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R. Brun (CERN-DD), M. Caillat (LAPP-Annecy), M. Maire (LAPP-Annecy),
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Abstract

The electromagnetic processes included in the GEANT3 Monte Carlo program are reviewed. We present the results of calculations on the longitudinal and radial development of electromagnetic showers for several materials and energies: iron, 200 MeV; lead glass 1 GeV; BGO, 10 GeV and 50 GeV. These results are compared with the same calculations using the EGS3 code.

1. Introduction

The size and the complexity of high energy physics detectors are switfly increasing together with the new accelerators, and the calorimeters are becoming essential components of these detectors.

These features have two implications (amongst other things) on the computer simulation of such detectors:

- the need to develop shower simulation programs as complete and accurate as possible
- the need to handle complex geometries.

Sophisticated and efficient shower programs exist now, both for electromagnetic [1,2] and hadronic [3] processes. However, those codes have a rather poor geometrical part: it is not easy to describe a complete experimental set up within the context of such programs.

On the other hand, GEANT [4] is a system of detector description and simulation tools which was designed in order to provide a framework:

- to handle any kind of particle in any kind of medium within a unified tracking system
- to define complex geometries as simply and automatically as possible.

The GEANT structure has been made highly modular, allowing a continuous upgrading of the geometry system and of the physics routines. In particular the simulation of the physics processes has been carefully isolated from the tracking part.

As a consequence, it been possible to incorporate the physics generation routines originating from several hadronic shower programs (CASCADE,

GHEISHA, TATINA) within the GEANT framework, while the tracking of the particles remains under the GEANT control.

EGS [1] is a very well optimized program for the simulation and the tracking of electromagnetic showers. In such a code it is not easy to separate the generation of the physics processes from the tracking itself without destroying the overall efficiency.
In addition, the EGS code is written in MORTRAN, which is a powerful but non standard language creating portability problems in some laboratories.

Therefore, we decided to rewrite a set of electromagnetic process routines fully compatible with the GEANT system rather than to try to incorporate in it some pieces of the EGS code. In this way all the hadronic and electromagnetic processes are under the control of an unified tracking system.

Section 2 is a short review of all electromagnetic processes that have been taken into account in GEANT. A more detailed description of the formulae can be found in Appendix A which is, in fact, a subset of the GEANT3 write-up [4].

The EGS program gives impressive good agreement with available experimental data, from a few MeV up to 100 GeV, and for most types of calorimeter. Therefore, it is becoming a "standard" in the HEP community. We have chosen to compare systematically the GEANT results versus the EGS results for several materials (iron, BGO, lead glass) at several energies (200 MeV, 1 GeV, 10 GeV and 50 GeV). This study is presented in Section 3, and the conclusions in Section 4.

2. Electromagnetic processes in GEANT

We give here a very short summary of the electromagnetic processes that have been taken into account in the program. (For the details see Appendix A or [4]). Most of the methods used in the simulation are very similar to those in EGS. We have chosen different approaches for some of the interactions, mainly because we wanted to have a unified simulation program (without a separate pre-processing part) and we wanted to improve the timing too.

The main features of the two programs are compared in the Table 1, and the rest of this section briefly explains the differences between EGS and GEANT.

	EGS	GEANT
Medium	Any chamical element compound or mixture	Same
Energy region	0.1 MeV - 100 GeV	Same
Pair production	Bethe-Heitler theory with Coulomb and empirical corrections Average angle	Semiempirical total cross-section, others are same
Compton scattering	Klein-Nishina formula	Empirical total, Klein-Nishina differential cross-section
Photoeffect	Cross-section from Storm, Israel $E_e=E_\gamma-E_k(Z)+m$	Empirical cross-section (fitted to Storm's data) $E_e=E_\gamma-E_k(Z)+m$
Bremsstrahlung	Bethe-Heitler with with Coulomb and empirical corrections Average angle Continuous energy loss from soft photons	Semiempirical total cross-section, others are same
δ-rays	Moller-Bhabha cross-sections	Same
e^+e^- annihilation	Heitler's formula	Same
Ionisation	Restricted stopping power formula of Berger and Seltzer	Same
Multiple scattering	Moliere theory, lateral deviation in position neglected	Gaussian distribution, lateral deviation calculated

Table 1

Comparison of EGS and GEANT
(e.m. physics)

Pair production

For this process EGS uses empirical total cross section data at low energy (below 50 MeV) and the Coulomb corrected Bethe-Heitler formula at high energy (above 50 MeV). Instead of using a large amount of cross section data we have fitted an empirical formula to the data.

When sampling the energies of the positron/electron, EGS and GEANT both use the Bethe-Heitler differential cross section, but with a different "decomposition" of the formula. The result of this difference is that the sampling in GEANT is more efficient, especially at low energies. The difference in timing is 10-15 % at and above 50 MeV and it is a factor 3 at 5 MeV.

Compton scattering

EGS uses the well-known Klein-Nishina formula to compute the mean free path of a photon, GEANT works with an empirical formula, which gives better results for very low energy (below 1 MeV) photons in a material with high atomic number, Z.

Atomic photoeffect

In order not to read a lot of cross section data we have fitted an empirical formula to the data of Storm and Israel (which are used in EGS). This formula reproduces the data quite well above 0.1 MeV (but it does not reproduce the peaks of the photoelectric cross section below this energy).

Bremsstrahlung

We have fitted here a semi-empirical cross section and an energy loss formula to the "data" used in EGS. (These "data" are in fact calculated values from the Coulomb corrected Bethe-Heitler theory with empirical corrections.) These formulae reproduce these "data" well. Generating the photon spectrum we use a different decomposition of the differential cross section than that used in EGS. As a result we do not need take special care to avoid the infrared catastrophe and we have a slightly better timing (20-70 % depending on the energy and the photon cut).

Moller, Bhabha scattering (delta-rays)

These are treated in a very similar manner to EGS.

Multiple scattering

The two programs use different approximations in their treatment of multiple Coulomb scattering. EGS uses the Moliere theory, but it neglects the lateral deviations in the position of the particle. GEANT works with the Gaussian approximation and calculates the lateral deviations in the positions too (both of the programs correct the track length, of course).

3. Results of the comparison EGS / GEANT

We have compared the shower development in EGS and in GEANT for several materials and various energies, (see Table 2 below)

Material	Density gcm^{-3}	Radiat. length X_0 cm	Incident energy	Statistic
Iron	7.9	1.76	E_0 = 200 MeV	1000 showers
SF6 lead glass	5.2	1.66	E_0 = 1 GeV	200 showers
BGO	7.1	1.12	E_0 = 10 GeV	100 showers
BGO	7.1	1.12	E_0 = 50 GeV	100 showers

Table 2

The set up is shown in Fig. 1; for each material we considered a full cylinder of matter with the following dimensions:

$L = 20X_0$ (z-axis)

$R = 5X_0$ (r-axis)

In order to study the longitudinal and lateral development of the shower, we considered 2 kinds of bins (see Figs. 2 and 3) dividing the cylinder in 20 bins:

1) along z axis ; bin length = $1X_0$ (Fig. 2)

2) along r-axis ; bin length = $0.25X_0$ (Fig. 3)

Shower development

An incident photon with energy E_0 hits the centre of the cylinder (Fig. 1) and initiates an e.m. shower in the material.

The (total) energy cut offs were fixed to 0.1 MeV for photons and 1.511 MeV for electrons/positrons.

Starting at Figure 4 we show 4 groups of 6 figures corresponding to each test case:

- the profiles of the longitudinal energy deposition
- the corresponding cumulative values.
- the profiles of the radial energy deposition
- the corresponding cumulative values.
- the total energy deposited in the set-up
- the particle flow, i.e. the number of particles crossing a given plane perpendicular to the z-axis.

The error bars plotted are the shower fluctuations:

$$\left[1/N \sum_{i=1}^{N} (X_i - \bar{X})^2 \right]^{1/2}$$

The general features of the shower development are the same for the three energies presented here. Let us concentrate the discussion on the 1 GeV case. The mean values of the longitudinal energy profiles (Fig. 10)

$$\frac{1}{E_0} \frac{dE}{dZ}$$

are very close to each other and individual bins differ by $\simeq 0.3\%$, which is well within the energy fluctuations. This statement also stands for the radial development (Fig. 12). Of course, the fluctuations are rather large, especially within the few first radiation lengths. It is more significant to compare the cumulative deposited energy (Figs. 11 and 13) :

$$\frac{1}{E_0} \int_0^z \frac{dE}{dZ} dZ$$

Here again the mean values remain fully compatible and any discrepancies are within the shower fluctuations as can be seen from Table 3 below (extracted from Fig. 11)

	5 X_0	10 X_0	20 X_0
GEANT mean	39.44 %	83.34 %	97.14 %
EGS mean	39.25 %	82.43 %	96.77 %
GEANT - EGS	0.19 %	0.91 %	0.37 %
GEANT rms	19.9 %	11.9 %	1.5 %
EGS rms	19.3 %	11.6 %	1.5 %

Table 3

Cumulative deposited energy in SF6 lead glass

In Table 4 we give the GEANT-EGS difference of the total deposited energy over 20 radiation lengths, and the fluctuation on that energy.

	Iron 200 MeV	BGO 10 GeV	BGO 50 GeV
GEANT-EGS	+0.08 %	+0.67 %	+0.91 %
Fluctuations	1.7 %	1.6 %	2.2 %

Table 4

Total deposited energy over 20 r.l.

For completeness we show also the particle flow (Fig. 15). These distributions are extremely sensitive to the exact simulation mechanisms (physics and tracking). The very good agreement between the two programs give us confidence in the way all physics processes are generated.

Timing Considerations

As the time to simulate the development of an electromagnetic cascade increases linearly with the incident particle energy, this time can be characterized by just one number : namely the cpu time per GeV.

This time is nearly independent of the material, but strongly dependent on the energy cut off, of the tracking optimization, and on the way that the geometry is described.

In order to estimate the last factor, we have tested three geometries. The previous cylinder was divided in (20*20) cells as before, into (10*10) cells (i.e. $2X_0$ along the z-axis and $0.5X_0$ along the radius), and (1*1) cell (i.e. continuous medium without any internal boundaries).

In the latter case there are practically no geometrical computations, therefore we are testing the intrinsic time for the generation and the development of the shower. The results are shown in Table 5 for two sets of energy cut off.

There are considerable differences in time between the two programs. In the (1x1) case GEANT can fully exploit the so called "safety radius" algorithm. In the (20x20) case one can see the overhead introduced by the GEANT machinery to treat a trivial geometry whereas the EGS program is running with a specialized geometry routine for cylindrical volumes. The GEANT geometry package has been designed to handle very complex geometries (100000 volumes or more) and the ratio EGS/GEANT should increase substantially, especially after the optimisation of some critical routines.

Energy cut 0.1 MeV/1.511 MeV	(1*1) cell	(10*10) cells	(20*20) cells
GEANT	101 ms/GeV	204 ms/GeV	305 ms/GeV
EGS	215 ms/GeV	257 ms/GeV	345 ms/GeV
EGS / GEANT	2.13	1.26	1.13

Energy cut off 1. MeV/1.51MeV	(1*1) cell	(10*10) cells	(20*20) cells
GEANT	51 ms/GeV	100 ms/GeV	145 ms/GeV
EGS	186 ms/GeV	212 ms/GeV	270 ms/GeV
EGS / GEANT	3.64	2.12	1.86

Table 5

Time to develop an electromagnetic shower on the CERN SIEMENS 7890 machine.

4. Conclusions

An electromagnetic shower program has been developed within the GEANT3 framework. The physics processes taken into account are essentially the same as in the EGS program.

The agreement between these two programs are within the statistical shower fluctuations, both for the energy deposition profiles and for the particle flux.

The intrinsic time to develop a cascade is faster with GEANT than with EGS, and can still be improved by optimizing the overall GEANT system.

For the purpose of that comparison, we have used the version 3 of EGS. Our intention is to repeat the same exercise with EGS4, once a stable version is available. It is our understanding that EGS4 should not give substantial differences with EGS3 (cf note by W.R.Nelson) in the energy range investigated.

5. References

1) EGS, R.L. Ford, W.R. Nelson, SLAC-210, UC-32 (1978)

2) GAMMA, E. Longo, Univ. of Rome, Istituto di Fisica.

3) CASCADE84, A. Grant, CERN-EP.
GHEISHA, H. Fesefeldt, Aachen III.
TATINA, T. Baroncelli, Univ. of Rome.
HETC, T. Gabriel, ORNL.

4) GEANT3, R. Brun, et al. CERN, DD/EE/84-1.

FIGURE CAPTIONS

Fig. 1,2,3 : Geometrical configuration

Fig. 4 : Iron / 200 MeV longitudinal profile of the deposited energy
Fig. 5 : Iron / 200 MeV cumulative value of the deposited energy (longit)
Fig. 6 : Iron / 200 MeV radial profile of the deposited energy (scale: 0.25*x0)
Fig. 7 : Iron / 200 MeV cumulative value of the deposited energy (radial)
Fig. 8 : Iron / 200 MeV total energy deposition
Fig. 9 : Iron / 200 MeV particle flow

Fig.10 : Lead gl./1 GeV longitudinal profile of the deposited energy
Fig. 11 : Lead gl./1 GeV cumulative value of the deposited energy (longit)
Fig. 12 : Lead gl./1 GeV radial profile of the deposited energy (scale: $0.25*X_0$)
Fig. 13 : Lead gl./1 GeV cumulative value of the deposited energy (radial)
Fig. 14 : Lead gl./1 GeV total energy deposition
Fig. 15 : Lead gl./1 GeV particle flow
Fig. 16 : BGO / 10 GeV longitudinal profile of the deposited energy
Fig. 17 : BGO / 10 GeV cumulative value of the deposited energy (longit)
Fig. 18 : BGO / 10 GeV radial profile of the deposited energy (scale: $0.25*X_0$)
Fig. 19 : BGO / 10 GeV cumulative value of the deposited energy (radial)
Fig. 20 : BGO / 10 GeV total energy deposition
Fig. 21 : BGO / 10 GeV particle flow

Fig. 22 : BGO / 50 GeV longitudinal profile of the deposited energy
Fig. 23 : BGO / 50 GeV cumulative value of the deposited energy (longit)
Fig. 24 : BGO / 50 GeV radial profile of the deposited energy (scale: $0.25*X_0$)
Fig. 25 : BGO / 50 GeV cumulative value of the deposited energy (radial)
Fig. 26 : BGO / 50 GeV total energy deposition
Fig. 27 : BGO / 50 GeV particle flow

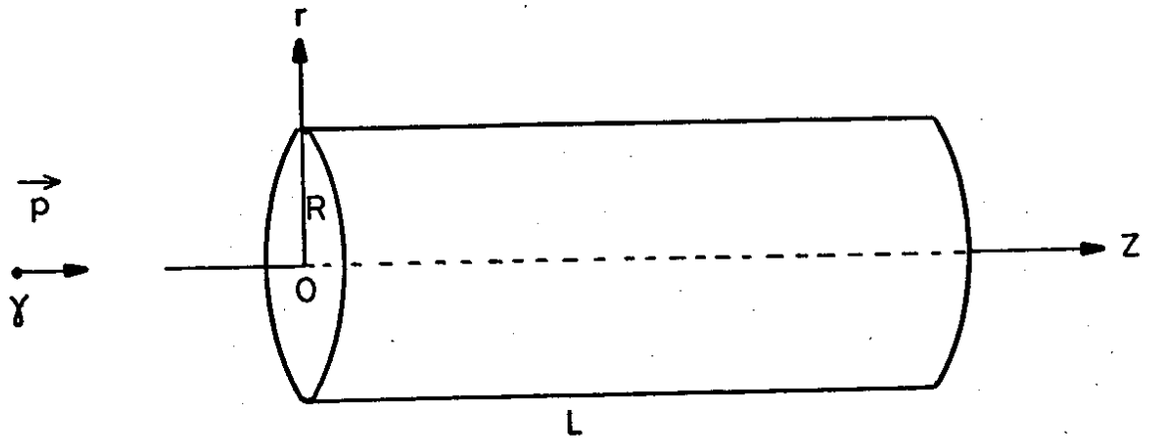


Fig. - 1

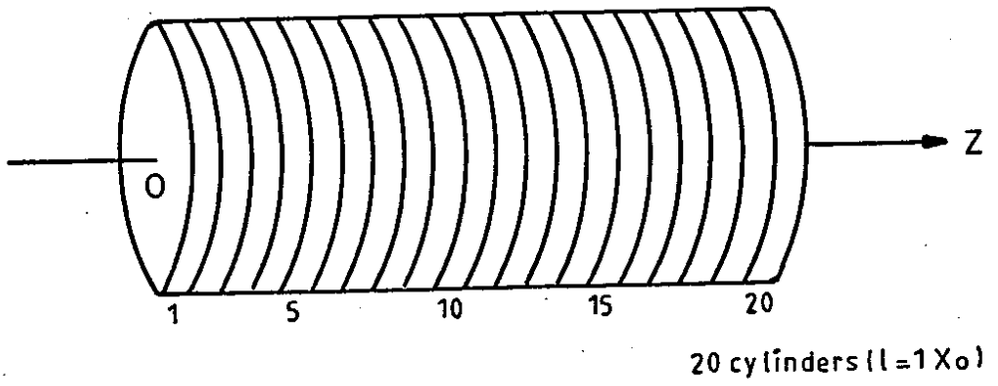


Fig. - 2

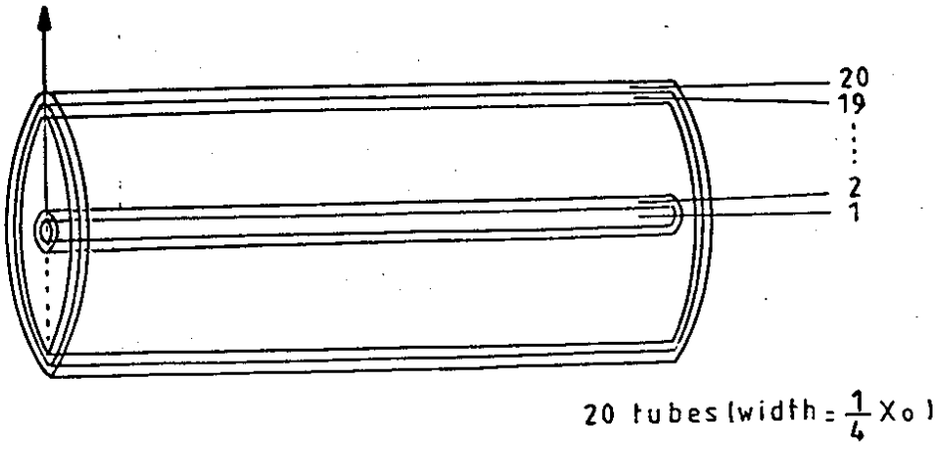
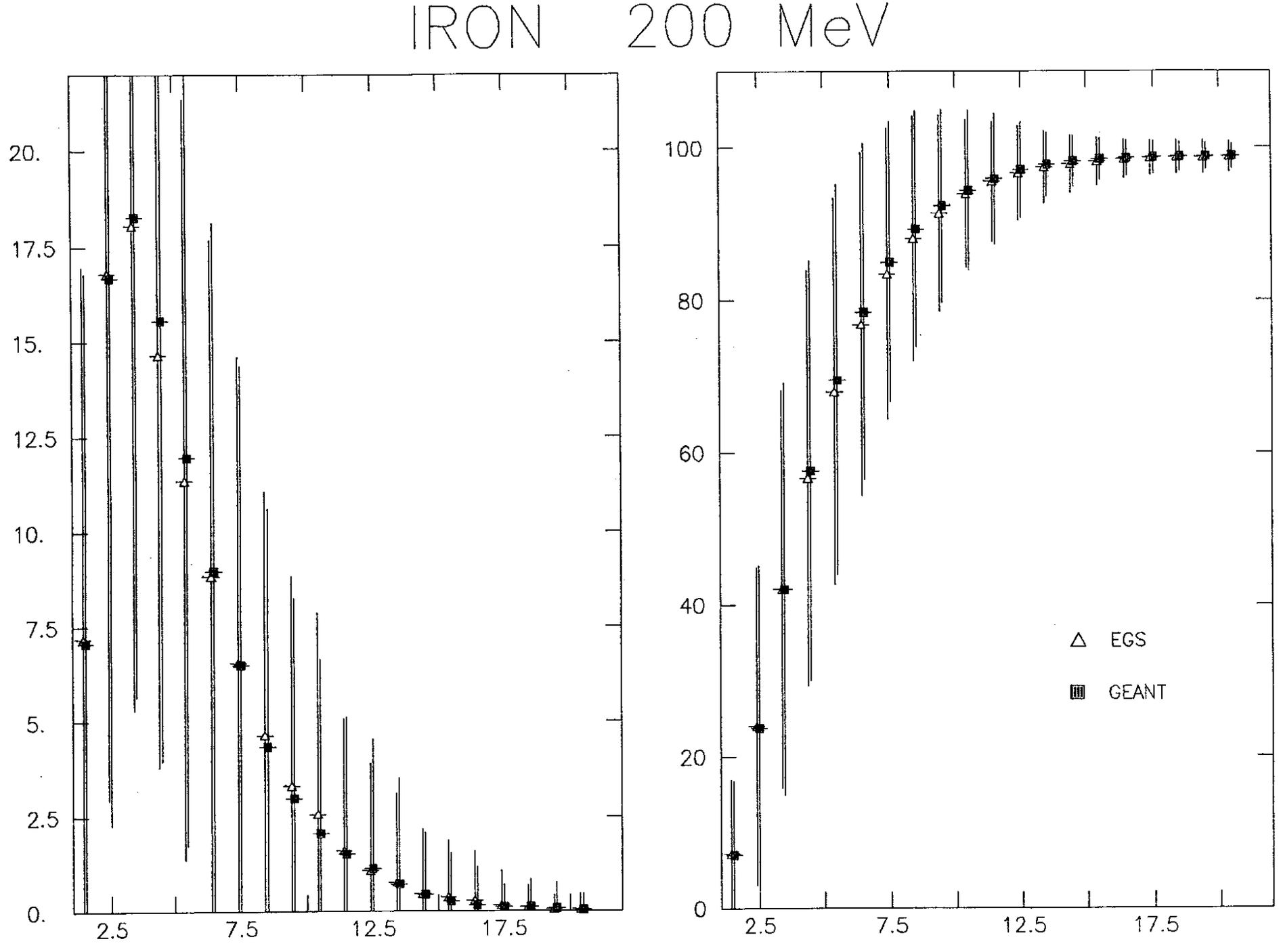


Fig. - 3



FIG, 4: LONGIT ENERGY DEPOSITION

FIG, 5: CUMUL LONGIT ENERGY DEP.

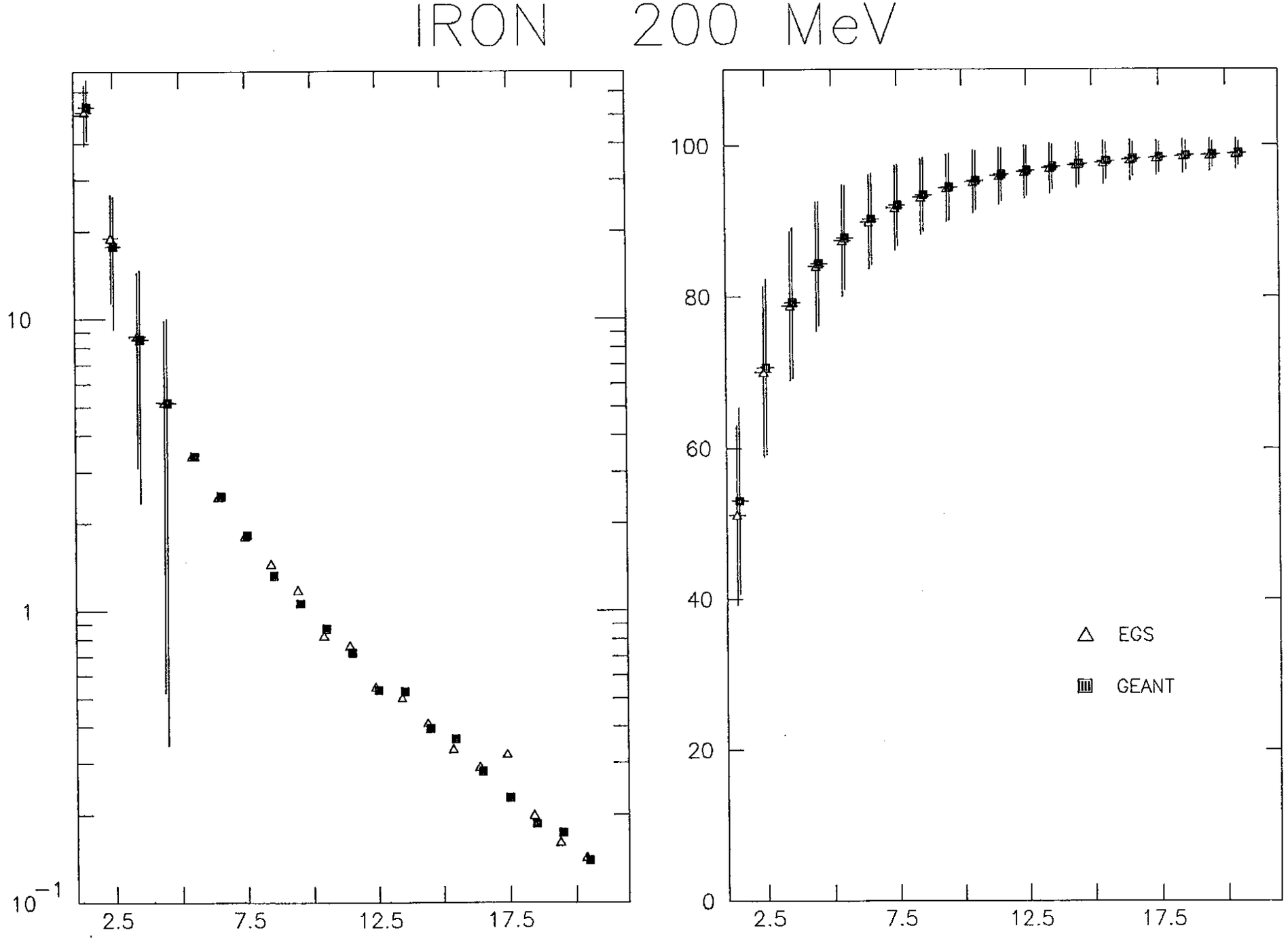


FIG. 6: RADIAL ENERGY DEPOSITION

FIG. 7: CUMUL RADIAL ENERGY DEP.

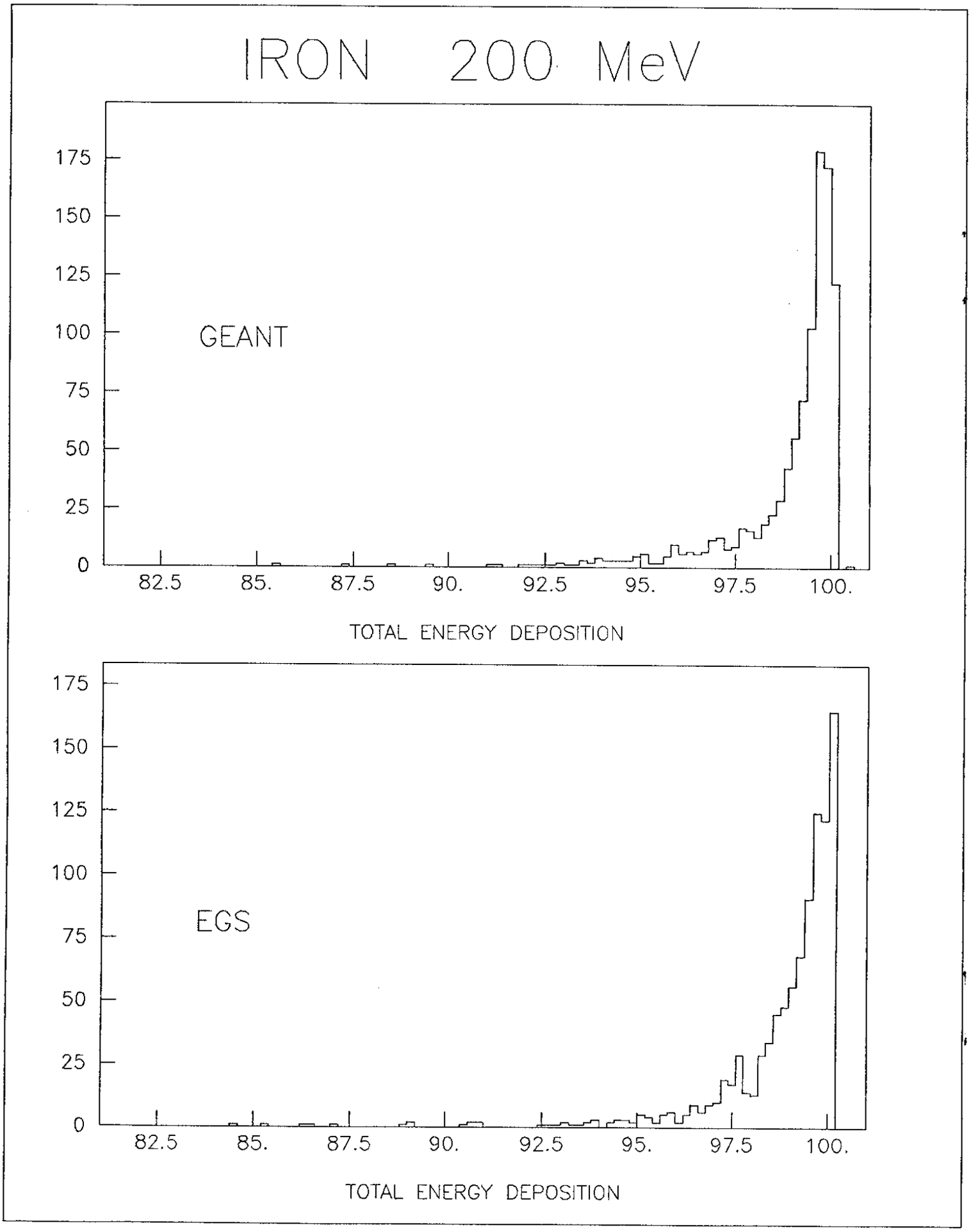

IRON 200 MeV
175
150
125
100
75
50
25
0
GEANT
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION
175
150
125
100
75
50
25
0
EGS
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION

FIG. 8

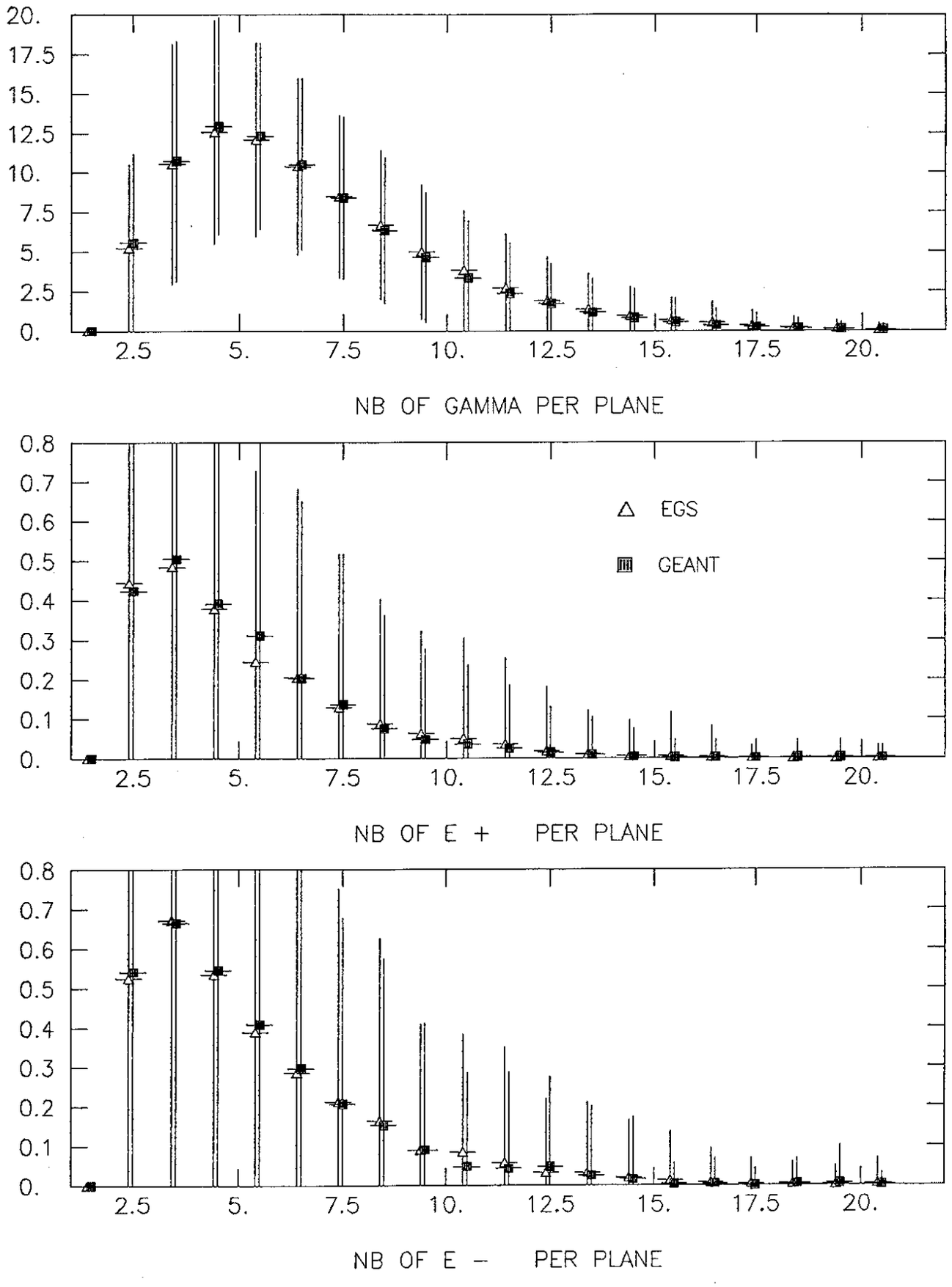

IRON 200 MeV
20.
17.5
15.
12.5
10.
7.5
5.
2.5
0.
2.5
5.
7.5
10.
12.5
15.
17.5
20.
NB OF GAMMA PER PLANE
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
EGS
GEANT
NB OF E + PER PLANE
NB OF E − PER PLANE

FIG. 9

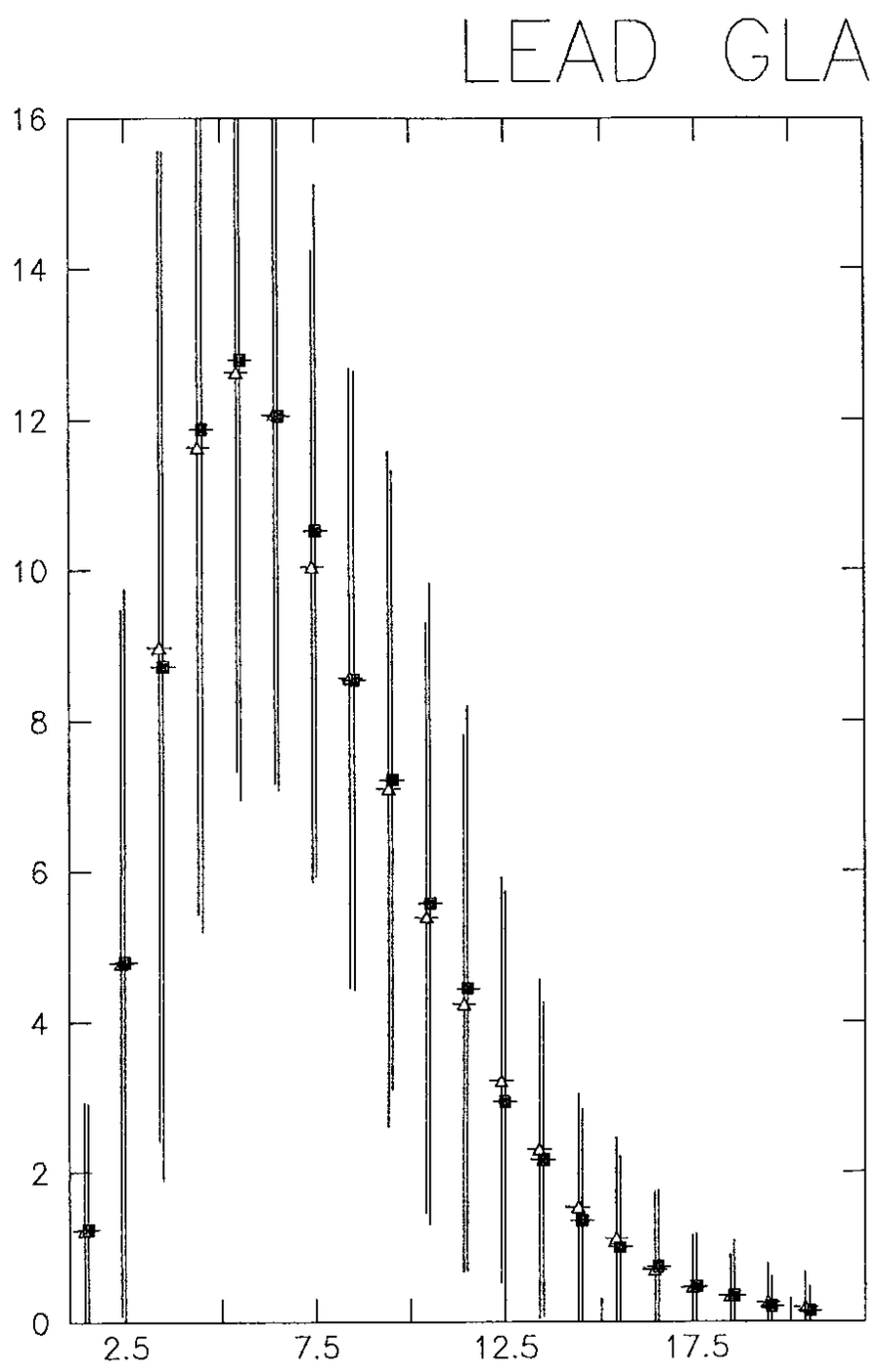


FIG. 10: LONGIT ENERGY DEPOSITION

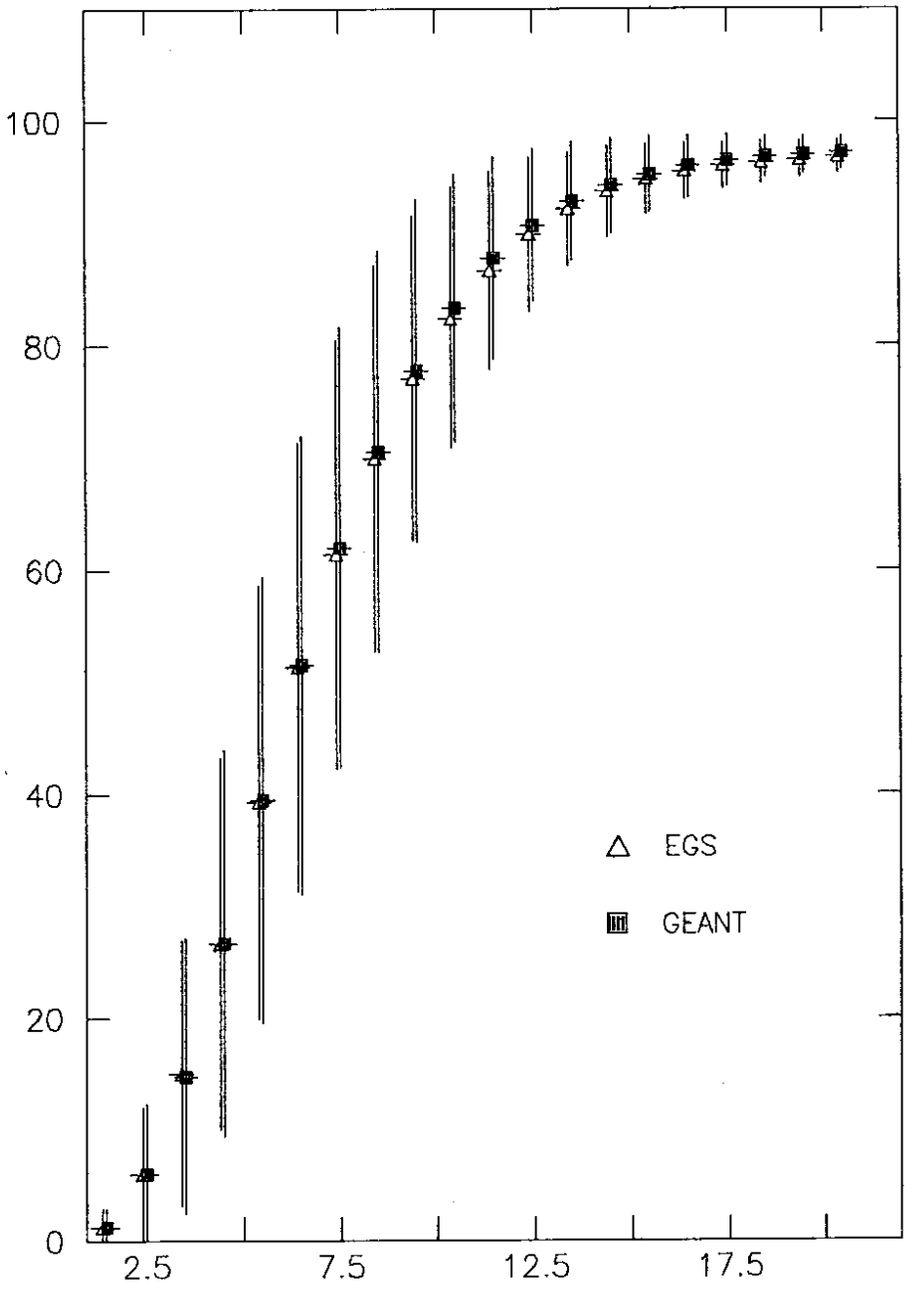


FIG. 11: CUMUL LONGIT ENERGY DEP.

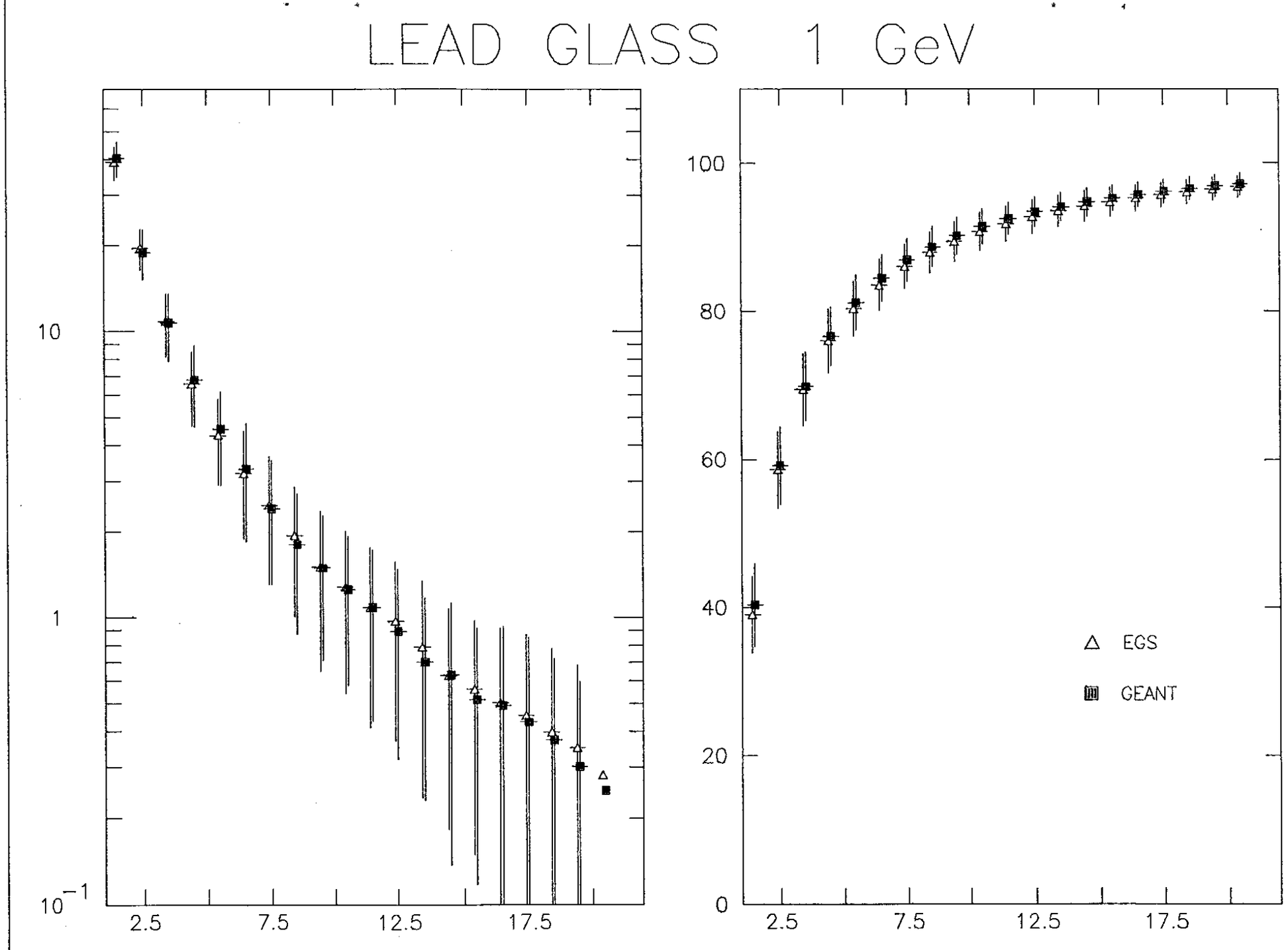


FIG. 12: RADIAL ENERGY DEPOSITION

FIG. 13: CUMUL RADIAL ENERGY DEP.

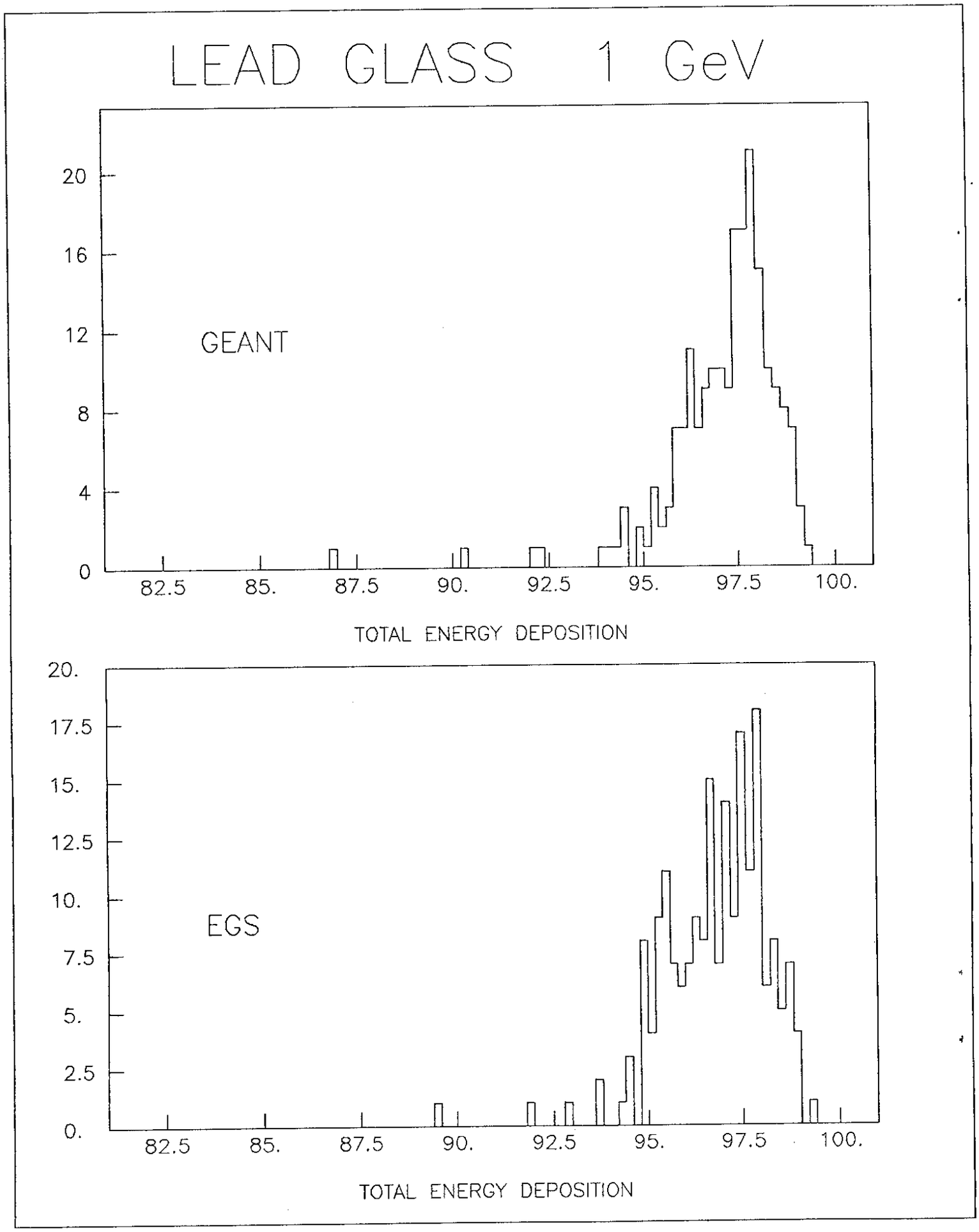

LEAD GLASS 1 GeV
GEANT
20
16
12
8
4
0
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION
EGS
20.
17.5
15.
12.5
10.
7.5
5.
2.5
0.
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION

FIG. 14

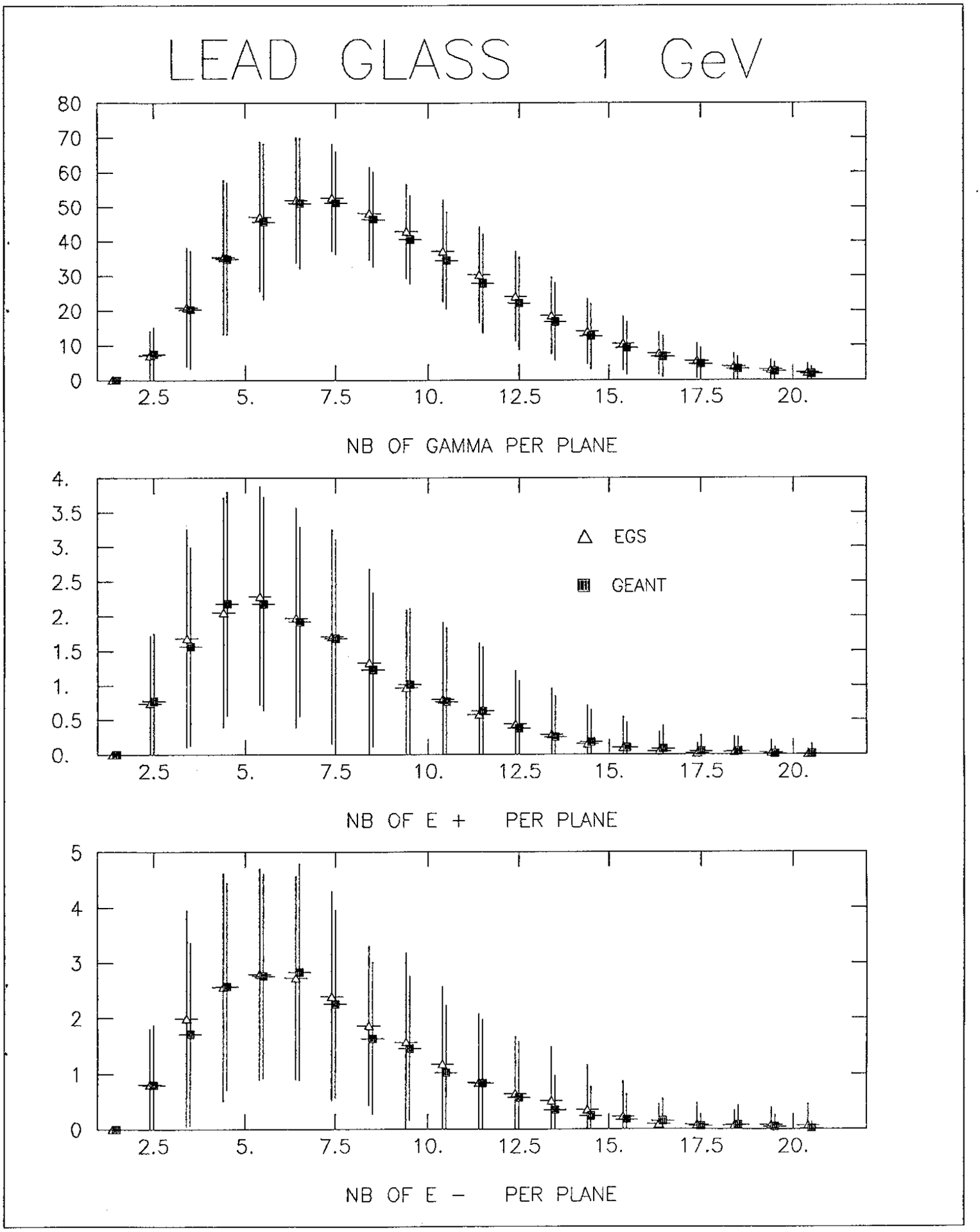

LEAD GLASS 1 GeV
80
70
60
50
40
30
20
10
0
2.5
5.
7.5
10.
12.5
15.
17.5
20.
NB OF GAMMA PER PLANE
4.
3.5
3.
2.5
2.
1.5
1.
0.5
0.
EGS
GEANT
NB OF E + PER PLANE
5
4
3
2
1
0
NB OF E − PER PLANE

FIG. 15

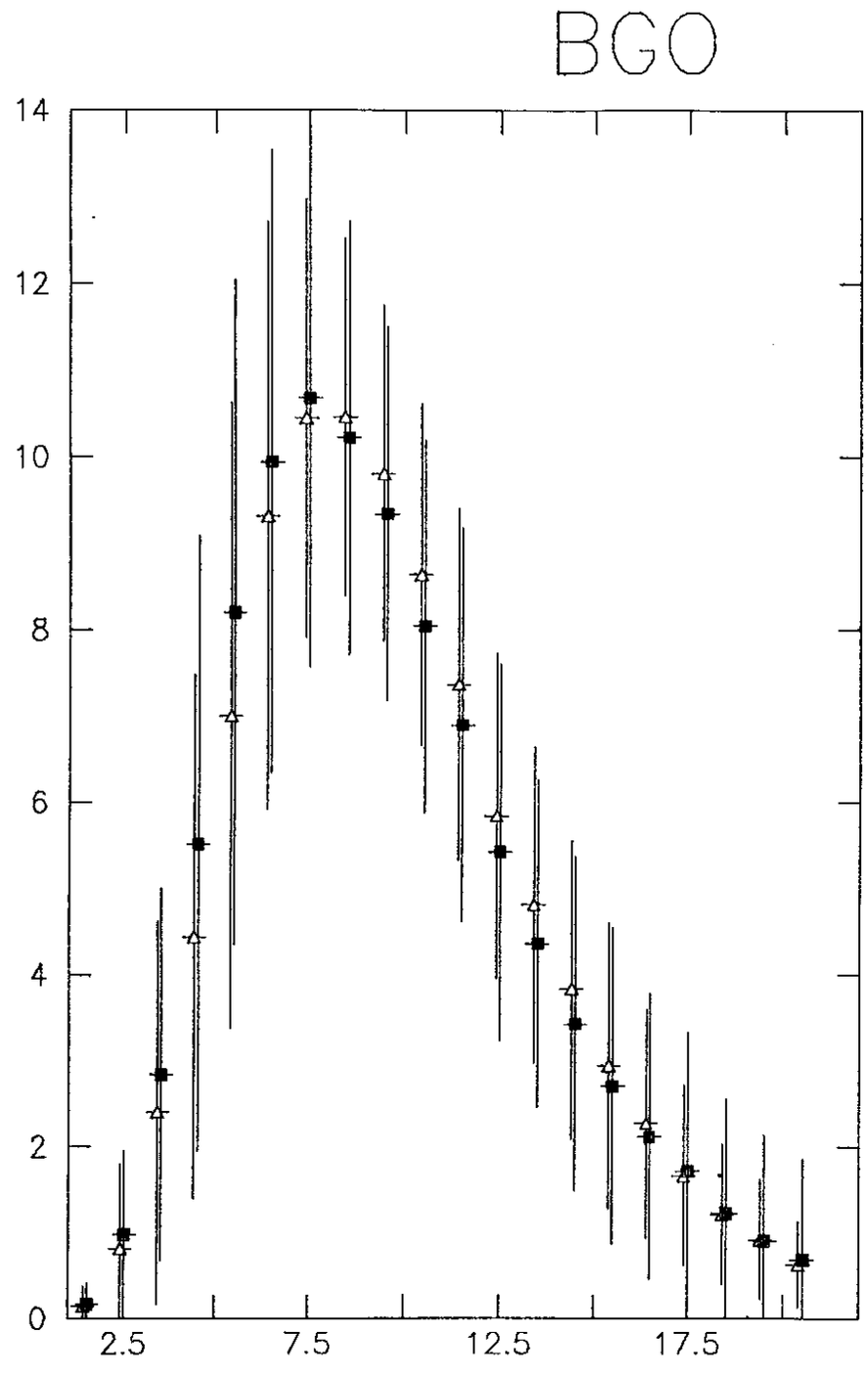


FIG. 16: LONGIT ENERGY DEPOSITION

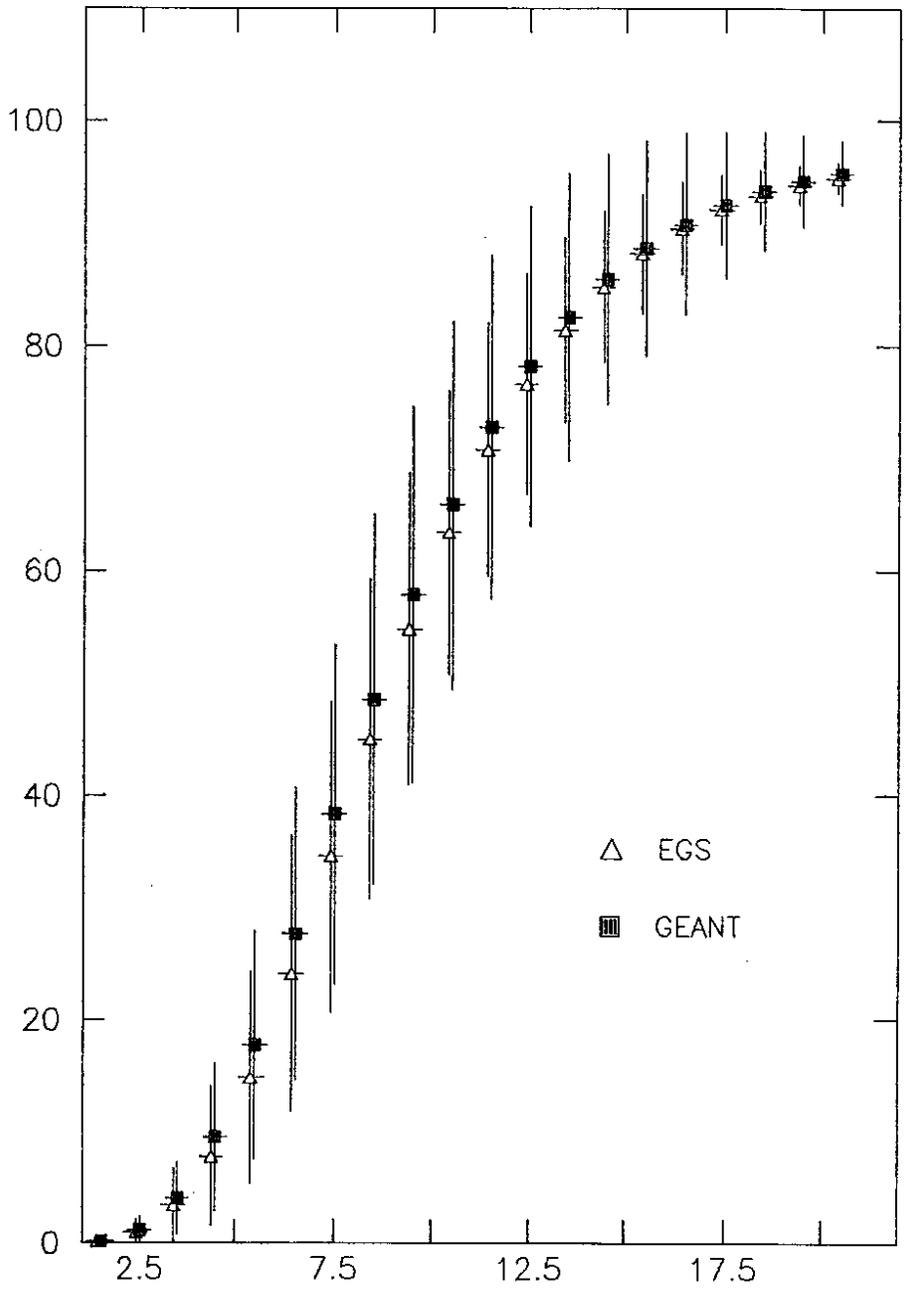


FIG. 17: CUMUL LONGIT ENERGY DEP.

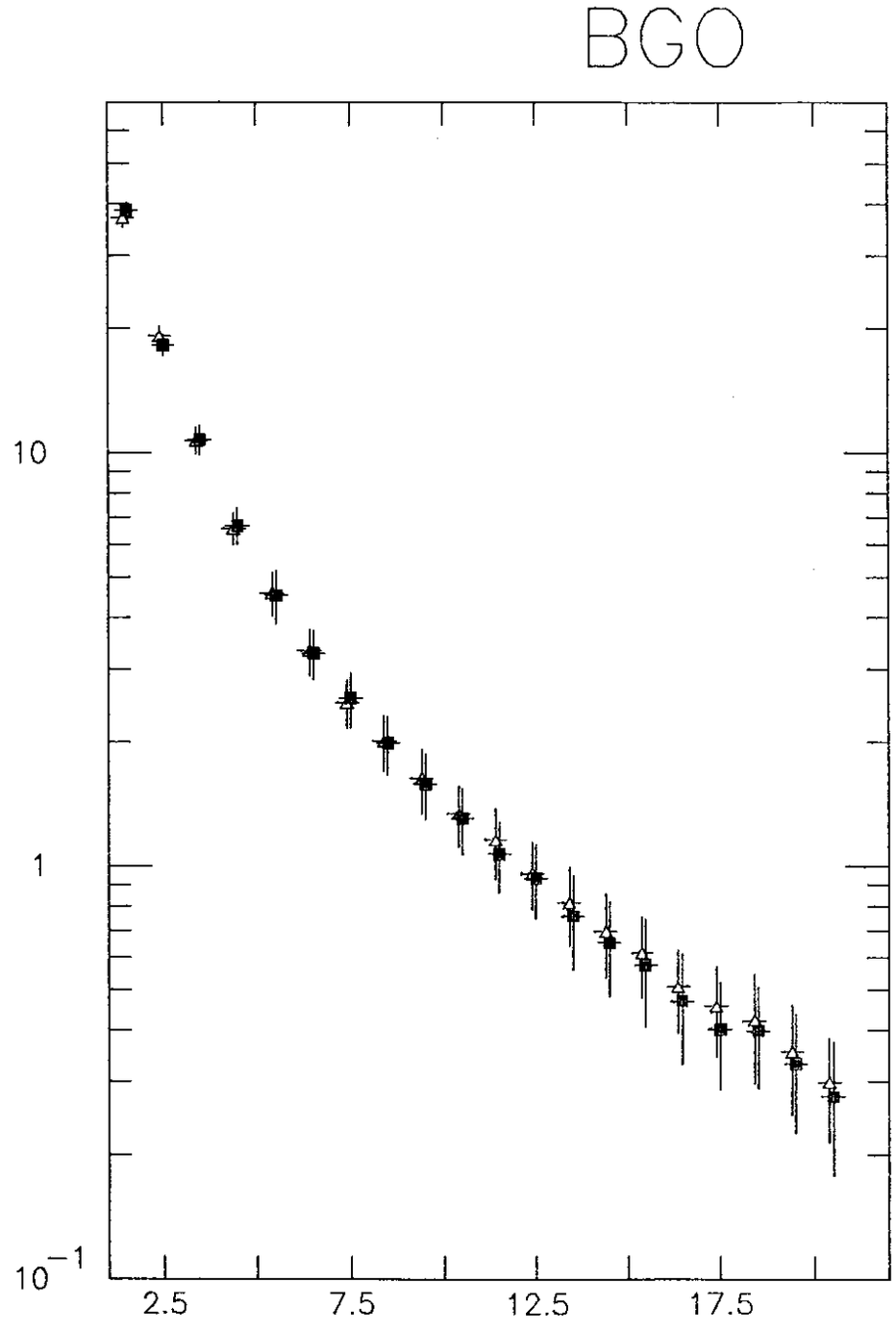


FIG. 18: RADIAL ENERGY DEPOSITION

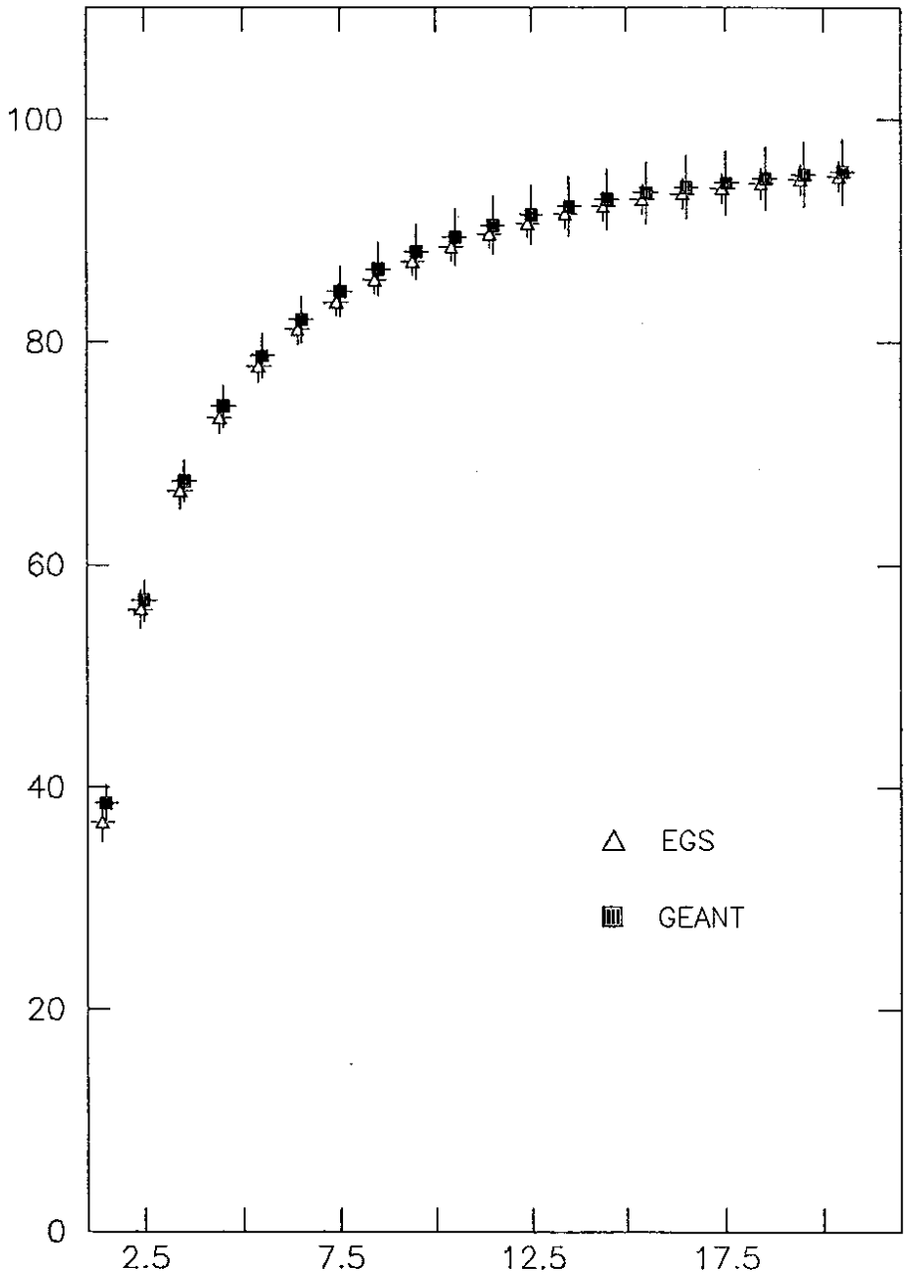


FIG. 19: CUMUL RADIAL ENERGY DEP.

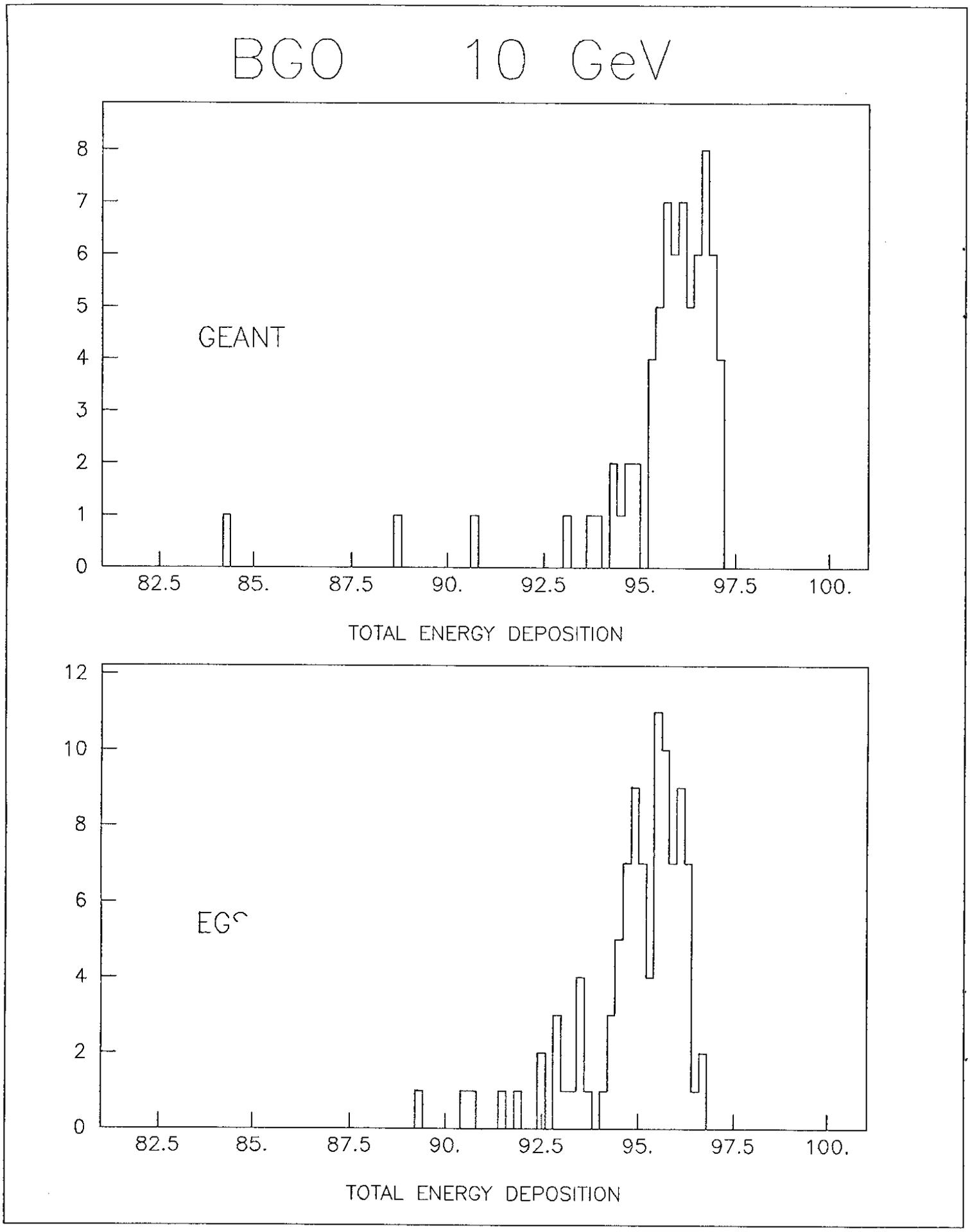

BGO 10 GeV
GEANT
0
1
2
3
4
5
6
7
8
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION
EGS
0
2
4
6
8
10
12
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION

FIG. 20

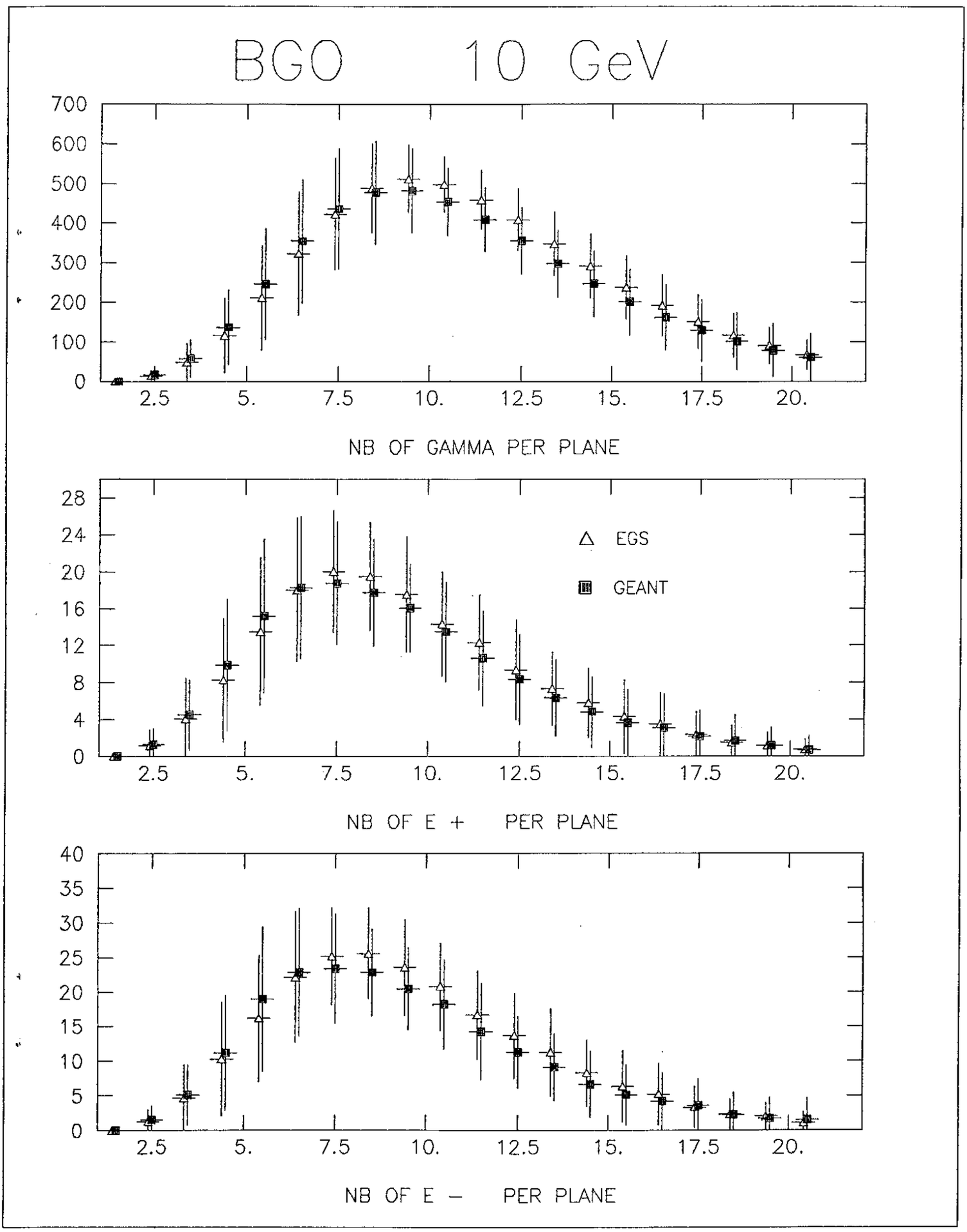

BGO 10 GeV
700
600
500
400
300
200
100
0
2.5
5.
7.5
10.
12.5
15.
17.5
20.
NB OF GAMMA PER PLANE
28
24
20
16
12
8
4
0
EGS
GEANT
NB OF E + PER PLANE
40
35
30
25
20
15
10
5
0
NB OF E − PER PLANE

FIG. 21

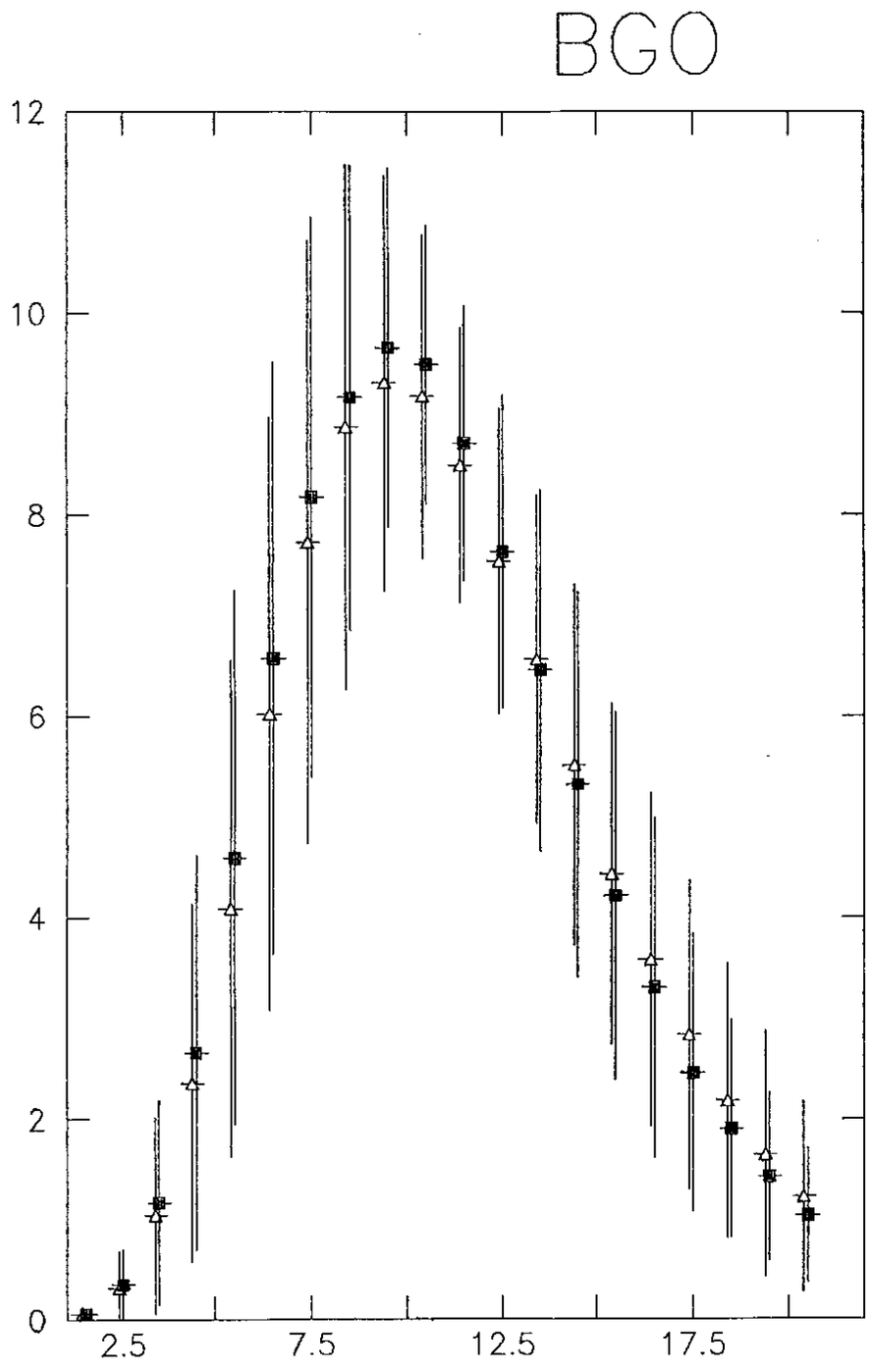


FIG. 22: LONGIT ENERGY DEPOSITION

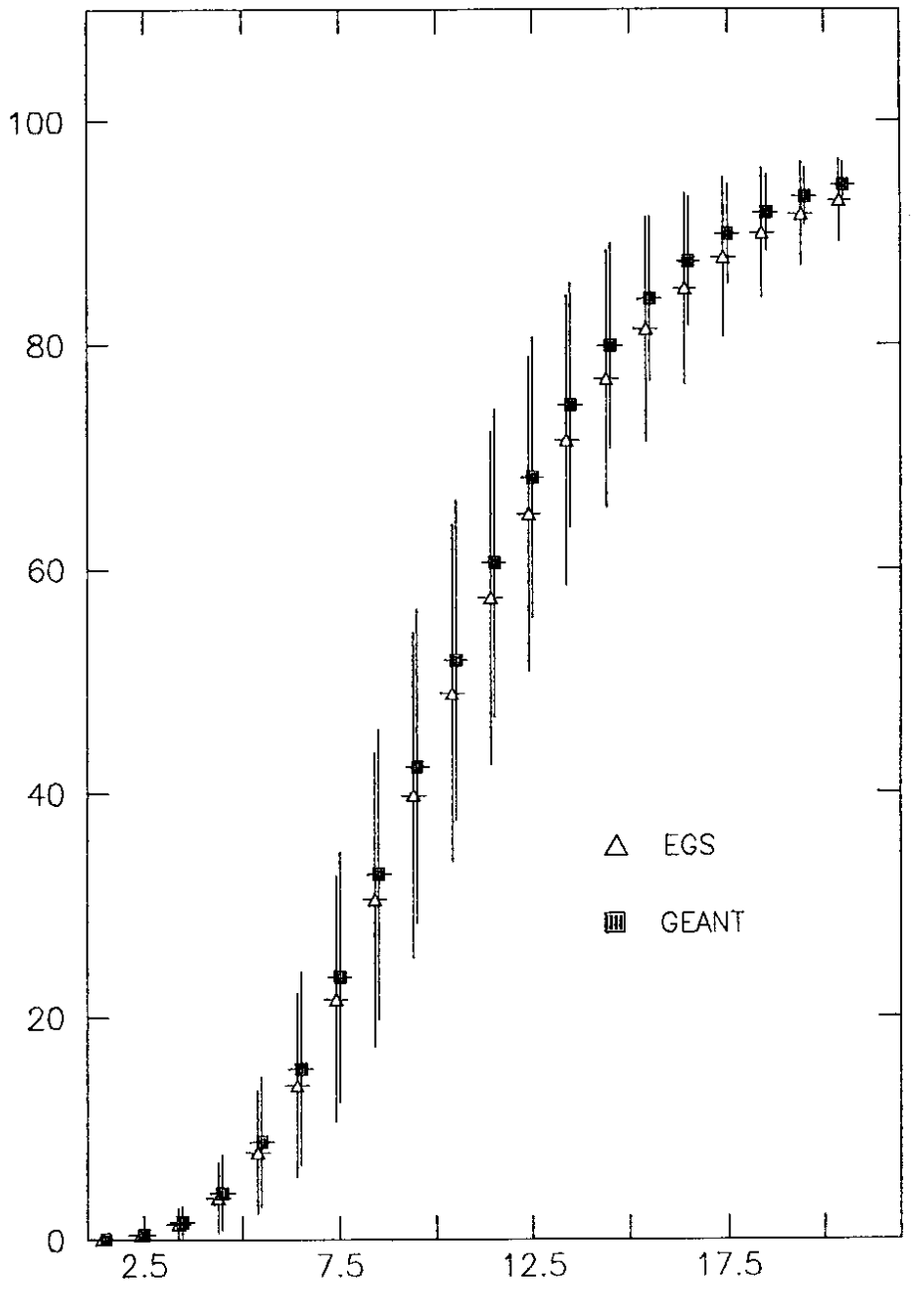


FIG. 23: CUMUL LONGIT ENERGY DEP.

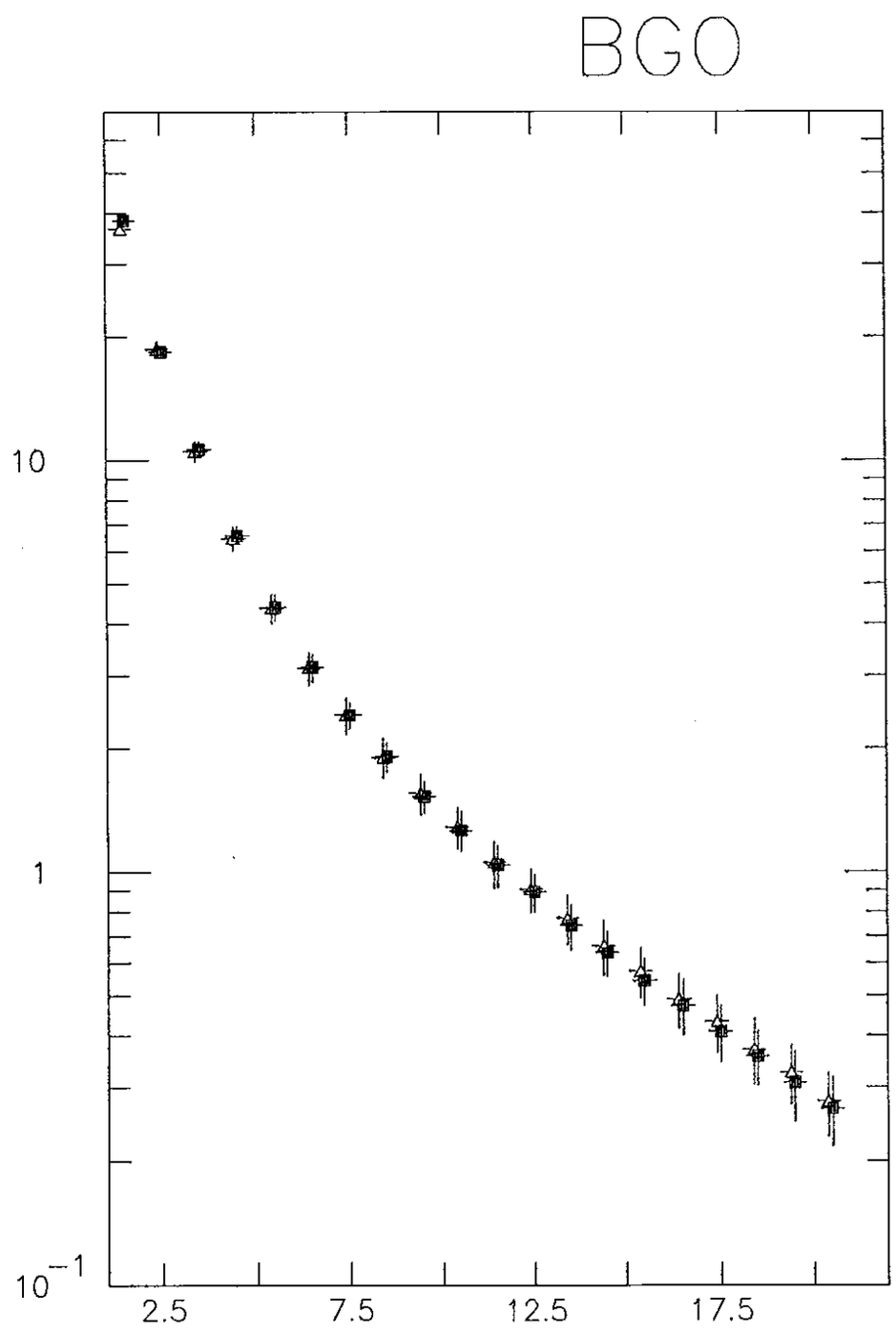


FIG. 24: RADIAL ENERGY DEPOSITION

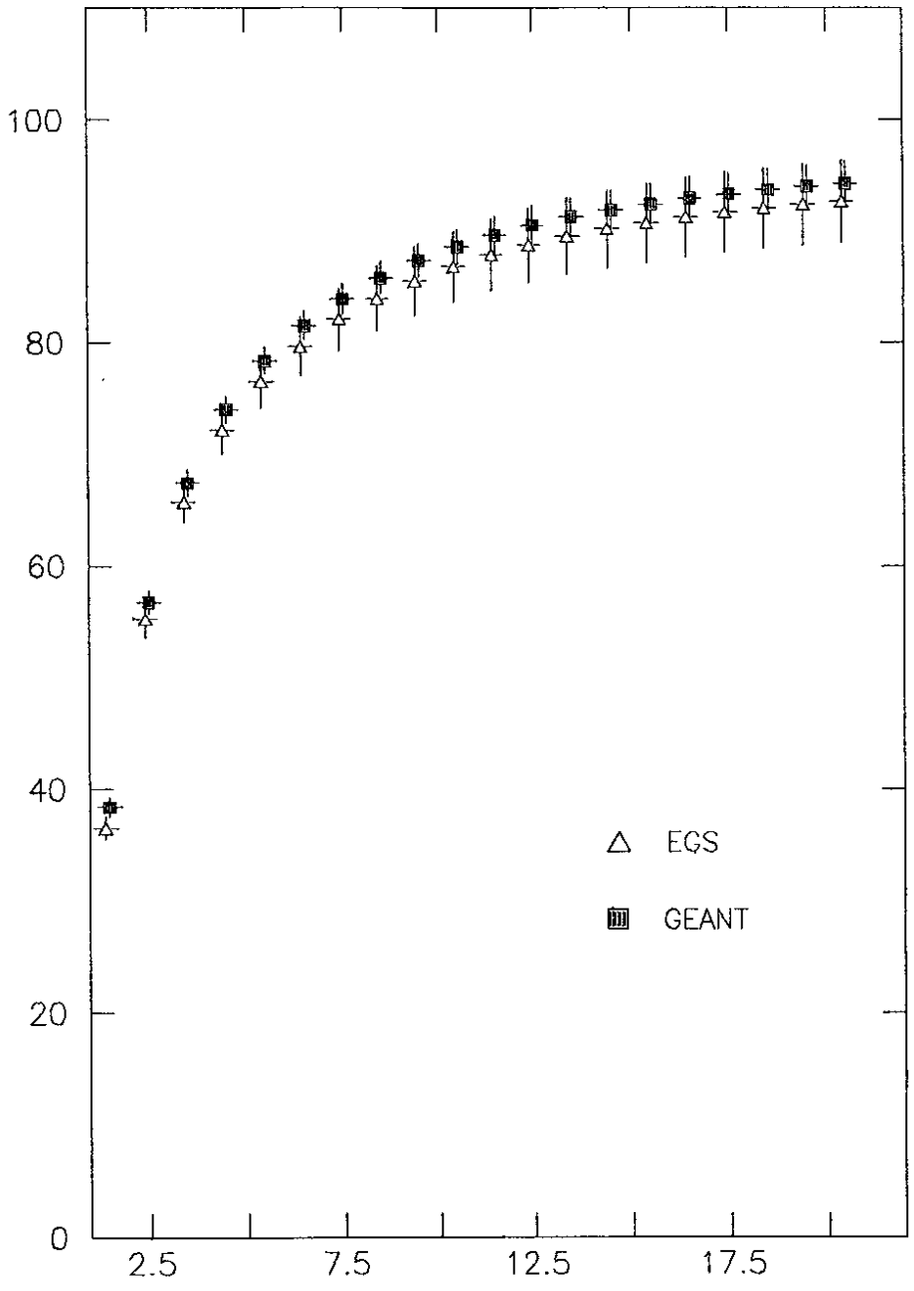


FIG. 25: CUMUL RADIAL ENERGY DEP.

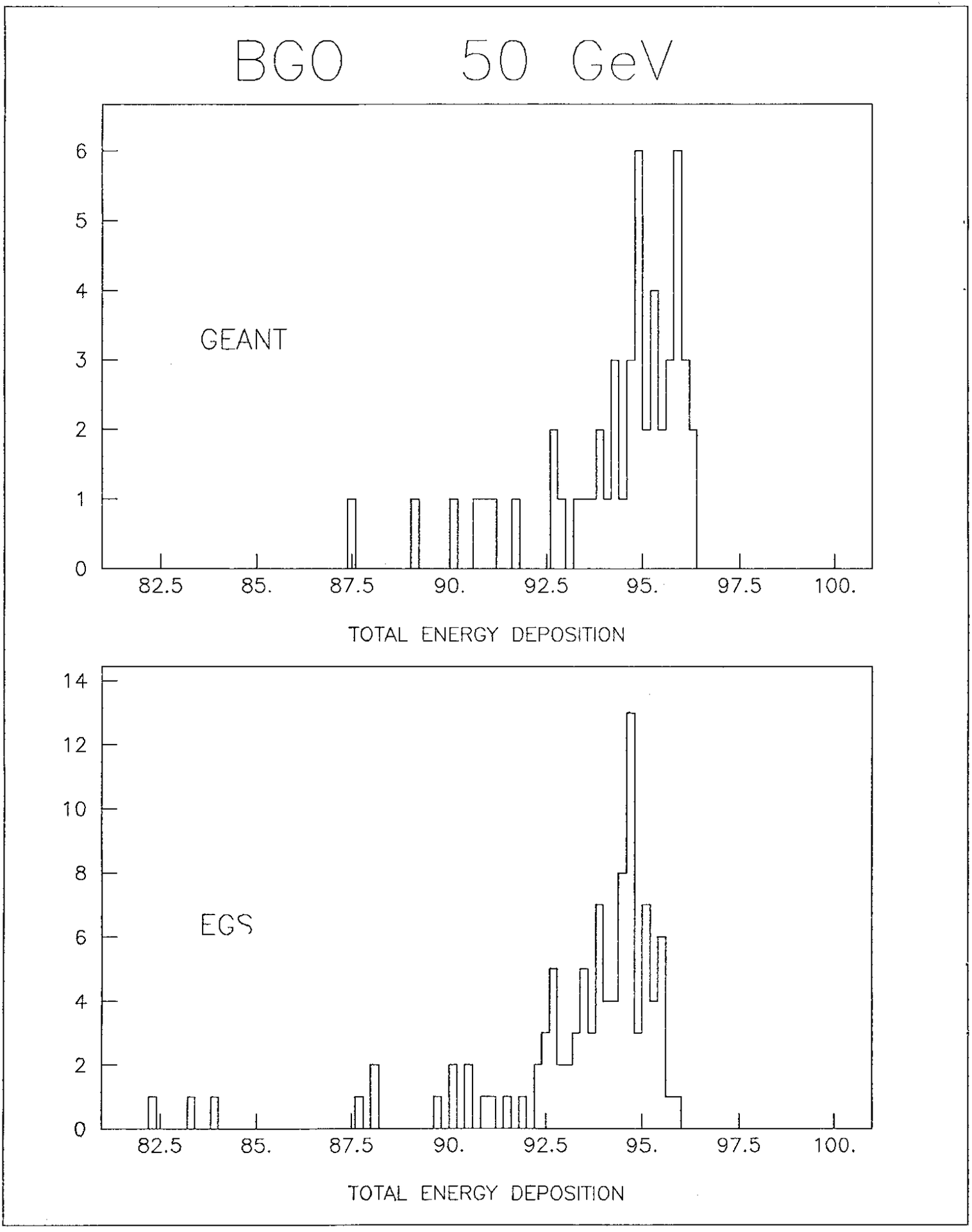

BGO 50 GeV
GEANT
0
1
2
3
4
5
6
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION
EGS
0
2
4
6
8
10
12
14
82.5
85.
87.5
90.
92.5
95.
97.5
100.
TOTAL ENERGY DEPOSITION

FIG. 26

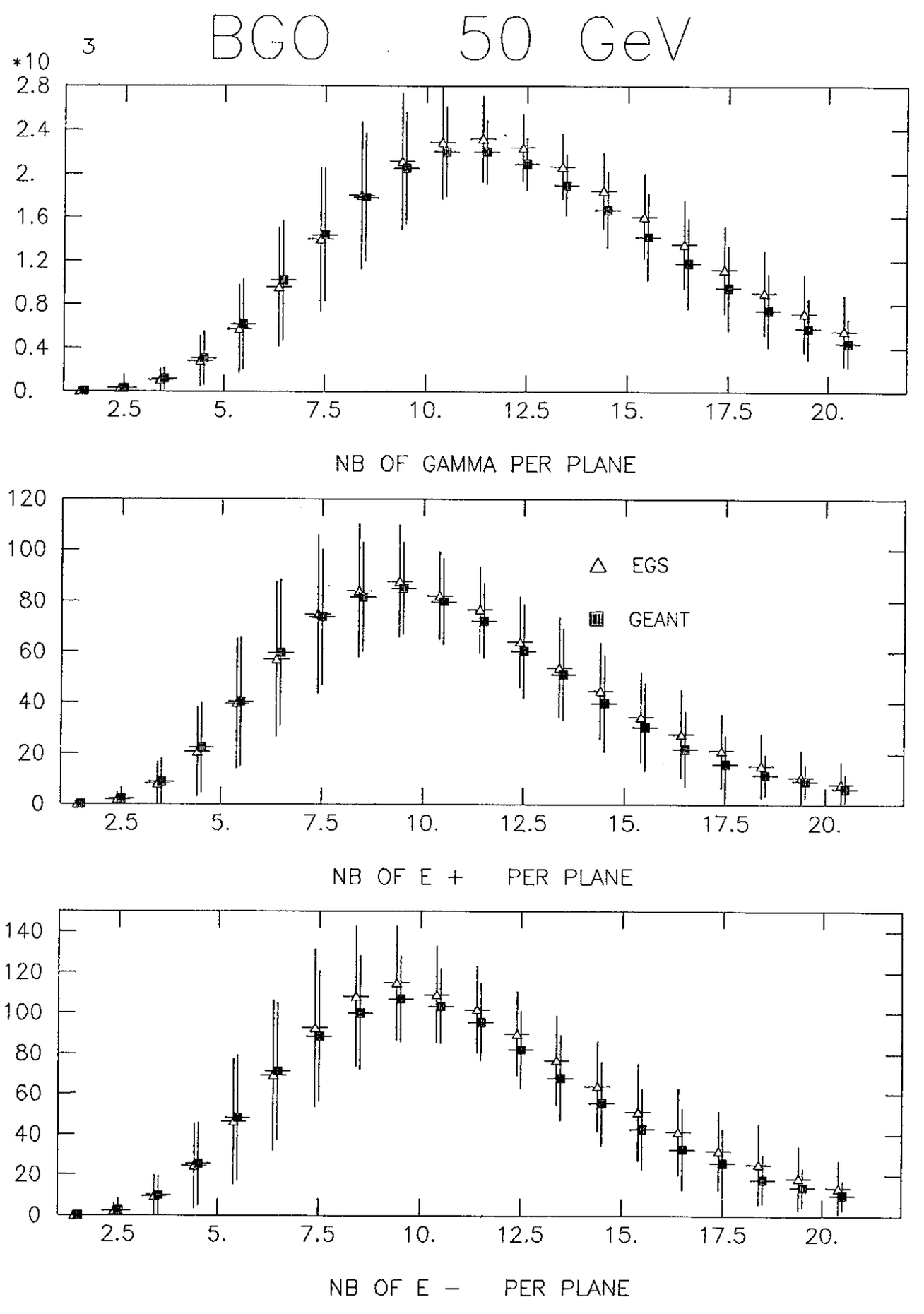

BGO 50 GeV
*10 3
2.8
2.4
2.
1.6
1.2
0.8
0.4
0.
2.5
5.
7.5
10.
12.5
15.
17.5
20.
NB OF GAMMA PER PLANE
120
100
80
60
40
20
0
EGS
GEANT
NB OF E + PER PLANE
140
120
100
80
60
40
20
0
NB OF E − PER PLANE

FIG. 27

Appendix A

Author(s) : R.Brun, G.N.Patrick
Origin : GEANT2/3

Submitted: 01.06.83
Revised: 01.10.84

Standard Material definition and related routines

CALL GMATE

Stores the following standard material constants in the data structure JMATE.

Material	No.	A	Z	Density	Radiat L	Absorb Len
Hydrogen	1	1.010	1.000	0.071	865.000	790.000
Deuterium	2	2.010	1.000	0.162	757.000	342.000
Helium	3	4.000	2.000	0.125	755.000	478.000
Lithium	4	6.940	3.000	0.534	155.000	120.600
Beryllium	5	9.010	4.000	1.848	35.300	36.700
Carbon	6	12.010	6.000	2.265	18.8	49.900
Nitrogen	7	14.010	7.000	0.808	44.500	99.400
Neon	8	20.180	10.000	1.207	24.000	74.900
Aluminium	9	26.980	13.000	2.700	8.900	37.200
Iron	10	55.850	26.000	7.870	1.760	17.100
Copper	11	63.540	29.000	8.960	1.430	14.800
Tungsten	12	183.850	74.000	19.300	0.350	10.300
Lead	13	207.190	82.000	11.350	0.560	18.500
Uranium	14	238.030	92.000	18.950	0.320	12.000
Air	15	14.610	7.300	1.205E-3	30423	6750.000
Vacuum	16	1.E-16	1.E-16	1.E-16	1.E+16	1.E+16

Note : If the user does not need all the above materials, or needs more materials, or wants to override the standard, the routine GSMATE can be called instead to define materials one at a time.

All data, except for nuclear absorption lengths, are taken from M. Aguilar-Benitez*. The absorption lengths, kept provisionally for backward compatibility, are no more used. They are recomputed at execution time.

* M. Aguilar-Benitez et al, Review of Particle Properties, Rev. Mod. Phys. 56(1984).

Author(s) : R.Brun, M.Maire, J.Allison
Origin : R.Brun, M.Maire

Submitted: 01.06.83
Revised: 01.10.84

Mixtures and Compounds

CALL GSMIXT(IMATE,NAMATE,A,Z,DENS,NLMAT,*WMAT*)

Defines mixture or compound IMATE as composed by the basic NLMAT materials defined through the arrays A, Z and WMAT.

If NLMAT > 0 then WMAT contains the proportion by weights of each basic material in the mixture.

If NLMAT < 0 then WMAT contains the number of atoms of a given kind in the molecule of the compound. In this case, WMAT in output is changed to relative weights.

Mixtures of compounds can also be defined:

IMATE	user material (mixture) number 0<IMATE<100
NAMATE	mixture name (ended by $)
A	array of atomic weights
Z	array of atomic numbers
DENS	density in gr/cm3
NLMAT	see above
WMAT	see above

The radiation length is computed according the EGS manual*,

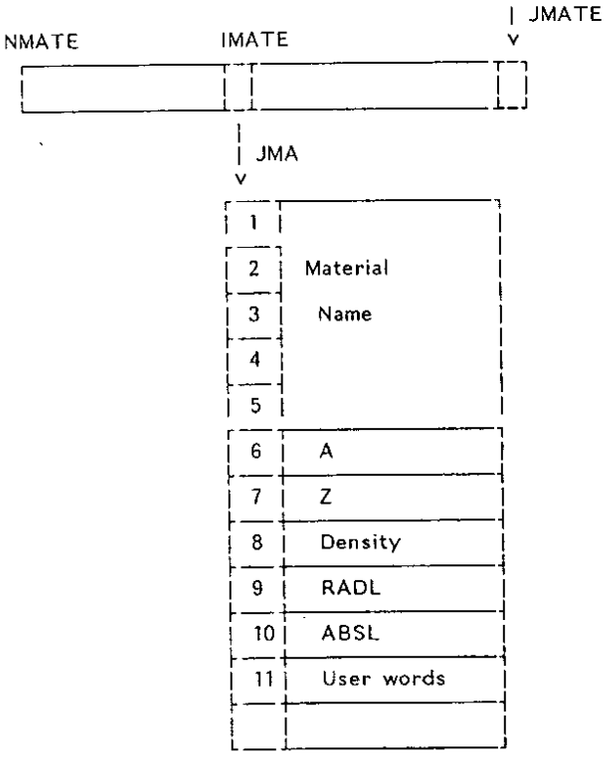
The subroutine GHMIX called by GSMIXT works out an effective atomic weight for a material with NLMAT elements. The criterion is that the hadronic interaction length of a 5 GeV/c pion is correct. Errors on the calculated hadronic interaction length for other momenta and other particles are less than 1% in most cases. For details see Memorandum OPAL/0037N/JA.

* EGS manual, SLAC-210 UC-32, June 78, formula 2-6-8 (37).

Author(s) : GEANT3
Origin : GEANT2

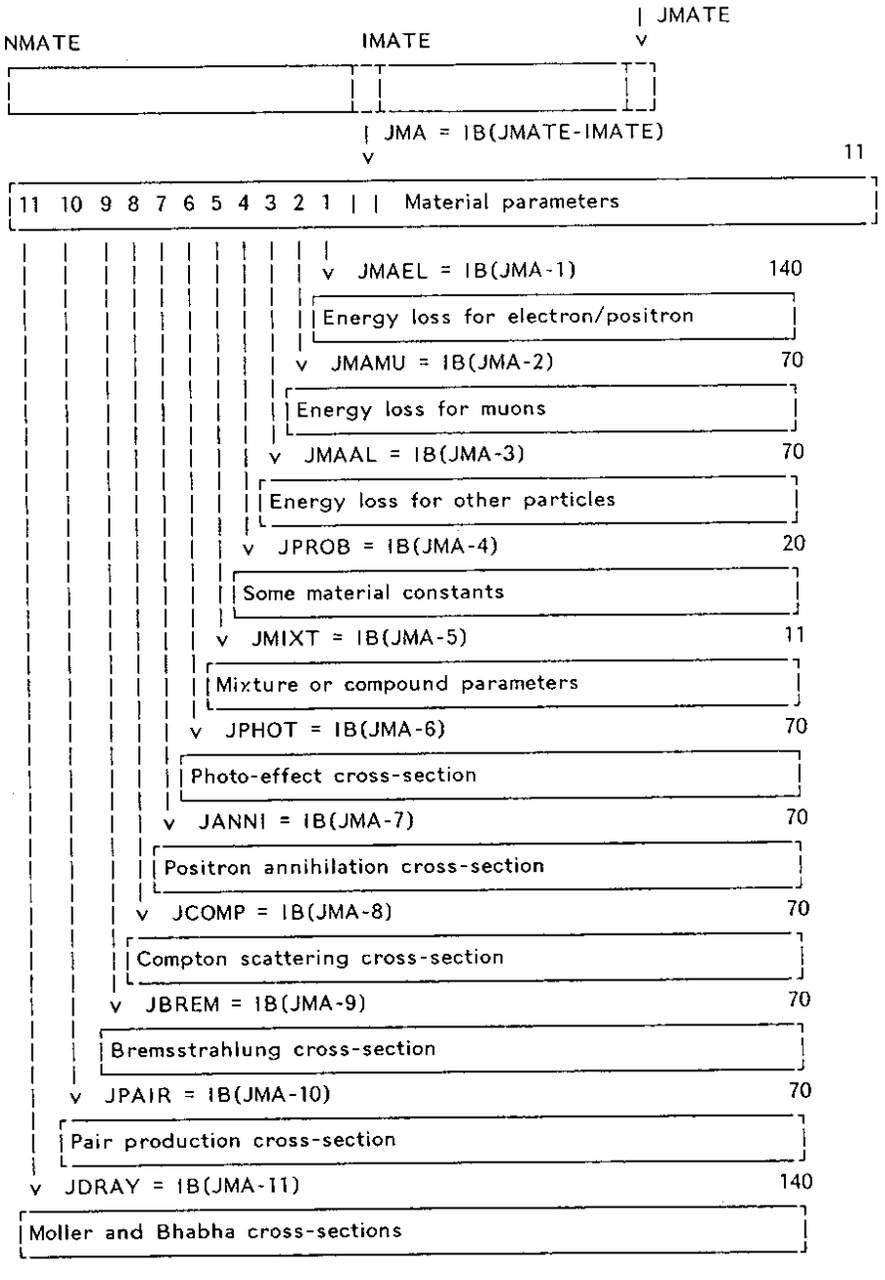
Submitted: 01.11.78
Revised: 01.10.84

The Material data structure JMATE



JMA = IB(JMATE-IMATE) pointer to material IMATE

When the subroutine GPHYSI is called at initialisation time the following banks are created for each material (tabulation of energy loss and cross-section).



Author(s) : M. Maire
Origin : GEANT3

Submitted: 24.09.84
Revised: 01.10.84

Introduction to the section PHYS

1. The physics processes

The processes currently implemented in GEANT3 can be classified as follows:

- Decays in flight
- Multiple scattering
- Continuous electromagnetic processes
- Discrete electromagnetic processes
- Hadronic interactions.

The fist two are controlled by the routines GDECAY which generates the decay products and GMUL which computes the change in position and angle due to multiple scattering over a given tracking step, respectively. The others are reviewed separately in the following paragraphs.

For convenience the particles are given a 'tracking type' in GEANT, depending on their interaction with matter:

ITRTYP =	1	'photon' like
	2	'electron' like
	3	neutral hadrons (and neutrinos!)
	4	charged hadrons
	5	'muon' like

The paragraph 5 gives a summary of the physics processes activated for each type of particle with the momentum range of validity when relevant.

GPHYSI controls the initialisation of the various processes.

2. Continuous electromagnetic processes

GPROBI	Initializes material 'constants' used for computing the probability of various interactions.
GELOSI	Control routine to fills DE/DX tables for energy loss due to ionisation.
GDEDXE	Calculates energy loss due to ionisation for charged particles other than electrons and positrons.
GELIEL	Calculates energy loss due to ionisation for electrons and positrons. As Moller and Bhabba scattering are treated as discrete processes the energy loss due to ionisation is a function of the electron kinetic energy cut-off DCUTE below which these processes are treated as a continuous energy loss. DCUTE is initialized to 1 Mev in /GCDRAY/ by GINIT and may be modified as second variable of the data card DRAY.
GELOSB	Initializes Bremsstrahlung cross-section and fills energy loss tables for Bremsstrahlung. Both formulae depend on the photon energy cut-off BCUTE below which Bremsstrahlung is treated as a continuous energy loss. BCUTE is initialized to 1 Mev in /GCBREM/ by GINIT and may be modified as third variable of the data card BREM.
GELOSP	Fills DE/DX tables for energy loss by direct pair production for high energy muons. The corresponding discrete process is not considered.
GELOSS	Computes at tracking time the total energy loss over a given step by interpolation through the relevant energy loss tables.

3. Discrete electromagnetic processes

For the simulation of each given discrete physics process three tasks have to be performed:

- The evaluation of the step length. This is computed from the updated probability for the occurrence of the process
  (GEANT routines GPxxxx).

- After transport of the particle, the generation of the final state particles
  (GEANT routines Gxxxxx).

- If the particle survives after the interaction, recompute the probability.

It should be outlined that the evaluation of the step length is made independently for each process which can occur, the final step size being the minimum of all and the process finally considered being the corresponding one.

GPHOTI	Tabulates cross-section for photo-electric effect at initialisation time
GPPHOT	Computes distance to next photo-electric interaction.
GPHOT	Simulates photo-electric mechanism
GCOMPI	Tabulates cross-section for Compton intersection at initialisation time
GPCOMP	Computes distance to next Compton scattering interaction
GCOMP	Simulates Compton scattering
GPAIRI	Tabulates cross-section for pair production at initialisation time
GPPAIR	Computes distance to next electron pair production by photons
GPAIR	Simulates electron pair production by photons
GBREMI	Tabulates cross-section for Bremsstrahlung at initialisation time
GPBREM	Computes distance to next hard Bremsstrahlung by electrons
GBREM	Simulates hard Bremsstrahlung by electrons. - see cut-off BCUTE in routine GELOSB above
GDRAYI	Tabulates cross-section for delta-ray at initialisation time
GPDRAY	Computes distance to next interaction point for delta rays (i.e. Moller or Bhabba scattering)
GDRAY	Simulates delta rays (Moller or Bhabba scattering) - see cut-off DCUTE in routine GELIEL above

GANNII	Tabulates cross-section for positron annihilation at initialisation time
GPANNI	Computes distance to next positron annihilation
GANNI	Simulates positron annihilation in flight
GANNIR	Simulates positron annihilation at rest.

4. Hadronic processes

For TATINA:

GHSIG/M	Returns absorption cross-section in millibarns for a given particle of given momentum. Interpolation in tables compiled by R. Barlow (Manchester), 31/01/84
GPHADR	Computes distance to next interaction point for hadronic processes
GHTATI	Simulates hadronic interactions, program TATINA (T. Baroncelli)

For CASCADE84:

GPHADR	Computes distance to next interaction point for hadronic processes
GHCASC	Simulates hadronic interactions, program CASCADE84 (A. Grant) CASCADE84 is available on a separate PAM file.

For GHEISHA:

GPGHEI	as GPHADR
GHEISH	Simulates hadronic interactions, program GHEISHA (H. Fesefeldt) GHEISHA is available on a separate PAM file.

5. Summary of physics processes

PROCESS	ROUTINES	ITRTYP	RANGE
Decay in flight	GDECAY	3,4	
Multiple scattering	GMUL	2,4,5	
Soft electromagnetic:			
Continuous energy loss by ionisation	GELOSI GELIEL GDEDX	2,4,5	0.1 Mev / 1000 Gev
by Bremsstrahlung	GELOSB	2,5	
by direct pair production	GELOSP	5	
Discrete electromagnetic:			
Photo-electric effect	GPPHOT GPHOT	1	0.1 /15 Mev
Compton scattering	GPCOMP GCOMP	1	0.5 /100 Mev
Pair production	GPPAIR GPAIR	1	1 Mev/100 Gev
Bremsstrahlung	GPBREM GBREM	2	
Delta rays (Moller, Bhabba)	GPDRAY GDRAY	2	
Positron annihilation	GPANNI/R GANNI GANNIR	2	
Discrete hadronic:			
TATINA	GPHADR GHTATI	3,4	
CASCADE84	GPHADR GHCASC	3,4	
GHEISHA	GPGHEI GHEISH	3,4	

6. Switch ON/OFF the physical processes

It is possible to scwitch on/off each individual process, totally or partially, by setting a relevant flag to 0, 1, or 2. This can be done via data cards (see BASE 040).

Below are listed the keyword for data card, the flag name, and the resulting action.

Keyword Data card	Flag name /common/	OFF=0	ON=1	2
	IDECAY /GCFLAG/	NO decay in flight	part. stopped ISTOP=1	
	IMUL /GCFLAG/	NO multiply scattering		
	ILOSS /GCFLAG/	NO continuous energy loss		
PHOT	IPHOT /GCPHOT/	NO photo elect. effect	phot stopped ISTOP=1	photon stopped no elec. generat ISTOP=2
COMP	ICOMP /GCCOMP/	NO compton effect	phot NOT stop ISTOP=0	phot NOT stopped no elect gener. ISTOP=0
PAIR	IPAIR /GCPAIR/	NO pair production	phot. stopped ISTOP=1	phot. stopped NO pair generat. ISTOP=2
BREM	IBREM /GCBREM/	NO brem	elec NOT stop ISTOP=0	elec NOT stopped no phot generat ISTOP=0
DRAY	IDRAY /GCDRAY/	NO delta ray	elec NOT stop ISTOP=0	elec NOT stopped no secondary gen ISTOP=0
ANNI	IANNI /GCANNI/	NO positron annihilation	positron stop ISTOP=1	posit. stopped no phot. gener. ISTOP=2
HADR	IHADR /GCHADR/	NO hadronic interaction	hadron stopped ISTOP=1	hadron stopped no secondaries gen. ISTOP=2

Author(s) : R. Brun
Origin : Same

Submitted: 14.03.84
Revised: 16.01.85

Initialisation of Soft Physics Processes

CALL GPHYSI

Initializes the GEANT physics processes.

(to be documented).

Subroutines Called

GELOSI, GELOSB, GELOSP, GPROBI.

Author(s) : G.N. Patrick
Origin : GEANT2

Submitted: 12.03.82
Revised: 01.10.84

Initialisation of Ionisation Energy Loss

CALL GELOSI

GELOSI initialises the simulation of continuous charged particle energy loss due to ionisation, by filling an average proton DE/DX table for each material defined within GEANT.

All input information is obtained from the GEANT material banks, JMATE. All output information is written to specially constructed banks in the JMATE data structure.

GELOSI is called from the initialisation routine GPHYSI.

The resulting tables are designed for subsequent use by GELOSS (see entry in this write-up) which scales the energy loss to the current track, and corrects it's momentum on a step-by-step basis.

1. Method

The basic Bethe-Bloch(1) formula is employed together with modifications for the density effect and shell corrections.

$$\frac{DE}{DX} = D\,\frac{Z}{A}\,\frac{1}{\beta^2}\left[\frac{1}{2}\ln\left(\frac{2m_e c^2\eta^2 E_m}{I^2}\right) - \beta^2 - \frac{\delta}{2} - \frac{C_e}{Z}\right]$$

where,

$D = \frac{4\pi N}{A} r_e^2 m_e c^2 = 0.000307$ GeV cm²/g(2)

m_e = electron mass
Z = atomic number of medium
A = atomic weight
$\eta = \beta\gamma$

I is the average ionisation potential of the atom in question. There exists a variety of phenomenological approximations for this, and the following quoted by Bricman et al (2) has been adopted in GELOSI:

$$I = 16(Z)^{0.9} \text{ eV}$$

This somewhat arbitrary choice does not, however, represent a serious source of error since I only enters into the logarithmic term of the equation.

E_m is the maximum transferable energy to a free electron by the incident projectile:

$$E_m = \frac{2m_e c^2\eta}{1+2m_e/M(\eta^2+1)^{0.5}+(m_e/M)^2}$$

where, M = projectile mass.

δ is a correction term which takes account of the reduction in energy loss due to the so-called density effect. This becomes important at high energy because media have a tendency to become polarised as the incident particle velocity increases. As a consequence, the atoms in a medium can no longer be considered as isolated. To correct for this effect the formulation of Sternheimer (3) is used:

$$\delta(\eta) \quad \begin{cases} = 0 & X < X_0 \\ = 4.606X + C + a(X_1-X)^m & X_0 < X < X_1 \\ = 4.606X + C & X \geq X_1 \end{cases}$$

where, $4.606X = \ln(\eta^2)$

X_0, X_1, a, m and C are constants which depend on the medium and are calculated as below:

$$C = -2\ln(I/h\nu_p)-1$$

where

ν_p = plasma frequency of medium $= (ne^2/\pi m_e)\ s^{-1}$

n = number electrons / cm³

$$a = \frac{4.606(X_a - X_0)}{(X_1 - X_0)^m}$$

where, $X_a = \frac{-C}{4.606}$

I < 100 eV : $X_1 = 2.0$, m = 3.0

i) $X_0 = 0.2$ for $-C < 3.681$
ii) $X_0 = -0.326C-1.0$ for $-C > 3.681$

I ≥ 100 eV : $X_1 = 3.0$, m = 3.0

i) $X_0 = 0.2$ for $-C < 5.215$
ii) $X_0 = -0.326C-1.5$ for $-C > 5.215$

C_e/Z is a shell-correction term which accounts for the fact that at low energies (light elements) or for heavy elements (all energies) the probability of particle-electron collisions at deep electronic layers (K, L, etc.) is negligible. Barkas (4) has published a semi-empirical formula which is applicable to

all materials, and this is utilised by GELOSI:

$$C_e(I,\eta) = (0.42237\eta^{-2}+0.304\eta^{-4}-0.00038\eta^{-6})10^{-6}I^2 + (3.858\eta^{-2}-0.1668\eta^{-4}+0.00158\eta^{-6})10^{-9}I^3$$

where the units are MeV if I is expressed in eV !

However, this formula breaks down at very low energies and is thus only applied if $\eta > 0.13$.

2. Description of Banks

For each material the proton DE/DX function is stored as a table at 26 discrete values of proton kinetic energy, T. The following data can be accessed after the call to GELOSI:

JM	= IB(JMATE-I)	pointer to I'th material	
JEL1	= IB(JM-1)	pointer to DE/DX bank	for e^+/e^-
JEL2	= IB(JM-2)	" " "	for μ^+/μ^-
JEL3	= IB(JM-3)	" " "	for all others particles

3. Accuracy

GELOSI in conjunction with GELOSS (see entry in this write-up) has been tested against energy loss tables (5,6) for various materials in the energy range 0 - 25 GeV. Typical discrepancies are as follows:

Beryllium	:	1.1 % at 0.05 GeV
		0.02 % at 25 GeV
Hydrogen	:	1.5 % at 0.05 GeV
		12.1 % at 25 GeV
Water	:	8.1 % at 0.05 GeV
		4.4 % at 6 GeV

4. Restrictions

1) This routine used in conjunction with GELOSS is strictly only valid for particles much heavier than the electron. For electrons, comparisons have been made with the modified figure of Messel and Crawford (7) which appears in Bricman et al. (2). Over the energy range 0 - 100 MeV an average discrepancy of ≃ 47% was observed, although the qualitative energy dependence of the DE/DX function was correct. This is, of course, not unexpected because a special form of the DE/DX equation should be used for electrons. However, this accuracy may be good enough for many purposes because (a) Bremsstrahlung losses dominate above 10 MeV, (b) above 100 MeV ionisation losses are negligible, (c) below 10 MeV other losses (Moller, Bhabha) scattering should also be accounted for.

2) For e^-/e^+, see routine GELIEL.

3) Although the routine works, with varying accuracy, for any material, the user must take care when adding mixtures or compounds to the GEANT material banks (JMATE). It is essential that the specified physical properties (Z, A, ρ, etc.) are correctly weighted and averaged: see routine GSMIXT [CONS 110]

5. References

1) H.A. Bethe : Ann. d. Phys. 5 (1930) 325
Zeits f. Phys. 76 (1932) 293
M.S. Livingston & H.A. Bethe Rev. Mod. Phys. 9 (1937) 245
F. Bloch Zeits. f. Phys. 81 (1933) 363

2) C. Bricman et al. : Rev. Mod. Phys. 52 (1980) 1

3) R.M. Sternheimer : Phys. Rev. 88 (1952) 851
Phys. Rev. 103 (1956) 511

4) W.H. Barkas : UCRL - 10292 (August 1962)

5) C. Serre : CERN 67-5 (March 1967)

6) C. Richard - Serre : CERN 71-18 (Sept. 1971)

7) H. Messel & D.F. Crawford : Electron - Photon Shower Distribution Function Tables for Lead, Copper and Air Absorbers, Pergamon Press (1970) 18.

Author(s) : L. Urban
Origin : Same

Submitted: 26.10.84
Revised: 11.01.85

Energy loss by ionisation for electron/positron

CALL GELIEL

The routine GELIEL gives the energy loss/centimeter due to ionisation for electrons and positrons.

Input : E energy of the particle

Output: GELIEL -dE/dx in GeV/cm.

GELIEL is automatically called from the routine GELOSI.

1. Method

The ionisation energy loss, -dE/dx, is calculated by the formula of Berger and Seltzer [1] which is based on the Bethe-Block formula [2] (see [3] too).

$$-\frac{dE}{dx} = \frac{2\pi r_0^2 mn}{B^2}\left[\ln\frac{2(\tau+2)}{(I/m)^2} + F^{\pm}(\tau,\Delta) - \delta\right] \qquad [1]$$

where,

$\gamma = E/m$; $B^2 = 1-1/\gamma^2$; $\tau = \gamma-1$

$$\tau_c = \frac{Ec-m}{m}$$

E_c energy cut for $e^{\pm}$

τ_{max} maximum possible energy transfer/electron mass

$\tau_{max} = \tau$ for e^+ , $\tau/2$ for e^-

$\Delta = \min(\tau_c, \tau_{max})$

n electron density of the medium
I average mean ionisation energy
δ density effect correction.

The functions $F^{\pm}$ are given by

$$F^+(\tau,\Delta) = \ln(\tau\Delta) - \frac{B^2}{\tau}\left[\tau+2\Delta - \frac{3\Delta^2\gamma}{2} - (\Delta - \frac{\Delta^3}{3})\gamma^2 - (\frac{\Delta^2}{2} - \tau\frac{\Delta^3}{3} + \frac{\Delta^4}{4})\gamma^3\right] \qquad [2]$$

$$F^-(\tau,\Delta) = -1-B^2+\ln[(\tau-\Delta)\Delta] + \tau/\tau-\Delta + \left[\frac{\Delta^2}{2} + (2\tau+1)\ln(1-\frac{\Delta}{\tau})\right]\frac{1}{\gamma^2} \qquad [3]$$

where, $\gamma = 1/(\gamma+1)$

The density effect correction is calculated as [4]

$$\delta = \begin{cases} 0 & \text{if } x < x_0 \\ 2\ln 10\cdot X + C + a(X_1-X)^m & x_0 \le x \le x_1 \\ 2\ln 10\cdot X + C & x_1 < x \end{cases} \qquad [4]$$

where $x = \ln(\gamma^2-1) / 2\ln 10$

The quantities n, I and the parameters of the density effect correction (x_0, x_1, C, a, m) are computed in the routine GPROBI, but we give the corresponding formulae here.

The electron density of the medium, n, can be written as

$$n = \frac{N\rho Z}{A} \quad \text{for elements} \qquad [5]$$

$$n = \frac{N\rho\sum_i p_i Z_i}{\sum_i p_i A_i} \quad \text{for compounds/mixtures}$$

where

N	Avogadro's number
$Z(Z_i)$	atomic number (of i-th component) of the medium
$A(A_i)$	atomic weight (of i-th component) of the medium
ρ	density of the material
p_i	proportion by number of the i-th element in the material (for a mixture p_i can be calculated as $p_i = w_i/A_i$ where w_i the corresponding proportion by weight).

The average mean ionization energy can be calculated as

$$I = (16\cdot Z^{0.9})10^{-9} \text{ GeV} \qquad [6]$$

for a chemical element. In the case of a compound or mixture the average value

$$I = \exp[\Sigma_i p_i Z_i \ln(Z_i)/\Sigma_i p_i Z_i]$$

is used (1,3,4).

The density effect correction parameters can be computed (for condensed medium, 4) as

$$\bar{C} = -C=1+2\ln \frac{I}{28.8\sqrt{\rho(\Sigma_i p_i Z_i/\Sigma_i p_i A_i)10^{-3}}}$$

$m = 3$

$$X_a = \frac{\bar{C}}{2\ln 10}$$

$$a = \frac{2(\ln 10)(X_a - X_0)}{(X_1 - X_0)^m}$$

I	$\bar{C}$	X_0	X_1
$<10^{-7}$	<3.681	0.2	2
$<10^{-7}$	≥3.681	-0.326C-1	2
$\geq 10^{-7}$	<5.215	0.2	3
$\geq 10^{-7}$	≥5.215	-0.326C-1.5	3

2. References

1) M.J. Berger, S.M. Selzer, NASA-SP-3012 (1964).

2) H.A. Bethe, Ann. Phys. 5 (1930) 325.
F. Bloch, Z. fur Phys. 81 (1933) 363.

3) R.L. Ford, W.R. Nelson, SLAC-210, UC-32 (1978).

4) R.M. Sternheimer, Phys. Rev. B3 (1971) 3681.

Author(s) : L. Urban
Origin : GEANT2

Submitted: 10.12.84
Revised: 16.01.85

Initialisation of Soft Bremsstrahlung

CALL GELOSB

GELOSB initialises: (a) the simulation of continuous (soft) energy loss due to Bremsstrahlung and (b) the simulation of discrete (hard) Bremsstrahlung by electrons (see also entry GBREMI).

Commons Used

GCBREM

where,

BCUTE cut-off photon energy (GeV) below which electron Bremsstrahlung is treated as a continuous energy loss and above which it is considered a discrete process
BCUTM can be changed by data card BREM.
SIGMAH, SIGMAL, DEDXH, DEDXL are arrays contained the coefficients of the cross section-energy loss formulas (at output)

GELOSB is automatically called by GPHYSI.

1. Method

Semiempirical formulas [1] are used to calculate the Bremsstrahlung cross section and the energy loss due to soft Bremsstrahlung. The differential cross section per atom for the Bremsstrahlung can be written as [2].

$$\frac{d\sigma(Z,E,k)}{dk} = \frac{A(Z,E)r_0^2\alpha Z[Z+\xi(z)]}{k}\{[1+(E'^2/E^2)][\Phi_1(\delta)-F(Z)] - {}^2/_3(E'/E)[\Phi_2(\delta)-F(Z)]\} \quad [1]$$

where,

E,E' are the energies before and after the Brems process
K photon energy
r_0 classical electron radius
α fine structure constant
Z atomic number of the medium

$$\xi(Z) = \frac{\ln(1440Z^{-2/3})}{\ln(183Z^{-1/3})-fc(Z)} \quad fc(Z) \quad \text{Coulomb correction}$$

$\Phi_1(\delta)$ screening functions

$$F(Z) = \begin{cases} 4/3\ln Z & E<0.05\text{GeV} \\ 4/3\ln Z+4fc(Z) & E\geq 0.05\text{GeV} \end{cases}$$

$A(Z,E)$ is an empirical correction factor (see entry GPPAIR).

In order to simulate the discrete Brems and the energy losses due to soft photons we need the quantities (k_c is the cutoff energy)

$$\sigma(Z,E,k_c) = \int_{k_c}^{E-m} dk \frac{d\sigma(Z,E,k)}{dk} \qquad [2]$$

and

$$E_l(Z,E,k_c) = \int_{0}^{k_c} dk \frac{d\sigma(Z,E,k)}{dk} \cdot k \qquad [3]$$

we have chosen the following parametrisation for σ and E_l

$$\sigma = Z*[Z+\xi(Z)]*[\ln(183Z^{1/3})-f_c(Z)]* F_\sigma \qquad [4]$$

$$E_l = Z*[Z+\xi(Z)]*[\ln(183Z^{1/3})-f_c(Z)]* F_l \qquad [5]$$

where,

a. $E\geq E_0 = 0.05\text{GeV}$ (high energy)

$$F_\sigma = \ln(E/k_c)\ [(c1+c2v+c3v^2) + (c4+c5v+c6v^2)/x \qquad [6]$$
$$+(c7+c8v+c9v^2)y/x + (c10+c11v+c12v^2)/2x + (c13+c14v+c15v^2)y/x]$$

$$F_l = k_c * [\text{ the same functional form as above }] \qquad [7]$$

b. $E\leq E0$, low energy

$$F_\sigma = F_\sigma^{he}(Z,E=E_0,k_c)\ \{[d1+(d2+d3v+d4v^2+d5v^3)y$$
$$+(d6+d7v+d8v^2+d9v^3)y^2 + (d10+d11v+d12v^2+d13v^3)y^3] \qquad [8]$$
$$+Z*[(d14+d15v+d16v^2+d17v^3)y + (d18+d19v+d20v^2+d21v^3)y^2$$
$$+(d22+d23v+d24v^2+d25v^3)y^3]\}$$

$$F_l = F_l^{he}(Z,E=E_0,k_c) \quad \{\text{as above}\} \qquad [9]$$

and

$X = E/E_0$

$Y = \ln x$

$v = \ln(E_0/k_c)$

(h.e. = high energy formula)

It can be noticed that for a fixed value of the cutoff energy we have less parameters (5 and 7 in the high and low energy region resp.) and that the formulas are very similar to the pair production cross section formula (see entry GPPAIR).

The linear parameters c_i, d_i have been determined from a least squares fit where we have used the numerically calculated integrals [2], [3] as data points. The formulas give a good description of the "data" in the region

$$5 \lesssim z \lesssim 100$$
$$0.001\ \text{GeV} \lesssim E \lesssim 100\ \text{GeV}$$
$$0.02\ \text{MeV} \lesssim k_c \lesssim \text{MeV}$$

The parameters and the errors of the formulas are given in the tables below.

High energy

i	c_i^σ	c_i^E
1	2.86945 e-3	3.16988 e-3
2	2.96195 e-5	2.25198 e-7
3	-1.27624 e-6	-2.00801 e-8
4	-1.01700 e-3	-2.94850 e-4
5	2.04059 e-4	9.93583 e-5
6	-1.21638 e-5	-8.20938 e-6
7	-7.05793 e-4	3.19133 e-5
8	1.30959 e-4	-2.94725 e-6
9	-7.32482 e-6	2.33261 e-7
10	-8.34548 e-4	-4.94659 e-4
11	8.84740 e-5	4.75015 e-5
12	-5.16483 e-6	-3.90750 e-6
13	-1.63347 e-3	-1.38333 e-3
14	5.15663 e-6	-4.81533 e-5
15	-4.48653 e-8	4.06024 e-6

low energy

i	D_i^{σ}	D_i^{E}
1	1.04724	1.10182
2	1.91891	5.84887 e-1
3	-7.23990 e-1	-2.75899 e-1
4	1.09973 e-1	6.17671 e-2
5	-5.67750 e-3	-4.26711 e-3
6	6.14075 e-1	6.44148 e-2
7	-3.44923 e-1	-5.01329 e-2
8	6.22575 e-2	2.20108 e-2
9	-3.51035 e-3	-1.99463 e-3
10	3.30758 e-2	6.71936 e-2
11	-3.14272 e-2	-2.76054 e-2
12	7.05950 e-3	5.93621 e-3
13	-4.44824 e-4	-4.39020 e-4
14	-7.66292 e-4	2.02537 e-3
15	-6.72520 e-5	-7.69405 e-4
16	5.81004 e-5	1.12267 e-4
17	-5.02147 e-6	-6.87182 e-6
18	-9.59296 e-4	3.20839 e-3
19	2.83092 e-4	-1.02555 e-3
20	-9.40777 e-7	1.33871 e-4
21	-1.89274 e-6	-7.21580 e-6
22	-1.57014 e-4	9.45172 e-4
23	7.63065 e-5	-3.35890 e-4
24	-6.18695 e-6	4.12272 e-5
25	-2.39779 e-8	-1.95517 e-6

2. Errors (in %)

Z	E≥E0		E≤E0	
	$\frac{\Delta\sigma}{\sigma}$	$\frac{\Delta E_l}{E_l}$	$\frac{\Delta\sigma}{\sigma}$	$\frac{\Delta E_l}{E_l}$
10	<2.6	<1.8	<6.3	<10.5
≥20	<1.5	<0.8	<6.3	<10.5

After the photon cut-off energy is fixed GELOSB calculates the parameter combinations independent of the photon energy and Z:

SIGMAH(1) = c1+c2v+c3v²
SIGMAH(2) = c3+c4v+c5v²

SIGMAL(1) = d1
SIGMAL(2) = d2+d3v+d4v²+d5v³

for the cross section formula and the corresponding combinations for the energy loss formula too. With this kind of initialisation the cross section and

the energy loss can be calculated (in the routines GPBREM and GELOSS) more easily and quickly.

3. References

1) L. Urban, Modification in the electromagnetic part of the program GEANT, LEP3 internal report, 28/08/78.

2) R.L. Ford, W.R. Nelson, SLAC-210, UC-32, 1978.

Author(s) : G.N. Patrick
Origin : GEANT2

Submitted: 14.06.82
Revised: 01.10.84

Initialisation of Energy Loss due to direct Pair production

CALL GELOSP

GELOSP initialises the simulation of continuous energy loss due to direct pair-production by high energy muons. This is achieved by filling DE/DX tables as a function of energy.

All input information is obtained from the GEANT material banks, JMATE. All output information is written to specially constructed banks in the JMATE data structure.

GELOSP should be called from the user-supplied initialisation routine once only.

1. Description of Banks

JM	= IB(JMATE-I)	pointer to I'th material	
JEL1	= IB(JM-1)	pointer to DE/DX bank	for e^+/e^-
JEL2	= IB(JM-2)	" " "	for μ^+/μ^-
JEL3	= IB(JM-3)	" " "	for all others particles

Author(s) : G.N. Patrick
Origin : GEANT2

Submitted: 13.02.82
Revised: 01.10.84

Computation of Energy Loss during Tracking

CALL GELOSS(P,DSTEP,DP*,DEDX*)

Compute energy loss for the current particle in /GCKINE/ with momentum P when travelling in the current medium /GCTMED/ a distance DSTEP.

The momentum loss is returned in DP.

The energy loss in DEDX.

Energy loss tables have been filled at initialisation time by the routines GELOSI, GELOSB, GELOSP.

IB(JMA-1) contains the total energy loss for
electrons (bins 1 to 70)
positrons (bins 71 to 140)

IB(JMA-2) contains the total energy loss for muons

IB(JMA-3) contains the ionisation energy loss for all other particles.

It should be noted that before GELOSS is used the initialisation routine GPHYSI should have been called to set up the DE/DX tables.

Author(s) : R.Brun, M.Hansroul
Origin : Same

Submitted: 01.10.83
Revised: 01.10.84

Generates Multiple Scattering

CALL GMUL(VECT,STEP,VOUT*)

STEP Step size
VECT Initial coordinates, direction cosines, momentum

VOUT Output coordinates, direction cosines, momentum

The call to this routine is automatically selected by the GEANT tracking routines GTELEC,GTHADR,GTMUON . The algorithm is the following.

- first the particle position and direction are transformed into a local frame of reference in which the new Z axis coincides with the direction of the particle;

- two random deviation angles are computed in two orthogonal projections (i.e. in the new planes XZ and YZ). Each angle is obtained from a gaussian distribution with zero mean value and standard deviation given by the well known formula

$$ASD = 0.015*(SQRT(L/LR)/P + ALOG10(L/LR)/9)$$

where P is the momentum in GEV/C
L is the length of material traversed
LR is the radiation length of the material

- similarly, the deviations of the particle position are computed in the same two orthogonal projections. However, the correlations between the position and the corresponding angle deviation is taken into account;

let RAX be the random deviation angle in the XZ projection
RPX be the random number obtained from a gaussian distribution of zero mean and unit standard deviation.

Then the deviation of the position is calculated by the relation

$$DX = 0.5*(RAX + RPX/SQRT(3))*L*ASD$$

A similar relation holds for the YZ projection;

- then the modified position and direction of the particle are transformed back from the local frame into the original reference frame.

Author(s) : G.N.Patrick, L.Urban
Origin : GEANT2

Submitted: 07.12.84
Revised: 16.01.85

Interaction point for Discrete Brems by Electrons

CALL GBREMI

CALL GPBREM

During the tracking stage of GEANT the routine GPBREM is used to estimate the remaining distance before an electron radiates a photon via discrete Bremsstrahlung.

The relevant cross-section is extrapoled from tables filled by the routine GBREMI at initialisation time.

Input: via COMMON/GCTRAK/

Output: via COMMON/GCTRAK/

SBREM distance (cm) remaining until Bremsstrahlung interaction.

GPBREM is automatically called by the tracking routine GTELEC.

1. Method

The mean free path, λ, for an electron to radiate a photon via Bremsstrahlung is given by

$$\lambda = \frac{1}{\Sigma} \qquad [1]$$

where, Σ is the macroscopic cross-section (1/cm). This macroscopic cross section can be written as

$$\Sigma = \frac{N\rho\sigma(Z,E,k_c)}{A} \qquad [2a]$$

$$\Sigma = \frac{N\rho\sum_i p_i\sigma(Z_i,E,k_c)}{\sum_i p_i A_i} \qquad [2b]$$

for a compound or mixture, where

N	Avogadro's number
$Z(Z_i)$	atomic number of the material (i-th component of the material)
$A(A_i)$	atomic weight of the material (i-th component)
ρ	density of the material
σ	total cross section per atom for discrete (photon energy = k_c) Bremsstrahlung
p_i	proportion by number of the i-th element in the material $(p_i\text{-}w_i)A_i$ where w_i is the corresponding proportion by weight
k_c	photon cut-off energy.

To calculate the cross selection a semiempirical cross section formula is used[1] (see entry for GELOSB).

This cross-section is tabuled at initialisation time as a function of the medium and of the energy by routine GBREMI (see JMATE data structure). The energy binning is set within the array ELOW (COMMON CGLOSS) in the routine GPHYSI.

λ may, of course, change along an electron's trajectory (say x_0 to x) due to (a) medium changes, or (b) continuous energy losses. The number of mean free paths traversed, N_λ, is thus strictly given by

$$N_\lambda = \int_{x_0}^{x} dx/\lambda(x) \qquad [3]$$

If N_R is a random variable denoting the number of mean free paths from a given point until the next interaction, then it can be shown that N_R has the distribution

$$P(N_R < N_\lambda) = 1-\exp(-N_\lambda) \qquad [4]$$

N_λ can therefore be sampled by

$$N_\lambda = -\ln(R1) \qquad [5]$$

where, R1 is a random number between 0 and 1.

To account for the variation of σ with track length due to continuous energy losses, σ and λ are recalculated on each entry to GPBREM.

[1] L. Urban, Modification in the e.m. part of the program GEANT, LEP3 internal report, 28/08/78.

Author(s) : G.N. Patrick, L. Urban
Origin : Same

Submitted: 26.10.84
Revised:

Simulation of discrete Bremsstrahlung by electrons

CALL GBREM

GBREM generates electron Bremsstrahlung by treating it as a discrete process. Two alternative techniques are used to sample the energy of the radiated photon.

Input : via COMMON GCTRAK

Output: via COMMON GCKING.

GBREM itself is automatically called from the tracking routines if, and when the parent electron reaches its radiation point during the tracking stage of GEANT. For this purpose another routine, GPBREM, exists, which calculates the remaining distance to the interaction after each step.

1. Method

If IBMODE=1 (in COMMON GCBREM) a crude high-energy approximation [1] is used for the frequency distribution of the radiated photon energy, E_γ

$$E_\gamma = ELO \cdot (E/ELO)^R \qquad [1]$$

where, ELO maximum of BCUTE (the photon energy cut-off) and 0.0001 GeV (to avoid infra-red catastrophe)

E parent electron energy
R random number ($0 \leq R \leq 1$)

If IBMODE=2, a modified version of the random number techniques of Butcher and Messel is used (see Refs 2 and 3, and, entries for GPAIR).

The Coulomb corrected Bethe-Heitler differential cross section for production of a photon of energy ϵE by an electron of primary energy E is given as [3]:

$$\frac{d\sigma(Z,E,\epsilon)}{d\epsilon} = \frac{r_0^2 \alpha Z[Z+\xi(Z)]}{\epsilon} \{[1+(1-\epsilon)^2][\Phi_1(\delta)-F(Z)] - {}^2/_3(1-\epsilon)[\Phi_2(\delta)-F(Z)]\} \qquad [2]$$

where, δ is the usual screening variable

$$\delta = \frac{136m}{Z^{1/3}E} \frac{\epsilon}{1-\epsilon} \qquad [3]$$

all the other quantities have been described in the entry for GPAIR.

The kinematically allowed region for ϵ

$$\epsilon_c = \frac{k_c}{E} \leq \epsilon \leq 1 - \frac{m}{E} = \epsilon_m \qquad [4]$$

where, k_c is the photon cut off energy below which the Bremsstrahlung is treated as a continuous energy loss (BCUTE in the program).

The cross section [2] can be decomposed as (the normalisation is not important!)

$$\frac{d\sigma}{d\epsilon} = \sum_{i=1}^{2} \alpha_i f_i(\epsilon) g_i(\epsilon) \qquad [5]$$

where

$$\alpha_1 = F_{10} \ln\frac{\epsilon_m}{\epsilon_c}$$

$$\alpha_2 = F_{20} \frac{3}{8} \frac{\epsilon_m^2-\epsilon_c^2}{1-\epsilon_c}$$

$$f_1(\epsilon) = \frac{1}{\epsilon \ln\frac{\epsilon_m}{\epsilon_c}} \frac{1}{\epsilon}$$

$$f_2(\epsilon) = \frac{2}{\epsilon_m^2-\epsilon_c^2} \epsilon$$

$$g_1(\epsilon) = \frac{F_1}{F_{10}} \frac{1-\epsilon}{1-\epsilon_c}$$

$$g_2(\epsilon) = \frac{F_2}{F_{20}}$$

and

$$F_1(\delta) = 3\Phi_1(\delta)-\Phi_2(\delta)-2F(Z)$$

$$F_{10} = F_1(\delta_{min})$$

$$F_2(\delta) = 2\Phi_1(\delta)-2F(Z)$$

$$F_{20} = F_2(\delta_{min})$$

$$\delta_{min} = \frac{136m}{Z^{1/3}E} \frac{\epsilon c}{1-\epsilon c}$$

is the minimum value of the variable δ.

Using the decomposition [5] the sampling of ϵ can be done by (r_0, r_1, r_2, etc. a set of random numbers; $0 \leq r_i \leq 1$)

1) selecting the integer i (1 or 2)

   if $r_0 \leq BPAR = \alpha_1/(\alpha_1+\alpha_2)$ i=1

   if $r_0 > BPAR$ i=2

2) sampling ϵ from $f_i(\epsilon)$

   $\epsilon = \epsilon c(\epsilon m/\epsilon c)^{r_1}$ i=1

   $\epsilon = \sqrt{\epsilon c^2 + r_1(\epsilon m^2 - \epsilon c^2)}$ i=2

3) calculating the rejection function $g_i(\epsilon)$ and

   if $r_2 > g_{i(\epsilon)}$ rejecting ϵ, starting again from 1

   if $r_2 \leq g_i(\epsilon)$ accepting ϵ.

As in the case of the pair production after sampling ϵ from $f_i(\epsilon)$ we have to check that

$$\delta = \frac{136m}{Z^{1/3}E} \frac{\epsilon}{1-\epsilon c} \leq \delta_{max} = \exp\left(\frac{21.12-F(Z)}{4.184}\right) - 0.952$$

and if this is not the case, we have to start again from step 1.

The decomposition [5] used here is more simple and more effective than that is used in the earlier versions of GEANT and in EGS, and the method outlined above has no convergence problems.

After the successful sampling of ϵ, GBREM generates the polar angles of the radiated photon with respect to an axis defined along the parent electron's momentum. The azimuthal angle Φ is generated isotropically and Θ is assigned the approximate average value given by Marmier and Sheldon [4]

$$\Theta = m/E$$

This information is used to calculate the momentum vector of the radiated photon, transform it to the GEANT coordinate system and store the result into /GCKING/. Also, the momentum of the parent electron is corrected.

2. Restrictions

1) Effects due to breakdown of the born approximation at low energies are ignored.

2) Target materials composed of compounds or mixtures are treated identically to elements using the effective atomic number Z calculated in GSMIXT (This restriction does not hold when computing the mean free path in GPBREM!).

3. References

1) R.W. Williams : Fundamental Formulas of Physics Vol. 2, ed. D.H. Menzel, Dover Pubs. Inc (1960) 550.

2) J.C. Butcher & H. Messel : Nucl. Phys. 20 (1960) 15

3) R. Ford & W. Nelson : SLAC-210, UC-32 (June 1978).

4) P. Marmier & E. Sheldon : Physics of Nuclei and Particles Vol. 1, Academic Press (1969) 140.

Author(s) : G.N.Patrick, L.Urban
Origin : GEANT2

Submitted: 10.11.84
Revised: 16.01.85

Interaction point for e^+e^- pair production by photons

CALL GPAIRI
CALL GPPAIR

During the tracking stage of GEANT the routine GPPAIR is used to estimate the remaining distance before a photon interacts to produce an e^+e^- pair.

Input: via COMMON GCTRAK, GCKINE

Output: via COMMON/GCTRAK/

SPAIR distance (cm) remaining until $\gamma \rightarrow e^+e^-$ interaction.

The relevant cross-section is extrapoled from tables filled by the routine GPAIRI at initialisation time.

1. Method

The mean free path, λ, for a photon to interact via pair production is given by

$$\lambda = 1/\Sigma \qquad [1]$$

where, σ is the macroscopic cross-section (1/cm). This quantity is given by

$$\Sigma = \frac{N\rho\sigma(Z,E_\gamma)}{A} \qquad [2a]$$

$$\Sigma = \frac{N\rho\sum_i p_i\sigma(Z_i,E_\gamma)}{\sum_i p_i A_i} \qquad [2b]$$

in the case of a compound or mixture, where

N	Avogadro's number
$Z(Z_i)$	atomic number of the material (i-th component of the material)
$A(A_i)$	atomic weight of the material (i-th component)
ρ	density of the material
σ	total cross section per atom for discrete (photon energy = k_c) Bremsstrahlung
p_i	proportion by number of the i-th element in the material $(p_i-w_i)A_i$ where w_i is the corresponding proportion by weight

The pair production total cross section is given (in barn/atom) by the semiempirical formula of L. Urban (see Ref. 1) as

$$\sigma(Z,E_\gamma) = Z[Z+\xi(Z)][\ln(183/Z^{1/3})-f_c(Z)]\left[c_1+\frac{c_2+c_3y}{x}+\frac{c_4+c_5y}{zx}+\frac{c6}{z^2x}\right] \qquad [3a]$$

for $E_\gamma \geq E_0 = 0.05$GeV (high energy case) and

$$\sigma(Z,E_\gamma) = \sigma^{he}(Z,E_0)[d_1+y(d_2+d_3y+d_4y^2)+Zy(d_5+d_6y+d_7y^2) +Z^2y(d_8+d_9y+d_{10}y^2)] \qquad [3b]$$

for $E_\gamma \leq E_0$ (low energy case), where (see Ref. 2)

$$\xi(Z) = \frac{\ln(1440/Z^{2/3})}{\ln(183/Z^{1/3})-f_c(Z)},$$

$f_c(Z)$ the Coulomb correction function

$f_c(Z) = a*(1/(1+a)+0.20206-0.0369*a+0.0083*a^2-0.002*a^3)$

$a = (\alpha*Z)^2$

$x = E/E0$

$y = \ln(x)$

and h.e. stands for high energy [3b].

The parameters c_i d_i have been taken from a least squares fit to the data (see Ref. 3)

i	ci	di
1	1.77335 e-3	1.00053
2	-6.24597 e-4	3.71977 e-1
3	-3.14699 e-4	-1.55704 e-2
4	-1.15638 e-3	-1.16654 e-2
5	-8.74558 e-4	-3.13118 e-3
6	7.84871 e-4	-1.60056 e-3
7	-	-2.02953 e-4
8	-	2.85252 e-5
9	-	1.84522 e-5
10	-	2.90179 e-6

The errors of the formula [3a] and [3b] are given below

$\frac{\Delta\sigma}{\sigma}$ (%)

Z	low energy	high energy
1	<70 (<20 for E>0.01)	<8
2	<15 (<8 for E>0.005)	<8
5	<2.8	<3.2
10	<2.8	<2.0
20	<2.7	<1.5
30	<2.2	<1.0

The accuracy seems to be quite good for Z>2, but the formula can be used for Z=1,2 too, because in this case the ratio σ pair production/σ total is small.

E(GeV)	σpp/ σtot (%) Z=1	Z=2
0.0015	0.03	0.05
0.002	0.12	0.24
0.005	1.7	3.0
0.01	6.1	9.6
0.02	16	23
0.05	38	49

This cross-section is tabuled at initialisation time as a function of the medium and of the energy by routine GPAIRI (see JMATE data structure). The energy binning is set within the array ELOW (COMMON CGLOSS) in the routine GPHYSI.

λ may, of course, change along the photon's trajectory (say x_0 to x) as it moves from one medium to another. The number of mean free paths traversed, N_λ, is thus strictly given by

$$N_\lambda = \int_{x_0}^{x} dx/\lambda(x) \qquad [4]$$

If N is a random variable denoting the number of mean free paths from a given point until the next interaction, then it can be shown that N_R has the distribution

$$Pr(N_R < N_\lambda) = 1-\exp(-N_\lambda) \qquad [5]$$

N_λ can therefore be sampled by

$$N = -\ln(R1) \qquad [6]$$

where, R1 is a random number uniformly distributed between 0 and 1.

2. References

1) L. Urban, Modification in the electromagnetic part of the program GEANT, LEP3 internal report, 28/08/78.

2) R.L. Ford, W.R. Nelson, SLAC-210, UC-32, 1978.

3) J.H. Hubbell, H.A. Gimm, I. Overbo, Jou. Phys. Chem. Ref. Data 9, 1023 (1980).

Author(s) : G.N.Patrick, L.Urban
Origin : GEANT2

Submitted: 12.12.84
Revised:

Simulation of e^+e^- Pair production by Photons

CALL GPAIR

GPAIR generates photon pair production by using a modified version of the random number techniques of Butcher and Messel (see Ref. 1) to sample the secondary electron/positron energies from the Coulomb corrected Bethe-Heitler differential cross-section.

Input: via COMMON/GCTRAK/

Output: via COMMON/GCKING/

GPAIR is automatically called, if, and when, the parent photon reaches its decay point during the tracking stage of GEANT. For this purpose another routine, GPPAIR, exists, which calculates the remaining distance to the interaction after each step.

1. Method

We give a very short summary of the random number technique used here (see [1], [2]). The method is a combination of the composition and rejection Monte Carlo methods.

Suppose we wish to sample x from the distrbution f(x) and the (normalized) probably density function can be written as

$$f(x) = \sum_{i=1}^{n} \alpha_i f_i(x) g(x) \qquad [1]$$

where,

$\alpha_i > 0$
$f_i(x)$ density functions (normalised correctly)
$0 \leq g_i(x) \leq 1$

According to this method x can be chosen by

1) selecting a random integer i ($1 \leq i \leq n$) with probability of selecting i being proportional to α_i

2) selecting a value x' from the distribution $f_i(x)$

3) calculating $g_i(x')$ and accepting $x = x'$ with selection probability of

$g_i(x')$ (if x' is rejected we have to start again from step 1).

It can be shown that this scheme is a correct one and the mean number of tries to accept a value is

$$\sum_i \alpha_i$$

In practice we have a good method of sampling from the distribution f(x) if

- all the subdistributions $f_i(x)$ can be sampled easily
- the rejection functions $g_i(x)$ can be evaluated easily/quickly
- the mean number of tries is not too large.

Thus the different possible decompositions of the distribution f(x) are not equivalent from the point of view of the practice (e.g. they can be very different in computational speed) and it can be very useful to make some kind of optimization.

A remark of pratical importance: if our distribution is not normalized ($\int f(x)dx = C > 0$; $C \neq 1$), the method can be used in the same manner, the only difference is that the mean number of tries in this case is given by

$$\sum_i \alpha_i / C$$

The Coulomb corrected Bethe-Heitler differential cross section for a photon of energy of energy E to produce an electron pair one of which has an energy ϵE is given as [3]:

$$\frac{d\sigma(Z,E,\epsilon)}{d\epsilon} = \frac{r_0^2 \alpha Z^2 [Z+\xi(Z)]}{E^2} \{[\epsilon^2+(1-\epsilon)^2][\Phi_1(\delta)-F(Z)] + {}^2/_3\epsilon(1-\epsilon)[\Phi_2(\delta)-F(Z)]\} \qquad [2]$$

where, $\Phi_i(\delta)$ are the screening functions depending on the screening variable δ

$$\delta = \frac{136m}{Z^{1/3}E} \frac{1}{\epsilon(1-\epsilon)} \qquad [3]$$

$$\Phi_1(\delta) = 20.867 - 3.242\delta + 0.625\delta^2 \quad \left.\right\} \; \delta \leq 1$$
$$\Phi_2(\delta) = 20.209 - 1.930\delta - 0.086\delta^2 \quad \left.\right\}$$
$$\Phi_1(\delta) = \Phi_2(\delta) = 21.12 - 4.184 \ln(\delta+0.952) \quad \delta > 1 \qquad [4]$$

$$F(Z) = \begin{cases} {}^4/_3 \ln Z & E < 0.05\,GeV \\ {}^4/_3 \ln Z + 4f_c(Z) & E \geq 0.05\,GeV \end{cases} \qquad [5]$$

(The definition of $\xi(Z)$ and the Coulomb correction $f_c(Z)$ can be found at the entry for GPPAIR.)

The kinematical range for the variable ϵ is

$$m/E \leq \epsilon \leq 1 - m/E \qquad [6]$$

The cross section is symmetric with respect to the interchange of ϵ with $1-\epsilon$, so we can restrict ϵ to lie in the range

$$\epsilon_0 = m/E \leq \epsilon \leq 1/2$$

After some algebraic manipulations we can decompose the cross section as (note: the normalisation is not important):

$$\frac{d\delta}{d\epsilon} = \sum_{i=1}^{2} \alpha_i f_i(\epsilon) g_i(\epsilon) \qquad [7]$$

where

$$\alpha_1 = \frac{(1/2-\epsilon_0)^2}{3} F_{10}$$

$$\alpha_2 = 1/2\ F_{20}$$

$$f_1(\epsilon) = \frac{3}{(1/2-\epsilon_0)^3} (\epsilon-1/2)^2$$

$$f_2(\epsilon) = \frac{1}{1/2-\epsilon_0} \cdot 1$$

$$g_1(\epsilon) = \frac{F_1}{F_{10}}$$

$$g_2(\epsilon) = \frac{F_2}{F_{20}}$$

$$F_1 = F_1(\delta) = 3\Phi_1(\delta)-\Phi_2(\delta)-2F(Z)$$

$$F_{10} = F_1(\delta_{min})$$

$$F_2 = F_2(\delta) = 3/2\Phi_1(\delta)+1/2\Phi_2(\delta)-2F(Z)$$

$$F_{20} = F_2(\delta_{min})$$

and

$$\delta_{min} = \frac{136m}{Z^{1/3}E} \cdot 4$$

is the minimal value of the screening variable (δ). It can be seen easily that the functions $f_i(\epsilon)$ are normalized and that the functions $g_i(\epsilon)$ are "valid" rejection functions.

Therefore, given a set of uniformly distributed random numbers r0, r1, r2, ... ($0 \leq ri \leq 1$.) we can sample the variate ϵ (x in the program) by

1) selecting the integer i (1 or 2)

$$BPAR = \frac{\alpha_1}{\alpha_1+\alpha_2} = \frac{DSIG1}{DSIG1+DSIG2}$$

if $r_0 <$ BPAR i=1

if $r_0 \geq$ BPAR i=2

2) sampling ϵ from $f_i(\epsilon)$. This can be performed by the following expressions

$\epsilon = 0.5[1-(0.5\epsilon_0)(1-r_1)^{1/3}]$ i=1

$\epsilon = \epsilon_0+(1/2-\epsilon_0)r_1$ i=2

3) calculating the rejection function $g_i(\epsilon)$ and rejecting/accepting ϵ

if $r2 > g_i(\epsilon)$ reject ϵ, return to step a.
if $r2 \leq g_i(\epsilon)$ accept ϵ.

It should be mentioned that we need a step just after sampling ϵ in the step b. The point is that the cross section formula goes to negative values at sufficiently large δ. The cross section must not be allowed to go negative, so this imposes an upper limit for δ

$$\delta_{max} = \exp\left((21.12-F(Z))/4.184\right) - 0.952$$

If we get a δ value using the sampled ϵ so that $\delta > \delta_{max}$, we have to start again from the step a.

After the successful sampling of ϵ, GPAIR generates the polar angles of the electron/positron with respect to an axis defined along the direction of the parent photon. The azimuthal angle Φ is generated isotropically and Θ is assigned the approximate average value given by Marmier and Sheldon (see Ref. 4) as

$$\Theta = m/E$$

Using this information together with the momentum conservation enables the momentum vectors of both decay products to be calculated and transformed into the GEANT coordinate system.

The choice of which particle in the pair is the electron/positron is made randomly.

2. Restrictions

1) Effects due to the breakdown of Born approximation at low energies are ignored (but the Coulomb correction is included now).

2) As suggested by Ford and Nelson (3), for very low energy photons ($E \lesssim 2.1$ MeV) the electron energy is approximated by sampling from a uniform distribution over the interval $m \rightarrow {}^1/_2E$. The reason for this suggestion is that the sampling method used in EGS and in the earlier GEANT versions becomes progressively more inefficient as the pair threshold is approached. This is not true for the sampling method outlined above (the efficiency of the method pratically does not depend on the photon energy), but we have chosen to keep this approximation.

3) Target materials composed of compounds or mixtures are treated identically to elements (this is not the case when computing the mean free path in GPPAIR!) using the effective atomic number computed in the routine GSMIXT. It can be show that the error of this type of treatment is small and can be neglected.

4) The differential cross-section implicitly accounts for pair production in both nuclear and atomic electron fields. However, triplet production is not generated, and the recoil momentum of the target nucleus/electron is assumed to be zero.

3. References

1) J.C. Butcher & H. Messel : Nucl. Phys. 20 (1960) 15

2) J.M. Hammersley, D.S. Handscomb : Monte Carlo methods, J. Wiley et Sons Inc. New York, 1964.

3) R. Ford & W. Nelson : SLAC-210, UC-32 (June 1978).

4) P. Marmier & E. Sheldon : Physics of Nuclei and Particles Vol. 1, Academic Press (1969).

Author(s) : G.N.Patrick, L.Urban
Origin : Same

Submitted: 26.10.84
Revised: 16.01.85

Interaction point for compton scattering

CALL GCOMPI

CALL GPCOMP

1. Method

The mean free path, λ, for a photon to interact via Compton scattering is given by

$$\lambda = \frac{1}{\Sigma} = \frac{A}{N.\rho.\sigma(Z,E)} \qquad [1]$$

where

N	Avogadro's number
Z,A	atomic number and weight of the medium
ρ	density of the medium
σ	total cross section per atom for the compton scattering.

For the total cross section an empirical cross section formula is used[1] which produces the cross section data rather well down to 0.05-0.1 MeV.

$$\sigma(Z,E) = Z\left[1-\frac{\alpha Z}{\sqrt{E}}\right]\left[p_1\frac{\ln(1+E)}{E} + \frac{p_2+p_3E+p_4E^2}{1+aE+bE^2+cE^3}\right] \text{ bam/atom} \qquad [2]$$

where,
E = energy of the photon MeV
α = 0.00051 a = 65 b = 360 c = 180
p_1 = 1.32730E-1, p_2 = 2.31752E-2
p_3 = 3.66816E+1, p_4 = 3.95904E+1

This cross-section is tabuled at initialisation time as a function of the medium and of the energy by routine GCOMPI (see JMATE data structure). The energy binning is set within the array ELOW (COMMON CGLOSS) in the routine GPHYSI.

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[1] L. Urban, Modification in the e.m. part of the program GEANT, LEP3 Internal Report, 28/08/84.

Author(s) : G.N. Patrick
Origin : Same

Submitted: 26.09.83
Revised: 03.10.83

Simulation of Compton Scattering

CALL GCOMP

GCOMP generates the Compton Scattering of a photon from an atomic electron by using the random number techniques of Butcher and Messell(1) to sample the scattered photon energy from the Klein-Nishina formula(2).

All input/output information is routed through specially constructed GEANT common blocks.

Input:

via COMMON/GCTRAK/....	VECT(1)	x	
	(2)	y	
	(3)	z	
	(4)	P_x/P	> incident
	(5)	P^x/P	photon
	(6)	P^y/P	
	(7)	P^z	

Output:

via COMMON/GCKING/....	GKIN(1,1)	P_x	
	(2,1)	P^x	
	(3,1)	P^y	> recoil
	(4,1)	P^z	electron
	(5,1)	particle	
			type (3)
via COMMON/GCTRAK/....	VECT(4)	P_x/P	
	(5)	P^x/P	scattered
	(6)	P^y/P	photon
	(7)	P^z	

Compton scattering is selected in GEANT by means of the FFREAD data card COMP.

If COMP > 0 Compton scattering is enabled
≤ 0 " " disabled.

When selected, GCOMP is automatically called from the GEANT gamma tracking routine GTGAMA. Alternatively, the user can call GCOMP at the appropriate point in his own utility routine.

1. Method

A complete account of the Monte-Carlo methods used would be outside the scope of this write-up, and only the basic formalism is outlined. The interested user is recommended to refer to the publications of Butcher and Messel (1), Messel and Crawford (3), and Ford and Nelson (4) for more details.

The quantum-mechanical Klein-Nishina differential cross-section is given by Ford & Nelson (4) as

$$\Phi(E,E') = \frac{X_0 n \pi r_0^2 m_e}{E^2} \left[\frac{1}{\varepsilon} + \varepsilon \right] \left[1 - \frac{\varepsilon \sin^2\tau}{1+\varepsilon^2} \right]$$

here, E = energy of the incident photon
E' = energy of the scattered photon
ε = E'/E
m_e = electron mass
n = electron density
r_0 = electron radius
X_0 = radiation length

Assuming an elastic collision, the scattering angle τ is defined by the Compton formula

$$E' = \frac{E m_e}{m_e + E(1-\cos\tau)}$$

Using the combined "composition" and "rejection" Monte-Carlo methods described in the GPAIR entry, we may set:

$$f(\varepsilon) = \left[\frac{1}{\varepsilon} + \varepsilon \right] = \sum_{i=1}^{2} \alpha_i f_i(E) \qquad \text{for } \varepsilon_0 < \varepsilon < 1$$

$$g(\varepsilon) = \left[1 - \frac{\varepsilon \sin^2\tau}{1+\varepsilon^2} \right] \qquad \text{rejection function}$$

$$\alpha_1 = \ln(1/\varepsilon_0)$$

$$\alpha_2 = (1-\varepsilon_0^2)/2$$

$$f_1(\varepsilon) = \frac{1}{\ln(1/\varepsilon_0)} \left(\frac{1}{\varepsilon} \right)$$

$$f_2(\varepsilon) = \frac{2\varepsilon}{1-\varepsilon_0^2}$$

The value of ε corresponding to the minimum possible scattered photon energy (backward scattering) is given by:

$$\varepsilon_0 = \frac{1}{1+2E/m_e}$$

In practice, therefore, given a set of random numbers R0,R1,...,Rn, GCOMP samples the variate ε by:

(a) Deciding which element of the $f(\varepsilon)$ distribution to sample from

$$\alpha_T = (\alpha_1+\alpha_2).R0$$

If $\alpha_1 \geq \alpha_T$ select $f_1(\varepsilon)$

$\alpha_2 < \alpha_T$ select $f_2(\varepsilon)$

(b) Sampling ε from the distributions corresponding to f_1 and f_2. For f_1, this is simply achieved by

$$\varepsilon = \varepsilon_0 e^{\alpha_1.R1}$$

For f_2, we change variables and use

ε' = AMAX1(R2,R3) for $E/m \geq (E/m+1).R4$

ε' = R5 for all other cases

Then, $\varepsilon = \varepsilon_0+(1-\varepsilon_0)\varepsilon'$

(c) Calculating $\sin^2\tau$ and testing the rejection function

$$t = m_e(1-\varepsilon)/E'$$

$$\sin^2\tau = \text{AMAX1}(0.,t(2-t))$$

If R6 $>$ $g(\varepsilon)$ reject ε

R6 $\leq$ $g(\varepsilon)$ accept ε

After the successful sampling of ε, GCOMP then generates the polar angles of the scattered photon with respect to an axis defined along the parent photon. The azimuthal angle Φ is generated isotopically and Φ is as defined above.

This information is then used to calculate the momentum vector of the scattered photon and recoil electron, and to then transform them into the GEANT co-ordinate system.

2. Restriction

1) The differential cross-section is strictly only valid for those collisions in which the energy of the recoil electron is large compared with its binding energy (which is ignored). However, as pointed out by Rossi (5), this has a negligible effect because of the small number of recoil electrons produced at very low energies.

3. References

1) J.C. Butcher & H. Messel : Nucl. Phys. 20 (1960) 15

2) O. Klein & Y. Nishina : Z. f. Physik 52 (1929) 853.

3) H. Messel & D.F. Crawford : Electron - Photon Shower Distribution Function Tables for Lead, Copper and Air Absorbers, Pergamon Press (1970) 3.

4) R. Ford & W. Nelson : SLAC-210 UC-32 (June 1978).

5) B. Rossi High Energy Particles, Prentice-Hall Inc. (1952) 77-79.

Author(s) : L. Urban
Origin :

Submitted: 26.10.84
Revised: 16.01.85

Computes the interaction point for Moller/Bhabha scattering

(δ-ray production)

CALL GDRAYI

CALL GPDRAY

During the tracking stage of GEANT the routine GPDRAY is used to estimate the remaining distance before an electron or positron interacts to produce a δ-ray (δ-ray = electron of energy E_e > m*DCUTE, DCUTE in COMMON GCDRAY).

Input : via COMMON GCTRAK

Output: via COMMON GCTRAK; SDRAY is the distance (in cm) remaining until Moller/Bhabha scattering.

GPDRAY is automatically called fro the GEANT tracking routine GTELEC.

The relevant cross-section is extrapoled from tables filled by the routine GDRAYI at initialisation time.

1. Method

The mean free path, λ, for an electron/positron to interact via Moller/Bhabha scattering is given by

$$\lambda = \frac{1}{\Sigma} = \frac{A}{N.\rho.\sigma(Z,E,Ec)} \qquad [1]$$

where

N	Avogadro's number
ρ	density of the material
Z,A	atomic number and weight of the material
σ	total cross-section of the δ-ray production
Ec	energy cut below which the scattering is treated as a continuous energy loss (ionisation) and above which it is considered as a discrete process (Ec = m*DCUTE, the kinetic energy cut DCUTE in COMMON GCDRAY).

The total cross-section are given by ([1],[2])

$$\sigma(Z,E,Ec) = \frac{2\pi r_0^2 mZ}{B^2(E-m)}\left[\frac{(\gamma-1)^2}{\gamma^2}\left(\frac{1}{2}-x\right)+\frac{1}{x}-\frac{1}{1-x}-\frac{2\gamma-1}{\gamma^2}\ln\frac{1-x}{x}\right] \qquad [2]$$

for Moller (e^-e^-) and

$$\sigma(Z,E,Ec) = \frac{2\pi r_0^2 mZ}{(E-m)}\left[\frac{1}{B^2}\left(\frac{1}{x}-1\right)+B_1 x+B_2(1-x) - \frac{B_3}{2}(1-x^2)+\frac{B_4}{3}(1-x^3)\right] \qquad [3]$$

for Bhabha scattering (e^+e^-),

where,

$$\gamma = \frac{E}{m}; \qquad B^2 = 1-\frac{1}{\gamma^2}$$

$$x = \frac{Ec-m}{E-m}; \qquad y = \frac{1}{\gamma+1}$$

$$B_1 = 2-y^2 \; ; \quad B2 = (1-2y)(3+y^2)$$

$$B_3 = (1-2y)^2+(1-2y)^3 \; ; \quad B_4 = (1-2y)^3$$

The formulae [2] and [3] give the total cross-section of the scattering above the threshold energies

$$E^{thr}_{Moller} = 2Ec-m \qquad [4]$$

$$E^{thr}_{Bhabha} = 2Ec$$

This cross-section is tabuled at initialisation time as a function of the medium and of the energy by routine GDRAYI (see JMATE data structure). The energy binning is set within the array ELOW (COMMON CGLOSS) in the routine GPHYSI.

The number of mean free paths traversed from a point x_0 to x can be written as

$$N_\lambda = \int_{x_0}^{x} \frac{dx}{\lambda(x)} \qquad [5]$$

If N_R is a random variable denoting the number of mean free paths from a given point until the next interaction, N_R has the distribution

$$P\{N_R < N_\lambda\} = 1-\exp(-N_\lambda) \qquad [6]$$

so N_λ can be sampled by

$$N_\lambda = -\ln(R_1) \qquad [7]$$

where R_1 is a random number uniformly distributed in the interval (0,1).

2. References

1) H. Messel, D.F. Crawford, Electron-Photon Shower Distribution Function, Pergamon press (1970).

2) R.L. Ford, W.R. Nelson, SLAC-210, UC-32 (1978).

Author(s) : L. Urban
Origin : Same

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Revised:

Simulation in the Delta ray production

CALL GDRAY

GDRAY generates the discrete Moller-Bhabha scattering (δ-ray production).

Input : via COMMON GCTRAK

Output: via COMMON GCKING.

The routine is automatically called from the GEANT electron/positron tracking routine GTELEC, when an electron/positron reaches its interaction point.

1. Method

The differential cross section of the δ ray production can be written as [1],[2]:

$$\frac{d\sigma}{d\epsilon} = \frac{2\pi Z r_0^2 m}{B^2(E-m)} \left[\frac{(\gamma-1)^2}{\gamma^2} + \frac{1}{\epsilon}\left(\frac{1}{\epsilon} - \frac{2\gamma-1}{\gamma^2}\right) + \frac{1}{1-\epsilon}\left(\frac{1}{1-\epsilon} - \frac{2\gamma-1}{\gamma^2}\right) \right] \qquad [1a]$$

for the electron/electron (Moller) scattering and

$$\frac{d\sigma}{d\epsilon} = \frac{2\pi Z r_0^2 m}{(E-m)} \left[\frac{1}{B^2\epsilon^2} - \frac{B_1}{\epsilon} + B_2 - B_3\epsilon + B_4\epsilon^2 \right] \qquad [1b]$$

for the positron-electron (Bhabha) scattering, where

E energy of the incident particle

$$\gamma = \frac{E}{m}\ , \quad B^2 = 1 - \frac{1}{\gamma^2}$$

Z atomic number of the medium

$$y = \frac{1}{\gamma+1}\ , \quad B_1 = 2-y^2\ , \quad B_2 = (1-2y)(3+y^2)$$

$$B_3 = (1-2y)^2+(1-2y)^3\ , \quad B_4 = (1-2y)^3$$

$$\epsilon = \frac{E'-m}{E-m}$$

where E' is the energy of the scattered electron (of the lower energy in the case of e^-e^- scattering).

The kinematical limits for the variable ϵ are:

$$\epsilon_0 = \frac{Ec-m}{E-m} \leq \epsilon \leq \frac{1}{2} \qquad (e^-e^-) \qquad [2a]$$

$$\epsilon_0 \leq \epsilon \leq 1 \qquad (e^+e^-) \qquad [2b]$$

Apart from the normalization, the cross section can be written as:

$$\frac{d\sigma}{d\epsilon} = f(\epsilon).g(\epsilon) \qquad [3]$$

where

$$f(\epsilon) = \frac{\epsilon_0}{1-2\epsilon_0}\,\frac{1}{\epsilon^2}$$

$$g(\epsilon) = \frac{4}{9\gamma^2-10\gamma+5}\left[(\gamma-1)^2\epsilon^2-(2\gamma^2+2\gamma-1)\,\frac{\epsilon}{1-\epsilon}+\frac{\gamma^2}{(1-\epsilon)^2}\right]$$

in the case of the e^-e^- scattering and

$$f(\epsilon) = \frac{\epsilon_0}{1-2\epsilon_0}\,\frac{1}{\epsilon^2}$$

$$g(\epsilon) = \frac{B_0-B_1\epsilon+B_2\epsilon^2-B_3\epsilon^3+B_4\epsilon^4}{B_0-B_1\epsilon_0+B_2\epsilon^2_0-B_3\epsilon^3_0+B_4\epsilon^4_0}$$

in the case of e^+e^- scattering. Here $B_0 = \gamma^2/(\gamma^2-1)$, all the other quantities have been defined above.

GDRAY samples the variate ϵ by:

1) sampling ϵ from $f(\epsilon)$

2) calculating the rejection function $g(\epsilon)$ and accepting the sampled ϵ with a probability of $g(\epsilon)$.

After the successful sampling of ϵ, GDRAY generates the polar angles of the scattered electron with respect to an axis defined by the incident particle. The azimuthal angle Φ is generated isotropically, Θ is calculated from the energy-momentum conservation. This information is the used to calculate the energy and momentum of both scattered particles and to transform them into the GEANT coordinate system.

2. References

1) H. Messel, D.F. Crawford, Electron-Photon Shower Distribution Function, Pergamon press (1970).

2) R.L. Ford, W.R. Nelson, SLAC-210, UC-32 (1978).

Author(s) : L. Urban
Origin : Same

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Computes the interaction point for e^+ annihilation

CALL GANNII

CALL GPANNI

During the tracking stage of GEANT the routine GPANNI is used to estimate the remaining distance before a positron annihilates.

Input : via COMMON GCTRAK, GCKINE

Output: via COMMON GCTRAK; SANNI is the distance (in cm) remaining until $e^+e^- \rightarrow 2\gamma$ interaction.

The relevant cross-section is extrapoled from tables filled by the routine GANNII at initialisation time.

1. Method

The mean free path, λ, for a positron to interact via annihilation is

$$\lambda = \frac{1}{\Sigma} = \frac{A}{N.\rho.\sigma(Z,E)} \qquad [1]$$

where,

Σ	macroscopic cross-section
N	Avogadro's number
Z,A	atomic number and weight of the medium
ρ	density of the medium
$\sigma(Z,E)$	total annihilation cross-section per atom.

For the annihilation cross-section the formula of Heitler (see Refs 1 and 2) is used.

The total cross-section are given by ([1],[2])

$$\sigma(Z,E,Ec) = \frac{Z\pi r_0^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln(\gamma+\sqrt{\gamma^2-1}) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right] \qquad [2]$$

where, r_0 classical radius of the electron

$\gamma = E/m$

This cross-section is tabuled at initialisation time as a function of the medium and of the energy by routine GANNII (see JMATE data structure). The energy binning is set within the array ELOW (COMMON CGLOSS) in the routine GPHYSI.

As the mean free path may change along the positron's trajectory as it moves from one medium to another, the number of mean free paths traversed, N_γ, is given by

$$N_\gamma = \int_{x_0}^{x} \frac{dx}{\lambda(x)} \qquad [3]$$

If N_R is a random variable denoting the number of mean free paths from a given point until the next interaction, N_R has the distribution

$$P(N_R < N_\gamma) = 1-\exp(-N_\gamma) \qquad [4]$$

so N_γ can be sampled by

$$N_\gamma = -\ln(R_1) \qquad [5]$$

where, R_1 is a random number uniformly distributed in the interval (0,1).

In case of compound/mixture the effective atomic number and weight (evaluated in GSMIXT) is used computing the cross-section. This treatment of the compounds/mixtures is correct in the processes where the cross-section depend on the atomic number Z linearly (annihilation, Moller and Bhabha scattering, Compton scattering).

2. References

1) W. Heitler, The Quantum Theory of radiation, Clarendon Press, Oxford (1954).

2) R.L. Ford, W.R. Nelson, SLAC-210, UC-32 (1978).

Author(s) : L. Urban
Origin : Same

Submitted: 26.10.84
Revised:

Simulates the two-photon e^+e^- annihilation

CALL GANNI

GANNI generates the two-photon annihilation of a positron.

Input : via COMMON GCTRAK

Output: via COMMON GCKING.

The routine is automatically called, if the positron reches its interaction point during the tracking (this interaction point is calculated by GPANNI, see the corresponding entry in this write-up).

1. Method

The differential cross section of the two-photon positron-electron annihilation can be written as [1],[2]:

$$\frac{d\sigma(Z,\epsilon)}{d\epsilon} = ma[S(a\epsilon)+S(a(1-\epsilon))] \qquad [1]$$

where, m electron mass

$\gamma = \frac{E}{m}$; E positron energy ; $a = \gamma+1$

$\epsilon = \frac{k}{E+m}$; K energy of the secondary photon of the lower energy

$$S(x) = C_1\left[-1+\frac{C_2}{x}-\frac{1}{x^2}\right]$$

$$C_1 = \frac{Z\pi r_0^2}{a(E-m)} \; ; \; c_2 = a+\frac{e\gamma}{a}$$

Z atomic number of the material.

The kinematical limits for the variable ϵ are given by

$$\epsilon_0 = \frac{1}{a+\sqrt{\gamma^2-1}} \leq \epsilon \leq 1/2 \qquad [2]$$

Due to the symmetry of the formula [1] in ϵ, the range of ϵ can be expanded from $(\epsilon_0, 1/2)$ to $(\epsilon_0, 1-\epsilon_0)$ and the second function S can be thrown away from the formula. Having done this the differential cross section can be decomposed (apart from the normalisation) as

$$\frac{d\sigma}{d\epsilon} = \underbrace{\frac{1}{\ln\frac{1-\epsilon_0}{\epsilon_0}}\frac{1}{\epsilon}}_{f(\epsilon)} \cdot \underbrace{\frac{(a^2+2a-2)-a^2\epsilon-1/2}{a^2-2}}_{g(\epsilon)} \qquad [3]$$

Using the expression [3] the secondary photon energy can be sampled by (r_0, r_1 random numbers).

1) sampling ϵ from $f(\epsilon)$

$$\epsilon = \epsilon_0 \exp\left[\ln\left(\frac{1-\epsilon_0}{\epsilon_0}\right) \cdot r_0\right]$$

2) computing the rejection function $g(\epsilon)$ and

if $r_1 \leq g(\epsilon)$ accepting ϵ
if $r_1 > g(\epsilon)$ accepting ϵ, starting again from 1.

After the successful sampling of ϵ, the photon energy is computed as

$$k = (E;m)\epsilon \qquad [4]$$

and then GANNI generates the polar angles of the photon with respect to an axis defined by the momentum of the positron. The azimuthal angle Φ is generated isotropically and Θ is computed from the energy-momentum conservation. Using this information enables the momentum vector of both photons to be calculated and transformed into the GEANT coordinate system.

2. Restrictions

1) The annihilation processes produced one or three or more photons are ignored, because these processes are far less likely than the two-photon annihilation (see Refs 2 and 3).

2) Calculating the process it is assumed that the atomic electron initially is free and at rest. This is a usual assumption used inn shower programs (see Ref. 2).

3) In the case of a compound or mixture target material the effective atomic number (computed in GSMIXT) is used.

3. References

1) W. Heitler, The Quantum Theory of radiation, Clarendon Press, Oxford (1954).

2) R.L. Ford, W.R. Nelson, SLAC-210, UC-32 (1978).

3) H. Messel, D.F. Crawford, Electron-Photon Shower Distribution Function, Pergamon press (1970).

Author(s) : L. Urban
Origin : Same

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Simulates the e^+e^- annihilation for low energy positrons

CALL GANNIR

The routine GANNIR treates the special case when a positron falls below cutoff momentum (CUTELE in COMMON GCCUTS) before annihilating[1]. In this case it is assumed that it comes to rest before annihiliting (see the tracking routine GTELEC). then GANNIR generates the photons, each of them has an energy k=m and the angular distribution is isotropic with gammas going in opposite direction.

Input : via COMMON GCTRAK

Output: via COMMON GCKING.

[1] R.L. Ford, W.R. Nelson, SLAC-210, UC-32 (1978).

Author(s) : L. Urban
Origin : Same

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Computation of the interaction point for atomic photoeffect

CALL GPHOTI

CALL GPPHOT

During the tracking stage of GEANT the routine GPPHOT is used to estimate the remaining distance before a photon interacts via photoeffect.

Input : via COMMON GCTRAK

Output: via COMMON GCTRAK; SPHOT is the distance (in cm) remaining until photoeffect.

GPPHOT is automatically called by the tracking routine GTGAMA.

The relevant cross-section is extrapoled from tables filled by the routine GPHOTI at initialisation time.

1. Method

The mean free path, λ, for a photon to interact via photoeffect is given by

$$\lambda = 1/\Sigma \qquad [1]$$

where, Σ macroscopic cross-section is given as

$$\Sigma = \frac{N\rho\sigma(Z,E_\gamma)}{A} \qquad [2a]$$

$$\Sigma = \frac{N\rho\sum_i p_i\sigma(Z_i,E_\gamma)}{\sum_i p_i A_i} \qquad [2b]$$

for the chemical element or compound/mixture respectively where,

N	Avogadro's number
$Z(Z_i)$	atomic number (of i-th component) of the medium
$A(A_i)$	atomic number (of i-th component) of the medium
ρ	density
σ	total cross-section

p_i	proportion by number of the i-th element in the material ($p_i=w_i/A_i$ where w_i the corresponding proportion by weight).

The cross-section of the photoeffect can be parametrized as [1]:

$$\sigma(Z,E_\gamma) = \frac{Z^a}{E_\gamma^b}\left[\frac{p_1}{Z}+\frac{p_2}{E_\gamma}+p_3+p_4Z+p_5E+p_6Z^2+p_7ZE+p_8E^2\right] \qquad [3]$$

(E_γ is in MeV, σ is in barn/atom).

This formula reproduces the cross-section data in the region

$$5 \le Z \le 100$$

$$E_{min} = \max[E_k(Z),0.1\text{MeV}] \le E_\gamma \le E_{0.01}(Z) \qquad [4]$$

where,

$E_K(Z)$ energy of K-edge,

$E_{p\%}(Z)$ energy, where σphoto/σtotal=p%

with the parameter values

a=4.800 b=3.325

i	p
1	5.28900E-9
2	-2.63159E-11
3	1.71558E-9
4	-1.90586E-11
5	1.88779E-9
6	6.26538E-14
7	-3.89566E-12
8	1.88382E-9

The errors of the formula [3] can be estimate as:

$$\frac{\Delta\sigma}{\sigma} \le 10\% \quad \text{for} \quad E_{min} \le E_\gamma \le E_{0.02}$$

$$\frac{\Delta\sigma}{\sigma} \le 20\% \quad \text{for} \quad E_{0.02} < E_\gamma \le E_{0.01} \qquad [5]$$

(cross-section data in [2] with a typical error of 10%).

The quantities $E_k(Z)$, $E_{0.01}(Z)$ can be estimated by the empirical expression:

$$E_K(Z) = Z^2[7.8462E-6+1.3291E-7Z-1.5346E-9Z^2+8.4302E-12Z^3] \text{ MeV} \quad [6]$$

$$E_{0.01}(Z) = Z^2[1.0158E-3+4.0194E-6Z+1.9809E-7Z^2-1.9132E-)Z^3] \text{ MeV} \quad [7]$$

these formulae are used in GPROBI and GPHOT (see the corresponding entries of this write-up).

This cross-section is tabuled at initialisation time as a function of the medium and of the energy by routine GPHOTI (see JMATE data structure). The energy binning is set within the array ELOW (COMMON CGLOSS) in the routine GPHYSI.

The mean free path may change along the photon's trajectory as it moves from one medium to another. The number of mean free path, N_γ is given by:

$$N_\lambda = \int_{x_0}^{x} \frac{dx}{\lambda(x)} \quad [8]$$

If N_R is a random variable denoting the number of mean free paths from a give point until the next interaction, than it can be shown that N_R has the distribution

$$P(N_R < N_\lambda) = 1-\exp(-N_\lambda) \quad [9]$$

so N_λ can be sampled by

$$N_\lambda = -\ln(R_1) \quad [10]$$

where R_1 is a random number uniformly distributed between 0 and 1.

2. Restrictions

1) Above 0.015 GeV a cut is imposed as the contribution from the photoeffect is negligible in this case.

2) The accuracy of the formula [3] is questionable below 0.1 MeV (the routine calculates the cross-section at 0.1 MeV for E_γ < 0.1 MeV).

3. References

1) L. Urban, Modifications in the e.m. part of the program GEANT, LEP3 internal report, 28/08/84.

2) H. Storm, H.I. Israel, Nucl. Data Tables, A7/1970/565.

Author(s) : L. Urban
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Simulation of atomic photoeffect

CALL GPHOT

The routine GPHOT generates the atomic photoeffect.

Input : via COMMON GCTRAK

Output: via COMMON GCKING.

GPHOT is automatically called by the tracking routines, if, and when, the photon reaches its interaction point (see GPPHOT).

1. Method

The simulation of the photoeffect is presently rather crude. If photoelectric interaction has occured a photoelectron is created with total energy

$$E = E_\gamma - E_k(Z) + m \quad [1]$$

where, $E_k(Z)$ is the K-edge energy

and with the same direction as the incident photon.