)^2(\sigma^2_{y_M}+\sigma^2_{y_A})\right]+\sigma^2_{z_M}+\sigma^2_{z_A}}}{\sqrt{(x_M-x_A)^2+(y_M-y_A)^2}}$$

$$\sigma_{\varphi_\Lambda} = \frac{\cos^2\varphi_\Lambda\sqrt{\sigma^2_{y_M}+\sigma^2_{y_A}+(\sigma^2_{x_M}+\sigma^2_{x_A})\operatorname{tg}^2\varphi_\Lambda}}{x_M-x_A}$$

If λ_Λ and φ_Λ can not be derived, we test the hypotheses described below, where the lambda direction is assumed to be unknown (Tests 3 and 4).

2.2. Calculation of errors.

In most cases we want to find out how many standard deviations an event disagrees from the explanation normal lambda decay. The event might be measured or generated. We have, however, the 8 "measured" quantities (m_i) with errors (σ_i). All wanted quantities (y_j) are calculated. We then vary all measured variables with a Monte Carlo programme (RANDOM) assuming that every one is normally distributed around the measured value with the given error. We replace m_i by $m_i + a_i \sigma_i$, where a_i is taken from a table of random normal standard deviations stored in RANDOM.

Eight new sets of m_i-values are generated (always starting from the measured quantities) and all y_j-values are calculated for each set. The error of each y_j is derived from the distribution of the 8 new values around the first calculated one.

We do not allow errors in derived momenta to be smaller than 2 MeV and errors in derived angles to be smaller than 1° (programme ERROR).

2.3. Test of four hypotheses for normal lambda decay.

The four hypotheses come of course from combinations of the assumption that the lambda direction is either known or unknown, with the assumption that the pion decays directly at the production or after a range which is greater than 5 mm.

Test 1. Assumptions: 1) Lambda direction known.

2) The pion decays after more than 5 mm.

In this case (Fig. 2) we know that the measured direction of the negative track is identical with the direction of the pion.

We calculate and test the following four quantities:

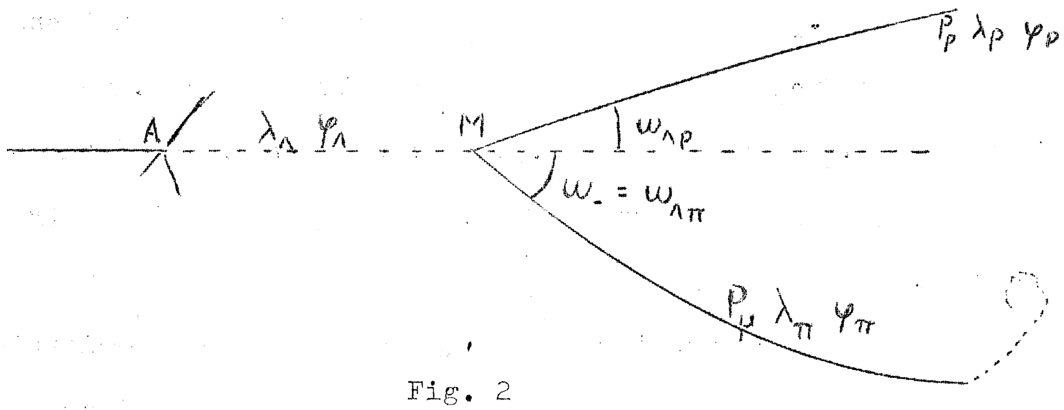

Fig. 2

α) The transverse momentum of the proton.

This should be less than 100 MeV/c for a normal lambda decay. If the observed transverse momentum exceeds the limit, we denote the excess, in units of standard deviations, by D_α.

β) The dip (β^-) of the negative track relative the lambda-proton plane.

The three tracks have to be coplanar for a normal lambda decay and we therefore expect β^- to be zero. The number of standard deviations from zero is denoted D_β. The dip is calculated from the formula:

$$\sin \beta^- = \frac{\begin{vmatrix} l_\Lambda & m_\Lambda & n_\Lambda \\ l_p & m_p & n_p \\ l_\pi & m_\pi & n_\pi \end{vmatrix}}{\sqrt{1 - (l_\Lambda l_p + m_\Lambda m_p + n_\Lambda n_p)^2}} \qquad (2)$$

where $l_i = \cos \lambda_i \cos \varphi_i$

$m_i = \cos \lambda_i \sin \varphi_i$

$n_i = \sin \lambda_i$

and the lambda direction is taken from M to A.

γ) The angle ($\omega_{\Lambda\pi}$) between the lambda and the pion.

The space angles between the lambda and each of the two tracks ($\omega_{\Lambda p}$ and ω_-) are calculated (programme SPACEV)

$$\cos \omega_{ij} = \cos(\varphi_i - \varphi_j)\cos \lambda_i \cos \lambda_j + \sin \lambda_i \sin \lambda_j \qquad (3)$$

Using the measured proton momentum and the calculated angle $\omega_{\Lambda p}$ we calculate the direction of the pion from a normal lambda decay (programme PION).

$$\cot \omega_{\Lambda\pi} = \frac{P_p \cos \omega_{\Lambda p} (M-2A) \pm E_p \sqrt{M^2 - 4M_\Lambda^2 A}}{2P_p A \sin \omega_{\Lambda p}} \qquad (4)$$

where $M = M_\Lambda^2 + M_p^2 - M_\pi^2$

$A = M_p^2 + P_p^2 \sin^2 \omega_{\Lambda p}$

We expect ω_- to be equal to one of the two solutions $\omega_{\Lambda\pi}$. The difference between ω_- and each of the two $\omega_{\Lambda\pi}$ (in units of standard deviations) is denoted as $D_{\gamma 1}$ and $D_{\gamma 2}$. The smallest one of these two is denoted D_γ.

δ) The momentum of the muon.

The momentum of the pion is calculated from the relation (programme PION).

$$P_\pi = \frac{P_p \sin \omega_{\Lambda p}}{\sin \omega_-} \qquad (5)$$

By letting the pion decay into a muon directly at angles of 0° and 180° we find (programme MUON) the maximum and minimum possible momentum of the muon.

$$(P_\mu)_{\substack{max \\ min}} = \frac{(M_\pi^2 + M_\mu^2) P_\pi \pm (M_\pi^2 - M_\mu^2) E_\mu}{2 M_\pi^2} \qquad (6)$$

If the measured muon momentum falls outside either limit, we calculate the distance to the nearest limit (D_δ standard deviations).

A tested event is in good agreement with the hypothesis normal lambda decay if all the quantities D_α, D_β, D_γ and D_δ are small.

We denote by D_1 the greatest one of these D-values.

Test 2. Assumptions: 1) Lambda direction known.

2) The pion decays directly.

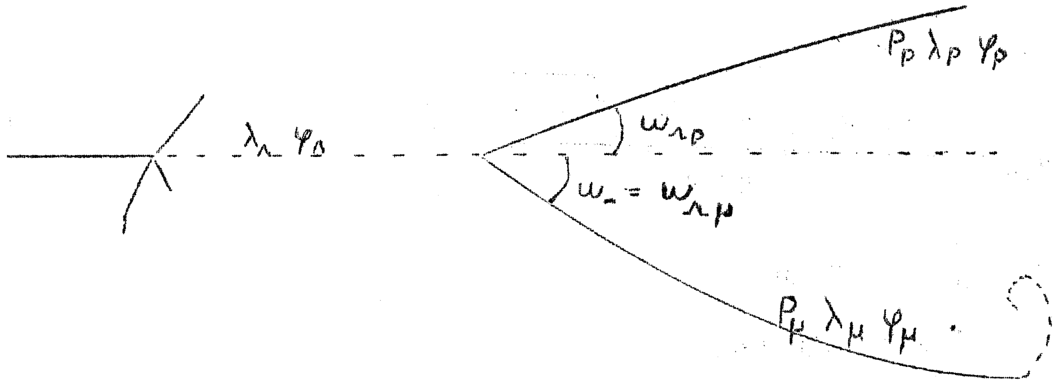

Fig. 3

The direction of the negative track is now identical with the direction of the muon. We do not observe the pion.

We test the following quantities:

α) The transverse momentum of the proton.

This is done as in Test 1, with the result D_α.

β) The space angle ($\omega_{\Lambda\mu}$) between the lambda and the muon.

This angle can be calculated from the measured proton data, the momentum of the muon and the dip angle (β^-) of the muon relative the lambda-proton plane (Eq. (2)). We have done it in the following steps.

The pion direction is calculated from Eq.(4), and the pion momentum is derived from Eq.(5), where ω_- is now replaced by $\omega_{\Lambda\pi}$. We have two pion solutions ($P_{\pi 1}, \omega_{\Lambda\pi 1}$) and ($P_{\pi 2}, \omega_{\Lambda\pi 2}$).

In Fig. 4 is assumed that the lambda decays in the paper plane. MC is the momentum vector of the derived pion. The muon from the pion decay must have a momentum vector, which starts in M and goes to any point on a momentum ellipsoid, which can be derived, and which is shown in the figure. We want now to fix the endpoint of the muon momentum vector.

As we know the magnitude of the muon momentum, we can cut the

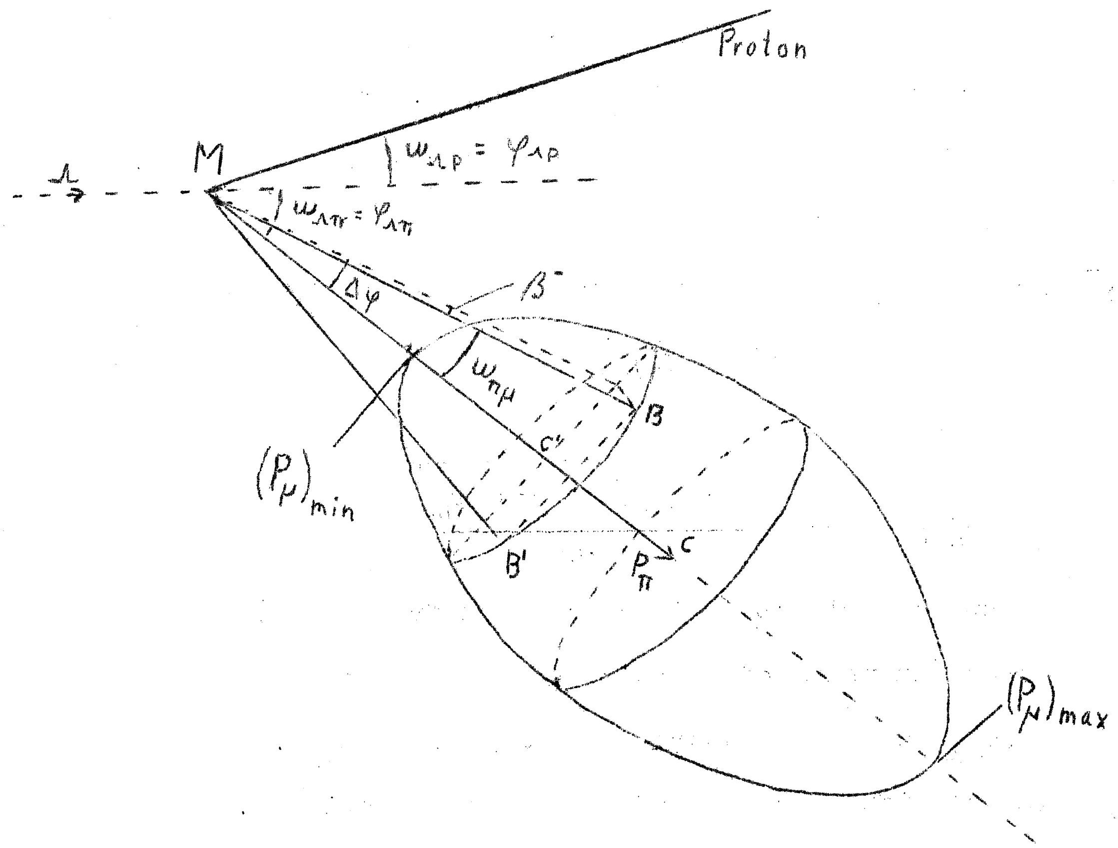

Fig. 4

ellipsoid and we find that the endpoint must be situated at a circle; the one with origin C′ in the figure. This circle is determined from the opening angle ($\omega_{\pi\mu}$) between the pion and the muon.

$$\cos \omega_{\pi\mu} = \frac{2E_\pi E_\mu - M_\pi^2 - M_\mu^2}{2 P_\pi P_\mu} \qquad (7)$$

The magnitude of the muon momentum has, however, to be between the two limits given in Eq.(6). If this condition is not fulfilled, we denote by $D_{\beta 1}$ (pion 1) and $D_{\beta 2}$ (pion 2) how many standard deviations outside the given limits the measured momentum is, and we further change the measured momentum to be equal to the nearest limit before we go on.

We know the dip angle (β^-) between the observed muon and the lambda-proton plane. There are two points on the circle (B and B′)

for which the muon tracks MB and MB′ will form the prescribed dip angle with the lambda decay plane.

In order to get any solution we must, however, have the condition $\beta^- \leq \omega_{\pi\mu}$ fulfilled. If this is not the case for an event, we denote by $D_{\gamma 1}$ (pion 1) and $D_{\gamma 2}$ (pion 2) the number of standard deviations β^- exceeds $\omega_{\pi\mu}$ and we further put β^- equal to $\omega_{\pi\mu}$.

The azimuthal angle ($\varphi_{\Lambda\mu}$) between the lambda and the derived muon is found from

$$\varphi_{\Lambda\mu} = \varphi_{\Lambda\pi} \pm \text{arctg} \frac{\sqrt{\cos^2\beta^- - \cos^2\omega_{\pi\mu}}}{\cos\omega_{\pi\mu}} \qquad (8)$$

and the space angle $\omega_{\Lambda\mu}$ is calculated from Eq.(3). We denote the difference (in units of standard deviations) between the derived angle $\omega_{\Lambda\mu}$ and the measured one ω_- with $D_{\delta 11}$, $D_{\delta 12}$, $D_{\delta 21}$, and $D_{\delta 22}$ (2 solutions for each of the 2 pion solutions).

An event is in good agreement with the tested hypothesis if all the D-values in one of the following four rows are small. (Each row represents one possible decay)

1)	D_α	$D_{\beta 1}$	$D_{\gamma 1}$	$D_{\delta 11}$
2)	D_α	$D_{\beta 1}$	$D_{\gamma 1}$	$D_{\delta 12}$
3)	D_α	$D_{\beta 2}$	$D_{\gamma 2}$	$D_{\delta 21}$
4)	D_α	$D_{\beta 2}$	$D_{\gamma 2}$	$D_{\delta 22}$

We define the best row to be the one which has the lowest maximum D-value, and we denote this maximum value by D_2.

An observed event is compatible with the explanation normal lambda decay if either D_1 or D_2 is small. The smallest one of these are denoted D_{12}. D_{12} is therefore the number of standard deviations with which the best possible solution (out of up to 6 possible) disagrees from the hypothesis normal lambda decay with the direction of the lambda known.

Test 3. Assumptions: 1) Lambda direction unknown.

2) The pion decays after more than 5 mm.

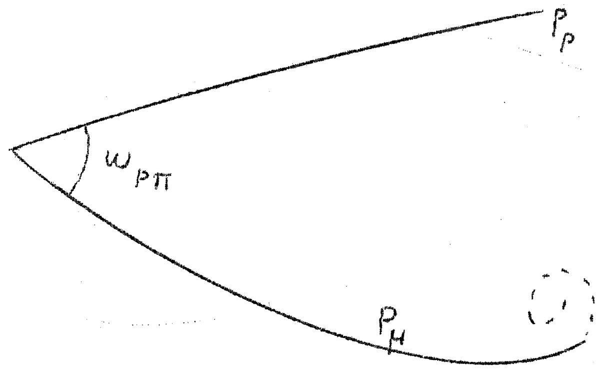

Fig. 5

We know the proton momentum and the angle between the proton and the pion. We want to calculate and test the momentum of the muon.

From the momentum of the proton and the angle $\omega_{p\pi}$ we calculate the momentum of the pion (programme FREELA)

$$P_\pi = \frac{1}{2\,A}\left\{MP_p \cos\omega_{p\pi} \pm E_p\sqrt{M^2 - 4M_\pi^2 A}\right\} \qquad (9)$$

where

$$M = M_\Lambda^2 - M_p^2 - M_\pi^2$$

$$A = M_p^2 + P_p^2 \sin^2\omega_{p\pi}$$

For each P_π we calculate the maximum and minimum momentum of the muon (Eq.(6)).

We compare the observed muon momentum with the calculated limits. If the observed momentum falls outside the limits for both solutions, we donote by D_3 the number of standard deviations to the nearest limit for the best solution.

Test 4. Assumptions: 1) Lambda direction unknown.

2) The pion decays directly.

We now observe the proton and muon momenta and the angle between these particles. Starting from the two momenta we want to calculate possible values of the opening angle $\omega_{p\mu}$.

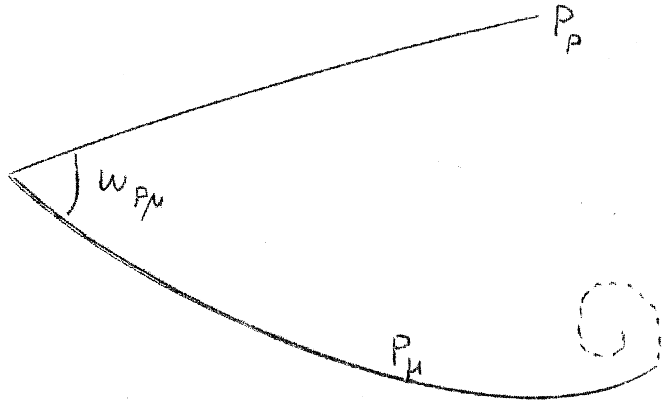

Fig. 6

The maximum and minimum possible value of the pion momentum can be calculated from both P_μ and P_p.

From P_μ we get

$$(P_\pi)_{\substack{max \\ min}} = \frac{1}{2M_\mu^2}\left\{P_\mu(M_\pi^2 + M_\mu^2) \pm E_\mu(M_\pi^2 - M_\mu^2)\right\} \tag{10}$$

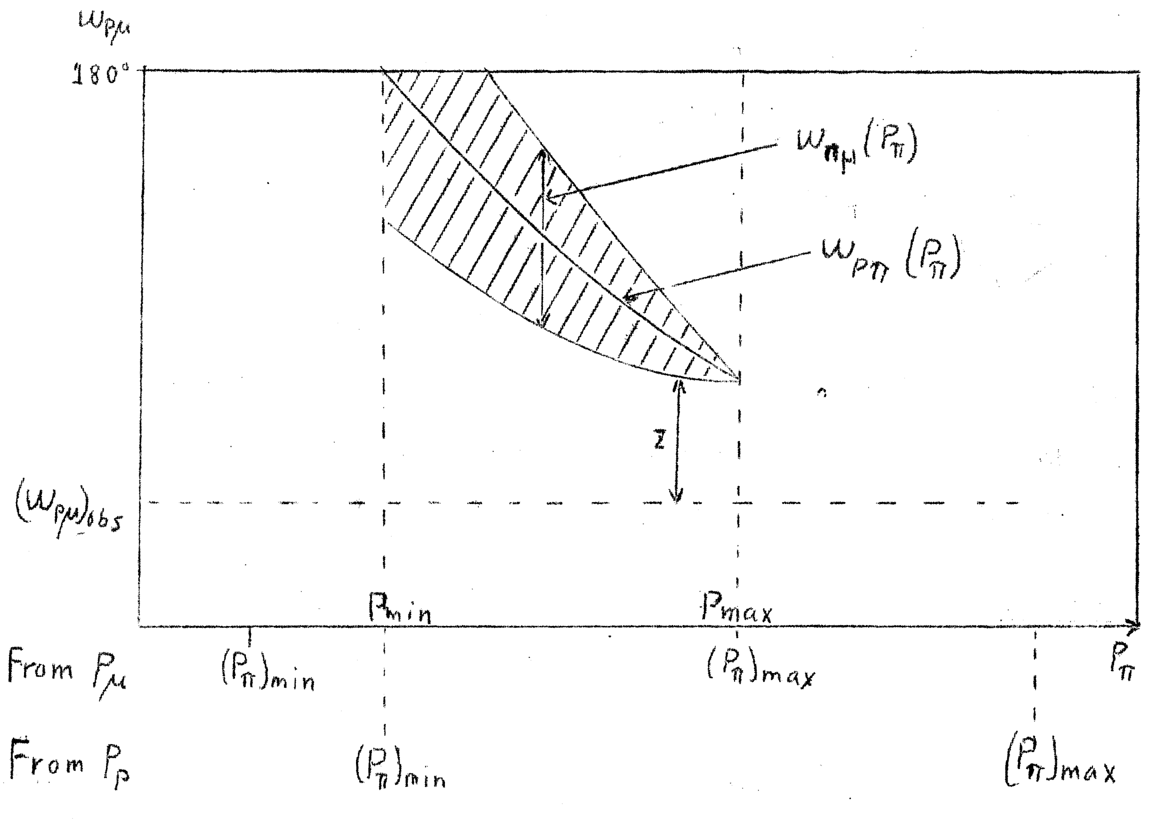

Fig. 7

From P_p we derive

$$(P_\pi)_{\substack{max \\ min}} = \frac{1}{2M_p^2} \left\{ E_p \sqrt{M^2 - 4M_p^2 M_\pi^2} \pm M P_p \right\} \tag{11}$$

where $M = M_\Lambda^2 - M_p^2 - M_\pi^2$

The pion momentum has to be between the greatest $(P_\pi)_{min} = P_{min}$ and the smallest $(P_\pi)_{max} = P_{max}$ (Fig. 7).

The angle $\omega_{p\mu}(P_\pi)$ must be between the limits

$$\omega_{p\mu}(P_\pi)_{\substack{max \\ min}} = \omega_{p\pi}(P_\pi) \pm \omega_{\pi\mu}(P_\pi) \tag{12}$$

where

$$\cos \omega_{p\pi}(P_\pi) = \frac{2E_p E_\pi - M_\Lambda^2 + M_\pi^2 + M_\mu^2}{2P_p P_\pi} \tag{13}$$

and

$$\operatorname{tg} \omega_{\pi\mu}(P_\pi) = \frac{M_\pi^2 - M_\mu^2}{\sqrt{4M_\mu^2 P_\pi^2 - (M_\pi^2 - M_\mu^2)}} \tag{14}$$

We vary P_π between P_{min} and P_{max} and calculate the distance (z) between the observed opening angle and the nearest limit of the kinematically possible values (shaded in Fig. 7).

The smallest distance z (measured in standard deviations) is denoted D_4.

For a normal lambda decay we have either D_3 or D_4 small. We denote the smallest one of these values by D_{34}.

D_{34} is thus the number of standard deviations an event is from the explanation normal lambda decay with the direction of the lambda unknown.

3. Monte Carlo programme giving normal lambda decays.

A few or our rolls have been carefully scanned for all types of lambda decays and all observed lambdas have been measured and sent through GRIND. Momenta and cordinates of apeces for about 100 of these lambdas are used as starting values in our Monte Carlo programmes. We have also used a sample (~ 50) measured in Paris.

We choose an arbitrary angle ($\omega^*_{\Lambda p}$) between the proton and the lambda in the center of momentum system of the lambda (programme ANGLE).

$$\omega^*_{ij} = \arccos \left\{ \frac{99 - 2k}{100} \right\} \tag{15}$$

where k is a random number between 0 and 99; taken from a table in programme RAND.

The transformation from the CM system to the lab. system is done in the programme CMSLAB.

$$\omega_{ij} = \mathrm{arctg} \frac{\beta^*_j \sqrt{1 - \beta^2_{CM}} \sin^2 \omega^*_{ij}}{\beta_{CM} + \beta^*_j \cos \omega^*_{ij}} \tag{16}$$

where the particle i decays to j.

$\beta_{CM} = P_1/E_1$ is the velocity of the decaying particle and

$\beta^*_j = P^*_j/E^*_j$ is the velocity of the decay particle in the CM-system.

The momentum of the decay particle in the lab. system is

$$P_j = \frac{P^*_j}{\sqrt{1 - \beta^2_{CM}}} \; \frac{\beta_{CM} + \beta^*_j \cos \omega^*_{ij}}{\beta^*_j \cos \omega_{ij}} \tag{17}$$

From Eq. (16) and (17) we calculate the direction and the momentum of both the proton and the pion in the lab. system.

This part of the programme has been controlled in the following way. We generated 1000 lambda decays with a constant lambda momentum of 1 GeV/c. Different momentum and angular distributions of the generated events were plotted (Fig. 8) and found to be in excellent agreement with the theoretical distributions.

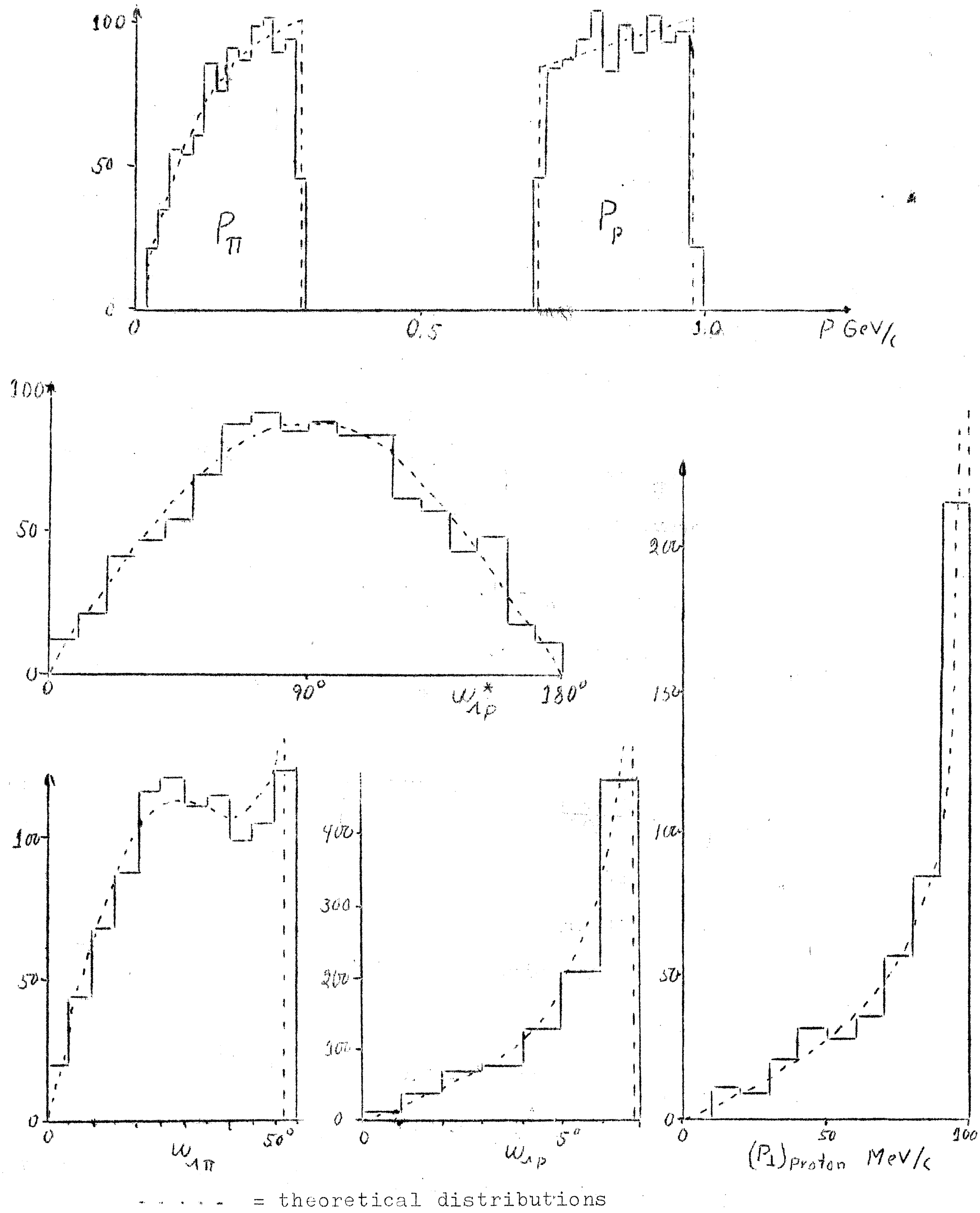

Fig. 8.

3.1. Generation of $\Lambda\pi\mu$ events where the pion decays after more than 5 mm (Type A).

We start from the generated normal lambda decays and let the pion decay in flight to a muon (programme PIDEC). We assume that the pion decays before the last 5 mm of its range. This cut off corresponds to a momentum of 51 MeV/c.

An event where the pion has lower momentum at the decay will always be in good agreement with the hypothesis normal lambda decay as the total length of the pion and the muon will differ very little from the potential path lenth of the pion.

We denote by t_{max} the time it takes (in the CM system) for a pion of momentum P_{π} to come down to the momentum 51 MeV/c. We have the empirical formula:

$$t_{max} = (-680\ P_{\pi}^3 + 420\ P_{\pi}^2 - 1.8\ P_{\pi} - 0.91) \times 10^{-9}\ \text{sec} \qquad (18)$$

where P_{π} is measured in GeV/c.

The pion is assumed to decay after a time $t_{max} - t$ where

$$t = -\tau \ln \left\{ 1 - \frac{2k+1}{200} \left(1 - e^{-\frac{t_{max}}{\tau}}\right) \right\} \qquad (19)$$

where τ is the lifetime of the pion and

k is an arbitrary number between 0 and 99.

The momentum of the pion at the decay is obtained from the empirical formula

$$(P_{\pi})_d = 0.0000393(t \times 10^9)^3 - 0.00123(t \times 10^9)^2 + 0.0230\ (t \times 10^9) + 0.051 \qquad (20)$$

The Six's criteria, which eliminates all pion-muon decays where the laboratory angle between the two particles is great, have to be fulfilled. This means, as a good approximation at our energies, that the muon has to be emitted at an CM-angle ($\omega^{*}_{\pi\mu}$) which is smaller than 39° or bigger than 167° relative to the direction of the pion. We choose the angle in the following way.

$$\omega^{*}_{\pi\mu} = \arccos\left\{ -1 - \frac{2(k-90)+1}{20} \cos 167 \right\} \qquad k = 90 - 99$$

$$= \arccos\left\{ \frac{2k+1}{180} \cos 39 \right\} \qquad k = 0 - 89 \qquad (21)$$

where k is an arbitrary number between 0 and 99.

The momentum of the muon in the laboratory system is now calculated by help of Eq. (17).

We are, however, not interested in the real muon momentum. We must in stead know what muon momentum corresponds to a track length which is equal to the sum of the muon track length (R_{μ}) and the distance (ΔR_{π}) the pion has passed before decaying.

We have one programme (RANGE) which calculates the range for a given momentum and vice versa. From this programme we find $R_{\mu} + \Delta R_{\pi}$ and also the apparent muon momentum corresponding to this track length.

We have now created a real $\Lambda\pi\mu$ event. The 8 input quantities (m_i) mentioned in Section 2.1. have been derived. We want to transform these variables to "experimental" values. We take the errors (σ_i) corresponding to m_i from one, arbitrary chosen, of our measured events.

We replace the 8 m_i values by $m_i + a_i \sigma_i$ where the a_i:s are random normal standard deviations stored in RANDOM.

The new set of m_i values is assumed to simulate measured quantities.

We require for a $\Lambda\mu$ candidate that the negative particle stops in the chamber. We have therefore to eliminate those of our created events which do not fulfil this condition.

A generated $\Lambda\pi\mu$ event is located in a plane (xy) which is parallell to the bottom of the chamber. The x-direction is parallell to the longest side of the chamber. A programme named ROT rotates the event an arbitrary angle ($\theta = \frac{2k+1}{100}\pi$; k=0...99) around the x-axis.

From the known coordinates of the lambda apex, the direction of the muon track, and the end coordinates of the chamber we calculate (programme RMA) the maximum observable muon path length. This is compared with the corresponding path length of the generated negative track (programme OUT). The calculations above are also performed for the proton.

If a proton leaves the chamber we have to increase the error in the "measured" momentum. This is done in POUT. In this programme is stored a table containing percentage errors in observed measured proton momenta for events where the proton has left the chamber. As the proton momentum now has a big error, we also have to change the "true" generated momentum to get a new "measured" momentum. This is done in the way described above.

3.2. Generation of $\Lambda\pi\mu$ events where the pion decays directly (Type B).

We start from the generated normal lambda decays. The muon from the pion decay can now be emitted in all directions in the CM system of the pion. We find the angle $\omega^*_{\pi\mu}$ from Eq. (15) and can calculate the muon direction and momentum in the laboratory system from Eq. (16) and (17).

Exactly in the same way as above (Section 3.1.) we transform a generated real event to a measured event. We also control if the proton and the muon tracks stop in the chamber and change the error and the value of the proton momentum if the proton leaves the chamber.

4. Monte Carlo programme giving Λ_μ decays.

We use the momenta and the apex coordinates for the unbiased sample of observed lambdas mentioned in Section 3 as starting values when generating Λ_μ events.

We have to make some theoretical assumptions:

a) The momentum distribution of the muon is obtained from the available non-invariant phase space (M.M. Bloch; Phys.Rev. 101, 796 (1956)).

$$d\rho_\mu \propto P_\mu^2(1-A)^2\left[3(M_\Lambda - E_\mu)^2(1+A) - P_\mu^2(1-A)\right] dP_\mu \qquad (22)$$

where

$$A = \frac{M_p^2}{(M_\Lambda - E_\mu)^2 - P_\mu^2}$$

and E_μ is the total energy of the muon.

In Fig. 9 we give the theoretical momentum distribution and the momentum distribution of 1000 generated muons (programme MUMOM).

b) The distribution of the space angle ($\omega^*_{\Lambda\mu}$) between the muon and the direction of the lambda is assumed to be isotropic in the CM system of the lambda. This is true because the lambda has spin 1/2 and is not longitudinally polarized. The angle $\omega^*_{\Lambda\mu}$ is obtained from Eq. (15).

c) The distribution of the space angle ($\omega^*_{\mu\nu}$) between the muon and the $\bar{\nu}$ is assumed to be isotropic in the CM system of the lambda. This is not true, but it is a good approximation to e.g. the V-A distribution. ($\omega^*_{\mu\nu}$ from Eq. (15))

d) The distribution of the azimuthal angle (Θ^*_ν) between the muon and the $\bar{\nu}$ is isotropic in the CM system of the lambda. This is true since the lambda is unpolarized. ($\Theta^*_\nu = \frac{2k+1}{100}\pi$; k=0...99)

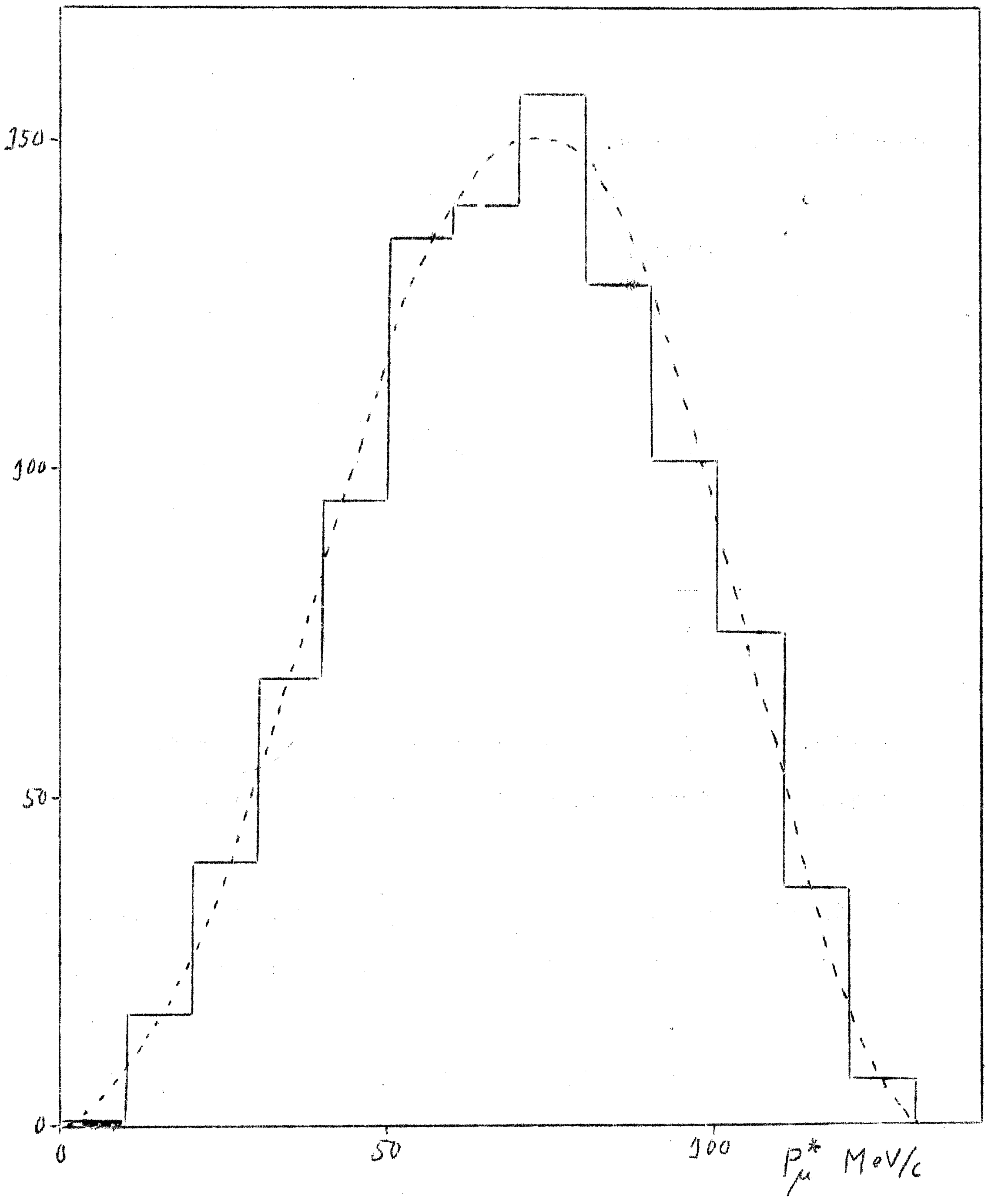

Fig. 9

The assumptions above perform us with data for the following variables: P^*_μ, $\omega^*_{\nu\mu}$, $\omega^*_{\mu\nu}$, and Θ^*_ν.

We can then calculate all other needed quantities.

The total energy (E^*_p) of the proton in the CM system is found from the relation:

$$E_p^* = \frac{P_\mu^{*2} + (M_\Lambda - E_\mu^*)^2 + M_p^2 + 2P_\mu^*(M_\Lambda - E_\mu^*) \cos \omega_{\mu\nu}^*}{2(M_\Lambda - E_\mu^* + P_\mu^* \cos \omega_{\mu\nu}^*)} \qquad (23)$$

The momentum of the neutrino is

$$P_\nu^* = M_\Lambda - E_p^* - E_\mu^* \qquad (24)$$

The angle ($\omega_{p\mu}^*$) between the proton and the muon in the CM system is obtained from the formula:

$$\omega_{p\mu}^* = \arccos \left\{ - \frac{P_p^{*2} + P_\mu^{*2} - P_\nu^{*2}}{2P_p^* P_\mu^*} \right\} \qquad (25)$$

The dip angle (λ_p^*) of the proton relative to the $\Lambda\mu$ plane is given by:

$$\lambda_p^* = \text{arctg} \left\{ - \frac{A \sin \Theta_\nu^*}{\sqrt{P_\mu^{*2} + A^2 \cos^2 \Theta_\nu^*}} \right\} \qquad (26)$$

where

$$A = \frac{P_\mu^* P_\nu^* \sin \omega_{\mu\nu}^*}{P_\mu^* + P_\nu^* \cos \omega_{\mu\nu}^*}$$

The plane angle ($\Theta_{\Lambda p}^*$) between the lambda and the proton in the CM system is found from

$$\Theta_{\Lambda p}^* = \omega_{\Lambda\mu}^* - \text{arctg} \left[\frac{A \cos \Theta_\nu^*}{P_\mu^*} \right] - \pi \qquad (27)$$

We can further calculate the space angle ($\omega_{\Lambda p}^*$) between the lambda and the proton from Eq. (3).

The momentum and direction of the muon (P_μ ; $\omega_{\Lambda\mu}$) and the proton (P_p ; $\omega_{\Lambda p}$) in the laboratory system can be calculated from the programme CMSLAB; Eq. (16) and (17) above.

These true $\Lambda\mu$ events can be transformed to observed events by the method described in Section 3.1. We finally select those events where the muon (and proton) stop inside the chamber, and we increase the error and change the momentum for protons leaving the chamber.

5. Results from Monte Carlo and test programmes.

5.1. Number of events and general statistics.

The following numbers of $\Lambda\mu$ candidates have been analysed by the programmes described.

Laboratory	Number of events with proton stopping	leaving or interact.	Number of events per laboratory
Bergen	47	18	65
CERN	82	33	115
Paris	78	40	118
Total numbers	207	91	298

Table 1.

We have generated about 5000 Monte Carlo events of each kind ($\Lambda\pi\mu$ of type A and B and $\Lambda\mu$). Some of the results below are, however, not received from this total number, and we will therefore for each investigation note the actual number of events.

From most of the generated events we find that the following fractions of particles stop inside the chamber.

Type of event	Percentage of particles stopping Muon	Proton	Both
$\Lambda\pi\mu$ type A	84.8	85.9	75.6
$\Lambda\pi\mu$ type B	91.1	84.3	78.5
$\Lambda\mu$	95.6	85.8	83.8

Table 2.

From the $\Lambda\pi\mu$ events of type A we further note

a) that 14.5 % of pions, which decay before the last 5 mm of their their potential path lengths, decay already before they have passed a range of 5 mm.

b) That about 4 % of all generated pions have a momentum which is less than 50 MeV/c (range less than 5 mm).

Some other remarcs:

1. Fraction of type A and type B events in our experimental sample.

We can now estimate how big fraction of our observed candidates are type A or type B events. Let us assume:

α) Measurements on the negative track gives the direction of the pion (muon) if the pion decays after (before) a range of 5 mm.

β) The Six criteria (elimination of pion decays (87 %) where the space angle pion-muon is great) can be applied down to a pion range of 3 mm.

We then have

$$\frac{\text{Number of type A events}}{\text{Number of type B events}} = \frac{k\, P_{>5}}{k\, P_{3-5} + P_{<3}} = \frac{0.13 \cdot 0.885}{0.13 \cdot 0.4 \cdot 0.145 + 0.6 \cdot 0.145} = 1.09$$

where k is the reduction of events by the Six criteria

P_L is the probability that a pion decays in the range interval L mm.

We conclude that we can expect about the same numbers of type A and type B events in our experimental sample.

This figure may be somewhat wrong from two reasons. Inclusion of pions which decay in the last five millimeters of their potential path lengths would increase the number of type A events. The interaction possiblity of pions would, on the other hand, decrease the number of type A events more than the number of type B events.

As we will find that the background from both types of events is about the same, we find it unnecessary to investigate more in detail how our observed events are divided between the two types.

2. Treatment of events with proton stopping or not stopping.

In about 88 % of our generated $\Lambda\pi\mu$ events with the muon stopping we find that the proton also stops.

In our experimental sample we find that the proton stops in 70 % only of all events. The difference is due to interaction of the proton. This reaction possibility is not included in the programmes. But a proton leaving the chamber or interacting will be subject to the same changes; the error in the momentum will change from 1 or 2 MeV/c to a big fraction of the true momentum. We can then apply the Monte Carlo results from events with proton leaving the chamber also on that fraction of our events where the proton interacts.

We therefore consider events with the proton stopping or leaving the chamber separately in the following analyses.

3. Momentum and angular distributions.

The programmes print out momentum and angular distributions for all particles (CM and laboratory system), both for primary generated events, for events stopping in the chamber and for events which disagree with more than a preset number of standard deviations from the explanation normal lambda decay.

They also print out distributions of D_1, D_2, D_3, D_4, D_{12}, and D_{34} defined in Section 2, and finally complete information of events which disagree by more than a preset number of standard deviations from the explanation normal lambda decay.

4. Overall control of the programmes.

All subroutines have been separately tested as far as possible. An overall test of the complete system of programmes is obtained in the following way.

That subroutine (RANDOM), which transforms the exact $\Lambda\pi\mu$ events (type A and B) into experimental events, is taken away. When sending the exact type A events through our test programmes we find, as expected, that all events are in complete agreement with tests 1 and 3 but not with 2 and 4. All events of type B are, on the other hand, in excellent agreement with tests 2 and 4 but disagree sometimes with the other two tests. This overall test has been performed on a sample of about 300 events of each type.

5.2. Lambda direction used (Tests 1 and 2).

The distributions of D_{12} for the three Monte Carlo programmes (proton stopping) and for all observed events (Bergen + CERN) are given in Fig. 10 and in Table 3.

Type	Number of events	Percentage of events D_{12} standard deviations					
		0-1	1-2	2-3	3-4	4-5	>5
$\Lambda\pi\mu$ type A	2814	61.7	25.5	7.9	1.9	1.1	1.9
$\Lambda\pi\mu$ type B	3505	58.4	27.1	8.6	2.6	1.3	2.0
$\Lambda\mu$	1688	35.0	28.4	14.5	6.8	5.5	9.8
Observed events	182	73.4	18.1	4.4	1.7	0.6	1.7

Table 3.

D_{12} is mainly the greatest number of standard deviations out of 2 or 3, which are supposed to be normally distributed. We therefore expect the D_{12} distributions for $\Lambda\pi\mu$ events to be broader then the normal distributions. We should for example have 10-15 % of our events outside 2 standard deviations in comparison with 5 % for a normal distribution. This is in agreement with the observed data.

But it is in disagreement, with what we expect, to have events outside 4 standard deviations. We have there about 3 % (= 200 events)

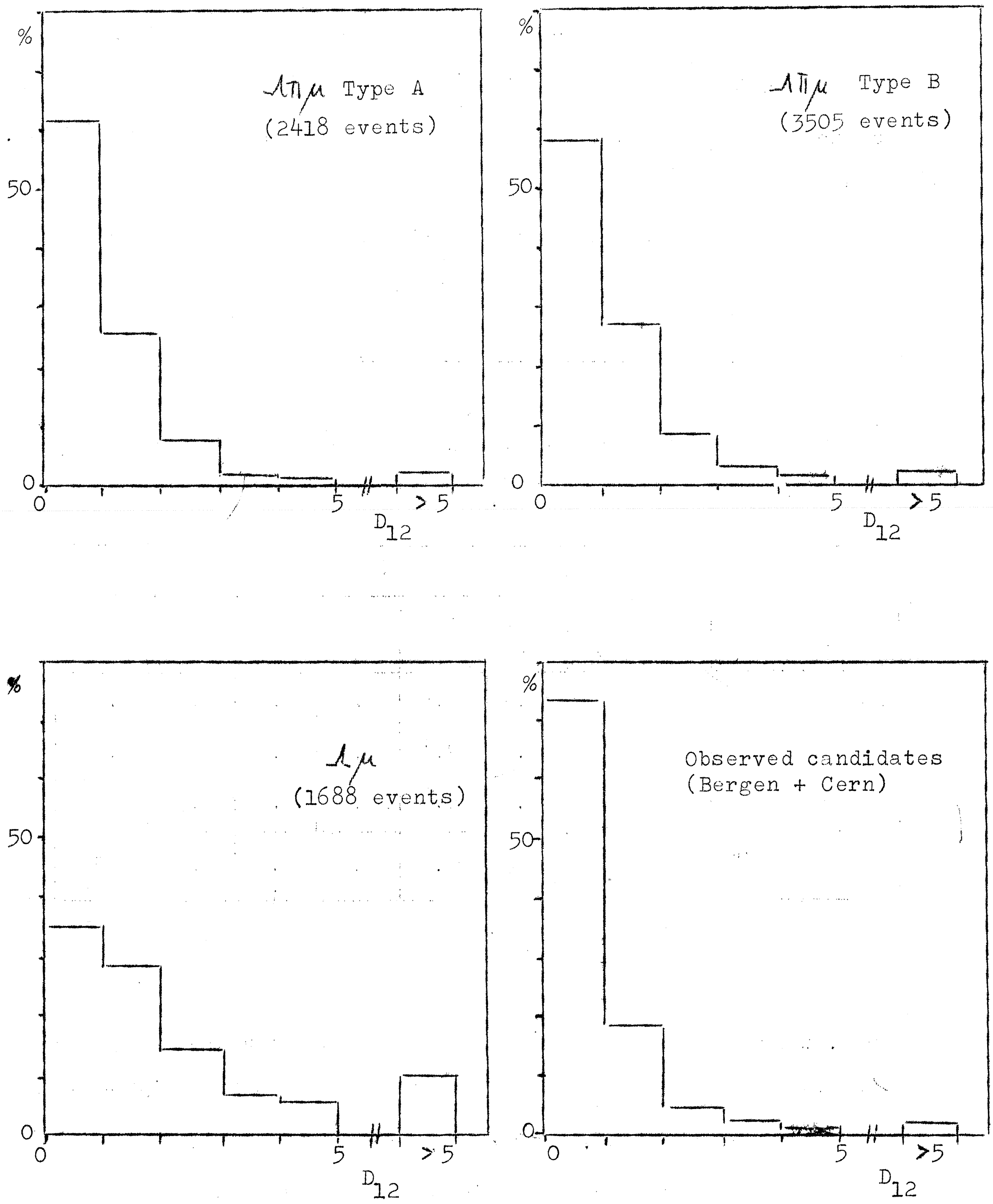

Fig. 10.

Distributions of D_{12} for events where the proton stops.

of our $\Lambda\pi\mu$ sample. The reason for this is not quite understood. It may be due to that some of our input quantities are not normally distributed. Or it may be so that we sometimes have events where the measured data are correlated in such a way that a unit action in some direction when changing the data from true values to experimental data transforms the whole event to such a bad condition that it is very hard to find the true explanation. Some kinematical catastrophic regions exist.

If this is the reason, we should expect the same to happen also for our observed events. And we find that the experimental distribution agrees very well with the two generated Monte Carlo distributions.

The $\Lambda\mu$ distribution contains of course more events, which disagree with the explanation normal lambda decay. We have e.g. 15.3 % outside 4 standard deviations. But the background of $\Lambda\pi\mu$ events is so high ($\sim$ 10 events outside 4 standard deviations for all the collaboration) that we conclude that it is impossible to use tests 1 and 2 to find $\Lambda\mu$ events.

This might not be too catastrophic. We have namely a type of background which is very difficult to el iminate in case we use the lambda direction. This is due to events where the lambda has scattered without giving rise to any visible tracks. The assumed lambda direction could disagree quite much from the true one in such cases, and the event might be taken as a lambda-muon event.

5.3. Lambda direction not used (Tests 3 and 4).

5.3.1. Proton stopping.

The different distributions of D_{34} are given in Fig. 11.

We find that the $\Lambda\pi\mu$ distributions are completely inside $\pm$ 3 standard deviations, while we have 12.6 % of the $\Lambda\mu$ distribution outside this limit. If we eliminate events within -2 and +3 standard deviations we find the detection efficiency of $\Lambda\mu$ events to be 15.0 % and the expected background from $\Lambda\pi\mu$ events 0.10 events.

The momentum of the neutrino is above 60 MeV/c, i.e. near the maximum possible value, for most of the $\Lambda\mu$ events which can be detected.

We have observed one event where D_{34} is less than -5. This can not be a normal lambda decay. We have also two events which have D_{34} greater than +1. We expect to have two background events in this region and have therefore assumed these two to be $\Lambda\pi\mu$ events.

5.3.2. Proton leaving the chamber.

The four distributions are found in Fig. 12. The big error in the proton momentum for events where the proton leaves the chamber (or interacts) makes it possible to explain almost all $\Lambda\mu$ events as possible $\Lambda\pi\mu$ events.

We conclude that we can not use $\Lambda\mu$ candidates where the proton does not stop in the chamber.

5.3.3. Test 3.

We want to give the results from test 3, for the case where the proton stops, separately. This test can be used quite easily without complex programmes, and it eliminates most of the $\Lambda\pi\mu$ background.

The distributions of D_3 are given in Fig. 13. We find that type A events are in good agreement with test 3. This is of course what one

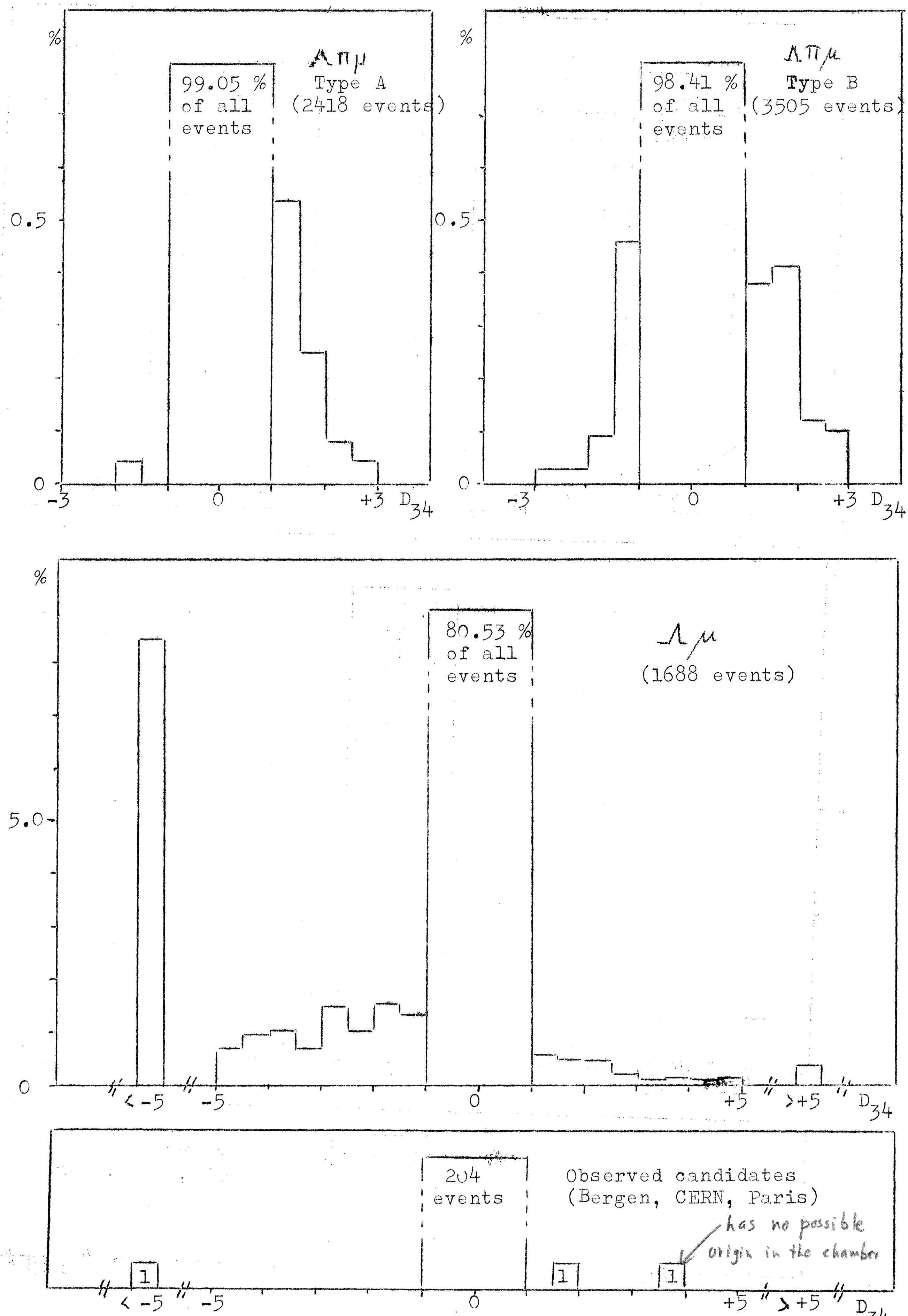

Fig. 11.

Distributions of D_{34} for events with stopping proton.

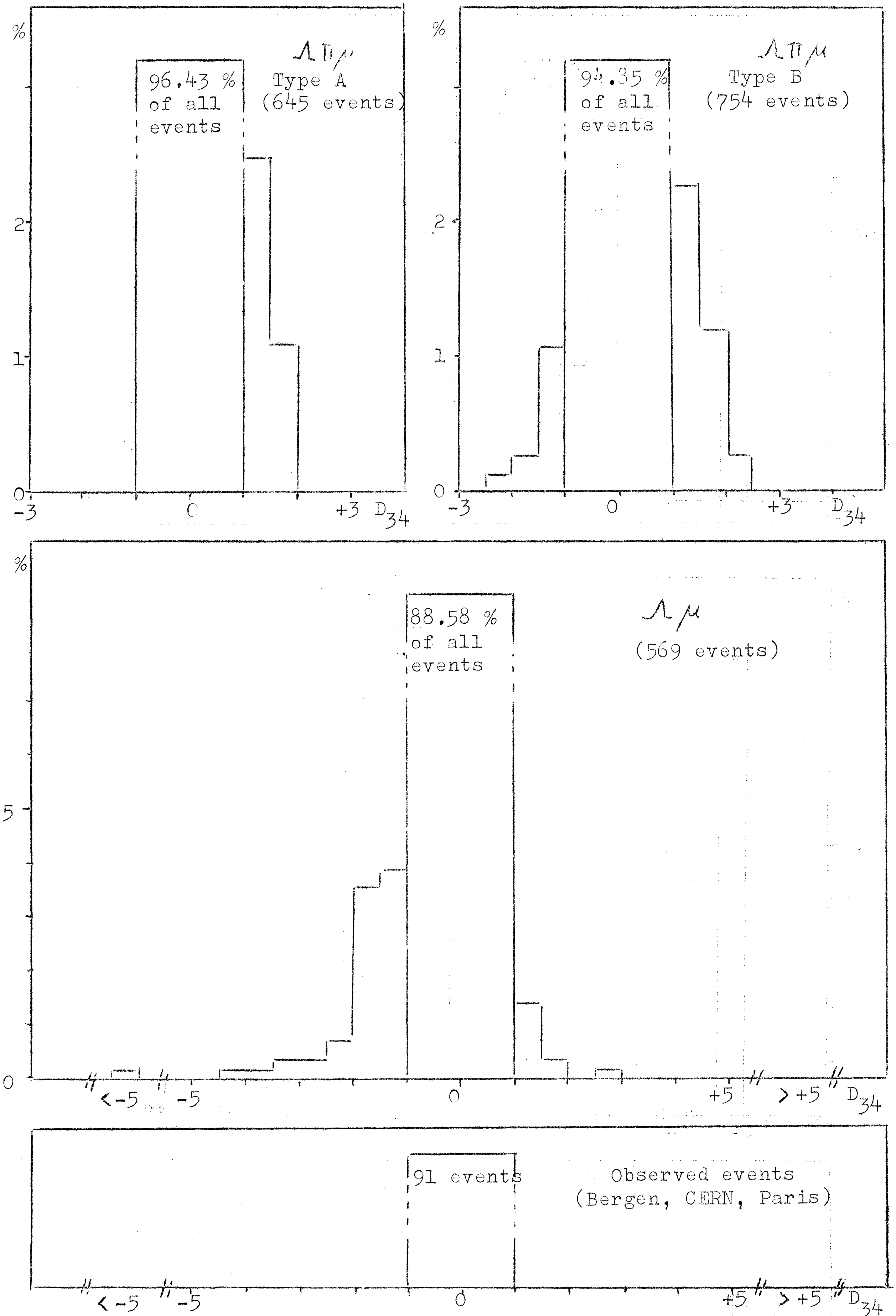

Fig. 12.

Distributions of D_{34} for events where the proton leaves the chamber.

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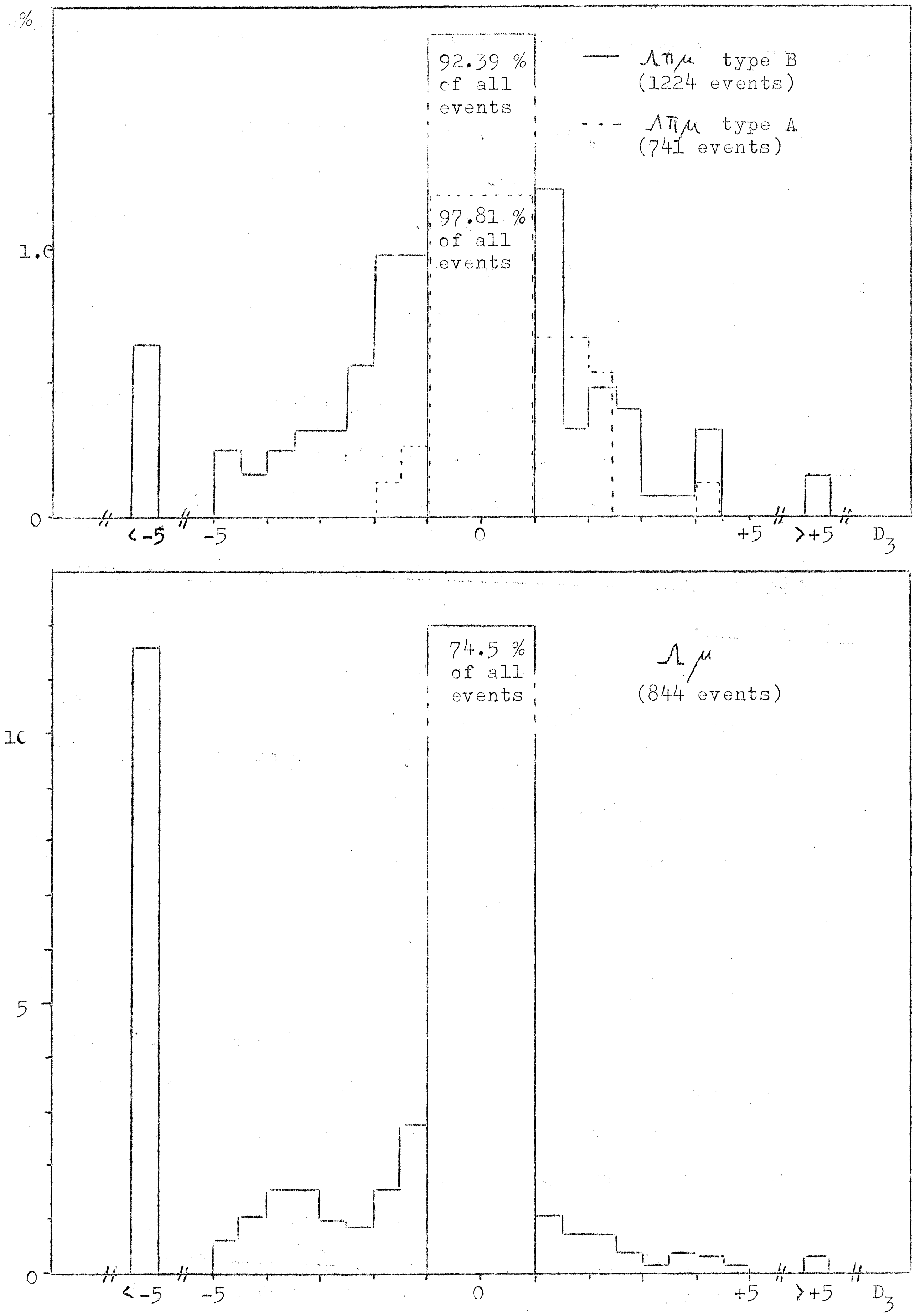

Fig. 13.

Distributions of D_3 for events with proton stopping.

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can expect as test 3 is just a test to see if the event can be explained as a type A event.

Type B events are, however, not always recognized as normal lambdas by test 3. We have e.g. 0.8 % (= 1 event) of all type B events outside 5 standard deviations. Outside one standard deviation we have 25.5 % of the $\Lambda\mu$ events and about 5 % of the normal lambdas ($\sim$10 expected events in our sample; 6 are observed by using test 3 only).

Test 3 is thus a good method to purify a big sample of $\Lambda\mu$ candidates to get a few events, which, however, have to be tested afterwards in some way, to find out if they may belong to type B.

6. $\Lambda\mu$ background due to scattering of the pion from a normal lambda decay.

If the pion from a normal lambda decay happens to be scattered near the production point (before a range of about 3 mm), it will be difficult to observe the scattering. Such a scattered pion, which decays to a muon, might simulate a $\Lambda\mu$ event.

To find out the background from such unobserved scatterings we proceed in the following way.

We choose a scattering angle (v^o) of the pion from a normal lambda decay (Section 3) and generate the azimuth angle (Θ) of the scattered pion by a Monte Carlo programme ($\Theta = \frac{2k+1}{200} 2\pi$; $k=0,1...99$) The new direction of the scattered pion is calculated. The pion is assumed to be scattered against a free proton and we therefore change the pion momentum from P_π to

$$P_s = \frac{M_p \beta_{CM}}{1 - \beta_{CM}^2 \cos^2 v} \left\{ \gamma_\pi \cos v + \sqrt{1 - \frac{\gamma_\pi^2 - \beta_{CM}^2}{1 - \beta_{CM}^2} \sin^2 v} \right\} \qquad (28)$$

where

$$\beta_{CM} = \frac{P_\pi}{M_p + \sqrt{M_\pi^2 + P_\pi^2}}$$

$$\gamma_\pi = \frac{M_p \sqrt{M_\pi^2 + P_\pi^2} + M_\pi^2}{M_p \sqrt{M_\pi^2 + P_\pi^2} + M_p^2}$$

The scattered pion is put into the Monte Carlo programme described in Section 3.1., which generates a $\Lambda\pi\mu$ event of type A. This scattered $\Lambda\pi\mu$ event is sent through the four test programmes (Section 2.3).

We generate 100-300 completely new events with the same scattering angle and can then determine how big fraction of these bad events would have been taken as $\Lambda\mu$ events according to our criteria above (D_{34} < -2 or > +3). This fraction, as a function of the scattering angle is shown in Fig. 14. It is based on 1660 events where both proton and muon stop.

We conclude that pions scattered less than or equal to 15° gives almost no background events (0 background events in 350 generated ones), and that about half of events where the pion is scattered more than 90° will contribute to the background.

We have to estimate how many scatterings in the first 3 millimeters of all our pion tracks we can expect to have in our sample of $\Lambda\mu$ candidates.

To be able to do this we have investigated 8.5 meters pion tracks (R_π > 2 cm) from an unbiased sample of normal lambda decays in the same rolls. The plane angle distribution ($\alpha \geq 3°$) of scattered pions is given in Fig. 15.

We have observed 10 events with plane angle greater than 15° and we have therefore about 15 events with space angle greater than

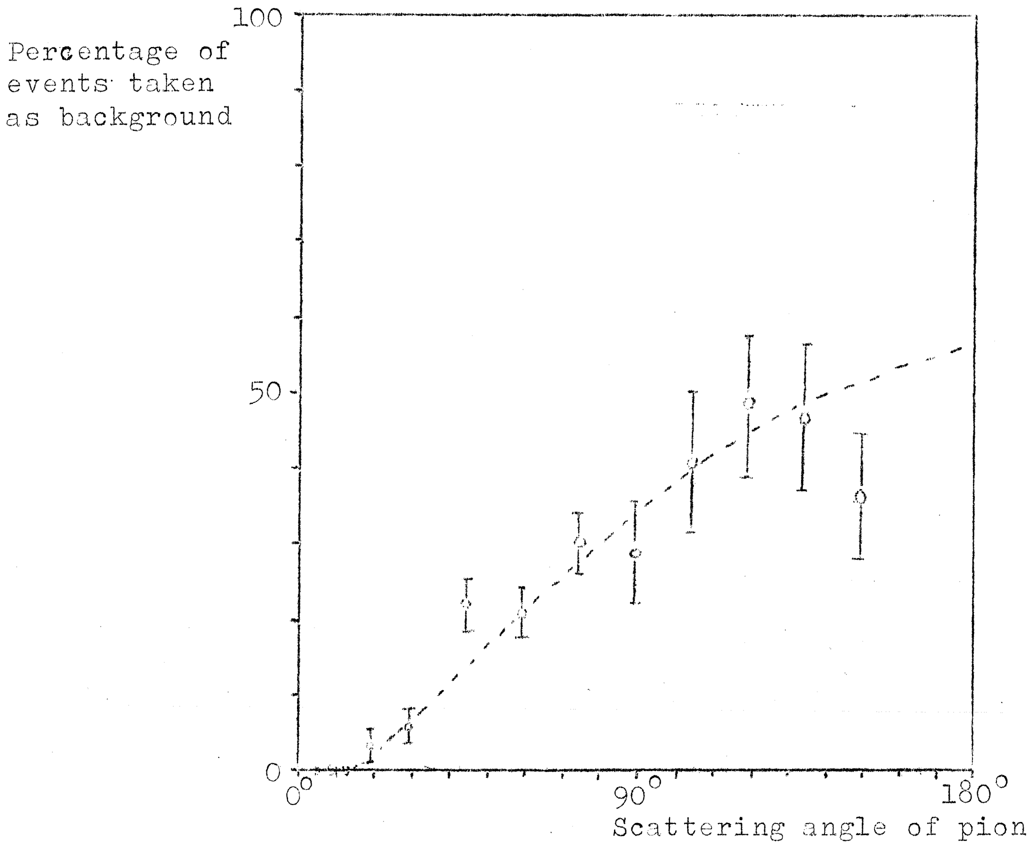

Fig. 14.

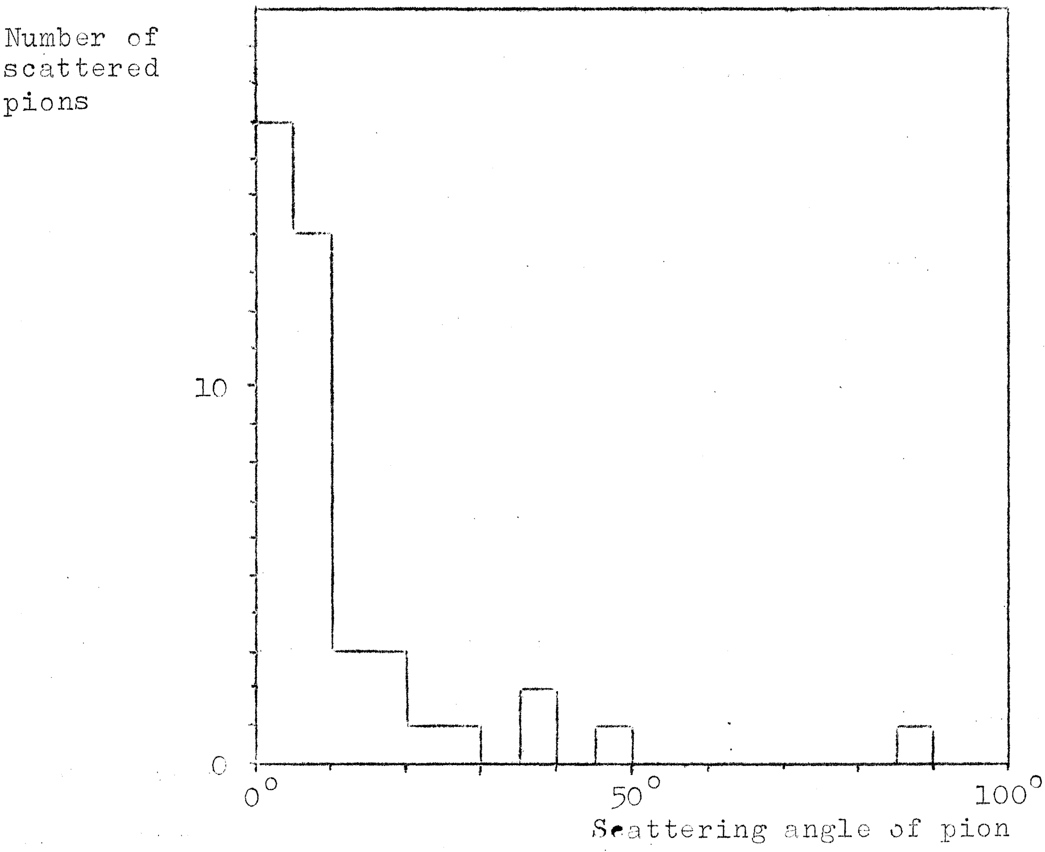

Fig. 15.

Plane angle distribution of scattered pions in 8.5 meters of pion tracks ($R_\pi > 2$ cm).

the given limit in a track length of 8.5 meters. Our 200 Λ_μ candidates with proton stopping correspond to 200 x 3 mm = 60 cm pion tracks where unobserved scatterings can have occured. We therefore expect to have about 1 big angle scattering, which should give a background of about 0.08 events according to Figures 14 and 15.

We want to note one special feature with the background events in Fig. 14. About 95 % of the events have D_{34} greater than +3 and only 5 % have D_{34} less than -2. The corresponding figures for our detectable Λ_μ events are 0.7 % and 14.3 % respectively. This means that if we restrict ourselves to taking only Λ_μ events where D_{34} is less than -2 (muon momentum lower than expected), we loose less than 1 % of our Λ_μ events, but have practically no background from scattered pions.

7. Summary.

i) We have written a number of FORTRAN programmes to test whether an observed Λ_μ candidate ($\Lambda \to p + \mu^- + \bar{\nu}$) can be explained as a normal lambda decay ($\Lambda \to p + \pi^-$) where the pion decays to a muon.

ii) The detection efficiency of Λ_μ events (defined to be that fraction of Λ_μ decays which can not be explained as normal lambda decays), has been estimated by help of Monte Carlo generated Λ_μ events.

We find that Λ_μ events where the proton leaves the chamber or interacts (big error in the proton momentum) are, with very few exceptions (1%), possible to explain as normal lambda decays.

For Λ_μ events where the proton stops inside the chamber we find that

a) 22.1 % of all generated events disagree by more than 3 standard deviations from the explanation normal lambda decay in case we use all "measurable" information from created events.

b) 12.6 % of our events fall outside the same limits in case we do not use the information about the lambda direction.

iii) To be able to determine how effective our programmes are in recognizing $\Lambda\pi\mu$ events, which have been subjected to measurement errors, we have written a Monte Carlo programme, which generates normal lambda decays and which transforms these true events to "measured" ones.

We find that $\Lambda\pi\mu$ events are effectively recognized as $\Lambda\pi\mu$ events (i.e. all generated events are within 3 standard deviations from the true explanation) in case we do not use the information about the lambda direction.

When using the extra constraints, which a knowledge of the lambda direction implies, we find that some of our $\Lambda\pi\mu$ events are not recognized as such. We have for example about three percente of our generated sample more than 4 standard deviations from the true explanation. This disagreement is probably due to the existence of some kinematically catastrophic regions and also to the fact that some of the measured variables are not normally distributed. The disagreement could probably be at least partly overcome if the error calculations were performed with still higher precision (but with the same method) as is used in the programmes.

iv) From ii) and iii) we conclude that a use of the better method (iia) to recognize $\Lambda\mu$ events (lambda direction known) leads to a high background of unrecognized $\Lambda\pi\mu$ events (about 10 events in our case) To avoid the $\Lambda\pi\mu$ background, therefore, we have to use the method in which one assumes the Λ direction to be unknown. This method gives a detection efficiency of 12.6 % with no background or 15.0 % with a background of 0.10 $\Lambda\pi\mu$ events in our case.
The second method has, in comparison with the first one, an other advantage. Λ's, which have been scattered without giving rise to visible tracks, will give a non negligible background, which is very difficult to eliminate, in the first method.

v) 207 $\Lambda\mu$ candidates with proton stopping (events from Bergen, CERN and Paris) have been analysed by the programmes. Two events were found to disagree with the explanation normal lambda decay. One of these has, however, no visible origin inside the chamber.

vi) The background from $\Lambda\pi\mu$ events where the pion is scattered very near the production point has been investigated by Monte Carlo programmes. We find that this background is about 0.08 events in our case.
Results from the $\Lambda\mu$ investigation, including determination of branching ratio and discussion of other types of backgrounds will soon be published.