IP-ASTP-10
December, 1987

$K \to \pi\pi\gamma$ and Chiral Perturbation Theory

Hai-Yang Cheng, S.-C. Lee, and Hoi-Lai Yu
Institute of Physics, Academia Sinica
Nankang, Taipei, Taiwan
Republic of China

ABSTRACT

Weak radiative decays $K_{L,S} \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$ are reexamined. The electromagnetic form factors and long-distance contributions to the direct photon emission are evaluated using the higher order effective chiral Lagrangian. We find that (1) the naive soft-pion theorem cannot be applied to the magnetic-type transition amplitude, (2) the short-distance contribution to $K_L \to \pi^+\pi^-\gamma$ is comparable to or even bigger than the long-distance one, (3) the $\Delta I = \frac{1}{2}$ enhancement persists in the decay $K^+ \to \pi^+\pi^0\gamma$, (4) to the order of $1/\Lambda_\chi^2$ (Λ_χ being the chiral-symmetry breaking scale) the direct photon emission amplitude does not receive a contribution from penguin operators, and (5) the $1/N_c$ expansion improves the discrepancy between theory and experiment.

I. Introduction

The radiative nonleptonic decay $K \to \pi\pi\gamma$ has been studied from time to time[1-5]. There are at least two motivations for this investigation: First, the process $K_L \to \pi^+\pi^-\gamma$ offers the interesting possibility of uncovering CP noninvariance at the decay-amplitude level. Second, it is very intersting to see if the $\Delta I = ½$ enhancement still persists in $K^+ \to \pi^+\pi^0\gamma$.

The decay $K \to \pi\pi\gamma$ consists of two pieces: inner bremsstrahlung and direct emission. The inner-bremsstbrahlung process in which a photon is radiated off from the charged mesons had been calculated in the past with predictions in excellent agreement with experiment (see, e.g. Ref. 3). On the other hand, it is very difficult to have a reliable estimate of the direct-emission (DE) amplitude. The long-distance contribution is usually treated within the framework of the vector-meson-dominance model. The main uncertainty arises from the very crude estimate of the photon-vector meson-pseudoscalar meson coupling. The short-distance effect is normally tackled using the effective weak Hamiltonian, current algebra and soft-pion theorems. While it was found by Malakian[2] that the short-distance contribution to $K_L \to \pi^+\pi^-\gamma$ is negligible, the calculation carried out by Lucio[3] however indicates that the short-distance effects are crucial. It is thus important to resolve this contradiction.

In this paper we wish to reexamine the $K \to \pi\pi\gamma$ processes from a different motivation, namely, the test of the recently proposed effective chiral Lagrangian through the measurement of DE rates of the radiative kaon decays. We notice that the gauge invariant DE amplitude is of third order in momenta. Within the context of chiral perturbation theory, this can only arise from the higher order weak Lagrangian. The long-distance contributions are generated by the anomalous Wess-Zumino terms. Athough the coupling constants of the higher-derivative chiral Lagrangian are unknown, we can apply the effective weak Hamiltonian and the vacuum-insertion method to calculate the short-distance effects. The electromagnetic form factors appearing in various hadronic matrix elements are then determined by the higher order Lagrangians. In the earlier references, for instance Ref. 3, all hadronic matrix elements are expressed in terms of the form factors F_V, F_A and f_+ via the soft-pion theorem. The values of the form factors are then taken from experiments.

Nowadays it is known that a complete next-order strong-interaction effective chiral Lagrangian with dimension-four operators including external gauge fields can be derived from a direct integration of both topological (Bardeen) and nontopological (spurious) chiral anomalies originating from quark loops. Using this higher order Lagrangian we are able to determine all relevant electromagnetic form factors in our calculations. We find out that the soft-pion technique cannot be applied to the magnetic transition (M1) amplitude (i.e.

the amplitude with the antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$). This means that in order to calculate the short-distance contribution to the M1 amplitude it is necessary to abandon the naive soft-pion theorem and appeal to the powerful chiral Lagrangian. Our computational formalism thus yields a more reliable estimate of both long-and short-distance effects.

This paper is organized as follows. In Sec. II we discuss the kinematics for $K \to \pi\pi\gamma$. Neglecting the momentum-transfer dependence in form factors we carry out the phase-space integrations. The relevant dimension-four chiral Lagrangian terms for $K \to \pi\pi\gamma$ are worked out in Sec. III. Sec. IV is devoted to the estimate of long-distance contributions. The effect of the $K - \eta'$ mixing is elaborated on. Short-distance effects are discussed in Sec. V. Numerical results and discussions are presented in Sec. VI. Our work is summarized in Sec. VII.

II. Kinematics

As mentioned in the Introduction, the radiative decay $K \to \pi\pi\gamma$ has two contributions: inner bremsstrahlung (IB) and direct emission (DE). In this paper we will focus on the latter case in which a photon is emitted from an internal state of the decay. Under Lorentz and gauge invariance, the invariant DE amplitude for the decay

$$K(k) \to \pi(p_1)\,\pi(p_2)\,\gamma(q)$$

has the form

$$A_{DE} = \tilde{\beta}\,\epsilon^{\mu\nu\rho\sigma} p^1_\mu p^2_\nu q_\rho \epsilon_\sigma + i\,\tilde{\gamma}\,[(p_1 \cdot \epsilon)(p_2 \cdot q) - (p_2 \cdot \epsilon)(p_1 \cdot q)] \tag{2.1}$$

where ϵ is the photon polarization vector. The first term on the r.h.s. of (2.1) corresponds to magnetic transitions whereas the second term is caused by electric transitions.

Neglecting the momentum dependence in the form factors $\tilde{\beta}$ and $\tilde{\gamma}$, we may numerically carry out the phase-space integrals and obtain the following DE decay rate

$$\Gamma_{DE} = \frac{\alpha m_K^7}{16\pi^2} \left(|\frac{\tilde{\beta}}{e}|^2 + |\frac{\tilde{\gamma}}{e}|^2 \right) h \tag{2.2}$$

where α is the fine-structure constant, $h = 4.83 \times 10^{-5}$ and 4.82×10^{-5}, respectively, for a 20 and 50 MeV photon-energy cutoff, corresponding to the experimental conditions of K_L and K_S radiative decays.[6] Experimentally, the DE rates of $K^+ \to \pi^+\pi^0\gamma$ are extracted in the charged-pion kinetic energy range of 55 to 90 MeV (Ref. 7). With this experimental condition we obtain $h = 2.13 \times 10^{-5}$ for $K^+ \to \pi^+\pi^0\gamma$. Eq (2.2) yields the following branching ratios for the direct emission[8]

$$\begin{aligned} B(K^+ \to \pi^+\pi^0\gamma)_{DE} &= (|\tilde{\beta}/e|^2 + |\tilde{\gamma}/e|^2)(1.32 \times 10^5\ GeV^6) \\ B(K^0_L \to \pi^+\pi^-\gamma)_{DE} &= (|\tilde{\beta}/e|^2 + |\tilde{\gamma}/e|^2)(1.33 \times 10^6\ GeV^6) \\ B(K^0_S \to \pi^+\pi^-\gamma)_{DE} &= (|\tilde{\beta}/e|^2 + |\tilde{\gamma}/e|^2)(2.28 \times 10^3\ GeV^6) \end{aligned} \tag{2.3}$$

Experimental results are listed in Table I.

III. The effective chiral Lagrangian

Within the framework of chiral perturbation theory the long-distance contributions to $K \to \pi\pi\gamma$ are generated by the usual Wess-Zumino terms. On the other hand, the following $\Delta S = 1$ weak chiral Lagrangians

$$\mathrm{Tr}(\lambda_6 F^{\mu\nu}_{R,L} D_\mu U^+ D_\nu U), \quad \mathrm{Tr}(\lambda_6 U^+ F^{\mu\nu}_L U F^R_{\mu\nu}) , \ldots$$

(The notation will be explained later) and the $\Delta S = 1$ Wess-Zumino terms are responsible for the short-distance effects. Unfortunately, the couplings of the higher order weak Lagrangians are unknown, hence we will employ the approach of the short-distance effective Hamiltonian and the method of vacuum insertion. We shall see that the form factors arising in various hadronic matrix elements are determined by the higher-derivative strong-interaction chiral Lagrangian coupled with external gauge fields.

It was realized in recent years that a complete next-order effective Lagrangian for strong interactions can be formally derived from the integration of the topologial and non-topological chiral anomalies. The integration of the proper (Bardeen) chiral anomaly gives rise to the Wess-Zumino (WZ) effective action[9], whereas the integration of spurious (non-topological) anomalies yields a unique quartic-derivative low-energy chiral Lagrangian for QCD.[10] Of course, this higher order Lagrangian may be derived from a different context, e.g. one-loop corrections to the nonlinear σ model.

In the following we write down the dimension-four chiral Lagrangian including external gauge fields A^L_μ and A^R_μ (Ref. 10)

$$\begin{aligned} L &= L_2 + L_4 \\ L_2 &= \frac{f_\pi^2}{8} \mathrm{Tr}\, D_\mu U^+ D^\mu U \\ L_4 &= \frac{1}{64\pi^2} \mathrm{Tr}\Big\{ (D_\mu U^+ D_\nu U)^2 + 2D^2 U^+ D^2 U - 2(L_\mu L^\mu)^2 + 2U^+ F^{\mu\nu}_L U F^R_{\mu\nu} \\ &\quad + 4(F^{\mu\nu}_R D_\mu U^+ D_\nu U + F^{\mu\nu}_L D_\mu U D_\nu U^+) \Big\} + L_{WZ} \end{aligned} \tag{3.1}$$

where

$$D_\mu U = \partial_\mu U + A^L_\mu U - U A^R_\mu$$

$$F^R_{\mu\nu} = \partial_\mu A^R_\nu - \partial_\nu A^R_\mu + [A^R_\mu, A^R_\nu]$$

$$F^L_{\mu\nu} = \partial_\mu A^L_\nu - \partial_\nu A^L_\mu + [A^L_\mu, A^L_\nu] \tag{3.2}$$

$$R_\mu = U^+ \partial_\mu U \quad , \quad L_\mu = \partial_\mu U U^+$$

$$U = \exp(i\, 2\phi^a \lambda^a / f_\pi) \quad , \quad \mathrm{Tr}(\lambda_a \lambda_b) = \delta_{ab}$$

$f_\pi \approx 130$ MeV is the pion decay constant, and $\phi = \phi^a \lambda^a$ describes the Goldstone boson fields π, K, η

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix} \tag{3.3}$$

The Wess-Zumino effective Lagrangian reads[9]

$$\begin{aligned} L_{WZ} = \frac{i}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr} \Big\{ &-(A^R_\mu R_\nu R_\rho R_\sigma + A^L_\mu L_\nu L_\rho L_\sigma) \\ &+ \frac{1}{2}(A^R_\mu R_\nu A^R_\rho R_\sigma - A^L_\mu L_\nu A^L_\rho L_\sigma) - A^R_\mu U^+ A^L_\nu U R_\rho R_\sigma \\ &+ A^L_\mu U A^R_\nu U^+ L_\rho L_\sigma + \partial_\mu A^R_\nu U^+ A^L_\rho U R_\sigma + \partial_\mu A^L_\nu U A^R_\rho U^+ L_\sigma \\ &+ (A^R_\mu \partial_\nu A^R_\rho + \partial_\mu A^R_\nu A^R_\rho + A^R_\mu A^R_\nu A^R_\rho + A^R_\mu U^+ A^L_\nu U A^R_\rho) R_\sigma \\ &+ (A^L_\mu \partial_\nu A^L_\rho + \partial_\mu A^L_\nu A^L_\rho + A^L_\mu A^L_\nu A^L_\rho + A^L_\mu U A^R_\nu U^+ A^L_\rho) L_\sigma \Big\} + \ldots \end{aligned} \tag{3.4}$$

where terms not relevant to our purposes are not included. From Eq. (3.1) it is evident that contributions from L_4 are suppressed by factors of order p^2/Λ_χ at low energies, where[11]

$$\Lambda_\chi = 2\pi f_\pi = 830 \text{ MeV} \tag{3.5}$$

is the scale of chiral symmetry breaking.[12]

Before proceeding two remarks are in order: (1) Explicit chiral-symmetry breaking adds a term $f_\pi^2 \mathrm{Tr}(M^+ U + MU^+)/8$ to L_2, where M is a meson mass matrix, and five mass-dependent terms to L_4 with five unknown coupling constants.[13] (2) The presence of the meson loop, which is necessary to preserve unitarity in the low-energy limit, amounts to renormalizing the "bare" coupling constants of L_4 and thus weakens the predictive power of the effective chiral Lagrangian. Nevertheless, the agreement between theory derived from the tree Lagrangian and experiment for various physical processes[14-17] implies that corrections to the soft-pion theorem are dominated by the dimension-four chiral Lagrangian L_4, as expected from the large-N_c argument. Therefore, we will consistently neglect the meson-loop effects in the present paper.

For physical applications discussed in this paper the external fields are identified with the photon A_μ and the left-handed $W_\mu^\pm$ boson fields

$$A_\mu^L = -i e A_\mu Q - 2i e \Lambda W_\mu \qquad (3.6)$$

$$A_\mu^R = -i e A_\mu Q$$

where

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}$$

and $\Lambda_{ij} = 1$ and vanishes otherwise for the quark current $\bar{q}_i \gamma_\mu (1-\gamma_5) q_j$. It should be stressed that the factor $(g/\sqrt{2}) \sin\theta_C$ (or $\cos\theta_C$) has been factored out in Eq. (3.6) since it will be absorbed in the effective weak Hamiltonian in Sec. V.

The magnetic-type transitions of $K \to \pi\pi\gamma$ are governed by the anomalous Wess-Zumino terms in Eq. (3.4). After a lengthy but straightforward calculation we find

$$\frac{e}{4\pi^2 f_\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\mu A_\nu W_\rho \mathrm{Tr}(3f_\pi \{\Lambda, Q\} \partial_\sigma \phi - 2 i\Lambda \{ \phi^Q, \partial_\sigma \phi \}$$
$$- 3i \{\Lambda, Q\} [\phi, \partial_\sigma \phi]) + \frac{ie}{\pi^2 f_\pi^3} \epsilon^{\mu\nu\rho\sigma} A_\mu \mathrm{Tr}(Q \partial_\nu \phi \partial_\rho \phi \partial_\sigma \phi) \qquad (3.7)$$

with

$$\phi^Q \equiv [Q, \phi] \quad .$$

The effect of the second and third terms in the Lagrangian L_4 [Eq. (3.1)] amounts to the

redefinition of decay constants f_π, f_K, f_η (Ref. 16). Hence we will not consider them in the ensuing discussion. Now Eq. (3.1) yields the results

$$-\frac{e}{4\pi^2 f_\pi^2} \Big\langle i f_\pi F^{\mu\nu} W_\mu \mathrm{Tr}(\Lambda \partial_\nu \phi^Q) + F^{\mu\nu} W_\mu \mathrm{Tr}(\Lambda^Q [\phi, \partial_\nu \phi])$$
$$+ 2W^{\mu\nu} A_\mu \mathrm{Tr}(\Lambda [\phi^Q, \partial_\nu \phi]) - 2W^\mu A^\nu \mathrm{Tr}(\Lambda^Q [\partial_\mu \phi, \partial_\nu \phi]) \Big\rangle \qquad (3.8)$$

which are relevant to the electric transitions of the radiative decay $K \to \pi\pi\gamma$ (Λ^Q being $[Q, \Lambda]$). We notice that the last two terms of (3.8) are not gauge invariant.

Finally we work out Eqs. (3.7) and (3.8) explicitly and retain those terms relevant to our later purposes

$$L = \frac{\delta}{4} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} W_\rho \Big\{ \partial_\sigma \pi^+ - 2\partial_\sigma K^0 + (\frac{4}{\sqrt{2}} \partial_\sigma \pi^0)_{11} + (-\frac{2}{\sqrt{2}} \partial_\sigma \pi^0)_{22} \Big\}$$
$$+ \frac{i\delta}{2f_\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} W_\rho \Big\{ -\frac{1}{\sqrt{2}} \pi^0 \overleftrightarrow{\partial}_\sigma K^+ + \frac{1}{2} \partial_\sigma (\frac{1}{\sqrt{2}} \pi^0 K^+ + \pi^+ K^0) +$$
$$+ \frac{1}{\sqrt{2}} \pi^0 \overleftrightarrow{\partial}_\sigma \pi^+ + (\pi^+ \partial_\sigma \pi^-)_{11} \Big\} - i\frac{\delta}{2} F^{\mu\nu} W_\mu \partial_\nu \pi^+ + \frac{\delta}{2f_\pi} F^{\mu\nu} W_\mu \Big\{ \frac{2}{\sqrt{2}} \pi^0 \overleftrightarrow{\partial}_\nu \pi^+$$
$$+ \frac{1}{\sqrt{2}} \pi^0 \overleftrightarrow{\partial}_\nu K^+ + \pi^+ \overleftrightarrow{\partial}_\nu K^0 \Big\} + \frac{ie}{\sqrt{2}\pi^2 f_\pi^3} \epsilon^{\mu\nu\rho\sigma} A_\mu \mathrm{Tr}(\partial_\nu \pi^0 \partial_\rho \pi^+ \partial_\sigma \pi^-$$
$$+ \frac{1}{\sqrt{3}} \partial_\nu \eta \partial_\rho \pi^+ \partial_\sigma \pi^- + \partial_\nu K^+ \partial_\rho K^- \partial_\sigma \pi^0) \qquad (3.9)$$

where $\delta = e/(2\pi^2 f_\pi)$, and the subscript 11 (22) denotes the contribution induced by the quark current $\bar{q}_{1(2)} \gamma_\mu (1-\gamma_5) q_{1(2)}$. The anomalous (normal) terms [i.e. terms with (without) the antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$] in Eq. (3.9) with one meson are even (odd) under charge conjugation, whereas the conclusion is opposite for the case with two mesons. It is straightforward to check that the naive soft-pion theorem cannot be applied to the anomalous terms as in the case of $\pi^0 \to \gamma\gamma$; for example, the anomalous contribution to $< \pi^0 \gamma | \bar{s}_L \gamma_\mu u_L | K^+ >$ is not related to that of $< \gamma | \bar{s}_L \gamma_\mu u_L | K^+ >$ by the usual soft-pion relation.

IV. Long-distance contributions

In principle the long-distance contributions to $K \to \pi\pi\gamma$ arise from the nonperturbative corrections to quark diagrams at the scale $\mu < 1$ GeV. However, they are not amenable to QCD perturbation theory. Therefore, the long-distance effect is generally treated in pole dominance model in which poles are the low-lying mesons $\pi, \eta, \eta', \ldots$ etc. In chiral perturbation theory the relevant pole diagrams for $K_L \to \pi^+\pi^-\gamma$ and $K^+ \to \pi^+\pi^0\gamma$ are depicted in Figs. 1 and 2. The $\phi\phi\phi\gamma$ vertex is induced by the Wess-Zumino Lagrangian and is given by the last term in Eq. (3.9). It is easily seen that the amplitude of Fig. 2c vanishes.

For $K^+(k) \to \pi^+(p^+)\pi^0(p^0)\gamma(q)$, the amplitude of Figs. 2a and 2b reads

$$A(K^+ \to \pi^+\pi^0\gamma)_{long} = -\epsilon^{\mu\nu\rho\sigma} p^+_\mu p^0_\nu q_\rho \epsilon_\sigma \frac{e}{\sqrt{2}\,\pi^2 f_\pi^3} \frac{1}{m_K^2 - m_\pi^2} [< \pi^+(k) | H_w | K^+(k) > - < \pi^+(p^+) | H_w | K^+(p^+) >] \tag{4.1}$$

Since the hadronic matrix element $< \pi^+(p) | H_w | K^+(p) >$ is proportional to p^2 (see, e.g. Ref. 18), the on-shell pole amplitude becomes

$$A(K^+ \to \pi^+\pi^0\gamma)_{long} = -\epsilon^{\mu\nu\rho\sigma} p^+_\mu p^0_\nu q_\rho \epsilon_\sigma \frac{e}{\sqrt{2}\,\pi^2 f_\pi^3} \frac{1}{m_K^2} < \pi^+(k) | H_w | K^+(k) > \tag{4.2}$$

The long-distance contribution to $K_L \to \pi^+\pi^-\gamma$ has been calculated by Lin and Valencia[5] with the result

$$\begin{aligned} A(K_L \to \pi^+\pi^-\gamma)_{long} &= \epsilon^{\mu\nu\rho\sigma} p^+_\mu p^-_\nu q_\rho \epsilon_\sigma \frac{e}{\sqrt{2}\,\pi^2 f_\pi^3} < \pi^0(k) | H_w | K_L(k) > \\ &\cdot \frac{1}{m_K^2 - m_\pi^2} \Bigg\{ 1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} \Big(\sqrt{\tfrac{1}{3}}(1+\xi)\cos\theta + 2\sqrt{\tfrac{2}{3}}\,\rho\sin\theta \Big) \\ &\cdot \Big[\sqrt{\tfrac{1}{3}}\Big(\frac{f_\pi}{f_8}\Big)^3 \cos\theta - \sqrt{\tfrac{2}{3}}\Big(\frac{f_\pi}{f_0}\Big)^3 \sin\theta \Big] + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_{\eta'}^2} \Big(\sqrt{\tfrac{1}{3}}(1+\xi)\sin\theta \\ &- 2\sqrt{\tfrac{2}{3}}\,\rho\cos\theta \Big) \Big[\sqrt{\tfrac{1}{3}}\Big(\frac{f_\pi}{f_8}\Big)^3 \sin\theta + \sqrt{\tfrac{2}{3}}\Big(\frac{f_\pi}{f_0}\Big)^3 \cos\theta \Big] \Bigg\} \end{aligned} \tag{4.3}$$

where f_8 and $f_0 \neq f_\pi$ if the SU(3)-breaking effect in $\eta_8, \eta_0 \to \pi\pi\gamma$ is included, ξ measures SU(3) breaking in relating the matrix element $< \eta_8 | H_w | K_L >$ to $< \pi^0 | H_w | K_L >$

$$< \eta_8 | H_w | K_L > = \sqrt{\tfrac{1}{3}}(1+\xi) < \pi^0 | H_w | K_L > \tag{4.4}$$

and θ is the mixing angle of the SU(3) octet η_8 and single η_0

$$\eta = \eta_8 \cos\theta - \eta_0 \sin\theta , \quad \eta' = \eta_8 \sin\theta + \eta_0 \cos\theta \tag{4.5}$$

with η, η' being the physical states. Since the matrix element $< \eta_0 | H_w | K_L >$ is not related to the K_L-π^0 transition by SU(3) symmetry, it is conventional to introduce a complex parameter ρ[19]

$$< \eta_0 | H_w | K_L > = -2\sqrt{\tfrac{2}{3}}\,\rho < \pi^0 | H_w | K_L > \tag{4.6}$$

If the $\Delta I = \frac{1}{2}$ $K \to \pi\pi$ amplitude is entirely responsible by the penguin interactions, then $\rho = 1$ as

$$< \eta_0 | O_5 | K_L > = -2\sqrt{\tfrac{2}{3}} < \pi^0 | O_5 | K_L > \tag{4.7}$$

where O_5 is the penguin operator given in Eq. (5.1).

At the tree level of L_{WZ}, $f_8 = f_0 = f_\pi$. One-loop corrections to the processes $\pi, \eta \to \pi\pi\gamma$ will modify f_8 and f_0, but, as we stated before, all loop effects will not be investigated throughout this paper. The one-loop calculation of ξ has been carried out in Ref. 20 with the result $\xi = 0.17$. However, ξ also receives contributions from the higher order Lagrangian L_4. For reason of consistency, we let $\xi = 0$. In realistic calculations performed in Sec. VI we put $f_8 = f_0 = f_\pi$ and $\xi = 0$, i.e. SU(3) breaking is not included.

Since the long-distance contribution to $K_L \to \pi^+\pi^-\gamma$ is very sensitive to the parameter ρ, it is important to have a reliable estimate of its value. Fortunately, the decay $K_L \to 2\gamma$ is dominated by the same poles, π, η and η'. Using $\theta = -20°$, as implied by the $1/N_c$ approach[13,21] and by the recent measurements of $\eta, \eta' \to 2\gamma$ rates, and the experimental results for $K_L \to 2\gamma$ we find[22]

$$\rho = (0.22 \pm 0.05) + 0.74\,\xi . \quad \text{or} \quad \rho = (0.63 \pm 0.05) + 0.74\,\xi \tag{4.8}$$

allowing a small (10^{-3}) imaginary part. The corresponding fraction of the $\Delta I = \frac{1}{2}$ amplitude due to the penguin interactions is 48% and 75%, respectively, for $\xi = 0$. It seems to us that the first solution $\eta = 0.22$ is more favored and plausible. It has been shown that perturbative penguin effects account for at most 5% of the $\Delta I = \frac{1}{2}$ transition[23]. Including nonperturbative effects to the penguin diagram (e.g. soft-gluon effects in "eye" graphs) will not likely enhance the penguin contribution up to 75% (i.e. $\rho = 0.63$).

Before ending this section we wish to stress that a priori there is no obvious reason that K_L, $K^+ \to \pi\pi\gamma$ should be dominated by long-distance contributions. The situation is different for a similar decay $K_L \to \gamma\gamma$: It is known that the long-distance effect is dominant in that mode. This is due to the fact that the quark-loop contribution to $K_L \to 2\gamma$ is dominated by u quarks.[24] Because of the smallness of the u-quark mass, the quark-loop effect is primarily of long-distance nature. As a consequence, the short-distance quark-loop contribution to $K_L \to \gamma\gamma$ is negligible. This argument cannot be directly taken over to the radiative decays of K_L and K^+.

V. Short-distance contributions

In this section we shall employ the effective weak Hamiltonian and the factorization method to study the short-distance contributions to $K \to \pi\pi\gamma$. The form factors appearing in the hadronic matrix elements are evaluated using the chiral Lagrangian described in Sec. III.

The effective $\Delta S = 1$ weak Hamiltonian has the form[25]

$$H_{eff} = -\sqrt{2}\, G_F \sin\theta_C \cos\theta_C \sum_{i=1}^{6} c_i O_i \tag{5.1}$$

with

$$\begin{aligned}
O_1 &= (\bar{s}d)(\bar{u}u) - (\bar{s}u)(\bar{u}d) \\
O_2 &= (\bar{s}d)(\bar{u}u) + (\bar{s}u)(\bar{u}d) + 2(\bar{s}d)(\bar{d}d) + 2(\bar{s}d)(\bar{s}s) \\
O_3 &= (\bar{s}d)(\bar{u}u) + (\bar{s}u)(\bar{u}d) + 2(\bar{s}d)(\bar{d}d) \\
O_4 &= (\bar{s}d)(\bar{u}u) + (\bar{s}u)(\bar{u}d) - (\bar{s}d)(\bar{d}d) \\
O_5 &= \bar{s}_L \gamma_\mu \lambda^a d_L (\bar{u}_R \gamma^\mu \lambda^a u_R + \bar{d}_R \gamma^\mu \lambda^a d_R + \bar{s}_R \gamma^\mu \lambda^a s_R) \\
O_6 &= \bar{s}_L \gamma_\mu d_L (\bar{u}_R \gamma^\mu u_R + \bar{d}_R \gamma^\mu d_R + \bar{s}_R \gamma^\mu s_R)
\end{aligned} \tag{5.2}$$

where $(\bar{q}_1 q_2) = \bar{q}_{1L} \gamma_\mu q_{2L}$, $q_{L,R} = \frac{1}{2}(1 \mp \gamma_5) q$, c_i are Wilson coefficients, and θ_C is the Cabibbo angle. The (V–A)(V+A) quark operators $O_{5,6}$ are induced from the penguin diagram, and O_4 is the only four-quark operator which is $\Delta I = \frac{3}{2}$. The short-distance effect due to the electromagnetic interactions $s \to d\gamma$ will not be considered here as it is negligible owing to the GIM cancellation. Indeed the coefficient of the operator $m_s \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} d_R$ was found to be very small[4].

To proceed we first evaluate the matrix elements of the (V–A)(V–A) operators $O_1 \ldots O_4$ and then turn to the penguin operators $O_{5,6}$. As an illustration let us consider the hadronic matrix element $\langle \pi^+\pi^-\gamma | O_1 | K^0 \rangle$

$$\begin{aligned}
\langle \pi^+(p^+)\pi^-(p^-)\gamma(q) | O_1 | K^0 \rangle &= (1 - \frac{1}{3}) \langle \pi^+\pi^-\gamma | (\bar{u}u) | 0 \rangle \langle 0 | (\bar{s}d) | K^0 \rangle \\
&+ (1 - \frac{1}{3}) \langle \pi^+\pi^- | (\bar{u}u) | 0 \rangle \langle \gamma | (\bar{s}d) | K^0 \rangle - (1 - \frac{1}{3}) \\
&\cdot \langle \pi^+\gamma | (\bar{u}u) | 0 \rangle \langle \pi^- | (\bar{s}d) | K^0 \rangle - (1 - \frac{1}{3}) \langle \pi^+ | (\bar{u}d) | 0 \rangle \\
&\cdot \langle \pi^-\gamma | (\bar{s}u) | K^0 \rangle
\end{aligned} \tag{5.3}$$

where the $\frac{1}{3}$ terms arise from the color mismatched currents. The matrix elements involving a photon can be directly read off from (3.9)[26]

$$\begin{aligned}
<\pi^+\pi^-\gamma \mid (\bar{u}u) \mid 0> &= -\frac{i\delta}{2f_\pi}\, \epsilon^{\mu\nu\rho\sigma}(p^+ - p^-)_\nu q_\rho \epsilon_\sigma \\
<\gamma \mid (\bar{s}d) \mid K^0> &= -\frac{1}{2}\,\delta\, \epsilon^{\mu\nu\rho\sigma} k_\nu q_\rho \epsilon_\sigma \\
<\pi^+\gamma \mid (\bar{u}d) \mid 0> &= \frac{1}{4}\,\delta(-\epsilon^{\mu\nu\rho\sigma} p^+_\nu q_\rho \epsilon_\sigma + i p^- \cdot q\, \epsilon_\mu - i p^+ \cdot \epsilon\, q_\mu\ ) \\
<\pi^-\gamma \mid (\bar{s}u) \mid K^0> &= \frac{i\delta}{4f_\pi}\,[\epsilon^{\mu\nu\rho\sigma}(k - p^-)_\nu q_\rho \epsilon_\sigma + i(q_\nu \epsilon_\mu - q_\mu \epsilon_\nu)(k + p^-)_\nu]
\end{aligned} \tag{5.4}$$

The remaining matrix elements are determined by the Lagrangian L_2 in (3.1)

$$\begin{aligned}
<0 \mid (\bar{s}d) \mid K^0> &= -<\pi^+ \mid (\bar{u}d) \mid 0> = \frac{1}{2}\, i f_\pi k_\mu \\
<\pi^+\pi^- \mid (\bar{u}u) \mid 0> &= \frac{1}{2}\,(p^+ - p^-)_\mu \\
<\pi^- \mid (\bar{s}u) \mid K^0> &= -\frac{1}{2}\,(k + p^-)_\mu
\end{aligned} \tag{5.5}$$

We notice that the second and third relations in (5.5) also can be obtained from the first matrix element via the soft-pion theorem. The inclusion of the higher order Lagrangian L_4 and chiral symmetry breaking will modify the kaon decay constant so that $f_\pi \to f_K$ and will introduce momentum-dependent form factors, for example[14,15]

$$\begin{aligned}
<\pi^-(p) \mid (\bar{s}u) \mid K^0(k)> &= -\frac{1}{2} f_+(t)\,(k + p)_\mu - \frac{1}{2} f_-(t)\,(k - p)_\mu \\
f_+(t) = 1 + \frac{t}{\Lambda_\chi^2} \quad &, \quad f_-(t) = \frac{f_K}{f_\pi} - 1 - \frac{k^2 - p^2}{\Lambda_\chi^2}
\end{aligned} \tag{5.6}$$

where $t = (k - p)^2$ and $\Lambda_\chi = 2\pi f_\pi$. However, for simplicity of ensuing calculations we will not consider the momentum-transfer dependence of the form factor f_+, keeping in mind that the contribution of the momentum dependence of f_+ is of order m_K^2/Λ_χ^2.

Substituting (5.5) and (5.4) into (5.3) yields the result

$$<\pi^+\pi^-\gamma \mid O_1 \mid K^0> = \frac{1}{4}\,(1 - \frac{1}{3})\,\delta\,(-3M - 2E) \tag{5.7}$$

with

$$M = \epsilon^{\mu\nu\rho\sigma} p^+_\mu p^-_\nu q_\rho \epsilon_\sigma \quad \text{and} \quad E = i\,[(p^+ \cdot \epsilon)(p^- \cdot q) - (p^+ \cdot q)(p^- \cdot \epsilon)]\,.$$

Likewise,

$$\begin{aligned}
<\pi^+\pi^-\gamma \mid O_{2,3} \mid K^0> &= \frac{1}{4}(1 + \frac{1}{3})\,\delta\,(-M + 2E) \\
<\pi^+\pi^-\gamma \mid O_4 \mid K^0> &= \frac{1}{4}\,(1 + \frac{1}{3})\,\delta\,(-7M + 2E) \\
<\pi^+\pi^0\gamma \mid O_1 \mid K^+> &= \frac{1}{4\sqrt{2}}(1 - \frac{1}{3})\,\delta\,(3M + 2E) \\
<\pi^+\pi^0\gamma \mid O_{2,3} \mid K^+> &= \frac{1}{4\sqrt{2}}(1 + \frac{1}{3})\,\delta\,(9M - 2E) \\
<\pi^+\pi^0\gamma \mid O_4 \mid K^+> &= \frac{1}{4\sqrt{2}}(1 + \frac{1}{3})\,\delta\,(3M - 2E)
\end{aligned} \tag{5.8}$$

We next turn to the penguin operators. It has been argued by Lucio[3] that contributions coming from $O_{5,6}$ vanish. Since the matrix elements of quark bilinears given in Ref. 3 are not correct, we present here a more rigorous derivation. Using the Fierz reordering theorem $(V+A) \to -2(S-P)$ and the relation

$$\lambda^a_{\mu\nu}\,\lambda^a_{\alpha\beta} = 2\delta_{\mu\beta}\delta_{\nu\alpha} - \frac{2}{3}\,\delta_{\mu\nu}\,\delta_{\alpha\beta} \tag{5.9}$$

we may recast the penguin operator O_5 to the form

$$O_5 = \frac{-32}{9}(\bar{s}_L u_R\, \bar{u}_R d_L + \bar{s}_L d_R\, \bar{d}_R d_L + \bar{s}_L s_R \bar{s}_R d_L) \tag{5.10}$$

Consider the matrix element $<\pi^+\pi^0\gamma \mid O_5 \mid K^+>$ in the vacuum-saturation approximation

$$\begin{aligned}
<\pi^+(p^+)\,\pi^0(p)\,\gamma \mid O_5 \mid K^+(k)> = \frac{-32}{9}\Big\{ &<\pi^+\pi^0\gamma \mid \bar{u}_R d_L \mid 0><0 \mid \bar{s}_L u_R \mid K^+> \\
&+ <\pi^+\pi^0 \mid \bar{u}_R d_L \mid 0><\gamma \mid \bar{s}_L u_R \mid K^+> + <\pi^+\gamma \mid \bar{u}_R d_L \mid 0> \\
&\cdot <\pi^0 \mid \bar{s}_L u_R \mid K^+> + <\pi^+ \mid \bar{u}_R d_L \mid 0><\pi^0\gamma \mid \bar{s}_L u_R \mid K^+>
\end{aligned}$$

$$+ <\pi^0\gamma \mid \bar{d}_R d_L \mid 0> <\pi^+ \mid \bar{s}_L d_R \mid K^+> + <\pi^0 \mid \bar{d}_R d_L \mid 0>$$

$$\cdot <\pi^+\gamma \mid \bar{s}_L d_R \mid K^+> \Big\} \tag{5.11}$$

The matrix elements of quark scalar and pseudoscalar densities are governed by the chiral representation[11,27]

$$\bar{q}_{Rj} q_{Li} = -\frac{1}{4} f_\pi^2 v U_{ij} - 2v \frac{f_\pi^2}{\Lambda_\chi^2} (D_\mu U D^\mu U^+ U)_{ij} \tag{5.12}$$

where

$$v = \frac{m_{\pi^-}^2}{m_u + m_d} = \frac{m_{K^-}^2}{m_s + m_u} = \frac{m_{K^0}^2}{m_d + m_s}$$

It leaves to the reader to prove that (Some of the quark bilinear matrix elements are given in Refs. 11 and 18)

$$<\pi^+\pi^0\gamma \mid O_5 \mid K^+> = i \frac{32}{9\sqrt{2}} \frac{v^2}{\Lambda_\chi^2} f_\pi (p^+ \cdot \epsilon) \tag{5.13}$$

Therefore, the penguin operator does not contribute to the direct photon emission since in order to produce the gauge and Lorentz invariant DE amplitude three factors of momentum are required. This conclusion also can be easily seen from the chiral representation of O_5

$$O_5 = -\frac{4}{9} f_\pi^4 \frac{v^2}{\Lambda_\chi^2} \mathrm{Tr}(\lambda_6 D_\mu U D^\mu U^+) \tag{5.14}$$

obtained by substituting (5.12) into (5.10)[28]. It is obvious that to the order of $1/\Lambda_\chi^2$ the penguin operator does not have enough derivatives to generate the DE amplitudes. Of course, the (V–A)(V+A) part of $O_{5,6}$ still has a contribution to the direct photon emission but it is suppressed by the smallness of the penguin coefficients (typically, $c_5 \sim -0.05$).

Noting that $K_{L,S} = \frac{1}{\sqrt{2}}(K^0 \pm \bar{K}^0)$ within the convention of the chiral Lagrangian and that the magnetic (electric) amplitude of $K^0 \to \pi^+\pi^-\gamma$ is opposite to (the same as) that of $\bar{K}^0 \to \pi^+\pi^-\gamma$ we obtain from Eqs. (5.1), (5.2) and (5.8)[29]

$$A(K_L \to \pi^+\pi^-\gamma)_{short} = G_F \sin\theta_C \cos\theta_C \frac{e}{4\pi^2 f_\pi} [-3c_1(1 - \frac{1}{3}) - (c_2 + c_3 + 7c_4)(1 + \frac{1}{3})] M$$

$$A(K_S \to \pi^+\pi^-\gamma)_{short} = G_F \sin\theta_C \cos\theta_C \frac{e}{2\pi^2 f_\pi} [c_1(1 - \frac{1}{3}) - (c_2 + c_3 + c_4)(1 + \frac{1}{3})] E$$

$$A(K^\pm \to \pi^\pm\pi^0\gamma)_{short} = G_F \sin\theta_C \cos\theta_C \frac{e}{8\pi^2 f_\pi} \Big\{ \pm 3[c_1(1 - \frac{1}{3})$$

$$+ (3c_2 + 3c_3 + c_4)(1 + \frac{1}{3})] M + 2[c_1(1 - \frac{1}{3}) - (c_2 + c_3 + c_4)(1 + \frac{1}{3})] E \Big\} \tag{5.15}$$

This together with the long-distance contributions (4.2) and (4.3) constitute the main results of this paper.

Owing to the zero contributions from penguin operators one may tempt to conclude that the $\Delta I = \frac{1}{2}$ enhancement is absent in the short-distance effect on $K^+ \to \pi^+\pi^0\gamma$. This agrument is however erroneous: First, it is known that perturbative penguin interactions (i.e. the penguin contributions at the scale $\mu \geqslant 1$ GeV) account for only ~5% of the $\Delta I = \frac{1}{2}$ $K \to \pi\pi$ amplitude.[18] Second, from (5.15) and (6.8) we see that the short-distance contribution to the $\Delta I = \frac{1}{2}$ amplitude of $K^+ \to \pi^+\pi^0\gamma$ dominates over the $\Delta I = \frac{3}{2}$ one and this becomes more manifest in the $1/N_c$ expansion addressed in the next section.

VI. Numerical results and discussions

Before embarking on a numerical calculation of the direct-emission rates, it is instructive to have a qualitative estimate. First, the IB rates are of order

$$\Gamma(K \to \pi\pi\gamma)_{IB} \sim \left(\frac{\alpha}{\pi}\right) \Gamma(K \to \pi\pi) \tag{6.1}$$

For $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$ decays, the IB contributions are suppressed by the $\Delta I = \frac{1}{2}$ rule and the smallness of CP violation, respectively. In chiral perturbation theory the DE amplitude, which is of third orders in momenta, is induced by the higher-order chiral Lagrangian L_4. As discussed in Sec. III, contributions of L_4 are suppressed by factors of p^2/Λ_χ^2. Consequently,

$$A_{DE}(K \to \pi\pi\gamma) \sim \left(\frac{m_K^2}{\Lambda_\chi^2}\right) A_{IB}(K \to \pi\pi\gamma) \tag{6.2}$$

and hence the branching ratio is expected to be

$$B(K \to \pi\pi\gamma)_{DE} \sim \left(\frac{\alpha}{\pi}\right)\left(\frac{m_K^2}{\Lambda_\chi^2}\right)^2 h \tag{6.3}$$

where h is a phase-space suppression factor given in Sec. II and is of order $(2-5) \times 10^{-5}$. Since the DE transition of $K^+ \to \pi^+\pi\gamma$ and $K_L \to \pi^+\pi^-\gamma$ is no longer subject to the $\Delta I = \frac{1}{2}$ rule and CP volation, respectively, so we expect that

$$B(K_S \to \pi^+\pi^-\gamma)_{DE} \sim \left(\frac{\alpha}{\pi}\right)\left(\frac{m_K^4}{\Lambda_\chi^4}\right) h \sim 10^{-8}$$

$$B(K_L \to \pi^+\pi^-\gamma)_{DE} \sim \left(\frac{\alpha}{\pi}\right)\left(\frac{m_K^4}{\Lambda_\chi^4}\right) h \frac{1}{\epsilon} \sim 10^{-5} \tag{6.4}$$

$$B(K^+ \to \pi^+\pi^0\gamma)_{DE} \sim \left(\frac{\alpha}{\pi}\right)\left(\frac{m_K^4}{\Lambda_\chi^4}\right) h \left(\frac{A_0}{A_2}\right)^2 \sim 4 \times 10^{-6}$$

where $A_0/A_2 = 22.2$ is the ratio of the $\Delta I = \frac{1}{2}$ to $\Delta I = \frac{3}{2}$ $K \to \pi\pi$ isospin amplitudes, and $\epsilon = 2 \times 10^{-3}$ characterizes the degree of CP noninvariance in the neutral kaon system. From (6.4) it is clear that the "normal" DE rates are of the order of 10^{-8}. If the experimental branching ratio of $K^+ \to \pi^+\pi^0\gamma$ lies in the range $10^{-5} - 10^{-6}$, it is then evident that the $\Delta I = \frac{1}{2}$ enhancement is operative in that decay.

Now we first give some numerical estimates of the long-distance contributions. The effective chiral Lagrangian of weak interactions leads to[18]

$$< \pi^0(p) \mid H_w \mid K_L(p) > \; = \; -4\frac{p^2}{f_\pi^2}\left(g_8 + g_{27}^{(1/2)} - 2g_{27}^{(3/2)}\right)$$

$$< \pi^+(p) \mid H_w \mid K^+(p) > \; = \; 4\frac{p^2}{f_\pi^2}\left(g_8 + g_{27}^{(1/2)} + g_{27}^{(3/2)}\right) \tag{6.5}$$

where g_8, $g_{27}^{(1/2)}$ and $g_{27}^{(3/2)}$ are the coupling constants of $\Delta S = 1$, $\Delta I = \frac{1}{2}$ octet and 27-plet and $\Delta I = \frac{3}{2}$ 27-plet interactions, respectively. From the experimental $K \to \pi\pi$ rates the coupling constants are determined to be[18]

$$(g_8 + g_{27}^{(1/2)})_{expt} = 0.26 \times 10^{-8}\ m_K^2 \quad , \; (g_{27}^{(3/2)})_{expt} = 0.86 \times 10^{-10}\ m_K^2 \tag{6.6}$$

Taking $\theta = -20°$, $f_8 = f_0 = f_\pi$, $\xi = 0$, $\rho = 0.22$, as discussed in Sec. IV, substituting (6.5) into (4.2) and (4.3), and noting $k^2 = m_K^2$ we find

$$K_L \to \pi^+\pi^-\gamma : \quad (\widetilde{\beta}/e)_{long} = 1.43 \times 10^{-6}\ \text{GeV}^{-3}$$

$$K^+ \to \pi^+\pi^0\gamma : \quad (\widetilde{\beta}/e)_{long} = -5.07 \times 10^{-6}\ \text{GeV}^{-3} \tag{6.7}$$

If we use $\rho = 1$ and the values of f_8, f_0 and ξ given in Ref. 5, we will obtain $(\widetilde{\beta}/e)_{long} = -3.63 \times 10^{-6}\ \text{GeV}^{-3}$ for $K_L \to \pi^+\pi^-\gamma$ (Ref. 30). However, we believe that $\rho = 1$, which corresponds to a $\Delta I = \frac{1}{2}$ enhancement entirely due to penguin interactions, is quite unrealistic and indeed inconsistent with $K_L \to 2\gamma$ data.

For short-distance contributions, we adopt the values[31]

$$c_1 = -2.38 \quad , \quad c_2 = 0.10 \quad , \quad c_3 = 0.084 \quad , \quad c_4 = 0.42 \tag{6.8}$$

for the Wilson coefficient functions c_i. The results of computations are presented in Table I. It is clear that the theoretical predictions of the branching ratios are too small by almost one order of magnitude when compared with experiment. This can be traced back to the fact that the factorization method fails to explain the $\Delta I = \frac{1}{2}$ rule. The conventional vacuum insertion can only account for ~20% of the $\Delta I = \frac{1}{2}$ $K \to 2\pi$ amplitude (for a detailed review, see Chap. 5 of Ref. 18). Moreover, all color-suppressed nonleptonic charm decays are predicted to be too small in the vacuum-saturation approximation. This calls for a modification of the traditional vacuum-insertion method.[32] In recent years the approach of the $1/N_c$ expansion (N_c being the number of colors) in which the Fierz-transformed terms are dropped to the leading order in $1/N_c$ reproduces successfully a bulk of the charm decay data[33]. Nevertheless, a detailed calculation indicates that even within the $1/N_c$ approach only 30% of the $\Delta I = \frac{1}{2}$ $K \to 2\pi$ transition is explained[18]. That is, the improvement over the discrepancy between theory and experiment is not significant in the case of the kaon; ~70% of the $\Delta I = \frac{1}{2}$ amplitude is still missing.

In Table I we also present the results in the $1/N_c$ expansion [This is equivalent to dropping all the $\frac{1}{3}$-terms in (5.15)]. We see that the improvement for $K_L \to \pi^+\pi^-\gamma$ is quite substantial. The remaining discrepancy comes from the fact that the $1/N_c$ approach can only explain a small part of the $\Delta I = \frac{1}{2}$ enhancement.

Several comments are in order.

(i) The DE amplitude of $K^+ \to \pi^+\pi^0\gamma$ is dominated by the long-distance magnetic transitions. There is a $\Delta I = \frac{1}{2}$ enhancement in both long- and short- distance contributions, as shown by Eqs. (5.15), (6.4) and (6.5), in contrast to the early claim made in Ref. 3. The theoretical calculation of short-distance effects in $K^+ \to \pi^+\pi^0\gamma$ by Lucio[3] is too large by a factor of 15 owing to the overestimate of the phase space (see Ref. 8 for a comment).

(ii) The long-distance contribution to $K_L \to \pi^+\pi^-\gamma$ is very sensitive to the choice of ρ and SU(3) breaking in various parameters. Our calculation using the realistic value of ρ indicates that although the long-distance effect is important the short-distance contribution is also quite appreciable. As a result, unlike $K_L \to 2\gamma$ the decay $K_L \to \pi^+\pi^-\gamma$ is not predominately of long-distance nature.

VII. Summary and conclusions

We have analyzed in this paper the direct photon emission in $K_L \to \pi^+\pi^0\gamma$ and $K_{LS} \to \pi^+\pi^-\gamma$ decays. Since the gauge- and Lorentz-invariant direct $K \to \pi\pi\gamma$ amplitude must have three factors of momentum, it is generated by the higher-order chiral Lagrangian within the context of chiral perturbation theory. Therefore, the measurement of direct radiative decays of K mesons offers a test of the effective dimension-four chiral Lagrangian L_4 at low energies, which can be uniquely determined from the integration of both topological and nontopological chiral anomalies.

From the effective Lagrangian L_4 we have worked out the relevant Lagrangian terms (3.7) – (3.9) for our purposes. We find that the soft-pion theorem cannot be applied to the amplitudes induced by the anomalous Wess-Zumino Lagrangian as in the case of $\pi^0 \to \gamma\gamma$. This means that one should not employ the conventional soft-pion technique and current algebra to deal with the magnetic transitions of $K \to \pi\pi\gamma$.

In the chiral-Lagrangian framework, the long-distance contribution to $K \to \pi\pi\gamma$ are dominated by the pole diagrams in which $\phi^3\gamma$ vertices are governed by the Wess-Zumino terms and poles are low-lying pseudoscalar mesons. The pole contribution to $K_L \to \pi^+\pi^-\gamma$ is very sensitive to SU(3) breaking and the parameter ρ which relates the $K-\eta$ matrix element to the $K-\pi$ transition. We utilize the realistic value $\rho \sim 0.22$ determined from the observed $K_L \to \gamma\gamma$ rates.

The short-distance effects are studied using the effective weak Hamiltonian and the hadronic matrix elements are evaluated with the vacuum-saturation method. Form factors f_V, f_A and f_+ are determined from the higher-order chiral Lagrangian L_4. To simply the calculation we have neglected the momentum-transfer dependence in the form factor f_+.

A simple qualitative argument yields the branching ratio $B(K \to \pi\pi\gamma) \sim (\alpha/\pi)(m_k^4/\Lambda_x^4)\, h$ with Λ_x being a chiral-symmetry breaking scale and h being a phase-space suppression factor. However, the direct-emission contribution to $K^+ \to \pi^+\pi^0\gamma$ is boosted by the $\Delta I = \frac{1}{2}$ enhancement, while the direct photon emission in $K_L \to \pi^+\pi^-\gamma$ is no longer subject to CP suppression.

Either from the explicit vacuum-insertion calculation or from the chiral representation of the penguin operator, it is shown that the DE amplitude does not receive contributions from penguin interactions to the order of $1/\Lambda_x^2$. The (V–A)(V+A) part of the penguin operator does contribute but it is suppressed by the smallness of the Wilson penguin coefficients. We accentuate that the zero contribution of penguin effects does not imply the absence of the $\Delta I = \frac{1}{2}$ enhancement in the decay $K^+ \to \pi^+\pi^0\gamma$: The perturbative penguin amplitude accounts for only ~5% of the $\Delta I = \frac{1}{2}$ $K \to 2\pi$ transition.

The fact that the experimental results are consistent with the aforementioned crude estimate of $B(K \to \pi\pi\gamma)$ implies a genuine $\Delta I = \frac{1}{2}$ enhancement in the decay $K^+ \to \pi^+\pi^0\gamma$. We find in our detailed calculations that this mode is dominated by the long-distance magnetic transitions and the observed DE rates cannot be accounted for by the short-distance effect alone. On the contrary, the long-distance contribution to $K_L \to \pi^+\pi^-\gamma$ is important but not necessarily dominates over the short-distance effect.

In the conventional vacuum-insertion method only ~20% of the $\Delta I = \frac{1}{2}$ $K \to 2\pi$ amplitude is explained. The motivation of understanding why is the color-suppressed channels of nonleptonic charm decays not suppressed as naively expected in the factorization approximation inspires the $1/N_c$ explanation (N_c being the number of colors). We find in this approach that the discrepancy between theory and experiment is improved, particularly for the decay $K_L \to \pi^+\pi^-\gamma$. Nevertheless, since 70% of the $\Delta I = \frac{1}{2}$ transition is still not accommodated in the $1/N_c$ approach, the existence of a deviation of the theoretical prediction from experiment, notably for $K^+ \to \pi^+\pi^0\gamma$, should not be a surprise. Presumably, the accuracy of the theory gets improved once the $\Delta I = \frac{1}{2}$ origin is well understood.

ACKNOWLEDGEMENT

This work was supported by the National Science Council of the Republic of China.

REFERENCES

1. S. Oneda, Y.S. Kim, and D. Korff, Phys. Rev. 136, B1064 (1964); S. V. Pepper and Y. Ueda, Nuovo Cimento 33, 1614 (1964); C. S. Lai and B. L. Young, Nuovo Cimento **52A**, 83 (1967); R. Rockmore, Phys. Rev. **D1**, 226 (1970); M. Moshe and P. Singer, Phys. Rev. **D6**, 1379 (1972); Phys. Lett. 51B, 367 (1974); R. Rockmore and T. Wang, Phys. Rev. D7, 3425 (1973).
2. Yu. P. Malakian, Nucl. Phys. B133, 145 (1978); Sov. J. Nucl. Phys. 26, 337 (1977).
3. J. L. Lucio M., Phys. Rev. D24, 2457 (1981).
4. M. McGuigan and A. I. Sanda, Phys. Rev. D36, 1413 (1987).
5. Y. C. R. Lin and G. Valencia, UMHEP-282 (1987).
6. Experiments for $K_S \to \pi^+\pi^-\gamma$: G. Burgun et al., Phys. Lett. **46B**, 481 (1973); H. Taureg et al., Phys. Lett. **65B**, 92 (1976); experiments for $K_L \to \pi^+\pi^-\gamma$: A. S. Carroll et al., Phys. Rev. Lett. **44**, 529 (1980).
7. Experiments for $K^+ \to \pi^+\pi^0\gamma$: R. J. Abrams et al., Phys. Rev. Lett. 29, 1118 (1972); K. M. Smith et al., Nucl. Phys. B109, 173 (1976).
8. Our phase-space integral for $K_{L,S} \to \pi\pi\gamma$ agrees with that of Lucio (Ref. 3). However, the Lucio's result for the $K^+ \to \pi^+\pi^0\gamma$ phase space is too large by a factor of 15 due to the fact that the power of $(1 + m^2/M^2 - 2E/M)$ in the denominator of D in Lucio's Eq. (A.5) should be 2 rather than 4.
9. See, e.g. N.K. Pak and P. Rossi, Nucl. Phys. **B250**, 279 (1985).
10. J. Balog, Phys. Lett. **149B**, 197 (1984); R. I. Nepomechie, Ann. Phys. (N.Y.) **158**, 67 (1984); K. Seo, Gifu Report No. GWJC-1, 1984 (unpublished); A. A. Andrianov, Phys. Lett. **157B**, 425 (1985); see also D. Ebert and H. Reinhardt, Nucl. Phys. **B271**, 188 (1986).
11. H. Y. Cheng, Phys. Rev. **D36**, 2056 (1987).
12. A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); J. Donoghue, E. Golowich, and B. Holstein, Phys. Rev. **D30**, 587 (1984).
13. J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
14. J. Balog, Nucl. Phys. **B258**, 361 (1985).
15. H. Y. Cheng, Phys. Rev. **D34**, 166 (1986).
16. A. A. Bel'kov and A. V. Lanev, Sov. J. Nucl. Phys. **45**, 2 (1987).
17. A. A. Bel'kov, D. Ebert, and V. N. Pervushin, Phys. Lett. **193B**, 315 (1987).
18. H. Y. Cheng, "Status of the $\Delta I = \frac{1}{2}$ rule in kaon decay", IUHEP-132 (1987).

19. J. Donoghue, E. Golowich, and B. Holstein, Phys. Lett. **135B**, 481 (1984); J. Donoghue and B. Holstein, Phys. Rev. D29, 2088 (1984).
20. J. Donoghue, B. Holstein, and Y.-C. R. Lin, Nucl. Phys. B277, 651 (1986).
21. J. Donoghue, B. Holstein, and Y.-C. R. Lin, Phys. Rev. Lett. **55**, 2766 (1985); G. Grunbery, Phys. Lett. **168B**, 141 (1986).
22. L. L. Chau and H. Y. Cheng, Phys. Lett. **195B**, 275 (1987); Phys. Rev. Lett. **54**, 1768 (1985).
23. For recent estimates, see Refs. 11 and 18.
24. E. Ma and A. Pramudita, Phys. Rev. **D24**, 2476 (1981).
25. A. I. Vainshtein, V.I. Zakharov, and M. A. Shifman, Sov. Phys. JETP **45**, 670 (1977).
26. The gauge-noninvariant terms in (3.8) cannot produce the gauge and Lorentz invariant direct-emission amplitude (2.1). The pole contribution to $<\pi\pi\gamma\,|(\bar{u}u)\,|\,0>$, for instance, is already taken into account in the long-distance effects.
27. T. N. Pham, Phys. Lett. **145B**, 113 (1984).
28. We like to emphasize that Eq. (5.14) is valid only under the factorization assumption.
29. The magnetic (electric) DE amplitude of $K^- \to \pi^-\pi^0\gamma$ is the same as (opposite to) that of $K^+ \to \pi^+\pi^0\gamma$.
30. Using the same values of the parameters given in Ref. 5, we find $B\,(K_L \to \pi^+\pi^-\gamma)_{DE} = 1.7 \times 10^{-5}$, to be compared with the result 5.7×10^{-5} obtained by Lin and Valencia. It seems to us that the phase space is overestimated by a factor of three in Ref. 5.
31. F. J. Gilman and M. B. Wise, Phys. Rev. D20, 2392 (1979).
32. H. Y. Cheng, Z. Phys. **C32**, 237 (1986).
33. A. J. Buras, J.-M. Gerard, and R. Ruckl, Nucl. Phys. **B268**, 16 (1986).
34. A recent experiment for $K^- \to \pi^-\pi^0\gamma$: V. N. Bolotov et al., in "Proceedings of the XXIII International Conference on High Energy Physics", edited by S.C. Loken (World Scientific, Singapore, 1987) p. 887.

Table I. Theoretical predictions for the direct-emission contributions to the decays $K^+ \to \pi^+\pi^0\gamma$ and $K_{L,S} \to \pi^+\pi^-\gamma$. Form factors $\tilde{\beta}/e$ and $\tilde{\gamma}/e$ are defined in Eq. (2.1) and are in units of 10^{-6} GeV^{-3}. Short-distance contributions are calculated using the vacuum-insertion method (I) and the $1/N_c$ expansion (II). Experimental results are taken from Refs. 6, 7 and 34.

Decay mode	$(\tilde{\beta}/e)_{long}$	$(\tilde{\beta}/e)_{short}$ (I)	$(\tilde{\beta}/e)_{short}$ (II)	$(\tilde{\gamma}/e)_{short}$ (I)	$(\tilde{\gamma}/e)_{short}$ (II)	B_{theor} (I)	B_{theor} (II)	B_{expl}
$K^+ \to \pi^+\pi^0\gamma$	−5.07	−0.22	−1.06	1.20	1.50	0.39×10^{-5}	0.53×10^{-5}	$(1.56 \pm 0.35) \times 10^{-5}$ $(2.3 \pm 3.2) \times 10^{-5}$ $(2.05 \pm 0.46) \times 10^{-5}$
$K_L \to \pi^+\pi^-\gamma$	1.43	0.30	2.02			0.40×10^{-5}	1.58×10^{-5}	$(2.89 \pm 0.28) \times 10^{-5}$
$K_S \to \pi^+\pi^-\gamma$				−2.4	−3.0	1.31×10^{-8}	2.05×10^{-8}	$< 4 \times 10^{-5}$

Figure Captions

Fig. 1 The long-distance pole diagram contributing to the direct photon emission of $K_L \to \pi^+\pi^-\gamma$.

Fig. 2 Long-distance pole contributions to the direct photon emission of $K^+ \to \pi^+\pi^0\gamma$.

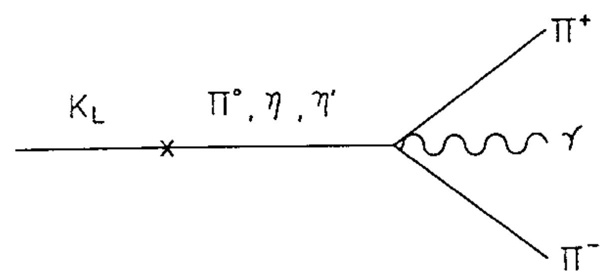

Fig. 1

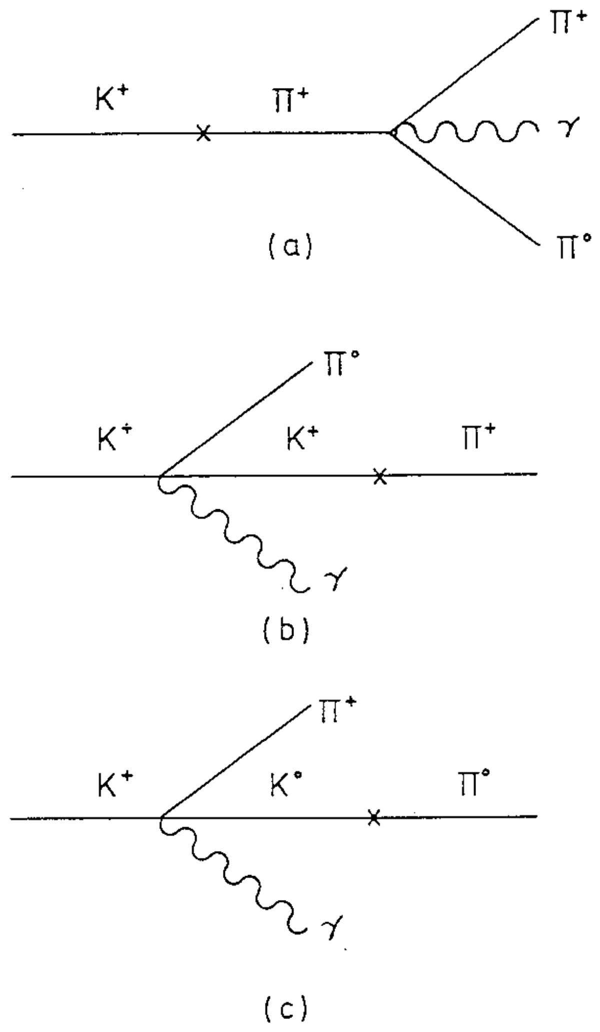

Fig. 2