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NUCLEON ELECTROMAGNETIC FORM-FACTOR IN THE TIME-LIKE REGION NEAR $\bar{N}N$ THRESHOLD

Moscow - 1988

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O.D. Dalkarov and K.V. Protasov

Nucleon Electromagnetic Form-Factor in the Time-like Region near $\bar{N}N$ Threshold

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Abstract

Nucleon electromagnetic form-factor in the time-like region near $\bar{N}N$ threshold is calculated. Its behavior is shown to be dominantely defined by the $\bar{N}N$ nuclear interaction in initial state. For this reason an investigation of nucleon form-factor in this energy region could be useful for independent extracting of the low energy $\bar{N}N$ parameters.

Introduction

Study of annihilation channels of the nucleon-antinucleon interaction at low energy is the object of intense interest for understanding the nature of the low energy baryon-antibaryon interaction. A strong nuclear attraction between nucleon and antinucleon is the common reason for large annihilation cross section and for existence of sufficiently narrow quasinuclear states simultaneously [1]. The presence of the subthreshold resonances (with the masses closed to 2M, where M is the nucleon mass) can be served as an additional cause of the enhancement of annihilation cross section near $\bar{N}N$ threshold in the states with definite quantum numbers. Therefore, an investigation of partial annihilation cross section is the source of information not only about the forces between $\bar{N}$ and N but about the proposed spectrum of the states also. The partial $\bar{N}N$ amplitudes will be studied in the future using phase shifts analysis of LEAR data. At present such investigation could be fulfield both from the data on antiprotonic atoms (scattering lengths) and due to analysing of $\bar{N}N$ annihilation in the states with definite quantum numbers. In the preprint we will discuss an annihilation process $\bar{p}p \to e^+e^-$ near $\bar{N}N$ threshold.

Annihilation into lepton pair $\bar{p}p \to e^+e^-$ can proceed only from $\bar{p}p$ states with photon quantum numbers $J^{PC} = 1^{--}$ (for slow antinucleons this corresponds to $^3S_1$-states with the isospin I = 0, 1). Now there are some data [2,3] on the cross section behaviour at small relative momentum between $\bar{p}$ and p. In particular, in the experiment [3] the relative probability of $\bar{p}p \to e^+e^-$ annihilation for $\bar{p}$ incident momentum 300 MeV/c was measured. It was found that $BR(e^+e^-) = (\bar{p}p \to e^+e^-)/(\bar{p}p \to \text{all}) = (3.2 \pm 0.9)\ 10^{-7}$. This is obviously a very large value if we compare $BR(e^+e^-)$ with the similar ratio for $\bar{p}p \rightarrow \pi^+\pi^-$ annihilation $BR(\pi^+\pi^-) = (3.2 \pm 0.3)\cdot 10^{-3}$, i.e. $BR(e^+e^-)/BR(\pi^+\pi^-) = 10^{-4} \sim \alpha^2$. The latter equality means that the proton form-factor $G(q^2)$ is close to unity at the boundary of the physical region with approach from the side of time-like transferred momenta q^2(i.e. for $-q^2 \gtrsim 4M^2$): $G(-q^2 = 4M^2) \sim 1$ (experimental value is equal to $G = 0.46 {+0.15 \atop -0.09}$ at laboratory momentum $p \simeq 300$ MeV/c, hereafter we assume $G_E(4M^2) = G_M(4M^2)$), q is the four-momentum and in the region in question $-q^2 = s$. If we try to approximate such behavior of G at $-q^2 \gtrsim 4M^2$ using usual dipole formula, the value G will be less by the order of magnitude than one experimentally measured. From our point of view such large value of G is at least an evidence for a strong nuclear attraction between $\bar{p}$ and p.

Large value of the electromagnetic form-factor G in the known vector dominant models (VDM) can be obtained by taking into account besides light ρ, ω, φ-mesons also heavy ρ', φ'-mesons [4, 5]. The coupling constants for these mesons with proton were in this case considered as free parameters. In our model the existence of the subthreshold resonances (i.e. bound $\bar{N}N$ quasinuclear states with masses close to 2M)was mentioned above to be additional reason for the form-factor increasing in the time-like region at $-q^2 > 4M^2$. This possibility was pointed out in ref. [6]. As for the existing of heavy vector mesons (like ρ' or φ') there is no direct experimental evidence for the quasinuclear nature of these resonances since being of the subthreshold type they cannot manifest themselves as bumps in $\bar{p}p$ "formation" experiments (arguments in favour for the quasinuclear nature of the resonances are that the multi-pion channels are dominant and that the resonant masses are close to 2M).

In this paper we use a coupled channel model for the calculation the $\bar{p}p \to e^+e^-$ annihilation cross section, i.e. correspondingly, an electromagnetic proton form-factor G in the time-like region near $\bar{N}N$ threshold. This model was used early for the description of low energy $\bar{p}p$ data [7]. As an input we consider the models well described the data in space-like region (i.e. ep-scattering): usual dipole formula and VDM taking into account an exchange by the light vector mesons (ρ, ω, φ) only. Initial state interaction which is a reason of large value of G causes in this case the energy behaviour of G also. The method for estimation of this interaction (nuclear and annihilative) was taken by the analogy with ref. [7], where the data on $\bar{N}N$ annihilation at low energy were fitted. We have calculated also the neutron electromagnetic form-factor and cross section for the inverse process $e^+e^- \to \bar{p}p$ in the region $-q^2 > 4M^2$.

The scheme of the exposition is as follows. In section 1 we formulate simple coupled channel model for description of electromagnetic proton form-factor G. Section 2 is devoted to the formulation of some analytical expressions for G near $\bar{N}N$ threshold which are valid if initial state $\bar{p}p$ interaction is dominant. In section 3 the results of numerical calculations are given. We consider also the possibility to compare these results with independent experimental $\bar{N}N$ data. In conclusion we emphasize the qualitative features of G behaviour at $-q^2 > 4M^2$ which are characterized for proposed here model.

1. The coupled channel model

For the calculation of the $\bar{p}p \to e^+e^-$ annihilation cross section the coupled channel model (CCM) was used. The channel 1 corresponds to $\bar{N}N$ system (interaction between $\bar{N}$ and N is due to one boson exchange which is described by the potential $V_{\bar{N}N}^{OBEP} = V_{11}$). Masses of proton and neutron were taken the same and were equal to M = 0.939 GeV. Channel 2 is annihilation one. The coupling between 1 and 2 channels is realized by short range Yukawa type potential (V_{12}). All of parameters for these channels are taken from [7] where the details of two channel model were given (in the framework of this model all $\bar{p}p$ low energy data were described). Third channel corresponds to e^+e^- system. It is connected only with $\bar{N}N$ channel by transition potential. This transition potential (V_{13}) has a form of δ-function since an amplitude for $\bar{p}p \to e^+e^-$ reaction (without initial state interaction) is a function of s only. Therefore, the potential corresponding to this amplitude will be proportional to $\delta(\vec{r})$.

The electromagnetic proton form-factor is connected with $\bar{p}p \to e^+e^-$ cross-section by the formula:

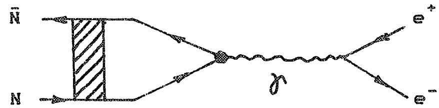
$$\sigma_{\bar{p}p \to e^+e^-} = \frac{\pi\alpha^2}{2\ M\ k}|G|^2$$

(here $G_E(4M^2) = G_M(4M^2) = G$, as mentioned above).

The calculations in CCM was made by the variable phase approach [8, 9]. The problem with δ-function potential is discussed in Appendix.

2. The influence of initial state $\bar{N}N$ interaction on the nucleon electromagnetic form-factor

Let's consider annihilation of slow antinucleons into e^+e^--pair. This process (in the first order on α) is associated with the diagram:



where the dashed block denotes the amplitude of the initial state interaction, the black circle corresponds to the form-factor G_0 which is assotiated with the singularities far from $\bar{N}N$ threshold (for instance, with ρ, ω, φ-poles in VDM). To take into account the effect of initial state interaction we consider from the beginning the diagram without interaction in the initial state. $\delta(\vec{r})$-potential is corresponded to this diagram (since this diagram does not depend on $\vec{q}$ -. three-dimentional momentum). If this potential is denoted by $V(\vec{r}) = V \cdot \delta(\vec{r})$ the transition amplitude for the diagram on figure painted above can be written:

$$T \sim \int \psi_{\bar{N}N}(\vec{r}) \; V(\vec{r}) \, \psi_{e^+e^-}(\vec{r}) \; d\vec{r}$$

Therefore the nucleon form-factor G has the form:

$$G = \left| \psi(0)_{\bar{N}N} \right| G_0$$

where G_0 is the form-factor corresponding to the diagram without initial state interaction. As for the concrete form $G_0 = G_0(s)$, in principle, it can be approximated by the various models well described the data in space-like region (dipole formula, VDM or from the calculation in the quark models [11]). Let's consider the factor $\psi(0)$ which can be expressed by $\psi(0) = 1./ f(-k)$, where $f(k)$ is the Jost function for $\bar{N}N$ system. Jost function (k) can be represented in the form [10]:

$$f(k) = \tau(k) \cdot e^{i\,\delta(k)}$$

(let's emphasize that this Jost function corresponds to S-wave since $\delta(\vec{r})$-potential extracts only this wave). Function $\tau(k)$ is symmetrical on k , $\delta(k)$ being antisymmetrical (for this reason as usually for S-matrix we have $S(k) = f(k)/f(-k) = e^{2i\,\delta(k)}$). Phase $\delta(k)$ is complex one since there are annihilation channels for $\bar{N}N$ system.

Therefore

$$|\psi_{\bar{N}N}(0)| = \frac{1}{|\tau(k)|} e^{-\,\mathrm{Im}\,\delta(k)}$$

Now we will consider only small energy in $\bar{N}N$ c.m. system. Let use an expansion of $\delta(k)$ in the scattering length approximation, i.e. $\delta(k) = a\cdot k$ (we neglected the higher on k terms in this expansion). It's need to emphasize that expansion of $\tau(k)$ includes only k^2 terms since $\tau(k) = \tau(-k)$. Therefore an expansion of G in power series in k has the form:

$$G = C_1(1 - \mathrm{Im}\,a\cdot k + C_2k^2 + \ldots) \qquad (1)$$

where C_1 and C_2 are the constants. No other term of the first order on k exist since all of other factors are a function of s, so they give only even series in k. The value of Ima corresponds to the imaginary part of S-scattering length. The term with Ima has a principal significance: it occurs only in the case of initial state interaction. Note that neither in VDM (for instance, [4]) nor in Regge-pole model [5] it is not principally possible to get the linear behaviour in k.

Only LEAR experiments now can give a direct answer on the question about the presence of linear k-term in G behaviour. If from the experiment a linear behaviour will be observed it could be a strong indication on the neccesity of taking into account an initial state interaction.

Moreover the experiments with form-factor measurements could be used as an independent source of information about Ima in addition to the investigation of the level shifts and widths of protonium. However, it's need to note that in the expansion of proton (neutron) form-factor in the power in k a slightly different value (not only Ima) will be included if the vertex is differed for various isospins in the above diagram. The correction isospin dependence of $\bar{N}N$ interaction is rather trivial and it is not important for obtained conclusion about the presence of linear term.

3. Numerical results

The enhancement coefficients for $\bar{N}N \rightarrow e^+e^-$ reaction (I = 0, 1) are shown in Figure 1. This factor is defined as

$$K = \sqrt{\frac{\sigma_{\bar{N}N \rightarrow e^+e^-}}{\sigma_{\bar{N}N \rightarrow e^+e^-}(V = 0)}}$$

where $\sigma_{\bar{N}N \rightarrow e^+e^-}$ is the cross section for $\bar{N}N \rightarrow e^+e^-$ reaction with initial state $\bar{N}N$ interaction, $\sigma_{\bar{N}N \rightarrow e^+e^-}(V = 0)$ is the same cross section but without one (both nuclear and annihilative into hadron channels interactions are switched off). The enhancement factor is shown as a function of $\bar{N}N$ c.m. relative momentum. The linear behaviour is seen to be revealed approximately up to $k \sim$ ~ 100 MeV/c in the enhancement factor, moreover the slope parameter is connected with the scattering lengths as it was shown in the previous section. The comparison with a calculation of the scattering lengths in the same CCM shows the following: $\mathrm{Im}\, a\, ^3S_1$ (I = 0) = 0.34 fm and $\mathrm{Im}\, a\, ^3S_1$ (I = 1) = 0.29 fm from [7]; $\mathrm{Im}\, a\, ^3S_1$ (I = 0) = 0.36 fm and $\mathrm{Im}\, a\, ^3S_1$ (I = 1) = 0.33 fm from slope parameters in Fig. 1, i.e. the approximation (1) is valid with 10% accuracy for $k \sim 100$ MeV/c.

In Fig. 2(a,b) the energy dependence of proton and neutron form-factors correspondingly in time-like region are shown. Experimental points are taken from ref. [2,3]. The behaviour of proton electromagnetic form-factor G is seen from the figures to be correct (within the experimental accuracy), but its absolute value depends strongly on the input parameter G_0, i.e. on the model used in space-like region. Moreover, the main energetic behaviour of G is given by the initial state interaction, because input form-factor G_0 is practically constant in the energy region discussed here (see Fig. 2(a) - dashed curve). As for the neutron electromagnetic form-factor, it's interesting to note that its value in the case of isotopic independent input G_0 parameter is very small (curve 2 on Fig. 2(b)). If one of the isotopic form-factors is dominant, the neutron form-factor will be closed to the proton one (for instance, curves 1 in Fg. 3(a) and (b)).

The inverse $e^+e^- \to \bar{N}N$ reaction is of the interest also. In principle, using this process one could measure a nucleon form-factor G just near $\bar{N}N$ threshold. A cross section for this reaction is connected with $\bar{N}N \to e^+e^-$ by the formula:

$$\sigma(e^+e^- \to \bar{N}N) = \left(\frac{k}{M}\right)^2 \sigma(\bar{N}N \to e^+e^-)$$

In Fig.3 the dependence of the value $\Delta R = \sigma(e^+e^- \to \bar{p}p)/\sigma_{\mu\mu}$ as a function of $\bar{p}p$ c.m. energy is shown ($\sigma_{\mu\mu}$ is the cross section for the $e^+e^- \to \mu^+\mu^-$ reaction).

Conclusion

The consideration given above can be summarized by the next way. The electromagnetic nucleon form-factor G behaviour near $\bar{N}N$ threshold is dominantely determined by the interaction in the inital state. So the annihilation cross section $\bar{N}N \to e^+e^-$ is enhanced, the enhancement coefficient being a fast increasing function with approaching to $\bar{N}N$ threshold. The exerimental observation of the linear G behaviour at small relative momentum in $\bar{p}p$ channel would be a significant indication of a determinative role of the interaction in the initial state.

The channel $\bar{N}N$ could affect significantly on a behaviour of ΔR for annhilaton $e^+e^- \to$ hadrons (the value ΔR is determined by the enhancement coefficient, which value is depended on the nuclear forces in $\bar{N}N$ channel).

The investigation of the electromagnetic nucleon form-factor in the nearthreshold region, as it was mentioned above, could served the source of the important information about $\bar{N}N$ interaction in the state with vector quantum numbers. This fact added to other data could be efectively used for determination of the true parameters of $\bar{N}N$ interaction and the nature of heavy vector mesons in the mass region 1.5 - 2 GeV.

The authors would like to thank Prof. I.S.Shapiro for useful discussions.

Appendix

The task was mentioned above to have been solved by the variable phase approach. The main difficulty in this case is to use this technique for δ-function potential. The problem with δ-function in the nondiagonal potential (the transition $\bar{N}N$ to e^+e^-) was solved analogically to the case, when δ-function were in diagonal interaction [9]: δ-function was considered as limit case of square well with zero width and unlimited depth. This procedure gives rise to changing the boundary condition at zero when we integrate the equation. The matrix equation has the form:

$$K'(r) = - [Y(r) + K(r)\,Z(r)]\;W(r)\;[Y(r) + Z(r)\,K(r)] \qquad (A.1)$$

where $K(r)$ is the reaction matrix for potential $V_{ij}(r)$, $W_{ij}(r) = \sqrt{M_i\,M_j}\cdot V_{ij}$; M_i is the particle mass in the channel i;

$$Y_{ij}(r) = \frac{1}{\sqrt{k_i}}\,j_l(k_i r)\cdot\delta_{ij},$$

$$Z_{ij}(r) = \frac{1}{\sqrt{k_i}}\,n_l(k_i r)\cdot\delta_{ij},$$

j_l, n_l are Bessel and Neuman functions (all channels are open); V_{ij} is the potential matrix: $V_{11} = V_{\bar{N}N}^{OBEP}$; $V_{12} = V_{21} = V_{ann}$, other matrix elements of V_{ij} are equal to zero.

The equation (A.1) was integrated with the boundary condition not $K(0) = 0$ (as it would be without δ-function potential), but $K(0) = K^0$, where

$$K^0_{ij} = C_0\,k_i k_j \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} ,$$

C_0 was normalized on the input form-factor.

We would like to note once more that the δ-function potential gives rise only to transition between states with orbital moment equal to zero, so we need Bessel and Neuman functions only for $l = 0$.

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Figure captions

Fig. 1. The enhancement coefficient (see text) for different isospins as a function of c.m. momentum in $\bar{p}p$ system.

Fig. 2(a). The proton electromagnetic form-factor as a function of c.m. energy in $\bar{p}p$ system. Data are taken from [2, 3]. Solid curve 1 corresponds to the calculations from this work using G_0 calculated in VDM with coupling constants for ρ, ω, φ-mesons from [7] (the curve is normalized on experimental value at k = 300 MeV/c); solid curve 2 is the calculation of G with G_0 from dipole formula; dashed curve corresponds to dipole fit.

(b). The same as in fig. 2(a) for neutron.

Fig. 3. The value $\Delta R = \sigma_{e^+e^- \to \bar{p}p} / \sigma_{\mu\mu}$ as a function of c.m. energy. Solid curve corresponds to present calculation with form-factor G (solid curve 1 in fig. 2(a)), dashed curve is the same without the interaction in initial state.

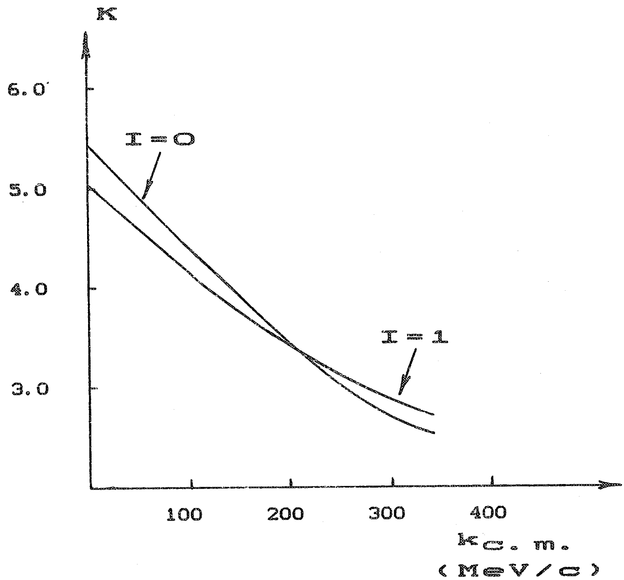
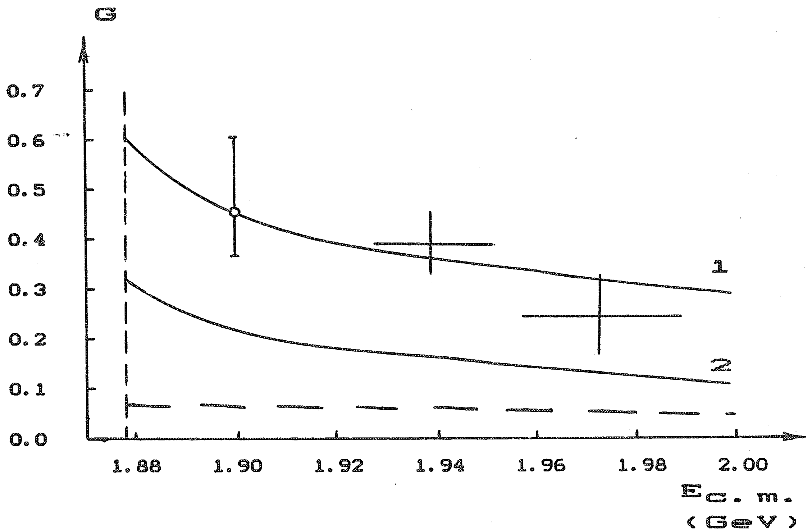


Fig. 1


G
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1
2
1.88
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1.96
1.98
2.00
Ec.m.
(GeV)

Fig. 2(a)

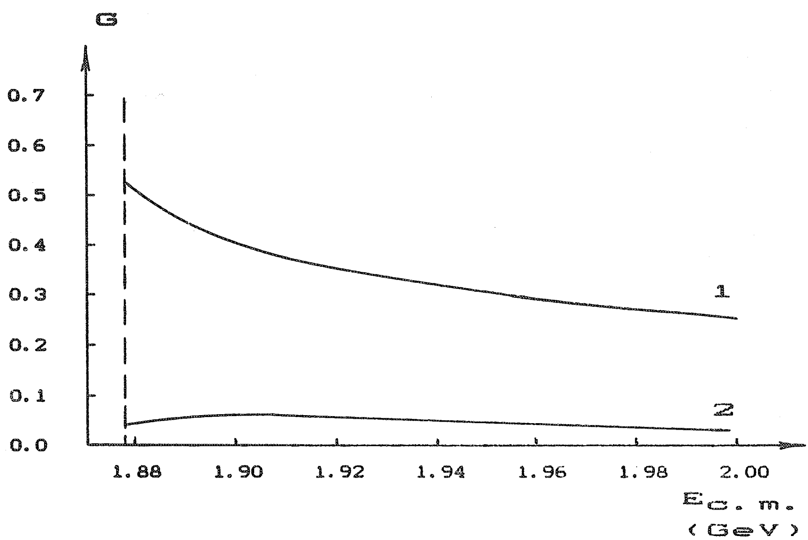
G
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Fig. 2(b)

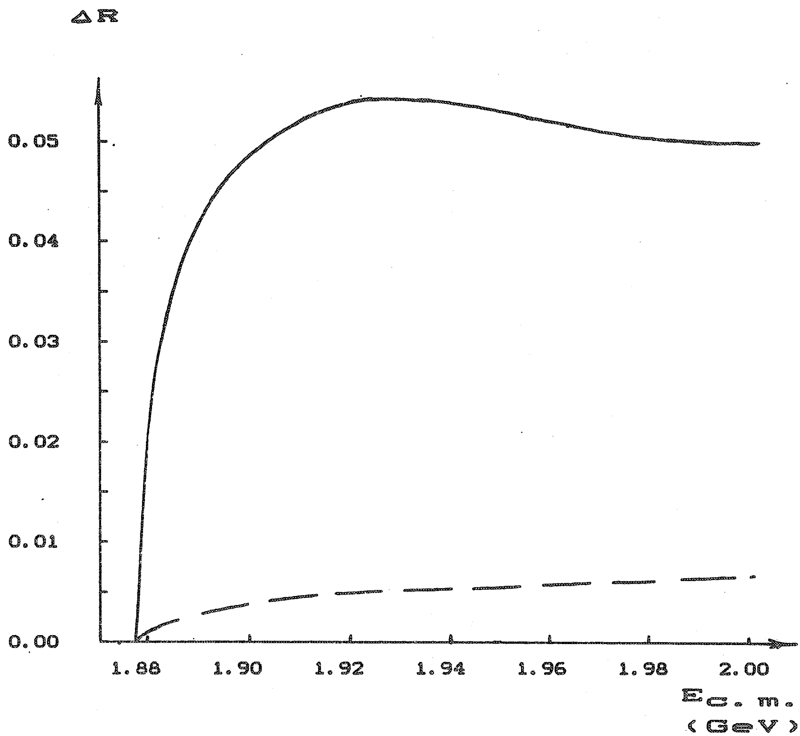

Fig. 3

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