CERN-TH.4104/85

ELECTROWEAK $SU(2)_L \times U(1)_Y$ GAUGE MODEL WITHOUT HIGGS FIELDS

J. Hošek*)

CERN - Geneva

A B S T R A C T

It is argued that a Higgsless $SU(2)_L \times U(1)_Y$ electroweak gauge model supplemented with a heavy vector boson interacting with all fermions of both chiralities can describe massive fermions and massive W and Z bosons with calculable masses. The dynamical mass generation is demonstrated explicitly using an effective $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian with a four-fermion interaction due to the heavy boson exchange. The present mechanism can be considered as a microscopic mechanism of mass generation, of which the Higgs one is a phenomenological manifestation. Masses of W and Z bosons are related to fermion masses by sum rules.

---

*) On leave of absence from Nuclear Physics Institute of the Czechoslovak Academy of Sciences, Řež, Czechoslovakia.

CERN-TH.4104/85

February 1985

1. - INTRODUCTION AND SUMMARY

Although the masses are basic characteristics of the elementary particles, their origin remains mysterious. The Higgs mechanism[1] of their generation should be considered phenomenological by definition: mass scale is fixed by non-zero vacuum expectation value of a scalar field, but each mass is determined by its own coupling constant which is independently renormalized. Hence, elementary particle masses can be described (parametrized) but cannot be calculated.

On the other hand, the Higgs mechanism is theoretically clean and phenomenologically very successful. It thus seems worthwhile to look for its microscopic foundation rather than for an alternative.

A microscopic theory underlying the Higgs one should, as one generally requires from microscopic theories, (1) reproduce good features of the phenomenological theory more economically, (2) be able to calculate parameters of the phenomenological theory, (3) provide new predictions lying outside the range of validity of the phenomenological theory, and (4) lead to the phenomenological theory by a controllable sequence of approximations.

A typical example of an interrelation of phenomenological and microscopic theory, hopefully relevant to our discussion, is the Ginzburg-Landau theory of superconductivity with its microscopic BCS counterpart. Relevance relies upon a commonly accepted view that the Higgs Lagrangian is a one-to-one relativistic translation of the Ginzburg-Landau Lagrangian[2]. What is then the relativistic translation of the BCS Lagrangian? This paper is devoted to a discussion of the properties of one such candidate.

We suggest to study the Higgsless standard $SU(2)_L \times U(1)_Y$ gauge model supplemented with a neutral vector boson C with a mass M coupled to all fermions of both chiralities with a strength h. Liberty is taken of assigning to different fermions different C hypercharges. These are pure numbers not undergoing renormalization. The model is operationally defined by its renormalizable (off-mass shell) perturbation expansion.

Dynamical mass generation is, however, a genuine non-perturbative phenomenon; chiral symmetry for fermions and gauge symmetry for gauge bosons guarantee masslessness in every order in perturbation theory. Since a systematic non-perturbative technique is not available in relativistic field theory without scalar fields[3], our considerations will only be plausibility arguments.

Our strategy is the following. Fermion-C boson interaction is treated non-perturbatively and it is argued that it is capable of generating fermion masses in terms of C-hypercharges. Standard electroweak interactions are considered as weak external perturbations. When switched off, the dynamical generation of fermion masses implies the appearance of three Nambu-Goldstone (NG) bosons with calculable couplings to fermions. When switched on, "would-be" NG bosons become longitudinal spin components of the originally massless gauge fields in accordance with the general Schwinger mechanism[4)]. As a result, masses of W and Z bosons are calculated in terms of masses of all fermions present in the theory.

When asking about the massiveness or masslessness of a theory with the only mass scale being the renormalization point μ [see Eq. (3)], it is natural to employ the renormalization group[5)]. It appears that our model is theoretically consistent with the assumption of dynamical mass generation only if we assume that the renormalized fermion-C boson coupling constant h lies in the domain of attraction of a non-trivial ultra-violet (UV) fixed point h^*_{UV} in the region of validity of perturbation theory[6)].

Obviously, this assumption[7)] cannot be disproved with present theoretical tools (i.e., with perturbation theory). An ultimate decision about the quality of this strong speculative assumption can be made only after the theory is more or less exactly solved[*)]. Results of our plausibility arguments presented in the following bona fide provide an indication of an a posteriori justification of our assumption.

In practice we analyze in this paper the fermion-C boson sector by applying the Nambu-Jona-Lasinio (NJL) technique[8)] to the effective four-fermion Lagrangian due to the C boson exchange. Consequently, concerning the calculability of fermion masses, we can speak only of matters of principle[9)]. With our assumption of the existence of h^*_{UV}, fermion-C boson dynamics resembles the dynamics of chiral symmetry breaking in QCD with quarks in different representations of $SU(3)_{colour}$[10)]. Different fermions f condense at different mass scales μ_f due to different C hypercharges $y_{Lf}y_{Rf}$. A renormalization group equation naturally provides large amplification of condensation mass scales as a response to small changes in C hypercharges.

---

*) In that case, however, renormalization group analysis becomes unnecessary[7)].

Since C hypercharges are pure numbers of the same order of magnitude, we can speak about the calculability of the fermion masses. We believe that the mechanism of the gauge boson mass generation, being quite general, can be trusted even in its present four-fermion form[11].

The paper is organized as follows. Section 2 is devoted to a discussion of the properties of the Lagrangian we suggest for dynamical mass generation. In Section 3, the dynamical fermion and gauge boson mass generation is explicitly elaborated using the NJL and Freundlich-Lurié techniques applied to an effective four-fermion interaction. Flavour mixing is discussed in Section 4, and Section 5 contains a comparison of the present approach with a canonical one.

2. - PROPERTIES OF THE LAGRANGIAN

The model we suggest to discuss is defined by its Lagrangian density

$$\begin{aligned}
\mathcal{L} = {} & \bar\psi_L i\gamma^\alpha(\partial_\alpha - ig\tfrac{1}{2}\vec\tau\vec A_\alpha + ig'\tfrac{1}{2}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)\psi_L + \\
& \bar\nu_R i\gamma^\alpha(\partial_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)\nu_R + \bar e_R i\gamma^\alpha(\partial_\alpha + ig'B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)e_R + \\
& \bar q_L i\gamma^\alpha(\partial_\alpha - ig\tfrac{1}{2}\vec\tau\vec A_\alpha - ig'\tfrac{1}{2}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)q_L + \\
& \bar u_R i\gamma^\alpha(\partial_\alpha - ig'\tfrac{2}{3}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)u_R + \bar d_R i\gamma^\alpha(\partial_\alpha + ig'\tfrac{1}{3}B_\alpha - ih\tfrac{1}{2}Y_H C_\alpha)d_R - \\
& -\tfrac{1}{4}(\partial_\alpha\vec A_\beta - \partial_\beta\vec A_\alpha + g\vec A_\alpha\times\vec A_\beta)^2 - \tfrac{1}{4}(\partial_\alpha B_\beta - \partial_\beta B_\alpha)^2 - \\
& -\tfrac{1}{4}(\partial_\alpha C_\beta - \partial_\beta C_\alpha)^2 + \tfrac{1}{2}M^2 C_\alpha C^\alpha .
\end{aligned} \tag{1}$$

It is the standard Glashow-Weinberg-Salam (GWS) model without Higgs sector, supplemented with a neutral vector boson C with a mass M interacting with all fermions of both chiralities with a coupling constant h. We take the liberty of assigning to different fermions different C hypercharges Y_H. For more fermion families, Lagrangian (1) is form-invariant. Weak interaction fermion eigenstates ψ_L, ν_R, e_R,... become columns of like fermions in family space and Y_H are diagonal non-degenerate real matrices.

1. - $\underline{SU(2)_L \times U(1)_Y}$ gauge invariance of the model (1) is transparent. It is this symmetry that prevents fermion and gauge boson bare mass terms being present in (1).

2. - Renormalizability. To be defined, the model has to be renormalizable. Although it is known that both the GWS model and QED with a massive photon are renormalizable theories, the problem of renormalizability of essentially their sum is non-trivial because of new triangular anomalies. Nevertheless, a two-parameter set of C hypercharges

$$Y_H \equiv (y(q_L), y(u_R), y(d_R); y(\psi_L), y(\nu_R), y(e_R))$$

$$Y_H = \alpha Y_H^{(1)} + \beta Y_H^{(2)} \tag{2}$$

is proved to correspond to an anomaly-free[12], hence renormalizable, theory. Here

$$Y_H^{(1)} = (1/3,\ 4/3,\ -2/3;\ -1,\ 0,\ -2)$$

$$Y_H^{(2)} = (0,\ 1,\ -1;\ 0,\ 1,\ -1).$$

3. - There is no genuine renormalization of the mass M[13]. The polarization tensor of the C field being transverse due to the corresponding current conservation contains one over-all logarithmic divergence. It is removed by a wave function renormalization constant of the C field, $M_R = M_0 Z_3^{\frac{1}{2}}(h_0, \Lambda/\mu)$. Since also the coupling constant is renormalized as $h_R = h_0 Z_3^{\frac{1}{2}}(h_0, \Lambda/\mu)$ due to the Ward identity which is valid here, we have $M_R = (M_0/h_0)h_R$. We may set

$$M_R = c\,\mu\, h_R\,, \tag{3}$$

where c is a theoretically arbitrary dimensionless parameter. It will become fixed only after identification of M_R and h_R with their physical values. In this respect, the Lagrangian (1) resembles purely massless theory, i.e., there is only one mass scale which is the renormalization point μ.

4. - C hypercharges Y_H are not renormalized. This trivial property is crucial to calculate the fermion masses. One can easily imagine that these C hypercharges become completely fixed, i.e. quantized, if the present scheme is embedded into a simple GUT group.

5. - Renormalization group argument[5]. For any dynamically generated mass in a theory with one mass scale [see footnote 10 of Ref. 14)], the formula

$$m = \mu f(h_R, c) \tag{4}$$

follows on dimensional grounds. The mass m, being physical, cannot depend upon where the theory is renormalized*):

$$\mu \frac{dm}{d\mu} = \left[\mu \frac{\partial}{\partial \mu} + \beta(h_R) \frac{\partial}{\partial h_R} \right] m = 0. \tag{5}$$

Here

$$\beta(h_R) = \mu \frac{\partial}{\partial \mu} h_R(h_0, \Lambda/\mu)\Big|_{\Lambda, h_0 \text{ fixed}} .$$

Equation (5) is easily solved:

$$m = \mu f(\hat{h}, c) \exp \left[\int_{h_R}^{\hat{h}} dx / \beta(x) \right] ,$$

where $\hat{h}$ is an arbitrary parameter chosen in such a way that β has no zeros between $\hat{h}$ and h_R.

For fixed μ, m is not analytic around fixed points of the renormalization group [those values of the coupling constant for which $\beta(h_R)=0$]. At an UV stable fixed point, m vanishes while at an infra-red (IR) stable fixed point it explodes. The immediate conclusion of such an analysis[5),6)] is that in the domain of attraction of the origin $h_R = 0$, asymptotically free theories can generate masses dynamically ($h_R=0$ is a perturbative UV stable fixed point), while infra-red free theories cannot ($h_R=0$ is a perturbative IR stable fixed point).

Our fermion-C boson interaction is certainly IR free[15)]. Hence, when looking for a dynamical mass generation in a theory like this, we simultaneously <u>assume that there is a non-trivial UV stable fixed point</u> h^*_{UV}[6),**)]. Taken literally, such a fixed point does exist in Abelian theory[16)]. But h^*_{UV} discussed here is rather of a non-perturbative origin. When the physical mass is found by some non-perturbative method which can be trusted, h^*_{UV} is reconstructed by using (5).

---

*) In the Landau gauge, which is known to be renormalization-group invariant[7)], gauge dependence can be ignored.

**) In the presence of other interactions, which is our case, a necessary condition is in fact weaker: IR zero must not be stable[6)]. However, we assume[7)] that fermion-C boson dynamics can be treated separately for all momenta.

3. - DYNAMICAL MASS GENERATION

In this section we demonstrate the dynamical mass generation by using the effective Lagrangian density which follows from (1) for momenta squared $\ll M^2$:

$$\mathcal{L}_{eff} = \mathcal{L}_{GWS} - \frac{h^2}{M^2} J^C_\alpha J^{C\alpha} . \tag{6}$$

This Lagrangian is again $SU(2)_L \times U(1)_Y$ gauge invariant, i.e., without bare mass terms of fermions and gauge bosons. Since we are going to use the non-perturbative Hartree-Fock-like procedure, a hope is that lack of perturbative renormalizability will not invalidate qualitative properties of the obtained results.

3.a. Fermion masses

For electrically charged fermions, only those terms in $J^C_\alpha J^{C\alpha}$ can yield to a $\langle \bar{f}_L f_R \rangle$ condensation (hence to fermion masses) which contain fields of opposite chiralities:

$$\begin{aligned}
\mathcal{L}_{NJL} = & -\frac{h^2}{2M^2} y(\psi_L) y(\nu_R) \bar{\psi}_L \gamma_\alpha \psi_L \cdot \bar{\nu}_R \gamma^\alpha \nu_R - \frac{h^2}{2M^2} y(\psi_L) y(e_R) \bar{\psi}_L \gamma_\alpha \psi_L \cdot \bar{e}_R \gamma^\alpha e_R - \\
& -\frac{h^2}{2M^2} y(q_L) y(u_R) \bar{q}_L \gamma_\alpha q_L \cdot \bar{u}_R \gamma^\alpha u_R - \frac{h^2}{2M^2} y(q_L) y(d_R) \bar{q}_L \gamma_\alpha q_L \cdot \bar{d}_R \gamma^\alpha d_R = \\
& = \frac{h^2}{M^2} y(\psi_L) y(\nu_R) \bar{\psi}_L \nu_R \cdot \bar{\nu}_R \psi_L + \frac{h^2}{M^2} y(\psi_L) y(e_R) \bar{\psi}_L e_R \cdot \bar{e}_R \psi_L + \\
& + \frac{h^2}{M^2} y(q_L) y(u_R) \bar{q}_L u_R \cdot \bar{u}_R q_L + \frac{h^2}{M^2} y(q_L) y(d_R) \bar{q}_L d_R \cdot \bar{d}_R q_L .
\end{aligned} \tag{7}$$

For neutrinos which can also have Majorana mass terms, the following Lagrangian is allowed as well

$$\begin{aligned}
\mathcal{L}_M = & \frac{h^2}{2M^2} y^2(\psi_L) \bar{\nu}^c_R \gamma_\alpha \nu^c_R \cdot \bar{\nu}_L \gamma^\alpha \nu_L + \frac{h^2}{2M^2} y^2(\nu_R) \bar{\nu}^c_L \gamma_\alpha \nu^c_L \cdot \bar{\nu}_R \gamma^\alpha \nu_R = \\
& = -\frac{h^2}{M^2} y^2(\psi_L) \bar{\nu}^c_R \nu_L \cdot \bar{\nu}_L \nu^c_R - \frac{h^2}{M^2} y^2(\nu_R) \bar{\nu}^c_L \nu_R \cdot \bar{\nu}_R \nu^c_L .
\end{aligned} \tag{8}$$

In (8), the superscript c abbreviates charge conjugation, $\psi^c = C\bar{\psi}^T$. A consistent treatment of the dynamical generation of neutrino masses deserves separate treatment. Here we assume for simplicity that the neutrinos acquire Dirac masses due to the interaction (7). The interaction (8) will not be taken into account in the following.

In rearranging $\mathcal{L}_{NJL}$ we have used the Fiertz transformation to make correspondence with the standard Higgs mechanism transparent. We may identify

$$\begin{aligned} \frac{1}{M^2}(\bar{e}_R \psi_L) &= \phi^{(e)} \\ \frac{1}{M^2}(\bar{d}_R q_L) &= \phi^{(d)} \end{aligned} \tag{9}$$

with two Higgs doublets with weak hypercharge y = +1 and

$$\begin{aligned} \frac{1}{M^2}(\bar{\nu}_R \psi_L) &= \phi^{(\nu)} \\ \frac{1}{M^2}(\bar{u}_R q_L) &= \phi^{(u)} \end{aligned} \tag{10}$$

with two other doublets with y = -1.

Fermion mass generation will be demonstrated for the electron, using the part of $\mathcal{L}_{NJL}$ which contains the composite doublet $\phi^{(e)}$:

$$\begin{aligned} \mathcal{L}^{(e)}_{NJL} &= \frac{h^2}{M^2} y(\psi_L) y(e_R) \bar{\psi}_L e_R \cdot \bar{e}_R \psi_L = \\ &= g_e [\bar{\nu}(1+\gamma_5)e.\bar{e}(1-\gamma_5)\nu + \bar{e}(1+\gamma_5)e.\bar{e}(1-\gamma_5)e], \end{aligned} \tag{11}$$

where

$$g_e = \frac{h^2}{4M^2} y(\psi_L) y(e_R). \tag{12}$$

Other parts of $\mathcal{L}_{NJL}$ give rise to masses of other fermions.

The NJL method[8] of dealing with interactions like (11) is a relativistic generalization of a self-consistent perturbation theory, which can also be formulated as a Hartree-Fock linearization procedure. We would like to stress that since in this approach one chooses the form of the physical ground state a priori at the beginning, the same results are obtained with the exact interaction $\mathcal{L}_{int} = -(h^2/M^2)J^C_\alpha J^{C\alpha}$ as with $\mathcal{L}_{NJL}$.

Both interactions lead to the self-consistency equation for the electron mass m_e:

$$1 - 8ig_e \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_e^2} = 1 - \frac{g_e}{4\pi^2} \int_{4m_e^2}^{\Lambda^2} \sqrt{1 - 4m_e^2/\varkappa^2}\, d\varkappa^2 = 0 . \tag{13}$$

For a non-trivial solution $m_e \neq 0$ to exist, g_e has to be positive. It is easy to find a C hypercharge set (2) such that $g_f > 0$ for all fermion species f.

Assuming the HF condensate field $\langle \bar{e}e \rangle$ to be different from zero, we get by linearizing interaction $-(h^2/M^2)J_\alpha^C J^{C\alpha}$ (or $\mathcal{L}_{NJL}$) another equivalent formula for the electron mass:

$$m_e = -2g_e \langle \bar{e}\, e \rangle . \tag{14}$$

At this point we use our assumption of the existence of a non-trivial UV fixed point h^*_{UV}. With this assumption, fermion-antifermion condensation is a low momentum phenomenon ($\mu < M$) and we may compare our model of dynamical fermion mass generation with chiral symmetry breaking in QCD.

In QCD one can argue[10),17)] that (i) the chiral symmetry breaking depends entirely on the strength of the quark-antiquark binding potential which is based on the one-gluon exchange approximation; (ii) the perturbation theory holds in the region of chiral symmetry breaking. It then follows[10)] that quarks in different representations R of $SU(3)_{colour}$ condense at widely separated mass scales μ_R due to their different colour charges $C_2(R)$ (quadratic Casimir operators).

Adapted to our case, this implies that different fermions f condense at very different mass scales μ_f [*)]

$$\langle \bar{f} f \rangle = \mu_f^3$$

due to their (slightly) different C hypercharges $y_{fL} y_{fR}$ [**)]. Using Eq. (14), we see that a very wide fermion mass spectrum is in principle easily understood (see the Table). We set $\frac{1}{2}h^2 y_{fL} y_{fR} = 10^{-1}$ for simplicity and $M = 10^6$ GeV. Such a high mass of the C boson is a necessity, since in the case of flavour mixing the

---

*) The anomalous dimension of $\bar{f}f$ [7)] is not considered in our qualitative analysis.

**) The rate $\mu_f/\mu_{f'}$ depends upon the character of a non-trivial zero of the β function that we assume[7)].

current J_α^C is not flavour-diagonal (see Section 4). Hence, the lowest condensation mass scale lies in the range $\mu \sim 100$ GeV.

3.b. Nambu-Goldstone bosons

If the electroweak long-range forces are switched off, the dynamical appearance of fermion masses implies a dynamical breakdown of the global $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$. Consequently, three massless spinless NG bosons must appear in the physical spectrum. We find them as massless poles in the fermion-antifermion scattering matrices calculated with (11) in the chain approximation[8].

The $\bar{\nu}e$ scattering matrix (ϕ_- component in the canonical Higgs approach) is given as

$$M_{\bar{\nu}e} = (1+\gamma_5)_i \frac{g_e}{1 - J_{o;m}(q^2)} (1-\gamma_5)_f \tag{15}$$

in accordance with Fig. 1. We set $m_e = m$ to simplify the notation. Here

$$J_{o;m}(q^2) = i g_e \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}(1-\gamma_5) S_F^o(p)(1+\gamma_5) S_F^m(p-q) = 1 - q^2 g_e I_{o;m}(q^2), \tag{16}$$

where

$$I_{o;m}(q^2) = \frac{1}{4\pi^2} \int_{4m^2}^{\Lambda^2} \frac{\sqrt{1-4m^2/\varkappa^2}}{\left(q^2 + \frac{1}{4}\varkappa^2\left(1+\sqrt{1-4m^2/\varkappa^2}\right)\right)^2} d\varkappa^2 . \tag{17}$$

In arranging (16), we have used the self-consistency equation (13). With (16) and (17), the $\bar{\nu}e$ scattering matrix acquires the desired form

$$M_{\bar{\nu}e} = (1+\gamma_5)_i I_{o;m}^{-1/2} \frac{1}{q^2} I_{o;m}^{-1/2} (1-\gamma_5)_f \tag{18}$$

of a massless spinless exchange. Constant $I_{o;m}^{-\frac{1}{2}}(0)$ is naturally interpreted as a fermion-charged NG boson coupling constant.

The scattering matrix $\bar{e}e$ ($\phi_0 + \phi_0^*$ component in the canonical Higgs approach) is evaluated analogously:

$$M_{\bar{e}e} = (i\gamma_5)_i \frac{2g_e}{1 - J_{m;m}(q^2)} (i\gamma_5)_f \, , \tag{19}$$

see Fig. 2. Here,

$$J_{m;m}(q^2) = ig_e \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\, i\gamma_5 S_F^m(p) i\gamma_5 S_F^m(p-q) = 1 - q^2 g_e I_{m;m}(q^2) \, , \tag{20}$$

where

$$I_{m;m}(q^2) = \frac{1}{4\pi^2} \int_{4m^2}^{\Lambda^2} \frac{\sqrt{1 - 4m^2/\varkappa^2}}{q^2 + \varkappa^2} d\varkappa^2 \, . \tag{21}$$

Hence

$$M_{\bar{e}e} = (i\gamma_5)_i I_{m;m}^{-1/2} \frac{2}{q^2} I_{m;m}^{-1/2} (i\gamma_5)_f \tag{22}$$

describes the exchange of a massless pseudoscalar meson. Its coupling to the electron field is $I_{m;m}^{-\frac{1}{2}}(0)$.

Considering explicitly only the interaction $\mathcal{L}_{NJL}^{(e)}$ in the chain approximation, we have found one component of the composite NG boson triplet. In general, we have

$$|NG(+)\rangle = \frac{1}{\sqrt{\sum_f m_f^2 I_{0;m_f}}} \left[m_\nu I_{0;m_\nu}^{1/2} |\bar{e}_L \nu_R\rangle - m_e I_{0;m_e}^{1/2} |\bar{e}_R \nu_L\rangle + \begin{pmatrix} \nu \\ e \end{pmatrix} \to \begin{pmatrix} u \\ d \end{pmatrix} \right]$$

$$|NG(-)\rangle = \frac{1}{\sqrt{\sum_f m_f^2 I_{0;m_f}}} \left[m_\nu I_{0;m_\nu}^{1/2} |\bar{\nu}_R e_L\rangle - m_e I_{0;m_e}^{1/2} |\bar{\nu}_L e_R\rangle + \begin{pmatrix} \nu \\ e \end{pmatrix} \to \begin{pmatrix} u \\ d \end{pmatrix} \right] \tag{23}$$

$$|NG(0)\rangle = \frac{1}{\sqrt{\sum_f m_f^2 I_{m_f;m_f}}} \left[m_\nu I_{m_\nu;m_\nu}^{1/2} |\bar{\nu} i\gamma_5 \nu\rangle + m_e I_{m_e;m_e}^{1/2} |\bar{e} i\gamma_5 e\rangle + \begin{pmatrix} \nu \\ e \end{pmatrix} \to \begin{pmatrix} u \\ d \end{pmatrix} \right].$$

3.c. Gauge boson masses

In Section 3.b, we have made an intermediate assumption of switching off the $SU(2)_L \times U(1)_Y$ gauge interactions. In reality, W, Z and A are present and two of them contribute also into fermion-antifermion scattering matrices we have considered in Section 3.b. We know in advance[4] what must happen: gauge bosons W and Z "eat" bound state NG bosons and become massive. To realize this programme explicitly in our model, we have to calculate fermion-gauge boson vertex functions, again in the chain approximation[11], using the interaction $\mathcal{L}^{(e)}_{NJL}$.

The vertex function Γ^α_W is (see Fig. 3) given as

$$\Gamma^\alpha_W = \frac{g}{2\sqrt{2}}\gamma^\alpha(1-\gamma_5) + \frac{g}{2\sqrt{2}}(1+\gamma_5)\frac{1}{1-J_{o;m}(q^2)}J^\alpha_{o;m}(q) =$$

$$= \frac{g}{2\sqrt{2}}\gamma^\alpha(1-\gamma_5) + \frac{g}{2\sqrt{2}}\frac{1}{q^2}q^\alpha m(1+\gamma_5), \qquad (24)$$

where

$$J^\alpha_{o;m}(q) = g_e\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}(1-\gamma_5)S^o_F(p)\gamma^\alpha(1-\gamma_5)S^m_F(p-q) = q^\alpha m\, g_e\, I_{o;m}(q^2). \qquad (25)$$

With Fig. 4 taken into account, the second term in Eq. (24) is understood as an effective coupling between the charged NG boson and the W boson. This coupling gives rise to the longitudinal part of the polarization tensor of the W boson, singular at $q^2 = 0$ with a residue $1/4\, g^2 m^2 I_{o;m}(0)$. Because of current conservation we know in fact the whole polarization tensor of the W boson. Hence[1),6),11)]

$$m_W^2 = \frac{1}{4}g^2 m^2 I_{o;m}(0)\,.$$

The vertex functions $\Gamma^\alpha_{A^3}$ and Γ^α_B are evaluated analogously (see Fig. 5):

$$\Gamma^\alpha_{A^3} = \frac{1}{4}g\gamma^\alpha(1-\gamma_5) + \frac{1}{4}g\gamma_5\frac{2}{1-J_{m;m}(q^2)}J^\alpha_{m;m}(q) =$$

$$= \frac{1}{4} g \gamma^\alpha (1-\gamma_5) + \frac{1}{4} g \frac{1}{q^2} q^\alpha 2m \gamma_5 \quad ,$$

$$\Gamma_B^\alpha = -\frac{1}{4} g' \gamma^\alpha (1-\gamma_5) - \frac{1}{2} g' \gamma^\alpha (1+\gamma_5) + \frac{1}{4} g' \frac{1}{q^2} q^\alpha 2m \gamma_5 \quad ,$$

where

$$J^\alpha_{m;m}(q) = g_e \int \frac{d^4p}{(2\pi)^4} \operatorname{tr} \gamma_5 S_F^m(p) \gamma^\alpha (1-\gamma_5) S_F^m(p-q) = q^\alpha m g_e I_{m;m}(q^2) .$$

Hence, a residue at the pole of the longitudinal part of the polarization tensor of neutral vector bosons is given by the matrix

$$\begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix} \frac{1}{4} m^2 I_{m;m}(0)$$

in the (A^3, B) basis. Its diagonalization leads to

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2) m^2 I_{m;m}(0)$$
$$m_A^2 = 0 \quad ,$$

where

$$Z_\alpha = -\cos\Theta_W A^3_\alpha + \sin\Theta_W B_\alpha \quad , \quad A_\alpha = \sin\Theta_W A^3_\alpha + \cos\Theta_W B_\alpha$$

and θ_W is the Weinberg angle, $tg\theta_W = g'/g$.

Considering all components of $\mathcal{L}_{NJL}$, i.e., considering a complete coupling of NG bosons (23) to the gauge bosons, we arrive at the sum rules

$$m_W^2 = \frac{1}{4} g^2 \sum_f m_f^2 \, I_{0;m_f}$$
$$m_Z^2 = \frac{1}{4}(g^2 + g'^2) \sum_f m_f^2 \, I_{m_f;m_f} \, . \qquad (26)$$

Comments

(i) The coupling constants $I^{-\frac{1}{2}}_{o;m_f}$ and $I^{-\frac{1}{2}}_{m_f;m_f}$ are calculable numbers in an ultimate theory and are to be compared with a quark-pion coupling constant of a dynamically broken chiral symmetry in QCD[17].

(ii) Addition of more fermion families without flavour mixing means that the summation index f in (26) runs over all the fermions present in the theory. Consequently, Eqs. (26) provide an upper bound on the heaviest fermion in the world. To saturate (26) by just one fermion f (with I = 1 for orientation) means $m_f \approx 250$ GeV (vacuum expectation value of the Higgs field in the standard approach).

(iii) There is no fundamental weak interaction mass scale.

(iv) The canonical ratio

$$m_W^2 / m_Z^2 \cos^2\theta_W = 1 \tag{27}$$

is slightly violated. For equal fermion masses in weak doublets it would be satisfied. Dependence of $I_{o;m}$ and $I_{m;m}$ upon fermion masses is rather weak.

(v) A parallel with the technicolour idea of gauge boson mass generation[18] is clear. The difference is that the technicolour force does not feel flavour, so that bound state technipions [to be compared with (23)] form an exact pseudoscalar SU(2) doublet and (27) holds.

4. - FLAVOUR MIXING

In this section we show how the mechanism of fermion and gauge boson mass generation is generalized for the case of more families with fermion mixing taken into account.

Fermion families i in the weak interaction basis are distinguished by C hypercharges

$$Y_{iH} = \alpha_i Y_H^{(1)} + \beta_i Y_H^{(2)} . \tag{28}$$

Four-fermion interactions responsible for generation of charged lepton and Dirac neutrino masses are

$$\mathcal{L}^{(e)}_{NJL} = \frac{h^2}{M^2} y_i(\psi_L) y_j(e_R) \left[\bar{\nu}_{iL} e_{jR} \cdot \bar{e}_{jR} \nu_{iL} + \bar{e}_{iL} e_{jR} \cdot \bar{e}_{jR} e_{iL} \right] \tag{29a}$$

and

$$\mathcal{L}^{(\nu)}_{NJL} = \frac{h^2}{M^2} y_i(\psi_L) y_j(\nu_R) \left[\bar{e}_{iL} \nu_{jR} \cdot \bar{\nu}_{jR} e_{iL} + \bar{\nu}_{iL} \nu_{jR} \cdot \bar{\nu}_{jR} \nu_{iL} \right], \tag{29b}$$

respectively. If we assume that the general non-diagonal mass term

$$\mathcal{L}_{mass} = - \bar{e}_{iL} \Sigma^{e}_{ij} e_{jR} - \bar{\nu}_{iL} \Sigma^{\nu}_{ij} \nu_{jR} + \mathrm{H.c.} \tag{30}$$

develops dynamically, it is determined by the self-consistency gap equation

$$m_a(e) \delta_{ab} - \frac{4ih^2}{M^2} V_{ac}(e_L) \int \frac{d^4p}{(2\pi)^4} \frac{m_c(e)}{p^2 - m_c^2(e)} V_{cb}(e_R) = 0 . \tag{31}$$

An analogous equation holds for $m_a(\nu)$, $V(\nu_L)$ and $V(\nu_R)$. In order to derive Eq. (31), we have introduced mass eigenstates of charged leptons $\ell_i = (e,\mu,\tau)$ and of neutrinos $n_i = (\nu_e, \nu_\mu, \nu_\tau)$:

$$\begin{aligned} \ell_L &= U(e_L) e_L \ , \quad \ell_R = U(e_R) e_R \\ n_L &= U(\nu_L) \nu_L \ , \quad n_R = U(\nu_R) \nu_R . \end{aligned} \tag{32}$$

Matrices U diagonalize the mass term (30)

$$\mathcal{L}_{mass} = - \bar{\ell}_L m(e) \ell_R - \bar{n}_L m(\nu) n_R + \mathrm{H.c.}, \tag{33}$$

that is,

$$m(e) = U(e_L) \Sigma^e U^+(e_R) \ , m(\nu) = U(\nu_L) \Sigma^\nu U^+(\nu_R) . \tag{34}$$

Matrices V are defined as follows:

$$\begin{aligned} V(e_L) &= U(e_L) y(\psi_L) U^+(e_L) \ , \quad V(e_R) = U(e_R) y(e_R) U^+(e_R) \\ V(\nu_L) &= U(\nu_L) y(\psi_L) U^+(\nu_L) \ , \quad V(\nu_R) = U(\nu_R) y(\nu_R) U^+(\nu_R) . \end{aligned} \tag{35}$$

Generation of the gauge boson masses will be demonstrated using a formal approach[19),20)]: $SU(2)_L \times U(1)_Y$ invariance of the Lagrangian implies Ward-Takahashi identities which must remain valid even in the presence of a symmetry breaking. This implies that the vertex parts Γ^α_W and Γ^α_Z must develop massless poles which correspond to the "would-be" NG bosons[21)]:

$$\Gamma^\alpha_W \underset{q\to 0}{=} \frac{1}{\sqrt{2}} g \bar{\nu}_L \gamma^\alpha e_L - \frac{1}{\sqrt{2}} g \frac{q^\alpha}{q^2} [\bar{\nu}_R \Sigma^\nu e_L - \bar{\nu}_L \Sigma^{e+} e_R] \tag{36}$$

and analogously for Γ^α_Z.

The form of the pole term in (36) determines the coupling of fermions to these NG bosons[20)]:

$$P = (N)^{-1/2} \left[(1-\gamma_5)\Sigma^\nu - (1+\gamma_5)\Sigma^{e+}\right]. \tag{37}$$

The normalization factor will be calculated in the following. The only thing to be calculated is the loop integral:

$$J^\alpha_{m(\nu);m(e)}(q) = \frac{g}{2\sqrt{2}} \int \frac{d^4p}{(2\pi)^4} \operatorname{tr} P^+ S^\nu(p) \gamma^\alpha (1-\gamma_5) S^e(p-q), \tag{38}$$

where S^ν and S^e are propagators of leptons with non-diagonal mass terms (30). They are easily diagonalized using (34). We regularize the integral (38) by setting Σ in the nominator of the fermion propagator in (38) to[22),23)]

$$\Sigma^{\nu,e}(p^2) = \frac{\sigma^{\nu,e} M^3}{M^2 - p^2}.$$

This regularization, besides being convenient, is quite physical. We have replaced the constant fermion mass term by the corresponding proper fermion self-energy part which is assumed to appear when the programme of dynamical fermion mass generation is realized by using Schwinger-Dyson equations[3),19),20),23)]. Whenever it is convenient, we use the property $m(f) = U(f_L)\sigma^f U^+(f_R)\cdot M$. The result is

$$J^\alpha_{m(\nu);m(e)}(q) = \frac{g}{2\sqrt{2}} (-iq^\alpha) N^{1/2}, \tag{39}$$

where

$$N = \left[(m(\nu)U(\nu_R)U^{+}(e_L))_{ij} \, (m(\nu)U(\nu_R)U^{+}(e_L))^{*}_{ij} \, I_{m_i(\nu);\, m_j(e)} \; + \; \nu \leftrightarrow e \right]$$

and

$$I_{m_i(\nu);m_j(e)} = \frac{1}{4\pi^2} \int_0^1 x \ln \left[\frac{(M^2 - m_j^2(e))x + m_j^2(e)}{(m_i^2(\nu) - m_j^2(e))x + m_j^2(e)} \right]^2 dx \,. \qquad (40)$$

The analogous calculation is easily repeated also for Γ^{α}_{Z}.

Having calculated $J^{\alpha}_{m(\nu);m(e)}(q)$ and $J^{\alpha}_{m(f);m(f)}(q)$, we add analogous contributions from quark mass terms Σ^{u}, Σ^{d} and we are ready to write down the final sum rules for m_W and m_Z:

$$m_W^2 = \frac{1}{4} g^2 \sum \Big\{ \left[(m(\nu)U(\nu_R)U^{+}(e_L))_{ij} (m(\nu)U(\nu_R)U^{+}(e_L))^{*}_{ij} \, I_{m_i(\nu);m_j(e)} + \nu \leftrightarrow e \right] +$$

$$\left[(m(u)U(u_R)U^{+}(d_L))_{ij} (m(u)U(u_R)U^{+}(d_L))^{*}_{ij} \, I_{m_i(u);m_j(d)} + u \leftrightarrow d \right] \Big\}$$

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2) \sum \Big\{ \left[(m(\nu)U(\nu_R)U^{+}(\nu_L))_{ij} (m(\nu)U(\nu_R)U^{+}(\nu_L))^{*}_{ij} \, I_{m_i(\nu);m_j(\nu)} + \nu \leftrightarrow e \right] +$$

$$+ \left[(m(u)U(u_R)U^{+}(u_L))_{ij} (m(u)U(u_R)U^{+}(u_L))^{*}_{ij} \, I_{m_i(u);m_j(u)} + u \leftrightarrow d \right] \Big\} \,. \qquad (41)$$

When the Lagrangian (1) is rewritten in terms of mass eigenstates, both for leptons and quarks, we see that: (i) the electromagnetic current and the weak neutral current remain intact; (ii) charged weak currents, leptonic and quark, contain unitary Kobayashi-Maskawa mixing matrices $U(\nu_L)U^{+}(e_L)$ and $U(u_L)U^{+}(d_L)$, respectively; (iii) the neutral current J^{C}_{α} becomes

$$J^{C}_{\alpha} = \frac{1}{2} \left[\bar{n}_L V(\nu_L)\gamma_\alpha n_L + \bar{n}_R V(\nu_R)\gamma_\alpha n_R + \bar{\ell}_L V(e_L)\gamma_\alpha \ell_L + \cdots \right] , \qquad (42)$$

where mixing matrices V are defined in (35). They are neither diagonal nor unitary. This property imposes a severe constraint on M, which we take as M = 1000 TeV[24].

5. - CONCLUSIONS

General principles of dynamically broken gauge symmetries are very attractive and provide us with a hope of reducing a number of free parameters of the standard model. If the standard GWS model without Higgs sector were capable of producing both fermion and gauge boson masses dynamically, this would definitely be the most economic possibility. Unfortunately, this is not the case[21),25)].

We suggest a minimal extension of the Higgsless GWS model defined by (1), and heuristically argue that it may be understood as a microscopic basis underlying the phenomenological Higgs approach.

1. - The model is operationally well defined. Electroweak interactions remain standard. Fermion masses, mixing angles and gauge boson masses are generated. The canonical ratio $m_W^2/m_W^2\cos^2\theta_W = 1$ is only slightly modified. Hence, good features of the standard model are reproduced.

2. - Fermion masses, mixing angles and gauge boson masses are calculable (in principle). There is only one free parameter h. Obviously, we count only those which undergo renormalization and which are theoretically undeterminable. C hypercharges can easily be quantized.

3. - There are some new features of the model (1) not shared by the standard model:

(i) There should exist heavy spinless bosons, both charged and neutral, with calculable couplings to fermions. Their existence is guaranteed by the fact that only one linear combination (23) (for a given charge) of the many-component composite Higgs doublets is absorbed by a gauge field as a "would-be" NG boson. Orthogonal combinations to those in (23) must remain in the physical spectrum. Being in a sense partners of W and Z bosons, they might have also masses*) comparable to m_W and m_Z. We point out that the existence of neutral heavy spinless bosons is phenomenologically postulated

---

*) This estimate is consistent with a lowest condensation scale $\mu = 100$ GeV (see the Table).

at present[26] in order to explain peculiar collider events[27]. When flavour mixing is taken into account, one has to assure that the effective Yukawa couplings of neutral heavy spinless bosons are not flavour-changing. Since quarks of different charges get masses from two different many-component composite doublets, it seems that the general criterion of Glashow and Weinberg[28] for diagonal fermion-neutral Higgs boson couplings is also satisfied here.

(ii) There is no fundamental weak interaction mass scale. The value of $(\sqrt{2}G_F)^{-\frac{1}{2}} \approx 250$ GeV is only a remnant of heavy fermions.

(iii) There should exist rare processes mediated by a C boson exchange. We restrict their strength to be of the same order as the same processes at one loop with W and Z exchanges.

(iv) Mixing angles of the right-handed fermions are observable. See Eqs. (41) and (42).

4. - At every stage of our calculations we are able to point out a corresponding step in the standard approach. This indicates that a formal derivation of the Higgs Lagrangian from a microscopic one, Eq. (1), can be done in the spirit of Gorkov's derivation[29] of the Ginzburg-Landau Lagrangian from BCS.

A renormalization-group analysis makes important predictions about the nature of the solutions of field theory[7]. We are trying at present to repeat the programme described here with the renormalization group implemented into the Schwinger-Dyson equations for the fermion propagator and the propagator of the field C[3].

ACKNOWLEDGEMENTS

The author thanks D. Nanopoulos for reading the manuscript and for useful discussions, and for its hospitality the TH Division at CERN where this work was completed.

μ (GeV)	100	10^3	10^4	10^5
m_f	100 eV	10^{-1} MeV	10^{-1} GeV	100 GeV

Table: Typical range of fermion masses due to different condensation mass scales.

REFERENCES

1) P.W. Higgs - Phys.Lett. 12 (1964) 132;
F. Englert and R. Brout - Phys.Rev.Lett. 13 (1964) 321;
G. Guralnik, C. Hagen and T. Kibble - Phys.Rev.Lett. 13 (1964) 585.

2) F. Englert - Lectures at Cargèse Summer Institute (1977).

3) H. Pagels - Phys.Rev. D21 (1980) 2336.

4) J. Schwinger - Phys.Rev. 128 (1962) 2425;
P.W. Anderson - Phys.Rev. 130 (1963) 439.

5) K. Lane - Phys.Rev. D10 (1974) 1353;
D.J. Gross and A. Neveu - Phys.Rev. D10 (1974) 3275.

6) R. Stern - Phys.Rev. D14 (1976) 2081.

7) K.G. Wilson - Phys.Rev. D3 (1971) 1818.

8) Y. Nambu and G. Jona-Lasinio - Phys.Rev. 122 (1961) 345.

9) K.D. Lane and M.E. Peskin - Lectures at XVme Rencontre de Moriond (1980).

10) W.J. Marciano - Phys.Rev. D21 (1980) 2452;
D. Lüst, E. Papantonopoulos and G. Zoupanos - Z.Phys. C25 (1984) 81;
J. Kogut, J. Polonyi, H.W. Wyld and D.K. Sinclair - Preprint ILL-(TH)-85-8 (January 1985).

11) Y. Freundlich and D. Lurié - Nucl.Phys. B19 (1970) 557.

12) M.S. Chanowitz, J. Ellis and M.K. Gaillard - Nucl.Phys. B128 (1977) 506.

13) C. Itzykson and J.-B. Zuber - Quantum Field Theory, McGraw Hill Inc., New York (1980).

14) S. Weinberg - Phys.Rev. D8 (1973) 3497.

15) S. Coleman and D.J. Gross - Phys.Rev.Lett. 31 (1973) 851.

16) E. de Rafael and J.L. Rosner - Ann.Phys.(N.Y.) 82 (1974) 369.

17) J. Govaerts, J.E. Mandula and J. Weyers - Phys.Lett. 130B (1983) 427.

18) S. Weinberg - Phys.Rev. D19 (1979) 1277;
L. Susskind - Phys.Rev. D20 (1979) 2619.

19) F. Englert, R. Brout and M.F. Thiry - Nuovo Cimento 43 (1966) 244;
A.A. Migdal and A. Polyakov - Zh.Eksp.Teor.Fiz. 51 (1966) 135.

20) R. Jackiw and K. Johnson - Phys.Rev. D8 (1973) 2386;
J. Cornwall and R. Norton - Phys.Rev. D8 (1973) 3338.

21) K. Huang and R. Mendel - Preprint MIT-CTP 876 (1980).

22) J.M. Cornwall - Phys.Rev. D22 (1980) 1452.

23) J. Hošek - Preprint JINR E2-83-657 (1983).

24) R.N. Cahn and H. Harari - Nucl.Phys. B176 (1980) 135;
S. Dimopoulos and J. Ellis - Nucl.Phys. B182 (1981) 505.

25) A. Carter and H. Pagels - Phys.Rev.Lett. 43 (1979) 1845;
R. Acharya et al. - Nuovo Cimento 60A (1980) 265;
R. Delbourgo and B.J. Kenny - J.Phys. G7 (1981) 417;
K.T. Mahanthappa and J. Randa - Phys.Lett. 121B (1983) 156.

26) H. Baer, K. Hagiwara and J. Ohnemus - Preprint MAD-PH-215 (1984);
J.L. Rosner - Preprint CERN-TH.4086/85 (1985).

27) UA1 Collaboration - Phys.Lett. 126B (1983) 398; 147B (1984) 241;
UA2 Collaboration - Phys.Lett. 129B (1983) 130.

28) S.L. Glashow and S. Weinberg - Phys.Rev. D15 (1977) 1958.

29) L.P. Gorkov - Soviet Phys. JETP 9 (1959) 1364.

FIGURE CAPTIONS

Fig. 1 Chain of graphs which gives rise to the charged NG boson.

Fig. 2 Chain of graphs which gives rise to the neutral NG boson.

Fig. 3 Chain of graphs for the W boson vertex function.

Fig. 4 Effective coupling of the charged NG boson with fermions and with the W boson.

Fig. 5 Effective couplings of the neutral NG boson with fermions and with neutral gauge bosons A^3 and B.

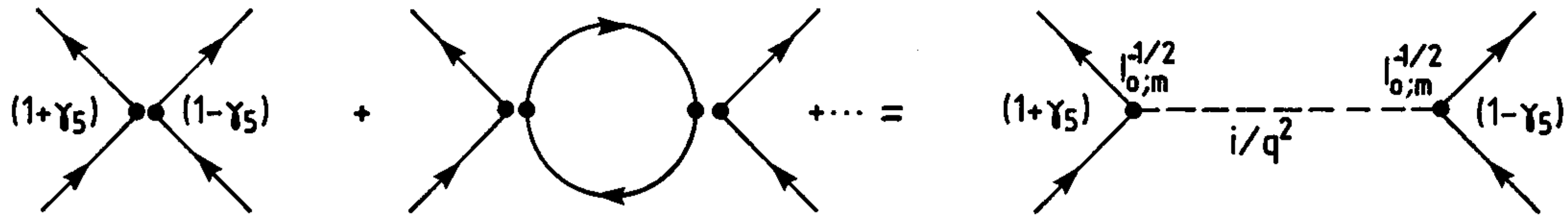

(1+γ5)
(1−γ5)
+
+ ··· =
(1+γ5)
l-1/2 o;m
i/q2
l-1/2 o;m
(1−γ5)

FIG. 1

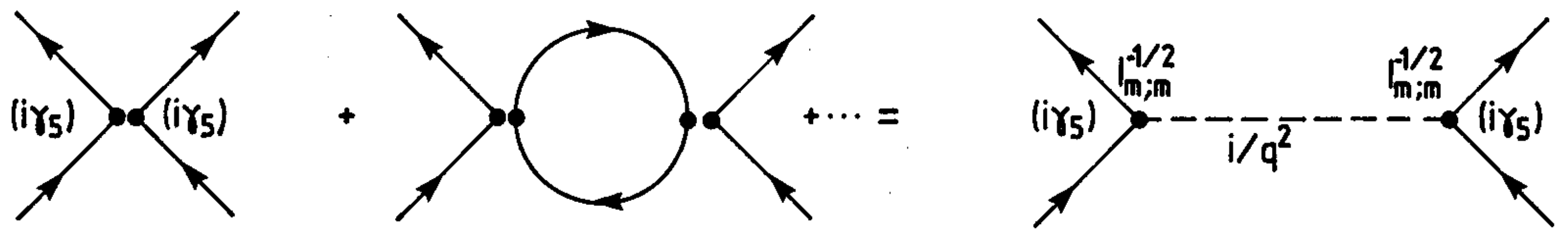

(iγ5)
(iγ5)
+
+ ··· =
(iγ5)
l-1/2 m;m
i/q2
l-1/2 m;m
(iγ5)

FIG. 2

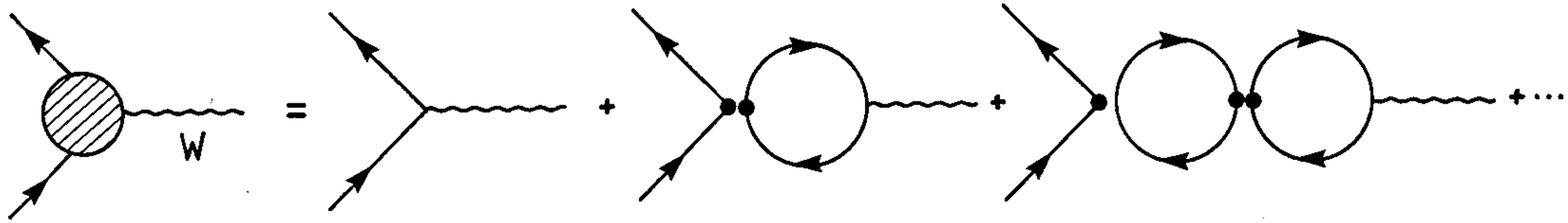


FIG. 3

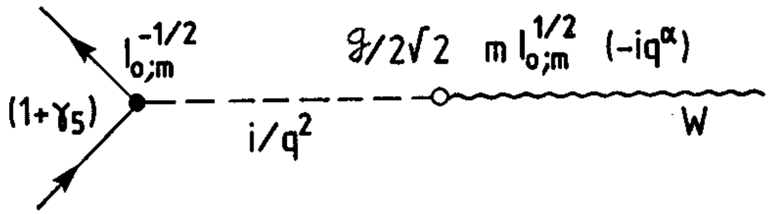
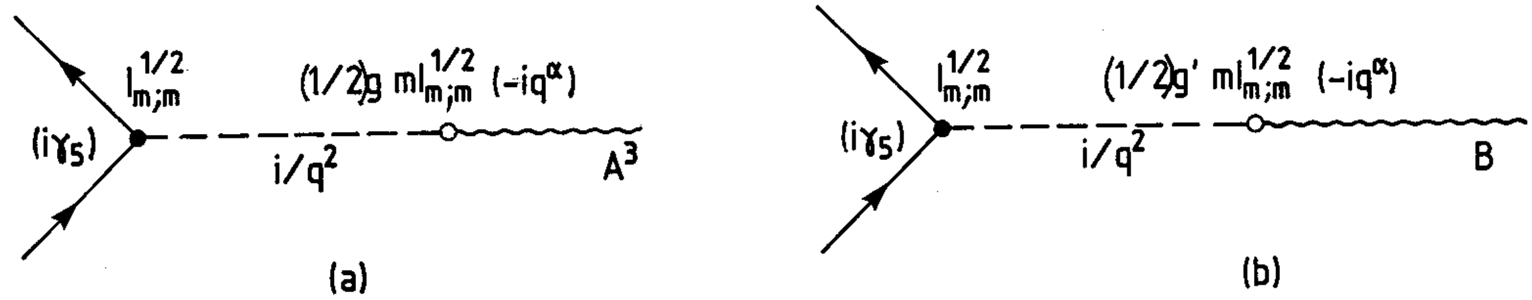


FIG. 4



(iγ5)
l1/2 m;m
(1/2)g ml1/2 m;m (-iqα)
i/q2
A3
(a)
(iγ5)
l1/2 m;m
(1/2)g' ml1/2 m;m (-iqα)
i/q2
B
(b)

FIG. 5