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Numbers of $\mu^{\pm}\pi^{\mp}$, $\mu^{+}\mu^{-}$ and $K^{+}K^{-}$ atoms and Coulomb pairs in the DIRAC experiment

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Abstract

Numbers of $\mu^{\pm}\pi^{\mp}$ and $\mu^{+}\mu^{-}$ atoms and Coulomb pairs are estimated from numbers of $\pi^{+}\pi^{-}$ atoms and Coulomb pairs produced in the DIRAC experiment. The statistics required for detection of $\mu^{\pm}\pi^{\mp}$ and $\mu^{+}\mu^{-}$ atoms have to be higher than presently available by two orders of magnitude. The relations between numbers of $K^{+}K^{-}$ atoms and Coulomb pairs and corresponding numbers of $K^{-}\pi^{+}$ atoms and Coulomb pairs are obtained.

Estimates of number of $\mu^{\pm}\pi^{\mp}$ atoms in the DIRAC experiment presented in this note are based on assumption that the methods of extraction of the $\pi^{+}\pi^{-}$ and $\mu^{\pm}\pi^{\mp}$ atomic signals are similar. Therefore one may assume that number of $\mu^{\pm}\pi^{\mp}$ atoms is determined (for the same statistics) by number $n(A_{2\pi})$ of already produced $\pi^{+}\pi^{-}$ atoms in the DIRAC experiment [1], and by ratio of average $\mu^{\pm}$ and $\pi^{\pm}$ multiplicities per inelastic collision (since $\pi^{\pm}$ in the $\pi^{+}\pi^{-}$ atomic pair is replaced by $\mu^{\pm}$). Besides it is necessary to account for difference in the Borh radiuses $a_B(\pi\pi)$ and $a_B(\mu\pi)$ of the $\pi^{+}\pi^{-}$ and $\mu^{\pm}\pi^{\mp}$ atoms since their production cross-sections are proportional to a_B^{-3} [2] (or to m_{12}^3, where $m_{12} = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the atomic particles). The difference in the values of E_A/M_A for different mesoatoms, where M_A and E_A are the atom mass and its total energy in the laboratory system, was ignored.

Therefore approximate estimate of the number of the $\mu^{\pm}\pi^{\mp}$ atoms is:

$$n(A_{\mu^{\pm}\pi^{\mp}}) = n(A_{2\pi}) \frac{\langle \mu^{\pm} \rangle}{\langle \pi^{\pm} \rangle} \frac{a_B(\pi\pi)^3}{a_B(\mu\pi)^3} \varepsilon_{\mu\pi} = n(A_{2\pi}) \frac{\langle \mu^{\pm} \rangle}{\langle \pi^{\pm} \rangle} \Big(\frac{2}{1 + m_\pi/m_\mu} \Big)^3 \varepsilon_{\mu\pi}, \tag{1}$$

where m_π and m_μ are the π and μ masses, and $\varepsilon_{\mu\pi}$ is the relative efficiency of the μ and π detection.

Similarly, the number of the $K^{\pm}\pi^{\mp}$ atoms amounts to

$$n(A_{K^{\pm}\pi^{\mp}}) = n(A_{2\pi}) \frac{\langle K^{\pm} \rangle}{\langle \pi^{\pm} \rangle} \frac{a_B(\pi\pi)^3}{a_B(K\pi)^3} \varepsilon_{K\pi} = n(A_{2\pi}) \frac{\langle K^{\pm} \rangle}{\langle \pi^{\pm} \rangle} \Big(\frac{2}{1 + m_\pi/m_K} \Big)^3 \varepsilon_{K\pi}, \tag{2}$$

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where $\varepsilon_{K\pi}$ is the relative apparatus efficiency of detecting $K^{\pm}\pi^{\mp}$ and $\pi^+\pi^-$ pairs. Notice that numbers of produced $\pi^+\pi^-$ and $K^{\pm}\pi^{\mp}$ atoms in [1] and [3] were obtained with similar within 5% statistics [4].

Average $\pi^{\pm}$ and $\mu^{\pm}$ meson and ρ^0, ω, ϕ, η meson resonance multiplicities per inelastic collision are presented in section 1. Estimates of expected number of $\mu^{\pm}\pi^{\mp}$, $\mu^+\mu^-$ and K^+K^- atoms and Coulomb pairs are given in section 2.

1. $\pi^{\pm}$, $\mu^{\pm}$, $K^{\pm}$ and ρ^0, ω, ϕ, η average multiplicities

Determination of average $\mu^{\pm}$ multiplicity per inelastic collision is based on assumption that the direct $\mu^{\pm}$'s are dominantly produced as $\mu^+\mu^-$ pairs. The production of low-mass e^+e^- and $\mu^+\mu^-$ lepton pairs (with masses up to $m_{pair} \approx m_{\phi}$) in hadronic interactions has been studied in many experiments during the last decades. For a long time it has remained unclear whether the production of these pairs can be described only by contributions from decays of the hadronic resonances ρ^0, ω, ϕ, η into lepton pairs and by Drell-Yan mechanism or there are some unknown unconventional sources possibly indicating "new physics". However about two decades ago it became clear that production of both e^+e^- and $\mu^+\mu^-$ pairs could be explained entirely satisfactory by pairs produced by hadronic resonance decays and there was no need to invoke any other low-mass source. The claims for an excess of low mass pairs in earlier experiments was due to too low estimates of resonance cross sections, especially for η production, and not always adequate knowledge of branching ratios. This was shown, for example, by the HELIOS experiment at the CERN SPS, where the production of low mass e^+e^- and $\mu^+\mu^-$ pairs was reported in pBe collisions at 450 GeV/c [5].

Therefore average $\mu^{\pm}$ multiplicity per inelastic collision, $\langle\mu^{\pm}\rangle$, can be calculated from corresponding ρ^0, ω, ϕ and η average multiplicities and known decay modes of these resonances into the final states with $\mu^+\mu^-$. Average particle and resonance multiplicities per inelastic collision in p-nuclear collisions at 24 GeV/c are not known. However average multiplicities in pp collisions represent their good estimates. This was shown, for example, by the NA22 experiment at the CERN SPS [6] from comparison of average ρ^0 multiplicities in π^+p-, π^+Al- and π^+Au-collisions at 250 GeV/c, found to be 0.46 ± 0.02, 0.45 ± 0.13 and 0.43 ± 0.21, respectively, or average $K^{*0}(890)$ multiplicity in K^+p- and K^+Al-interactions equal to 0.29 ± 0.03 and 0.34 ± 0.16. Clearly average resonance multiplicities per inelastic collision in hadron-proton and hadron-nuclear collisions are the same within errors.

Average ρ^0, ω, ϕ and η multiplicities in pp collisions at 24 GeV/c taken from [7] together with the branching fractions of these resonances into $\mu^+\mu^-$ pairs [8] are given in Table 1. As one can see from this Table, the average $\mu^{\pm}$ multiplicity per inelastic collision is $(5,83 \pm 0,81) \cdot 10^{-5}$. However the "long-lived" η gives no contribution to the Coulomb or atomic pairs. Without η contribution, the average $\mu^{\pm}$ multiplicity amounts to

$$\langle\mu^{\pm}\rangle = (3.04 \pm 0.62) \cdot 10^{-5}. \tag{3}$$

Average π^+ and π^- meson multiplicities have been also taken for pp collisions at 24 GeV/c. Their values were obtained from the π^+ and π^- meson inclusive cross-sections

Table 1: Average multiplicities of hadronic resonances per inelastic collision, $\langle Res \rangle$, in pp-interactions at 24 GeV/c [7], branching ratios of these resonances in the final states with $\mu^+\mu^-$ pair [8] and average $\mu^\pm$ multiplicities per inelastic collision, $\langle \mu^\pm \rangle$, for each of the resonances, for all them and for all but without η

Resonance	$\langle Res \rangle$	Branching$\cdot 10^{-5}$	$\langle \mu^\pm \rangle \cdot 10^{-5}$
ρ^0	0.114 ± 0.014	4.55 ± 0.28	0.57 ± 0.07
ω	0.105 ± 0.014	22.0 ± 5.1	2.31 ± 0.62
ϕ	0.0052 ± 0.0011	30.1 ± 2.0	0.16 ± 0.03
η	0.090 ± 0.012	31.6 ± 4.0	2.84 ± 0.52
All			5.83 ± 0.81
All(without η)			$3,04 \pm 0,62$

$\sigma(\pi^+) = (56.8 \pm 0.9)$ mb, $\sigma(\pi^-) = (33.8 \pm 0.6)$ mb and total inelastic cross section $\sigma_{inel} = (30.6 \pm 0.25)$ mb [9] and equal to $\langle \pi^+ \rangle = 1.680 \pm 0.040$ and $\langle \pi^- \rangle = 1.105 \pm 0.022$. However η and η' mesons give no contributions to mesoatoms and Coulomb pairs. Therefore subtracting their contributions obtained from their production rates evaluated in [7] and branching ratios into the modes with π^+ and π^- in the final states [8] one gets:

$$\langle \pi^+ \rangle = 1.641 \pm 0.040, \quad (4)$$
$$\langle \pi^- \rangle = 1.066 \pm 0.023. \quad (5)$$

Average π meson multiplicity is higher for p-nuclear collisions than for pp collisions by a factor 1.5-2.0 (see [10] for example). Therefore our estimates of numbers of the $\mu^\pm\pi^\mp$ atoms given below in (8) have to be, in fact, smaller by this factor.

Finally average K$^+$ and K$^-$multiplicities in pp collisions at 24 GeV/c were estimated in [11]:

$$\langle K^+ \rangle = 0.0766 \pm 0.0080, \quad (6)$$
$$\langle K^- \rangle = 0.0218 \pm 0.0022. \quad (7)$$

2. Estimates of numbers of $\mu^\pm\pi^\mp$, $\mu^+\mu^-$ and K$^+$K$^-$ atoms and Coulomb pairs

Assuming equal muon and pion detection efficiencies ($\varepsilon_{\mu\pi} = 1$), the expected numbers of $\mu^+\pi^-$ and $\mu^-\pi^+$ atoms can be obtained from (1) with the number of produced $\pi^+\pi^-$ atoms in the experiment [1], $n(A_{2\pi}) = 47171 \pm 904$, and average muon and pion multiplicities per inelastic collision (3)-(5):

$$n(A_{\mu^+\pi^-}) = 0.63 \pm 0.13, \quad n(A_{\mu^-\pi^+}) = 0.97 \pm 0.20 \quad (8)$$

or even less by a factor of 1.5-2.0 as just discussed. It is clear that for detection and studies of such atoms the required statistics have to be two orders of magnitude higher than the one available for studies of $\pi^+\pi^-$ and K$^\pm\pi^\mp$ atoms in DIRAC.

Since average multiplicity of $\mu^+\mu^-$ pairs per inelastic collisions is the same as for $\mu^{\pm}$, one arrives to the same conclusion that for observation of $\mu^+\mu^-$ atoms much higher statistics than presently accumulated by the DIRAC experiment is necessary.

Turning to estimation of number of the $\mathrm{K}^+\mathrm{K}^-$ atoms, notice first that K^- mesons are dominantly produced in the "central" collisions as $\mathrm{K}^+\mathrm{K}^-$ and $\mathrm{K}^0\mathrm{K}^-$ pairs[2] if one neglects the relatively small contribution from associated production of K^- mesons and strange antibaryons. This implies that in half of events with the K^- in the final state there is also the K^+ meson and that number of $\mathrm{K}^+\mathrm{K}^-$ atoms is close to 1/2 of $\mathrm{K}^-\pi^+$ atoms if the number of $\mathrm{K}^-\pi^+$ pairs in the final states is close to the number of $\mathrm{K}^+\mathrm{K}^-$ pairs.

Let us consider therefore the final states with $\mathrm{K}^+\mathrm{K}^-$ pairs in 4-prong (with 4 charged particles) and 6-prong events giving main contribution (88% for events with number of charged particles ≥ 4) to inelastic cross section for pp interactions at 24 GeV/c [9] and ignoring small contribution of double charge-exchange reactions (with two neutrons in the final states):

$$\begin{array}{ll} \mathrm{K}^+\mathrm{K}^- pp, & \mathrm{K}^+\mathrm{K}^-\pi^+ pn \\ \mathrm{K}^+\mathrm{K}^-\pi^+\pi^- pp, & \mathrm{K}^+\mathrm{K}^- 2\pi^+\pi^- pn. \end{array}$$

For two $\mathrm{K}^+\mathrm{K}^-$ pairs in these final states one has one $\mathrm{K}^-\pi^+$ pair in 4-prong events and three $\mathrm{K}^-\pi^+$ pairs in 6-prong events (with smaller cross section than for 4-prong events). Since numbers of the $\mathrm{K}^+\mathrm{K}^-$ and $\mathrm{K}^-\pi^+$ combinations are comparable, one arrives to the following estimate:

$$n(A_{\mathrm{K}^+\mathrm{K}^-}) = \frac{1}{2} n(A_{\mathrm{K}^-\pi^+}) \frac{a_B(K\pi)^3}{a_B(KK)^3} \varepsilon_{KK} = \frac{1}{2} n(A_{\mathrm{K}^-\pi^+}) \Big(\frac{m_\pi + m_K}{2m_\pi} \Big)^3 \varepsilon_{KK}, \tag{9}$$

where $n(A_{\mathrm{K}^-\pi^+}) = 188 \pm 21$ [3] is the number of produced $\mathrm{K}^-\pi^+$ atoms in the experiment as obtained from two-dimensional Q_T and Q_L distribution (Q_T and Q_L are the transverse and longitudinal projections of the relative momentum of two particles in their center-of-mass system) and ε_{KK} is the relative efficiency of detecting $\mathrm{K}^+\mathrm{K}^-$ and $\mathrm{K}^-\pi^+$ pairs.

Finally, estimates for numbers of the $\mu^{\pm}\pi^{\mp}$, $\mu^+\mu^-$ and $\mathrm{K}^+\mathrm{K}^-$ Coulomb pairs can be obtained similarly to estimates of numbers of atoms by removing the Borh radius factors in formulae (1) and (9):

$$n_{\mu^{\pm}\pi^{\mp}}(Coul) = n_{\pi^+\pi^-}(Coul) \cdot \frac{\langle \mu^{\pm} \rangle}{\langle \pi^{\pm} \rangle} \varepsilon_{\mu\pi} \tag{10}$$

and

$$n_{\mathrm{K}^+\mathrm{K}^-}(Coul) \approx \frac{1}{2} n_{\mathrm{K}^-\pi^+}(Coul) \varepsilon_{K\pi}, \tag{11}$$

where $n_{\pi^+\pi^-}(Coul)$ and $n_{\mathrm{K}^-\pi^+}(Coul)$ are the numbers of $\pi^+\pi^-$ and $\mathrm{K}^-\pi^+$ Coulomb pairs: $n_{\pi^+\pi^-}(Coul) = 16,09 \cdot 10^5$ for $Q_T < 4$ MeV/c, $Q_L < 15$ MeV/c and $n_{\mathrm{K}^-\pi^+}(Coul) = 2209$ for $Q_T < 4$ MeV/c, $Q_L < 20$ MeV/c [12].

[2]Contrary to K^+ mesons, dominantly produced at our energies in the fragmentation processes in association with strange baryons, so that $\langle \mathrm{K}^+ \rangle / \langle \mathrm{K}^- \rangle = 3.5 \pm 0.5$ [11].

With (3)-(5) and assuming $\varepsilon_{\mu\pi} = 1$, one obtains from (10):

$$n_{\mu^+\pi^-}(Coul) = 30 \pm 6 \quad \text{and} \quad n_{\mu^-\pi^+}(Coul) = 46 \pm 9$$

for $Q_T < 4$ MeV/c and $Q_L < 15$ MeV/c.

Expected number of $\mu^+\mu^-$ Coulomb pairs is smaller since average number of $\mu^+\mu^-$ pairs $\langle\mu^+\mu^-\rangle = \langle\mu^\pm\rangle$ and unknown average number of $\pi^+\pi^-$ pairs is higher than $\langle\pi^+\rangle$ and $\langle\pi^-\rangle$ due to obvious combinatorics.

For determination of numbers of K^+K^- atoms and Coulomb pairs according to (9) and (11) one should know the efficiency coefficient ε_{KK}.

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