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THE CATAMORPHY OF QUARK-LEPTON FAMILIES

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Abstract

Quark-lepton families appear to be limited in number. This can be understood if they are catamorphically related. This is illustrated by a simple geometrical *Ansatz* that gives the correct number of fermion fields per family and limits the number of families to three.

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Fermions appear to be organized into quark-lepton families of which we now recognize three, $(u, d;\ e, \nu_e)$, $(c, s;\ \mu, \nu_\mu)$ and $([t], b;\ \tau, \nu_\tau)$, each consisting of N=15 2-component fields of quarks and leptons: 3 coloured LH quark doublets, one electron-neutrino LH doublet and 7 RH singlets of the 6 coloured quarks plus the electron.

Present indications from laboratory experiments are that these quark-lepton families are limited to 6 in number, while comparison between the observed cosmical abundance of ^{4}He and the simplest expectations based on the Big Bang suggests a total of 3 or 4 such families but not more. We have no experimental clue as to why the particle families should be limited in number in this way. It is the object of this note to indicate that this limitation is a catamorphism and to illustrate this with a simple geometrical scheme which contains the correct number of fermion fields per family and limits the number of families to three.

Catamorphy is the lowest category of symmetry [1]; it specifies generic relationships between forms but also implies a natural limit to the degree to which those relationships can be pursued.

In the present context consider the following ***Ansatz***:

(i) The first quark-lepton family of N fields is represented by the most symmetrical circular arrangement of N identical regular n-gons linked just at corners;

(ii) the successor family to the N n-gons is represented by a similar circular arrangement, of N identical equal-sided almost-regular $(n+1)$-gons, attached to the outer rim of the n-gon arrangement; neighbouring corners of each $(n+1)$-gon are linked to just the corners of two adjacent n-gons; the $(n+1)$-gons are linked to each other just at other of their corners;

(iii) and so on.

The lowest n-gon for which the above *Ansatz* may be carried beyond (i) has n=5. The requirement in (i) of maximum symmetry implies that the bases of the pentagons be separated by the length of their sides; this yields N=15.

Application of (ii) and (iii) of the *Ansatz* now generates two further rings of 6-gons and 7-gons, respectively. The process cannot be carried beyond n=7 so there are just three families in all.

Appropriate systematic allocation of the fermion fields to the polygons combined with a simple counting prescription generates the Kobayashi-Maskawa matrix correctly to order λ^3 in the Wolfenstein expansion paramater λ ($\simeq \sin\theta_c \simeq 0.22$). Imposition of unitarity then yields, in standard notation, $A = 1$, $\rho\cos\delta = 1/2$. With these values the experimental $|\,\epsilon\,|$ and the magnitude of B^0-$\overline{B^0}$ mixing are simultaneously accounted for with $m_t \gtrsim 150$ GeV (using the value-ranges as customary for other relevant parameters [2]).

In this scheme the rate of $\mu \to e\gamma$ involves the suppression factor λ^{-24}, viz. 10^{16}, with corresponding factors of λ^{-20} and λ^{-12} for $\tau \to \mu\gamma$ and $\tau \to e\gamma$, respectively.

Catamorphy is not widely recognized in physics but a few examples are found in decorative art. Indeed, the 16th-century dome of the madrasa of Abdullah-khan in Bukhara displays, in elaborated form, precisely the 5,6,7-gon terminating sequence (there with N=16 as would, in our context, correspond to neutrinos of finite mass) that has been described here (see e.g. fig. 38 of ref. [3]).

References

[1] K.L. Wolf and R. Wolff, *Symmetrie* (Böhlau-Verlag, Münster-Köln, 1956).

[2] P. Krawczyk, D. London, R.D. Peccei and H. Steger, Nucl. Phys. B307 (1988) 19.

[3] E. Makovicky, Comp. and Maths. with Appls. 12B (1986) 949.