CERN-TH.5463/89

Towards the Measurement of the Gravitational Acceleration of Antimatter

DRAGAN HAJDUKOVIĊ *

CERN, Theory Division, Geneva, Switzerland

ABSTRACT

In order to measure the gravitational acceleration of antiprotons (and other long-lived charged particles) a simple method based on properties of the cycloidal pendulum is suggested. It may lead to extension of the method of CERN experiment PS196 to measure not only the inertial, but also the gravitational mass of the antiproton and to improvement of experiment PS200. In addition a cycloidal pendulum with neutrons (and antineutrons) is proposed.

* On leave from the Faculty of Natural Sciences and Mathematics, Montenegro, Yugoslavia

CERN-TH.5463/89
July 1989

Recently a number of papers have discussed the theoretical importance and experimental possibilities of the measurement of the gravitational acceleration of antimatter (see for example [1] and references therein). The methods used up to now for the measurement of the gravitational acceleration of ordinary matter are not appropriate for antimatter, so it is necessary to find methods of measurement adapted for antimatter. Such a method is suggested in the present paper.

As a first step let us find the form of a curve such that a particle moving along the curve because of the force of gravitation has a frequency of oscillation independent of amplitude. In order that frequency be independent of the amplitude the potential energy must be $U = ks^2/2$, where s is the length of the arc measured from the point of equilibrium and k is a constant. The kinetic energy is then $T = m_i\dot{s}^2/2$ (m_i being the inertial mass of a particle) and frequency $\omega = \sqrt{k/m_i}$ independent of the initial value of s. On the other hand in the gravitational field $U = m_i a y$, where y is the vertical coordinate and a is the gravitational acceleration. So, we have $m_i a y = ks^2/2$, *i.e.*

$$y = \frac{\omega^2}{2a}s^2 \tag{1}$$

Using Eq.(1) and the relation $ds^2 = dx^2 + dy^2$

$$x = \int \sqrt{\left(\frac{ds}{dy}\right)^2 - 1}\, dy = \int \sqrt{\frac{a}{2\omega^2 y} - 1}\, dy \tag{2}$$

Integration of (2) is possible with a change of variable

$$y = \frac{a}{4\omega^2}(1 - \cos\alpha) \tag{3}$$

which together with (2) finally leads to

$$x = \frac{a}{4\omega^2}(\alpha + \sin\alpha) \tag{4}$$

Equations (3) and (4) are parametric equations of a cycloid (Fig. 1) where the radius of the corresponding circle by which the cycloid is generated is $R = a/4\omega^2$, *i.e.*

$$a = 4R\omega^2 \tag{5}$$

Thus we have rederived a result known from textbooks of classical mechanics [3] that for a particle oscillating along a given cycloid, the frequency depends only on the gravitational acceleration and not on the amplitude (*i.e.* energy).

Now let us suppose that there is a tube in the form of a cycloid (Fig. 2) with a guiding magnetic field inside it with magnetic lines also in the form of a cycloid (one of them AOB is presented in Fig. 2). In such a tube a charged particle with sufficiently low energy ($E < m_i aR$ for a tube as in Fig.2) is in fact a cycloidal pendulum (of course the influence of stray electric fields on the particle must be small compared with gravitation and effects of an inhomogeneous magnetic field must be considered). I shall suggest two possibilities for using a cycloidal pendulum to measure the gravitational acceleration of antiprotons (and other long-lived charged particles). The first one is as follows. Let us have a particle which is oscillating along a part of cycloid AOB (Fig. 2) and which does not have enough energy to hit one of the detectors at points A and B. So, in order to detect a particle in A or B we must change its amplitude (*i.e.* energy) of oscillations. It may be done by resonance. Thus, detection of a particle at A or B is a sign that the proper frequency of the cycloidal pendulum is nearly the same as the known frequency used to obtain the effect of resonance. When we know the frequency and characteristic R of the cycloid, the gravitational acceleration can be calculated by Eq.(5). Let us note that the measurement of the inertial mass of antiprotons in experiment PS196 [4] is based on the use of resonance. So, it may be that the method of this experiment can be also adapted for the measurement of the gravitational acceleration of an antiproton oscillating as a cycloidal pendulum.

The other possibility is to launch particles from O (Fig. 2). A particle with energy $E < m_i aR$ will return to O after a time τ (half of a period T) which is the same for every launched particle, because of the properties of the cycloidal pendulum. This time of flight τ would be measured and then $\omega = 2\pi/T = \pi/\tau$, and in agreement with (5)

$$a = 4R\frac{\pi^2}{\tau^2} \tag{6}$$

In fact it is similar to the time of flight technique proposed in experiment PS200 [1] with a vertical drift tube. The principal advantage of the cycloidal pendulum is that the time of flight is the same for a set of particles with different velocities, whereas in a vertical tube it depends on initial velocity which is imprecisely known. In addition, in the variant with a cycloid the measurement can in principle be done with only one antiproton with sufficiently small energy, whereas with a vertical tube it is a statistical measurement and a large number of antiprotons is necessary [1]. The price for these advantages is the more complicated form of the required magnetic field and perhaps greater difficulty in the control of stray electric fields. A lot of work has to be done before we know if the advantages outweigh these technical difficulties.

In the short history of the efforts to measure gravitational acceleration of antimatter the antiproton has been considered as the principal candidate for such a measurement. To a certain extent it is by grace of existing facilities at LEAR and because of the ease of controlling the trajectory of charged particles. However, with charged particles there is always the problem of controlling stray electric fields [1]. For example, the gravitational force on an antiproton in the gravitational field of the Earth is of the same magnitude as the electric force on it in such a small electric field as 10^{-7} V/m (the situation with positrons is still worse because of smaller mass). Thus, it may be worthwhile to consider antineutrons in further planning of gravitational experiments. In fact I shall try to argue that the measurement of the gravitational acceleration of neutrons, based on the properties of a cycloidal pendulum, is realistic with existing experimental facilities. Such an experiment with neutrons could be a good first step towards a similar one with antineutrons. Let us note that there is a test of the weak equivalence principle with neutrons done by using neutron interferometry [5] but that technique would be difficult to apply to antineutrons.

Let us note a few facts about ultra low energy neutrons (in the present paper it means neutrons with kinetic energy $T_n < 10^{-7}$ eV). First, there are methods developed to obtain ultra low energy neutrons [6]. These neutrons have some unusual properties. The principal one is the total internal reflection of neutrons falling from vacuum onto a material surface at all incident angles. In fact the total internal reflection (which can be understood in the framework of the optical model of the nucleus [7]) appears when the kinetic energy of the neutron is smaller than a critical value T_c which depends only on the kind of material used for reflection (for example $T_c = 0.55 \times 10^{-7}$ eV for Al, and $T_c = 1.72 \times 10^{-7}$ eV for Cu). Due to this property, ultra low energy neutrons can be confined for a few hundred seconds in a closed volume or guided along a tube (see [8] for a review).

Another possibility to have some control of the movement of ultra low energy neutrons is based on the fact that their kinetic energy ($\approx 10^{-7}$ eV) is comparable with the energy of interaction $\vec{\mu}_n \cdot \vec{B}$ between the neutron magnetic moment $\vec{\mu}_n$ and a magnetic field $\vec{B}$ with a magnitude of about a few Tesla. So, for example, by using a non-homogenous magnetic field, neutrons have been successfully confined along the axis of a torus [9]. In any case it does not seem hopeless to restrict the movement of ultra low energy neutrons along a cycloid and to calculate acceleration by using the time of flight technique based on Eq.(6).

I would like to thank my experimental colleagues R. Hurst (WA82 experiment) and G. Gabrielse (PS196 experiment) from CERN and P. Cibin from my home institute for comments on the manuscript and discussion of technical details which are not discussed here but will be the matter of an elaborated paper. I am especially grateful to J. Ellis for hospitality in the CERN Theory Division.

REFERENCES

1. LEAR PS200 Collaboration, CERN/PSCC/86-2; PSCC/P94(1986).
2. D. Hajdukovič, CERN-TH 5378/89; to appear in Phys. Lett. B.
3. L. D. Landau and E. M. Lifshitz, *Mechanics*, Pergamon Press, Oxford, 1975.
4. LEAR PS196 Collaboration, CERN/PSCC/85-21/P83(1985).
5. J. Staudenman *et al.*, *Phys. Rev.* **21A** (1980) 1419.
6. A. Steyerl, *Nucl. Instrum. and Methods* **162** (1975) 461.
7. H. Feshbach, C. E. Porter, and V. F. Weisskopf, *Phys. Rev.* **23** (1963) 4.
8. R. Golub and J. M. Pendelburg, *Rep. Progr. Phys.* **42** (1979) 439.
9. K. S. Kugler, W. Paul, and V. Trunks, *Phys. Lett.* **72B** (1978) 422.

FIGURE CAPTIONS

1. Cycloid given by parametric equations (3) and (4).
2. A cycloidal pendulum. For details see the text.

Fig. 1

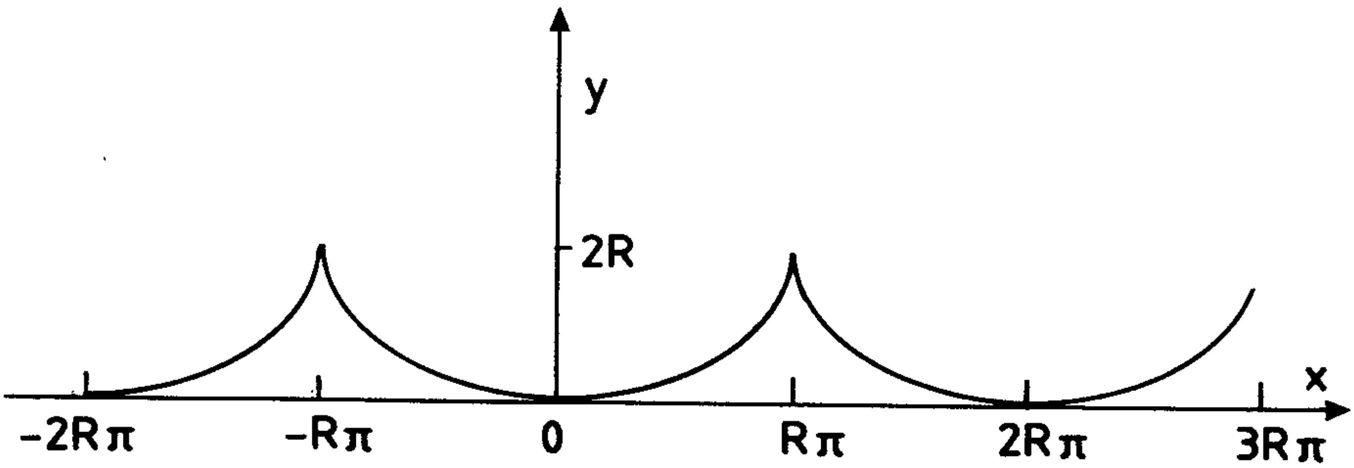

Fig. 2