NPA/Int. 66-5
Meyrin, 28th January 1966

NOTE ON THE COULOMB SCATTERING OF MUONS AND ELECTRONS FROM FLUORINE AND CHLORINE

In a previous paper by M. Paty and H. Yoshiki [1] (referred further to as I) the problem of the muon and pion scattering in freon has been discussed. In this note, the calculation of the nuclear form factor for the nucleus ^{19}F in I is presented in a more useful form. First, we discuss just this case. Secondly, we present new results for the nucleus ^{35}Cl. The element ^{35}Cl is important because of its presence in the light freon ($C_2 F_5 Cl$) utilised in the CERN heavy liquid bubble chamber (ref. I) for the current experiment measuring the muon polarisation of $K\mu_3$ in order to test the time reversability in the weak decay.

A knowledge of total Coulomb scattering cross sections for these elements is essential for the determination of mean free paths of light charged mesons in the mentioned liquid and thus for the analysis of bubble chamber data (cf. I).

In addition, we present useful expressions for calculating the nuclear form factor F of eq. (3) of I for the general case of spherical nucleus which are simple, and avoid complications of calculations of the type of Appendix 2 of I.

The energy transfer-integrated Coulomb scattering cross section represents the so-called Heisenberg sum rule or the energy-integrated response function of the target nucleus. It is the sum of the elastic and all inelastic nuclear scatterings. The said Born approximation cross section is a product of the single proton cross section multiplied by the sum rule form factor F. The Mott single proton cross section is modified so as to include the effect of the nuclear recoil (the finite nuclear mass) in the usual fashion [2]. Only the most important Coulomb interaction part is considered. The Rosenbluth (transverse) current interaction terms are important only at relatively higher momentum transfers K [3]. The angular distribution (differential cross section is then written as :

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{4\,k_o^2}\ \frac{\cos^2 \frac{1}{2}\theta}{\sin^4 \frac{1}{2}\theta} \left(1 + \frac{2\,k_o}{AM} \sin^2 \frac{1}{2}\theta\right)^{-1} . F \qquad (1)$$

where k_o is the incident electron (muon wave number, θ the scattering angle.

This problem has been discussed, e.g., by Drell and Schwartz [2], Gatto [4] and, for the inelastic part by Me Voy and Van Hove [3] and Ciocchetti and Molinari [5]. In the case of an independent particle model the form factor F can be written as :

$$F \equiv \left| \int \rho_o(1)\, e^{i \vec{K} \cdot \vec{r}_1}\, d\vec{r}_1 \right|^2 + (Z - S_o(K)) \qquad (2)$$

where $\rho_o(1) \equiv \sum_{i=1}^{Z} \psi_i^*(1)\, \psi_i(1)$ is the single proton density

and

$$S(K) \equiv \sum_{a,b=1}^{Z} \left| \int \psi_a^*(1)\, \psi_b(1)\, e^{i \vec{K} \cdot \vec{r}_1}\, d\vec{r}_1 \right|^2$$

The summations refer to the single proton states occupied in the ground state; K is the momentum transfer of the scattered particle. The first term in eq. (2) represents the so-called coherent part of the scattering ($\rightarrow Z^2$ for $K \rightarrow 0$) and the second one is the incoherent part ($\rightarrow Z$ for $K \rightarrow 0$).

We consider now a spherical nucleus whose ground state is given by a simple Slater determinant of j-j coupling wave functions. If the proton subshells are all closed, F can be put in the form :

$$F = \left| 4\pi \int_0^\infty j_o(Kr_1)\, \rho_o(r_1)\, r_1^2\, dr_1 \right|^2 + (Z - S_o(K)) \qquad (3)$$

where

$$S_o(K) = 4\pi \sum_{(a,b)} \sum_{\lambda} \langle l_a j_a \| Y_\lambda \| l_b j_b \rangle^2 \langle n_a l_a j_a | j_\lambda(Kr_1) | n_b l_b j_b \rangle^2 \qquad (4)$$

where

$$\langle l_a j_a \| Y_\lambda \| l_b j_b \rangle^2 = (4\pi)^{-1}\, \hat{\lambda}^2\, \hat{j}_b^2\, (j_b\, \lambda; -\tfrac{1}{2}\, 0 | j_a - \tfrac{1}{2})^2\, \tfrac{1}{2}\, (1 + (-)^{l_a + l_b - \lambda}) \qquad (5)$$

with $\hat{a}^2 = 2a + 1$; $\langle \, | j_\lambda | \, \rangle$ is a radial matrix element of $j_\lambda(Kr_1)$.

If the last proton subshell is only partly filled, all the elements of F involving that subshell are properly weighted by averaging and multiplying by an appropriate partial occupation factor.

In the case of ^{19}F the ninth proton is assumed to be 2 s 1/2. If we employ simple harmonic oscillator radial wave function neglecting, as usual, their j - dependence (spin-orbit coupling), we find :

$$F_F(x) = \left[e^{-x} \left(9 - \frac{8}{3} x + \frac{1}{6} x^2\right)^2 \right]_{coh} + \left[9 - e^{-x} \left(9 + \frac{16}{9} x^2 + \frac{1}{9} x^3 + \frac{1}{36} x^4\right) \right]_{incoh} \tag{6}$$

where "coh" and "incoh" denote the coherent and incoherent parts, respectively; thus :

$$F_F(x) = 9 + 72\, e^{-x} \left(1 - \frac{2}{3} x + \frac{25}{216} x^2 - \frac{1}{72} x^3\right) \tag{7}$$

where $x \equiv \frac{1}{2} b^2 K^2$; $b^2 = \nu^{-1}$ is the usual size parameter of a harmonic oscillator radial wave function. This result is to replace eq. (6') of I. On the other hand the numerical difference between our F_F of eq. (7) and that of eq. (6') of I is rather small ; our F_F is presented in fig. 1.

We apply now the same method to calculate $F_{Cl}(x)$ for the ^{35}Cl nucleus. The 16 protons in the ground state occupy the shells 1 s, 1 p, 1 d 3/2 and 2 s 1/2; the last add (valence) seventeenth proton occupies the 1 d 3/2 subshell and thus determines the ground state spin and parity $3/2^+$. A rather tedious computation yields the final result :

$$F_{Cl}(x) = \left[e^{-x} \left(17 - 8x + \frac{4}{5} x^2\right)^2 \right]_{coh} + \left[17 - e^{-x} \left(17 + \frac{539}{75} x^2 - \frac{2}{15} x^3 + \frac{343}{1050} x^4\right) \right] \tag{8}$$

Here x refers to the parameter b of ^{35}Cl fixed as equal to 1,819 fm (this corresponds to the harmonic oscillator spring constant with $\hbar\omega_o = 41\, A^{-1/3}$. [6]) $F_{Cl}(x)$ of eq. (8) is presented graphically in Fig. 1 together with $F_F(x)$, $F_C(x)$ and the light freon $F_{l.f}(x)$ as a function of the momentum transfer K. The values of the parameter b for ^{12}C and ^{19}F are also compatible with $\hbar\omega_o$ = 41. $A^{-1/3}$ of ref. [6].

It is then immediate to determine the muon mean free path in the light freon with the help of eq. (1). As one can conjecture from ref. [3] our results of Fig. 1 are probably not reliable for $K > 0.5\ fm^{-1}$. In the region of higher K one should include also the transverse current terms.

We express our thanks to Dr. C.A. Ramm for his kind interest in this problem.

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Figure Caption

Fig. 1 The nuclear form factors F_C (K) for ^{12} C, F_F (K) of eqs. (6) - (7), F_{Cl}(K) and $F_{l.c.}$(K) for light freon (C_2 F_5 Cl).

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