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ANALYTICAL TREATMENT OF THE SPIN-SPLITTER

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ANALYTICAL TREATMENT OF THE SPIN-SPLITTER

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Abstract: the spin-orbit coupling associated with the interaction of the magnetic moment of $p(\bar{p})$, circulating in a low energy storage ring, with appropriate field gradients of quadrupole elements in the machine, is studied analytically. The conditions for a separation of opposite spin states are discussed and methods are proposed for working at very low energies.

1. - INTRODUCTION

To obtain polarized antiprotons in LEAR[1)] a device called Spin-Splitter has been proposed[2)] on the basis of a method for separating opposite spin states in a storage ring by using the Stern-Gerlach effect of the inhomogeneous field of quadrupoles on the circulating particles[3)].

Although the effect of the field gradient on the magnetic moment is extremely small compared with the effect of the field on the charge of the circulating particles, the choice of specific conditions on the orbit and spin motion can produce a constructive build-up separation between opposite spin states in phase-space over many revolutions.

The Spin-Splitter in its basic configuration in a storage ring is a straight section equipped with two quadrupoles of opposite polarity interspaced with a solenoid that rotates the spin 180° around the beam axis (Fig. 1).

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For a realistic project to be implemented in LEAR, simulation calculations taking into account the specific characteristics of the machine are necessary[4].

The basic properties of a storage ring equipped with a Spin-Splitter can be however more trasparently illustrated through an analytical treatment characterized also by a broader generality of application[5].

The main approximations for this treatment are essentially a thin-lens description of the quadrupoles of the Spin-Splitter and the assumption of a fully compensated solenoid (for what concerns the coupling between horizontal and vertical oscillations).

Some of the results will take a very simple form in a smooth-machine approximation for the storage ring, although this simplification is not essential for the method to work.

2. - TRANSVERSE MOTION

Particles with opposite spin states are distinguished by a term $\vec{B}\cdot\vec{\mu}$ in the Hamiltonian.

In any magnetic structure with transverse fields $\vec{B} \equiv (B_x\ 0,\ B_z)$, neglecting the fringe fields, the Hamiltonian referring to both horizontal and vertical motions can be written as

$$H_\perp = \frac{1}{2\gamma\ m_p} (P_x^2 + P_z^2) - e\beta c\ A_y + U_{scalar} \tag{2.1a}$$

with

$$U_{scalar} = - \vec{\mu} \cdot \vec{B} = \mp \mu\ (S_x\ B_x + S_z\ B_z) \tag{2.1b}$$

where S_x, S_y, S_z are the components of the unit vector corresponding to the classical magnetic moment $\vec{\mu} = \mu\hat{s}$.

In particular, for a single sheer quadrupole:

$$A_y = 1/2\ G(x^2-z^2), \qquad \vec{B} \equiv (Gz,\ 0,\ Gx) \tag{2.2a}$$

while, for a single skew quadrupole:

$$A_y = G\,x\,z, \qquad \vec{B} \equiv (-Gx,\ 0,\ Gz) \tag{2.2b}$$

Bearing in mind that $p_x = p\,x' = (\beta\gamma\ m_p c)x'$ and $p_z = (\beta\gamma\ m_p c)z'$, eq. (2.12) is divided by $\beta^2\gamma\ m_p\ c^2$ giving rise to the reduced Hamiltonian

$$H = 1/2\ (x'^2+z'^2) - \frac{k^2}{2L_Q} \left| \begin{matrix} x^2-z^2 \\ 2\,x\,z \end{matrix} \right| \pm \frac{\eta'}{L_Q} \left| \begin{matrix} z\,S_x + x\,S_z \\ x\,S_x - z\,S_z \end{matrix} \right| \tag{2.3}$$

where $k^2 = (eG\ L_Q)/p = 1/f_Q$ and η' is the kick given by the quadrupole.

A kick, given by any transverse localized force $\vec{F}$ to the trajectory of a particle, results in a sudden variation of the trajectory slope. This change of direction, can be expressed as the ratio of the acquired transverse momentum-component $p_\perp$ to the total momentum p of the particle, i.e.

$$\eta' = p_\perp/p \simeq F\tau/p \tag{2.4}$$

(L is the length of the region where the force acts and βc is the velocity of the particle), $p = \beta\gamma\ m_p c$, $\gamma = (1-\beta^2)^{-1/2}$ and m_p is the (anti) proton mass; then:

$$\eta' = \frac{F\ L}{\beta^2\gamma\ m_p\ c^2} \tag{2.5}$$

In our case $\vec{F} = \vec{\nabla}(\vec{\mu}\cdot\vec{B}) = \mu G$ and therefore

$$\eta' = \frac{G\mu\ L_Q}{\beta^2\gamma\ m_p\ c^2} \tag{2.6}$$

with $\mu = 1.41 \times 10^{-26}\ JT^{-1}$, $m_p\ c^2 = 938$ MeV $= 1.503 \times 10^{-10}$ J.

Therefore $\beta^2\gamma\eta' = 9.38 \times 10^{-17}\ G\ L_Q$, which demonstrates that the lower the energy, the bigger the kick, for a fixed quadrupole.

For example, with $G = 20\ T\ m^{-1}$ and $L_Q = 0.5$ m, eq. (2.6) yields $\beta^2\gamma\ \eta' = 9.38 \times 10^{-16}$ rad; at low energy ($\gamma \simeq 1$, $\beta \simeq 0.1$) the kick is $\eta' \simeq 8.48 \times 10^{-14}$ rad.

For a sheer focussing quadrupole eq. (2.3) yields:

$$\begin{vmatrix} x \\ x' \\ z \\ z' \end{vmatrix} = \begin{vmatrix} \cos KL_Q & \frac{1}{K}\sin K\,L_Q & 0 & 0 \\ -K\sin KL_Q & \cos K\,L_Q & 0 & 0 \\ 0 & 0 & \cosh K\,L_Q & \frac{1}{K}\sinh K\,L_Q \\ 0 & 0 & K\sinh K\,L_Q & \cosh K\,L_Q \end{vmatrix} \begin{vmatrix} x_0 \\ x'_0 \\ z_0 \\ z'_0 \end{vmatrix} \pm \eta' \begin{vmatrix} 0 \\ s_z \\ 0 \\ s_x \end{vmatrix} \qquad (2.7a)$$

Taking into account the full action of the doublet formed by (F quadrupole) - Solenoid - (D quadrupole), eq. (2.7a) becomes

$$\begin{vmatrix} x \\ x' \\ z \\ z' \end{vmatrix} = \begin{vmatrix} 1-\frac{L_s}{f} & L_s & 0 & 0 \\ -\frac{L_s}{f^2} & 1+\frac{L_s}{f} & 0 & 0 \\ 0 & 0 & 1+\frac{L_s}{f} & L_s \\ 0 & 0 & -\frac{L_s}{f^2} & 1-\frac{L_s}{f} \end{vmatrix} \begin{vmatrix} x_0 \\ x' \\ z_0 \\ z'_0 \end{vmatrix} \pm \eta \begin{vmatrix} 0 \\ s_z \\ 0 \\ s_x \end{vmatrix} \qquad (2.7b)$$

assuming the thin-lens ($KL_Q<<1$) approximation and neglecting the coupling effects of the solenoid; η is an enhanced kick factor coming from the combined effect of a suitably chosen configuration for the splitter (see Appendix I).

Both eqs. (2.7a) and (2.7b) represent the affine transfer map, referring respectively to a single kick and to a double (enhanced) kick.

For one turn in the machine, the transfer map of the lattice is the affine transformation

$$\begin{vmatrix} x \\ x' \\ z \\ z' \end{vmatrix} = \begin{vmatrix} M_H & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & M_V \end{vmatrix} \begin{vmatrix} x_0 \\ x'_0 \\ z_0 \\ z'_0 \end{vmatrix} \pm \eta \begin{vmatrix} 0 \\ s_z \\ 0 \\ s_x \end{vmatrix} \qquad (2.7c)$$

where

$$M_H = \begin{vmatrix} \cos\mu_H + \alpha_H \sin\mu_H & \beta_H \sin\mu_H \\ -\gamma_H \sin\mu_H & \cos\mu_H - \alpha_H \sin\mu_H \end{vmatrix} = I \cos\mu_H + J\sin\mu_H = e^{J\mu_H} \qquad (2.8a)$$

$$M_V = \begin{vmatrix} \cos\mu_V + \alpha_V \sin\mu_V & \beta_V \sin\mu_V \\ -\gamma_V \sin\mu_V & \cos\mu_V - \alpha_V \sin\mu_V \end{vmatrix} = I \cos\mu_V + J\sin\mu_V = e^{J\mu_V} \qquad (2.8b)$$

$$\text{N.B.} \left[J = \begin{bmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{bmatrix}, \text{ with } J \times J = -I \right]$$

$$\mu_{H-V} = 2\pi\, Q_{H-V}, \text{ with } Q_{H-V} = \text{Horizontal-Vertical Betatron Tune} \qquad (2.8c)$$

Besides eq. (2.7c) can be written as $\vec{W} = M\,\vec{W}_0 + \vec{W}_\eta$ which, iterated n-times, yields:

$$\vec{W}_n = M^n\,\vec{W}_0 + \sum_{k=0}^{n-1} M^k\,\vec{W}_\eta \qquad (2.9)$$

where $\vec{W}_\eta$ is a four-dimensional vector of modulus η, with the sign of the spin-state.

3. - SPIN-PRECESSION IN $\vec{E}$ AND $\vec{B}$ FIELDS

The precession equation of a charged particle with polarization $\vec{P}$ is

$$d\vec{P}/dt = \vec{\Omega}_s \times \vec{P} \tag{3.1}$$

where the three-component vector $\vec{P}$ can be expressed as

$$\vec{P} = \psi^+ \vec{\sigma} \psi \tag{3.2}$$

in the quantum mechanics case of spin one-half particles; having in eq. (3.2)

$$\psi = \begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix} \tag{3.3a}$$

a two-component column vector, or spinor and $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ where

$$\sigma_x = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \quad \sigma_y = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}, \quad \sigma_z = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \tag{3.3b}$$

are the usual Pauli matrices.

Besides the vector $\vec{\Omega}_s$ in eq. (3.1) is given by[6)]

$$\vec{\Omega}_s = - \left[\frac{e}{\gamma m} (1+\gamma a)\vec{B} - (\gamma-1)a \frac{(\vec{B}\cdot\vec{v})\vec{v}}{v^2} + \gamma \left(a + \frac{1}{\gamma+1}\right) \frac{\vec{E}\times\vec{v}}{c^2}\right] \tag{3.4}$$

with

$$a = \frac{g-2}{2} = 1.7928 \quad (g = 5.5856) \tag{3.5}$$

for (anti) protons.

For $\vec{E} = 0$ and $\vec{B} = \vec{B}_{\parallel}$ (i.e. $(\vec{B}\cdot\vec{v})\vec{v} = v^2\vec{B}$), eq. (3.4) reduces to:

$$\vec{\Omega}_s = -e/\gamma m \, (1+a) \, \vec{B}_{\parallel} = - \, e/\gamma m \, g/2 \, \vec{B}_{\parallel} \tag{3.6}$$

Eq. (3.6) is useful to evaluate the amount $\emptyset$ of precession after crossing a solenoid of length L_s:

$$\emptyset = |\vec{\Omega}_s| t_{crossing} = |\vec{\Omega}| \frac{L_s}{\beta c} = g/2 \; \frac{e\, B_{\parallel}}{\beta\gamma} \frac{L_s}{mc} = g/2 \; \frac{e\, B_{\parallel}\, L_s}{p} \qquad (3.7)$$

On the other hand, if $\vec{B}\perp\vec{v}$ ($\vec{B}\cdot\vec{v} = 0$) and $\vec{E} \neq 0$, eq. (3.5) becomes:

$$\vec{\Omega}_s = - e/\gamma m \left[(1+a\gamma)\vec{B} + (a + \frac{1}{\gamma+1}) \frac{\vec{E}\times\vec{v}}{c^2} \right] \qquad (3.8)$$

which deserves some comments.

In order to proceed it is convenient to write down the motion equation of a charged particle in $\vec{E}$ and $\vec{B}$ fields:

$$\frac{d}{dt} (\gamma m\vec{v}) = e\; (\vec{E}+\vec{v}\times\vec{B}) \qquad (3.9a)$$

$$\gamma m \frac{d\vec{v}}{dt} = e\; (\vec{E}+\vec{v}\times\vec{B}) - \frac{d\gamma}{dt}\, m\vec{v} \qquad (3.9b)$$

but $\frac{d\gamma}{dt} = \frac{1}{mc^2} \frac{d}{dt} E_{tot} = \frac{1}{mc^2} \frac{d\, E_{kin}}{dt} = \frac{e\vec{E}\cdot\vec{v}\; dt}{mc^2}$, then eq. (3.9b) becomes

$$\gamma m \frac{d\vec{v}}{dt} = e\; (\vec{E}+\vec{v}\times\vec{B}) - \frac{e}{c^2}\; (\vec{v}\cdot\vec{E})\vec{v}$$

or

$$\frac{d\vec{v}}{dt} = \frac{e}{\gamma m} (\vec{E}+\vec{v}\times\vec{B}) - \frac{e}{\gamma m} \frac{(\vec{v}\cdot\vec{E})\vec{v}}{c^2} \qquad (3.10)$$

By considering that $(\vec{v}\cdot\vec{E})\vec{v}-v^2\vec{E} = (\vec{E}\times\vec{v})\times\vec{v}$ and that $c^{-2}-v^{-2} = -1/(\gamma^2-1)c^2$, eq. (3.10) can be re-written as:

$$\frac{d\vec{v}}{dt} = - \frac{e}{\gamma m} (\vec{B} + \frac{\vec{E}\times\vec{v}}{v^2}) \times \vec{v} + \frac{e}{\gamma m} \frac{(\vec{v}\cdot\vec{E})\vec{v}}{c^2(\gamma^2-1)} \qquad (3.11)$$

The first term on the right side of eq. (3.11) represents the modified (by $\vec{E}$) cyclotron frequency

$$\vec{\Omega}_c = - \frac{e}{\gamma m} (\vec{B} + \frac{\vec{E}\times\vec{v}}{c^2}) \tag{3.12}$$

while the second term

$$\delta\vec{v} = \frac{(\vec{v}\cdot\vec{E})\vec{v}}{\gamma(\gamma^2-1)V_p} = \frac{c^2 E}{\gamma^3 V_p} \cos \hat{vE} \frac{\vec{v}}{v} \qquad (V_p = \frac{m_p c^2}{e} = 938 \text{ MV}) \tag{3.13}$$

vanishes only when the field $\vec{E}$ is "exactly" perpendicular to the velocity $\vec{v}$ of the particle: the validity and the limitations of this statement will be discussed later.

In the hypothesis of $\vec{E} \perp \vec{v}$ eq. (3.11) reduces to

$$d\vec{v}/dt = - \vec{\Omega}_c \times \vec{v} \tag{3.14}$$

meaning that the motion of the particle is just a rotation, that is the typical motion inside a storage ring or circular accelerator.

The precession rate of the polarization $\vec{P}$, measured in a system which follows the particle, is

$$\vec{\Omega}_a = \vec{\Omega}_s - \vec{\Omega}_c = - \frac{e}{\gamma m} \left[a\gamma \vec{B} + \gamma (a - \frac{1}{\gamma^2-1}) \frac{\vec{E}\times\vec{v}}{c^2} \right] \tag{3.15}$$

which is also known as shifted-frequency.

The effect of the electric field can be minimized by setting

$$a - 1/(\gamma^2-1) = 0$$

or

$$\gamma = \sqrt{\frac{1+a}{a}} = 1.248 \quad (p = 0{,}701 \text{ GeV/c}) \tag{3.16}$$

("magic energy").

Anyway, either for the "magic energy" or for just $\vec{E} = 0$, eq. (3.15) gives:

$$\vec{\Omega}_a = -(e\vec{B}/\gamma m)(a\gamma) \qquad (3.17)$$

which allows to evaluate the spin precession, in the rotating system, as

$$\emptyset = |\vec{\Omega}_a| t = a\gamma \frac{eB}{\gamma m} t = a\gamma\, \Theta \qquad (3.18)$$

where Θ is just the azimuth-variation during the time t.

4. - SPINOR TRANSFORMATIONS AND CHOICE OF THE ENERGY

The spinor (3.3a) characterizing a particle, whose polarization $\vec{P}$ undergoes the precession (3.18), fulfill the transformation[7]

$$\psi(\Theta) = [I \cos \Phi/2 - i\, \sigma_z \sin \Phi/2]\, \psi(0) \qquad (4.1)$$

with $\Phi = a\gamma\Theta$, of course.

If the perturbation is inserted at a generic azimuth Θ, the trivial solution of the full revolution ($\Theta = 2\pi$ in eq. (4.1)) has to be split into two parts, described by the following matrices:

$$M_1 = I \cos \lambda\chi/2 - i\sigma_z \sin \lambda\chi/2 \qquad (4.2a)$$

$$M_2 = I \cos (1-\lambda)\, \chi/2\;\; i\sigma_z \sin(1-\lambda)\, \chi/2 \qquad (4.2b)$$

with

$$\lambda = \Theta/2\pi \qquad \text{and} \qquad \chi = 2\pi a\gamma \qquad (4.2c)$$

(Notice that $M_1M_2 = M_2M_1 = I \cos \pi a\gamma - i\sigma_z \sin \pi a\gamma$); then the matrix M_{pert} is inserted between M_1 and M_2, giving rise to

$$M = M_1\, M_{pert}\, M_2 \qquad (4.3)$$

In the case of our spin-splitter

$$M_{pert} = I \cos \varphi/2 - i\sigma_y \sin \varphi/2 \tag{4.4}$$

being φ the angle of precession around the direction of motion. Thus eq. (4.3) will yield:

$$M = I\, C_0 - i\sigma C_x - i\sigma_y C_y - i\sigma_x C_z \tag{4.5a}$$

with

$$C_0 = \cos \varphi/2 \cos a\gamma\pi = \cos\xi \tag{4.5b}$$

$$C_x = \sin \varphi/2 \sin a\gamma\ (\Theta-\pi) \tag{4.5c}$$

$$C_y = \sin \varphi/2 \sin a\gamma\ (\Theta-\pi) \tag{4.5d}$$

$$C_z = \cos \varphi/2 \sin a\gamma\pi \tag{4.5e}$$

The periodic solution $\vec{n}$, which is the spin equivalent to the betatron closed-orbit, has components

$$n_{x,y,z} = \pm \frac{C_{x,y,z}}{\sqrt{1-C_0^2}} = \pm \frac{C_{x,y,z}}{\sin\xi} \tag{4.6a}$$

or

$$n_x = \pm \frac{\sin a\gamma(\Theta-\pi)}{\sin\xi} \sin \varphi/2 \tag{4.6b}$$

$$n_y = \pm \frac{\cos a\gamma(\Theta-\pi)}{\sin\xi} \sin \varphi/2 \tag{4.6c}$$

$$n_z = \pm \frac{\sin a\gamma\pi}{\sin\xi} \cos \varphi/2 \tag{4.6d}$$

The main effect of the solenoid installed in the ring is that it acts as a Siberian Snake: it modifies the topology of spin motion in such a way that the spin transfer matrix, calculated at the azimuth opposite to the spin-splitter, is $-i\sigma_y$ (a rotation of π around the propagation axis).

It can be shown that in this direction the polarization is stable, if the closest depolarization resonances are weak enough; the other two spin components instead will be mixed in a time inversely proportional to the strength of the resonance.

Since the spin-splitter arrangement implies that $n_y = 0$ at $\Theta = \Theta_{\text{spin-splitter}} = 0$, eq. (4.6c) gives

$$a\gamma = \nu_s = K + 1/2 = \text{half-integer} \tag{4.7}$$

Then $C_0 = 0$, $\sin\xi = 1$, $\sin a\gamma(\Theta-\pi) = \sin a\gamma\pi = \pm 1$. besides, after setting

$$\varphi = \pi - 2\,\delta\varphi$$

eqs. (5, 6b,c,d) become:

$$\left.\begin{aligned} n_x &= \pm\cos\delta\varphi \qquad (4.8a)\\ n_y &= 0 \qquad (4.8b)\\ n_z &= \pm\sin\delta\varphi \qquad (4.8c)\end{aligned}\right\}\ \text{(at the spin-splitter)}$$

or, for $\delta\varphi = 0$ $(\varphi = \pi)$:

$$\left.\begin{aligned} n_x &= \pm 1 \qquad (4.9a)\\ n_y &= 0 \qquad (4.9b)\\ n_z &= 0 \qquad (4.9c)\end{aligned}\right\}\ \text{(at the spin-splitter)}$$

Notice that, for $\varphi = \pi$, $C_0 = C_z = 0$ however: then eqs. (4.6) become for $\Theta_{\text{spin-splitter}} = \Theta \neq 0$:

$$n_x = \pm\sin a\gamma\,(\Theta-\pi) \tag{4.10a}$$

$$n_y = \pm\cos a\gamma\,(\Theta-\pi) \tag{4.10b}$$

$$n_z = 0 \tag{4.10c}$$

The half-integer condition (4.7), and the trivial statement that γ cannot be smaller than 1, force the choice of the energy: in fact $\nu_s = 0.5$ would imply $\gamma = 0.28$, $\nu_s = 1.5$ would give $\gamma = 0.84$ (both unphysical) and at last $\nu_s = 2.5$ yields $\gamma = 1.394$ or $p = 0.912$ Gev/c. This would obviously affect the Stern-Gerlach beam separation (see 2.6): being now $\beta\gamma = 0.971$, versus $\beta\gamma = 0.107$ at $p = 0.100$ GeV/c, v_{sep} would decrease by a factor of 9.

5. - BUILD-UP OF SEPARATION

Considering the spin-separation in the vertical plane, the first term in the right-hand side of eq. (2.9) becomes $M_V^n = I \cos n\mu_V + J \cos n\mu_V$ and represents the stability of the vertical oscillations. Instead the second term, which represents the closed-orbit distortion due to the Stern-Gerlach kicks, reduces to

$$\sum_{k=0}^{n-1} M_V^k \begin{vmatrix} 0 \\ \eta \end{vmatrix} = \sum_{k=0}^{n-1} \begin{vmatrix} \beta_V \eta \sin k\mu_V \\ \eta(\cos k\mu_V - \alpha_V \sin k\mu_V) \end{vmatrix} = \begin{vmatrix} z_{co} \\ z'_{co} \end{vmatrix}_{\text{Spin-Splitter}} = \vec{w}_{co} \tag{5.1}$$

or

$$(z_{co})_{ss} = \beta_V \eta \sum_{k=0}^{n-1} \sin 2\pi k Q_V \tag{5.2a}$$

$$(z'_{co})_{ss} = \eta \left[\sum_{k=0}^{n-1} \cos 2\pi k Q_V - \alpha_V \sum_{k=0}^{n-1} \sin 2\pi k Q_V \right] \tag{5.2b}$$

After a quarter of betatron wave-length ($\alpha_V = 0$, $\mu_{advance} = \pi/2$), the closed-orbit vector is given by eqs. (5.2) times the matrix

$$\begin{bmatrix} 0 & \beta_V \\ -\frac{1}{\beta_V} & 0 \end{bmatrix}$$

or

$$(z_{co})_{Max} = \beta_v \eta \sum_{k=0}^{n-1} \cos 2 \pi Q_v = \beta_v \eta \frac{\sin n Q_v \pi \cos(n-1)Q_v\pi}{\sin Q_v\pi} \qquad (5.3a)$$

$$(z'_{co})_{corr} = - \eta \sum_{k=0}^{n-1} \sin 2 \pi k Q_v = - \eta \frac{\sin n Q_v\pi \sin(n-1)Q_v\pi}{\sin Q_v\pi} \qquad (5.3b)$$

when $\sin Q_v\pi \neq 0$.

For Q_v = integer eqs. (5.3a) and (5.3b) become respectively $n \beta_v\eta$ and 0.

Then, after n turns the maximal separation between the two closed-orbits, referring to the opposite spin states, is

$$\Delta z_M = 2 n \beta_v \eta \qquad (5.4)$$

In practice it is impossible to work exactly over an integer resonance with the extreme accuracy required. In fact, after several revolutions,(n-1) is undistinguishable from n; thus eq. (3.9a) becomes

$$\frac{z_{co}}{\beta_v\eta} \simeq \frac{\sin n Q_v \pi \cos Q_v \pi}{\sin Q_v \pi} = \frac{\sin 2 Q_v n \pi}{2 \sin Q_v \pi}$$

which, for $Q_v = k \pm \delta Q$ (k= integer), reduces to

$$\frac{z_{co}}{\beta_v\eta} \simeq \frac{\sin (2kn\pi \pm \pi n\delta Q)}{2 \sin (k\pi \pm \pi\delta Q)} = (-1)^k \frac{\sin(2\pi n\delta Q)}{2\sin(\pi\delta Q)} \qquad (5.5)$$

(NB. $(-1)^k$ is fixed once for ever, depending on the machine setting).

Since $|\sin 2\pi n \delta Q| = 1$, or $\delta Q = 1/4n$, and $\sin \pi\delta \simeq \pi\delta$, for δ small enough eq. (5.5) gives

$$\left| \frac{z_o}{\beta_v \eta} \right| \simeq \frac{1}{2\pi \delta Q} = (2/\pi)n$$

but with $\delta Q = 1/4n$, which is too a strict requirement.

Thus some tricks have to be conceived.

6. - SPIN-SPLITTER UPGRADES.

At this point we are in the condition to identify the major problems associated with the basic spin-splitter configuration:

- The choice of the energy is not completely free
- The machine should operate close to Q_v = integer
- The resonance condition is unsuitably strict.

In order to cope with these difficulties an additional device is introduced 6) which can have both functions of decoupling the energy of the machine from the half-integer condition (4.7) and the Stern-Gerlach separation build-up from any betatron resonance of the ring.

7. - ENERGY COMPENSATION SCHEME

In order to be able to work at any energy: in particular at p = 100 MeV/c which is the minimum attainable in LEAR, special energy-compensators have to be implemented. In fact, if the half-integer condition (4.7) is not fulfilled, a precession angle Φ results; i.e.

$$|\Phi| = \pi|f| = 2\pi\left|a\gamma - \frac{2k+1}{2}\right| \tag{7.1a}$$

or

$$|f| = |2a\gamma - (2k+1)| \tag{7.1b}$$

This precession angle can be compensated by a crossed $\vec{E}$-$\vec{B}$-fields "undeflecting" spinor rotator. The "undeflecting" condition is realized by setting the transverse electrostatic force opposite to the Lorentz force, (Fig. 2a):

$$e\,\vec{E} = -\,ev\times B \tag{7.2a}$$

or

$$\vec{E}\times\vec{v} = -\,v^2\vec{B} \tag{7.2b}$$

(Notice that the (7.2a) - related directions of $\vec{E}$ and $\vec{B}$ provide the sign of f).

If the magnetic and the electrostatic deflectors are just overlapped, it is convenient to use eq.(3.8) with the (7.2b) - condition, obtaining:

$$\vec{\Omega}_s = - \frac{e\vec{B}}{\gamma m} \frac{1+a}{\gamma} \tag{7.3}$$

Since no bending takes place (notice that by inserting eq. (7.2a) into eq. (3.12) we otain the trivial result $\vec{\Omega}_c = 0$), eq. (7.3) is slightly elaborated as follows:

$$|\vec{\Omega}_s| = \frac{d\Phi}{dt} = \frac{1+a}{\gamma} \frac{e\ B}{\gamma\ m} = \frac{Bc}{Vp} \frac{1+a}{\beta\gamma^2} \frac{ds}{dt} \tag{7.4}$$

having taken into account that

$$\frac{1}{\rho} = \frac{eB}{p} = \frac{eBc}{\beta\gamma\ m\ c^2} \text{ and } v = \beta c = \frac{ds}{dt}$$

From eq. (7.4) and eq. (7.2a) one obtains

$$\Phi = \frac{Bc\ L}{V_p} \frac{1+a}{\beta\gamma^2} = \frac{EL}{V_p} \frac{1+a}{(\beta\gamma)^2} \tag{7.5}$$

where L is the length of the crossed-fields "undeflector".

If the two magnetic and electrostatic regions are separated, like in Fig. 2b, one has to proceed in two steps:

i) magnetic deflectors ($\vec{E} = 0$: use eq. (3.18)):

$$\Phi_M = a\gamma \frac{\Theta}{2} + a\gamma \frac{\Theta}{2} = a\gamma\Theta \tag{7.6a}$$

ii) electrostatic deflector ($\vec{B} = 0$ in eq. (4.15)):

$$\vec{\Omega}_a = - \vec{\Omega}_c\ a\ \gamma\ (1 - \frac{a+1}{a\gamma^2}) \tag{7.6b}$$

with $\Omega_c = + \frac{e}{\gamma m} \frac{\vec{E}\times\vec{v}}{c^2}$, where the sign + takes into account the change of direction of the curvature. Then:

$$\Phi_E = -a\gamma\Theta(1- \frac{a+1}{a\gamma^2}) \tag{7.6c}$$

adding up eqs. (7.6a) and (7.6c) the result is

$$\Phi = \Theta \frac{1+a}{\gamma} \tag{7.6d}$$

Of course the magnetic deflection is

$$\Theta_M = 2 \frac{eB}{p} \frac{L_M}{2} = \frac{eB\ L_M}{\beta\gamma\ V_p}$$

while the electrostatic one is

$$\Theta_E = \frac{e\,E\,L_E}{\beta^2\gamma\; mc^2} = \frac{E\,L_E}{\beta^2\gamma V_p}$$

Inserting $|\Theta_M|=|\Theta_E| = \Theta$ ("underflecting" condition) into eq. (7.6d) we obtain:

$$\Phi = \frac{Bc\;L_M}{V_p}\;\frac{1+a}{\beta\gamma^2} = \frac{E\;L_E}{V_p}\;\frac{1+a}{(\beta\gamma)^2} \tag{7.7}$$

which coincide with eq. (7.5) if $L_M = L_E = L$.

The following general relations can be deduced from eq. (7.5):

$$\frac{BL}{\beta\gamma^2\;\Phi} = \frac{V_p}{(1+a)c} = 1.12 \text{ Tm} \tag{7.8a}$$

$$\frac{EL}{(\beta\gamma)^2\Phi} = \frac{V_p}{1+a} = 3.36\times10^8\text{V} \tag{7.8b}$$

For p = 100 MeV/c (γ = 1.006, $\beta\gamma \simeq \beta\gamma^2 \simeq$ 0.107), eq. (7.1b) gives f = 0.608 and eqs. (7.8) give BL $\simeq$ 0.23 Tm and EL = 7.11 MV.

8. - SPIN PRECESSION KICKS

In order to circumvent the difficulties arising from the requirement of working on an integer resonance (and with that level of accuracy!) alternate kicks of modulus δ are supplied to the particle polarization, by a pulsed crossed $\vec{E}$-$\vec{B}$ fields device, synchronized with the RF which bunches the antiproton beam and located, for the moment, at an arbitrary azimuth.

If the spin of the antiprotons forms an angle α with the motion-direction (y-axis) at the entrance of the first quadrupole of the spin-splitter, after one turn the precession angle is

$$(2k + 1/2)2\pi\; -\alpha+\delta = 4k\pi + \pi - \alpha + \delta$$

since α is reversed by the solenoid (bear in mind eqs (4.9)) and the crossed fields device has given a positive kick. After another turn the precession is

$$-4k\pi-\pi+\alpha-\delta+4k\pi+\pi-\delta = \alpha-2\delta$$

having now considered a negative kick, together with the usual 180° reversal. Iterating this procedure it is easy to show that the angles between the spin and the y-axis vary as follows

$$\alpha$$
$$-\alpha+\pi+\delta+4\pi$$
$$\alpha-2\delta$$
$$-\alpha+\pi+3\delta+4\pi$$

Since the Stern-Gerlach kick regards the projection of the spin on the x-axis, one has

$$\vec{w}_\eta = \beta_v \sin(\hat{sy}) \begin{vmatrix} 0 \\ \eta \end{vmatrix} \tag{8.1}$$

with $\sin(sy) = \sin\alpha$, $\sin(\alpha-\delta)$, $\sin(\alpha-2\delta)$, $\sin(\alpha-3\delta)$, and so on turn after turn.

In order to simplify our calculations, the matrix (2.8b) must be reduced to

$$M_v = \begin{vmatrix} \cos\mu_v & \beta_v \sin\mu_v \\ -\frac{1}{\beta_v}\sin\mu_v & \cos\mu_v \end{vmatrix} \tag{8.2}$$

i.e $\alpha_v = 0$ and $J = \begin{bmatrix} 0 & \beta_v \\ -\frac{1}{\beta_v} & 0 \end{bmatrix}$ or, if referred to the co-ordinate (z, β_v, z'),

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The simplified matrix (8.2) represents either a sheer approximation (smooth machine) or, more correctly, a situation where the vertical β-function has a maximum at the Spin-Splitter location, just for duly enhancing the Stern-Gerlach kicks.

Introducing the new variable

$$\tilde{z} = z - i\,\beta_v\, z' \qquad (i = \sqrt{-1}) \tag{8.3}$$

and applying the matrix (6.2) to all the couples of $z, \beta_v z'$ coordinates, we obtain after the 1st turn:

$$\tilde{z}_1 = (\cos\mu_v + i\sin\mu_v) z_0 - i(i\cos\mu_v - \sin\mu_v)\beta_v z_0' - i\beta_v \eta \sin(\alpha-\delta)$$

or

$$\tilde{z}_1 = e^{i\mu_v}\,\tilde{z}_0 - i\,\beta_v\,\eta\sin(\alpha-\delta)$$

after the 2nd turn:

$$\tilde{z}_2 = e^{2i\mu_v}\,\tilde{z}_0 - i\beta_v\eta\,[\sin(\alpha-2\delta) + e^{i\mu_v}\sin(\alpha-\delta)]$$

after the 3rd turn:

$$\tilde{z}_3 = e^{3i\mu_v}\,\tilde{z}_0 - i\beta_v\eta\left[\sin(\alpha-3\delta) + e^{i\mu_v}\sin(\alpha-2\delta) + e^{2i\mu_v}\sin(\alpha-\delta)\right]$$

and further iterating:

$$\tilde{z}_N = e^{Ni\mu_v}\,\tilde{z}_0 - i\beta_v\eta\sum_{h=1}^{N} e^{i(N-h)\mu_v}\sin(\alpha-h\delta)$$

or

$$\tilde{z}_N = e^{iN\mu_v}\left\{\tilde{z}_0 - i\beta_v\eta\left[\sin\alpha\sum_{h=1}^{N} e^{-ih\mu_v}\cos h\delta - \cos\alpha\sum_{h=1}^{N} e^{-ih\mu_v}\sin h\delta\right]\right\} \tag{8.4a}$$

or

$$\tilde{z}_N = e^{iN\mu_v}\left\{\tilde{z}_0 - \tfrac{1}{2}\beta_v\eta\left[\sin\alpha\left(\sum_{h=1}^{N} e^{-ih(\mu_v+\delta)} + \sum_{h=1}^{N} e^{-ih(\mu_v-\delta)}\right) + \right.\right.$$

$$\left.\left. + i\cos\alpha\left(\sum_{h=1}^{N} e^{-ih(\mu_v+\delta)} + \sum_{h=1}^{N} e^{-i(\mu_v-\delta)}\right)\right]\right\} \tag{8.4b}$$

If we want a blowing-up of the two closed-orbits, referring to the different spin-states, we must have either $\mu_v+\delta = 2\pi k$ or $\mu_v-\delta = 2\pi k$ (k = integer); chosing e.g.

$$\mu_v+\delta = 2\pi k \tag{8.5a}$$

$$\mu_v - \delta = 2\pi k - 2\delta \tag{8.5b}$$

follows immediately.

Besides, if $Q_v = k - \delta Q$, with δQ made as small as possible compatibly with the stability of the storage ring, one has from eqs. (8.5a) and (2.8c):

$$\delta = 2\pi\ \delta Q \tag{8.6}$$

Anyway, inserting eqs. (8.5) into eq. (8.4b) yields:

$$\tilde{z}_N = e^{iN\mu_v}\left\{\tilde{z}_0 - \tfrac{1}{2}\beta_v\eta\left[\sin\alpha\left(N + \frac{\sin(N+1)\delta}{\sin\delta}\, e^{iN\delta}\right) + i\cos\alpha\left(N - \frac{\sin(N+1)\delta}{\sin\delta}\right)e^{iN\delta}\right]\right\}$$

or

$$\tilde{z}_N = e^{iN\mu}\left\{\tilde{z}_0 + \tfrac{1}{2}\beta_v\eta\left[Ne^{-i\alpha} - \frac{\sin(N+1)\delta}{\sin\delta}\, e^{i(N\delta+\alpha)}\right]\right\} \tag{8.7}$$

since:

$$\sum_{h=1}^{N} e^{-ih(\mu_v+\delta)} = \sum_{h=1}^{N} e^{-ihk2\pi} = \sum_{h=1}^{N} 1 = N$$

$$\sum_{h=1}^{N} e^{-ih(\mu_v-\delta)} = \sum_{h=1}^{N} \left[\cos(2\pi hk - h2\delta) - i\sin(2\pi hk - h2\delta)\right]$$

$$= \Sigma\cos 2h\delta + i\,\Sigma\sin 2h\delta$$

$$= \frac{\sin(N+1)\delta}{\sin\delta}\,(\cos N\delta + i\sin N\delta)$$

For $\alpha = \pi/2$ eq.(8.7) becomes

$$\tilde{z}_N = e^{iN\mu_v}\left\{\tilde{z}_0 - \tfrac{1}{2}\, i\beta_v\eta\left[N + \frac{\sin(N+1)\delta}{\sin\delta}\, e^{iN\delta}\right]\right\} \tag{8.8}$$

or, recalling the definition (8.2), a particle with a certain spin state (i.e. with a defined sign of η), at the spin-splitter entrance after N turns, has the following coordinates:

$$z_N = z_0 \cos N\mu_v + \beta_v z'_0 \sin N\mu_v$$

$$z'_N = -\frac{z_0}{\beta_v} \sin N\mu_v + z'_0 \cos N\mu_v + \frac{1}{2} N\eta\left[1 + \frac{\sin(N+1)\delta}{\sin\delta} \frac{e^{iN\delta}}{N}\right]$$

which multiplied again by the matrix $\begin{bmatrix} 0 & \beta_v \\ -1/\beta_v & 0 \end{bmatrix}$ give the closed-orbit separation

$$\Delta z_{MAX} = N\, \beta_v \eta\, [1 + f(N)] \tag{8.9}$$

where f(N) is a cyclic function which fades off as the number N of revolutions increases.

Thus we have proven that, no matter how distant from an integer resonances the machine is, a building-up of the closed-orbit separation takes place, provided that the condition (8.6) is fulfilled.

This procedure allow us to create a resonant condition for the spin, via repeated Stern-Gerlach kicks, leaving out of account all the betatron resonances related to the motion in the ring of a charged particle.

In practice the spin is reversed each time the closed-orbit representative point crosses the z-axis in the (z,z') phase plane (see Fig. 3).

Now, taking into account eq. (8.6), eqs. (7.8) can be written as:

$$\frac{BL}{\beta\gamma^2 \delta Q} = \frac{4\pi\, V_p}{gc} = 7.04 \text{ Tm} \tag{8.10a}$$

$$\frac{EL}{(\beta\gamma)^2 \delta Q} = \frac{4\pi\, V_p}{g} = 2.11 \times 10^9 \tag{8.10b}$$

which for p = 0.1 GeV/c (again) and $\delta Q = 0.2$ (attainable now in LEAR or possibly smaller in the future) give

BL = 0.15 Tm (8.11a)

EL = 4.64 MV (8.11b)

which have to be modulated by the LEAR revolution frequency at p = 0.1 GeV/c: i.e. 405 KHz.

After a few turns of adjustment, trajectories of particles with opposite spin-states begin to part each other finding by themselves the due synchronism, giving rise to the separation (8.9) between the two closed orbits.

Even though these spin-states have opposite sign, the x-projection of the polarization varies according to the factor $\sin(\alpha - r\delta)$, or $\cos r\delta$ for $\alpha = \pi/2$. In practice, there will be two sets of particles, vertically separate at fixed azimuths according to eq. (8.9), with the spins of one a group lying on horizontal plane and oriented in some direction, while the spins of the other group lie again on the horizontal plane but are aligned in the opposite direction. Fig. 4 illustrates three possible orientations.

This situation can be exploited by pushing, e.g. via one-turn pulsed horizontal kick, the double-bumped beam into a vertical electrostatic (or pulsed, if necessary) separator, like the one shown in Fig. 5. Then a polarized beam, of unknown polarization, may be extracted (and even injected into another storage-ring!) while another beam will remain in the original ring, provided that a second pulsed horizontal kick is supplied to restore the correct orbit.

Once in one (or two) ring(s), it should be rather easy to check the beam polarization either via non-destructive methods, like the measure of intrabeam scattering (IBS) rate among-spin-aligned, or in a slightly destructive way by peeling the beam with a carbon-wire and detecting the scattering correlations.

Then, knowing the polarization and/or its time evolution, difficulties should no longer exist for fixing-adjusting-rotating the spin-state of the stored particles.

Of course the wire-peeling, and the IBS to a lesser extent, could be used also for triggering the pulsed kicker, mentioned above, in such a way as to have a beam splitting with a known spin-direction.

Anyway, all these experimental suggestions have to be further deepened and investigated, above all in view of that particular ring where the first trials will be accomplished.

9. - MISSED SPIN SPLITTER

Another way to avoid the drawbacks related to the integer-resonance condition consists of making the bunch to jump the spin-splitter, either by switching-off the solenoid or by-passing the whole system, each time the closed-orbit representative point crosses the z-axis in the (z-z') phase-plane as before.

Apart from the difficulties of implementing the hardware of this proposal, there is the further problem of synchronizing the z-axis crossing with the trigger of the jumping-device. Anyway this method should work in principle. In fact after $n = 1/(2\delta Q)$ turns, corresponding to the period during which the representative point remains in a half-phase-plane, we have:

$$\tilde{z}_n = e^{in\mu_v}\tilde{z}_0 + i\,\beta_v\eta\left[1+e^{i\mu_v}+e^{2i\mu_v}+\dots e^{(\pi-1)i\mu_v}\right]$$

then, having missed the spin-splitter, after other n-turns:

$$\tilde{z}_n = e^{2ni\mu_v}\tilde{z}_0+i\,\beta_v\eta\left\{\left[1+e^{i\mu_v}+\dots e^{(n-1)i\mu_v}\right]-\left[e^{ni\mu_v}+e^{(n+1)i\mu_v}+\dots e^{(2n-1)i\mu_v}\right]\right\}$$

iterating this process every n revolutions one obtains

$$\tilde{z}_N=e^{Ni\mu_v}+\tilde{z}_0+i\,\beta_v\eta\left[1+e^{i\mu_v}+e^{2i\mu_v}+\dots e^{(n-1)i\mu_v}\right]\cdot\left[1-e^{ni\mu_v}+e^{2ni\mu_v}-e^{3ni\mu_v}+\dots\right]$$

or choosing (e.g.) $N = 2mn$ = even integer:

$$\tilde{z}_N = e^{Ni\mu_v}\tilde{z}_0 + i\beta_v\eta\,\frac{(1-e^{ni\mu_v})^2}{1-e^{i\mu_v}}\,\frac{1-e^{m2ni\mu_v}}{1-e^{2mi\mu_v}} \tag{9.1}$$

having made use of the relation:

$$1+e^{ix}+e^{2ix}+\dots e^{(n-1)ix} = 1+a+a^2+\dots a^{n-1}$$

$$= \frac{1-a^n}{1-a}$$

$$= \frac{1-e^{nix}}{1-e^{ix}}$$

with $a = e^{ix}$.

But $\mu_v = 2\pi\ Q_v = 2\pi k+2\pi\delta Q = 2\pi k+\frac{\pi}{n}$, then

$n\ \mu_v = 2\pi\ n\ k+\pi$, $2n\ \mu_v = (2n\ k+1)\ 2\pi$, $m\ 2\ n\ \mu_v = m(2nk+1)\ 2\ \pi$

thus eq. (9.1) gives:

$$\tilde{z}_N = e^{Ni\mu}\,\tilde{z}_0 + 4\,m\,\beta_v\,\eta\,\frac{i}{1-e^{i\mu_v}} \qquad (9.2)$$

since:

$$1 - e^{ni\mu_v} = 1-(-1) = 2$$

$$\frac{1-e^{m2ni\mu_v}}{1-e^{2ni\mu_v}} = \frac{0}{0} = \frac{-m\;2\;ni}{-2ni} = m$$

Eq. (9.2) shows a growing-up of the closed-orbits separation proportional to N/2n.

10. - SPIN-PRECESSION LAG

A zero-order experiment can be performed, even using a coasting beam, to prove the validity of the method. It consists of checking the growth of the vertical beam dimensions over a huge number of revolution, by implementing a resonance of the periodic spin solution $\vec{n}$.

If the magnetic field in the solenoid does not have the due value to provoke a π precession of the spin, one has the $\vec{n}$-components shown in eqs. (4.8). This mean that turn after turn the precession lags, or preceeds, by an amount equal to $\delta\varphi$: then at the K-th crossing we have

$$\eta_k = \eta \cos k\,\delta\varphi \qquad (10.1)$$

hence

$$\tilde{z}_N = e^{iN\mu_v}\,\tilde{z}_{\sim} - i\beta_v\left[\eta_N + e^{i\mu_v}\,\eta_{N-1} + \ldots\, e^{(N-1)i\mu_v}\,\eta_1\right]$$

or

$$\tilde{z}_N = e^{iN\mu_v}\,\tilde{z}_0 - i\beta_v\,\eta\left[\cos N\,\delta\varphi + e^{i\mu_v}\cos(N-1)\delta\varphi + \ldots.\, e^{(N-1)i\mu_v}\cos\delta\varphi\right]$$

or

$$\tilde{z}_N = e^{iN\mu_v}\left\{\tilde{z}_0 - i\beta_v\,\eta\sum_{h=1}^{N} e^{-ih\mu_v}\cos h\,\delta\varphi\right\} \qquad (10.2)$$

which is just the diverging term in eq. (8.4a) when $\alpha = \pi/2$.

Opposite to the situation discussed for the spin precession kicks the spin states are in this case completely mixed, nevertheless giving rise to a broadening of the beam associated with the Stern-Gerlach kicks.

APPENDIX I

As mentioned in Section 1, the proposed Spin-Splitter consists of two quadrupoles with opposite polarity (F and D), interspaced by a solenoid. Then, in the thin lens approximation and neglecting (for the moment) any coupling between horizontal and vertical oscillations, one has the following behaviour:

1) after the F quadrupole:

$$\begin{vmatrix} z_1 \\ z_1' \end{vmatrix} = M_F \begin{vmatrix} z_0 \\ z_0' \end{vmatrix} + \begin{vmatrix} 0 \\ z_k' \end{vmatrix}_{(\text{1-st kick})} \tag{A1.a}$$

2) after the solenoid:

$$\begin{vmatrix} z_2 \\ z_2' \end{vmatrix} = M_S M_F \begin{vmatrix} z^0 \\ z_0' \end{vmatrix} + M_S \begin{vmatrix} 0 \\ z_k' \end{vmatrix}$$

3) after the D quadrupole:

$$\begin{vmatrix} z^3 \\ z_3' \end{vmatrix} = M_D M_S M_F \begin{vmatrix} z_0 \\ z_0' \end{vmatrix} + M_D M_S \begin{vmatrix} 0 \\ z_k' \end{vmatrix} + \begin{vmatrix} 0 \\ z_k' \end{vmatrix}_{(\text{2-nd kick})} \tag{A1.b}$$

where $z'_k = \eta' s_x$ is the elementary Stern-Gerlach kick in the quadrupole, with η' given by eq. (2.6).

(Notice that the 2nd kick in eq. (2.3b) has the same sign as the first kick, since the solenoid reverses the spin).

where

$$M_{\substack{F\\D}} = \begin{vmatrix} 1 & 0 \\ \pm \frac{1}{f} & 1 \end{vmatrix} , \; M_s \simeq M_o = \begin{vmatrix} 1 & L_s \\ 0 & 1 \end{vmatrix}$$

with

$$\frac{1}{f} = \frac{e\, GL_Q}{p} \simeq 30 \text{ m}^{-1}, \; L_Q = 0.5 \text{ m}, \; (L_S = 1 \text{ m}) \qquad \text{(A2)}$$

for $p = 100$ MeV/c and $G = 20$ T m^{-1}

Then eq. (A2) can be written as

$$\begin{vmatrix} z_{co} \\ z'_{co} \end{vmatrix} = \begin{vmatrix} z_3 \\ z'_3 \end{vmatrix} - M_D\, M_O\, M_F \begin{vmatrix} z_0 \\ z'_0 \end{vmatrix} = (M_D\, M_O + I) \begin{vmatrix} 0 \\ z'_k \end{vmatrix}$$

or

$$\begin{vmatrix} z_{co} \\ z'_{co} \end{vmatrix} = \begin{vmatrix} 2 & L_s \\ \frac{1}{f} & 2 + \frac{L_s}{f} \end{vmatrix} \begin{vmatrix} 0 \\ z'_k \end{vmatrix} = \begin{vmatrix} L_s\, z'_k \\ (2 + \frac{L_s}{f}) z'_k \end{vmatrix}$$

which demonstrates that, for high quadrupole-gradients and low momenta, the "optical -lever" due to the (quasi) free-flight in the solenoid, can enhance the double Stern-Gerlach kick as

$$\eta = z'_{co} = (2 + \frac{L_s}{f})\, z'_k \qquad \text{(A3)}$$

Again for $p = 100$ MeV/c, $L_s = 1$ m eq. (A3) yields a total kick of the order of $32\, z'_k \simeq 2.11\times10^{-12}$ rad.

The vertical separation, between two antiprotons with opposite spin-states, is given by the product of the angular kick (A3) times the betatron function β_v at the 2nd quadrupole of the Spin-Splitter. Then one can define as separation rate

$$v_{sep} = (2 + \frac{L_s}{f}) \frac{G_\mu\, L_Q\, \beta_v}{\tau_{rev}} = (2 + \frac{L_s}{f}) \frac{G_\mu\, L_Q\, \beta_v}{m_p c^2\, \tau_\infty} \frac{1}{\beta\gamma} = \frac{v_0}{\beta\gamma} \qquad \text{(A4)}$$

where

$$\tau_{rev} = \frac{\tau_\infty}{\beta}, \; \tau_\infty = \frac{C}{c} = \frac{\text{Ring Circumference}}{\text{Speed of Light}}$$

For Lear ($C = 78.54$ m, $\tau_\infty = 0.262$ μs, $\beta_V \simeq 10$ m) eq. (A4) gives $v_0 = 1.15\times10^{-6}$ ms^{-1} $\simeq$ 4mm/hour. Of course, the lower $\beta\gamma$ the faster the beam separation is built up in particular for $\beta\gamma = 0.107$ ($p = 100$ MeV/c), one obtains $v_{sep} \simeq 1.07\times10^{-5}$ ms^{-1} $\simeq$ 0.6/min $\simeq$ 4 cm/hour.

Notice how eq. (A3) gives an appreciable enhancement for $p \leq e\, G\, L_Q L_s \simeq 3$ GeV/c, becoming slowly equal to 2 as p increases, having kept the same values as in eq. (A2).

Of course, if the sequence of the quadrupole is inverted, the sign of f in eq. (A3) changes and the enhancing factor can be either reduced or lost (for $2 = -L_s/f$).

FIGURE CAPTIONS

Fig. 1 - The Spin-Splitter basic configuration.

Fig. 2a- Use of an E×B device to adjust the spin precession angle.

Fig. 2b- Separate electrostatic and magnetic deflectors, acting on the median horizontal plane.

Fig. 3 - Closed-orbit representative point evolution in the normalized phase space.

Fig. 4 - Three possible example of spin-orientation.

Fig. 5 - Sketch of an electrostatic deflector, acting vertically, capable of splitting the $p(\bar{p})$ beam in two beams with opposite spin-states.

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THE SPIN-SPLITTER

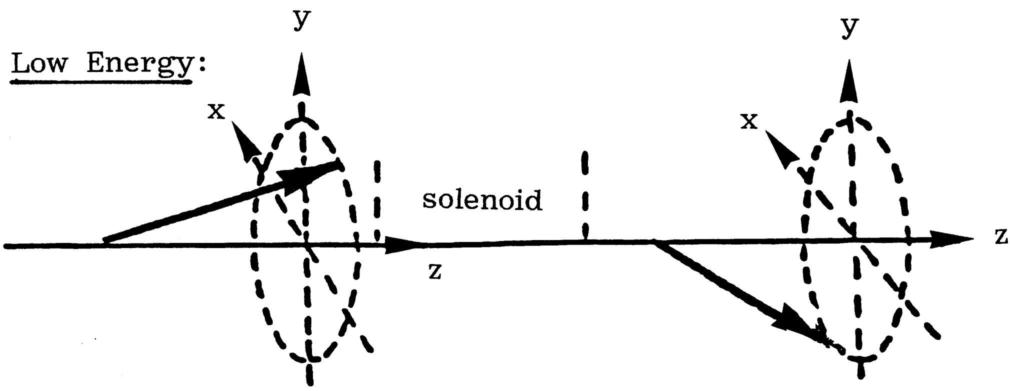

Fig. 1

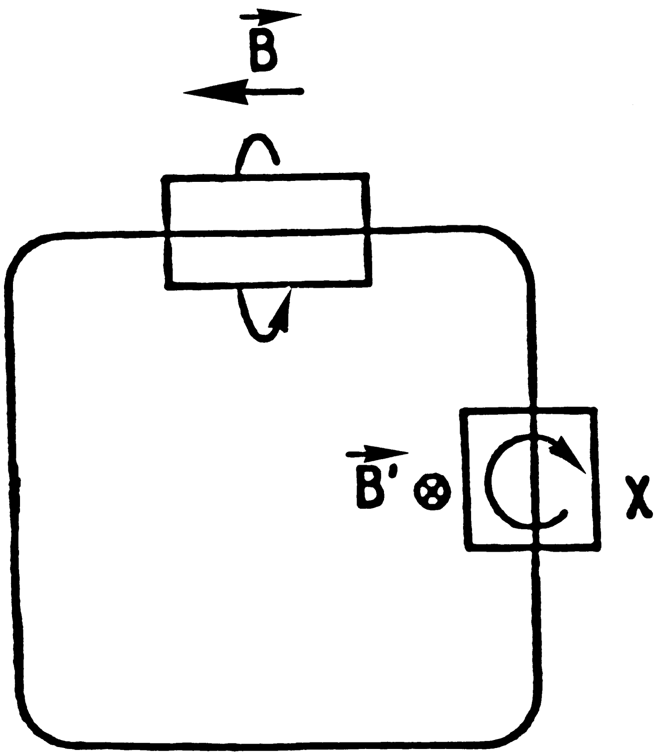

Fig. 2a

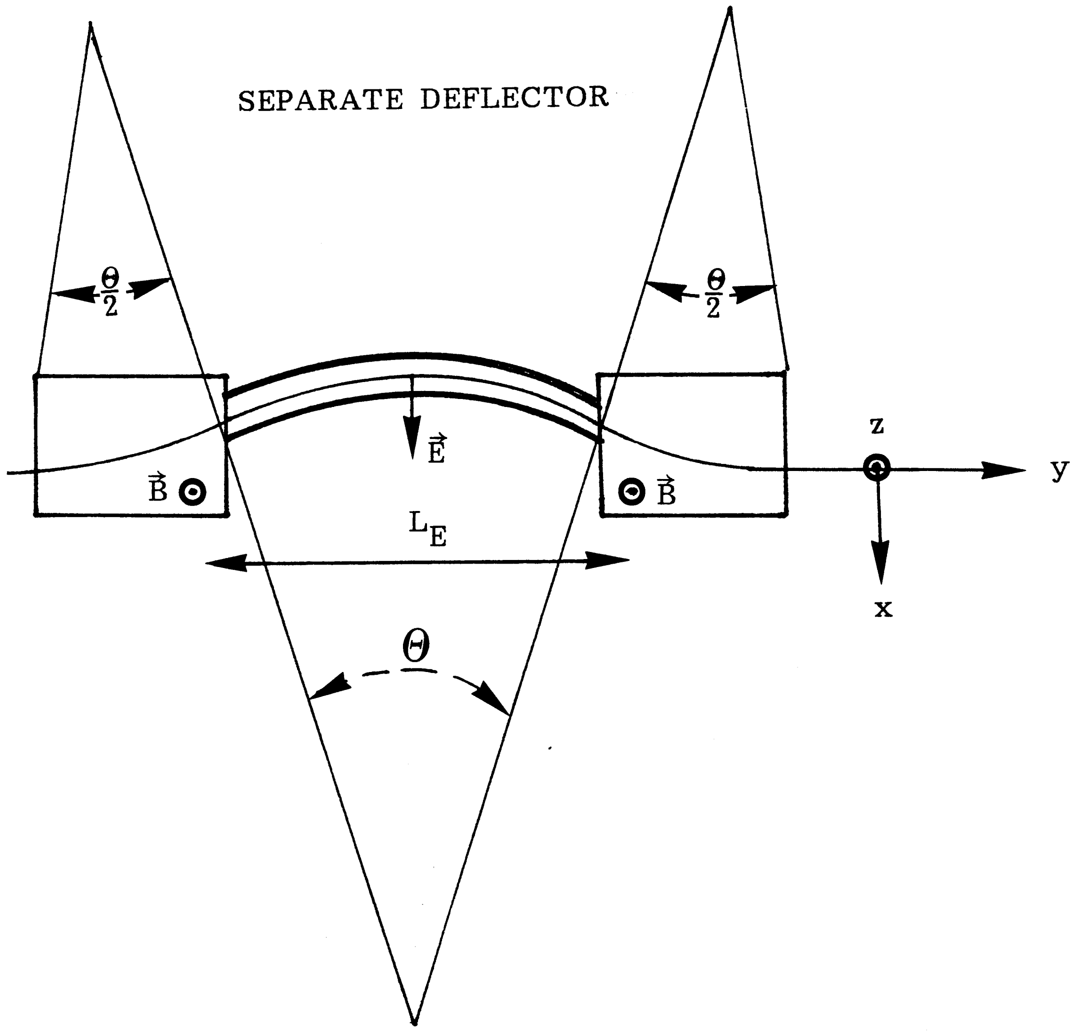

Fig. 2b

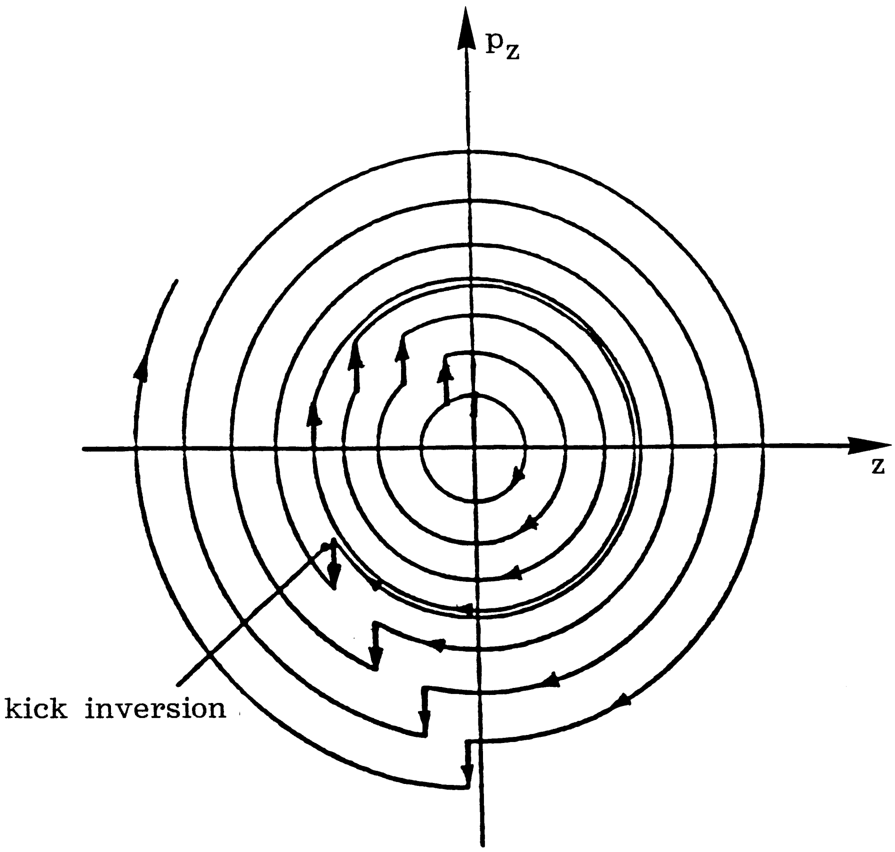

Fig. 3

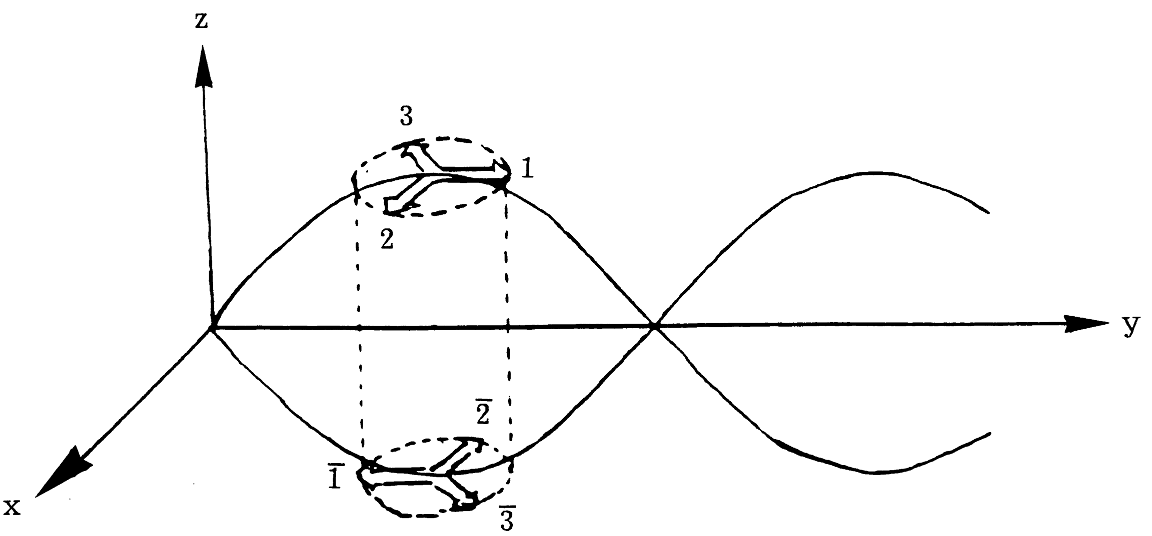

Fig. 4

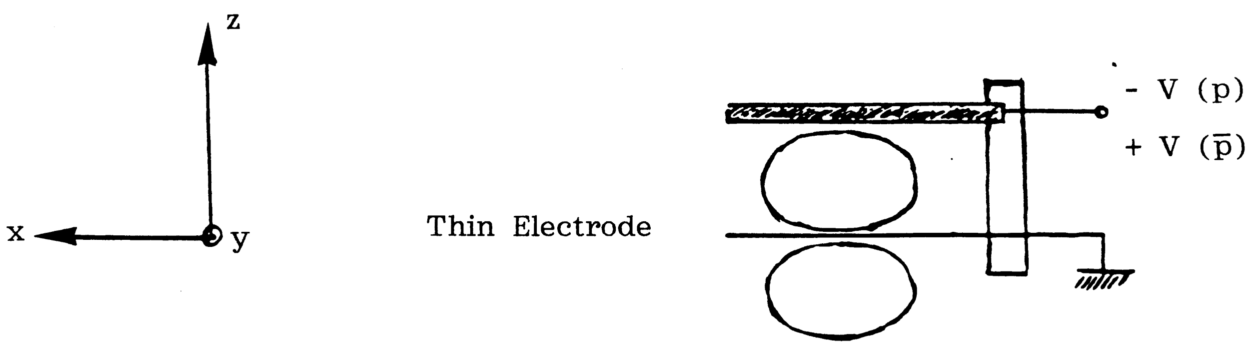

Fig. 5