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THE NO-EXCHANGE MODEL OF THE DOUBLE-SPIN-FLIP N-N AMPLITUDE

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ABSTRACT

We develop the no-exchange model (NEM) in which it is assumed that pions are not exchanged inside some distance R between the two nucleons. In the framework of this model we explain the OPE-δ procedure (or the "poor man's absorption model"). This model yields a quantitative description of the double-spin-flip helicity amplitude (ϕ_2) for elastic nucleon-nucleon scattering with R ≃ 0.7 fm.

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1. - INTRODUCTION

It is a well known fact that some elastic and inelastic amplitudes are dominated by the one pion exchange (OPE) modified contributions. As was observed by P.K.Williams[1)], the simplest and sometimes the most efficient correction, is the subtraction from the OPE amplitude of a term like δ_{J0} or terms of similar origin which appear in the form of polynomials in t (the momentum transfer squared) in the numerators of the OPE propagators.

To our knowledge there is no adequate explanation of this procedure - the so-called OPE - δ[1)] or poor man's absorption model[2)]. Usually one assumes strong absorption of the s-waves, which otherwise would often violate the unitarity limit.

Related to this is the idea that a better approximation than OPE is obtained by dropping the non-physical unitarity violating terms[3)]. Nevertheless a fundamental question is not being answered, namely why the above mentioned amplitudes are so dominated by the OPE - δ alone, even at very high energies[4),5)]. Here we would expect that the contributions from meson exchanges with higher spin (like the ρ and A_2 meson exchanges[5),7)]) will take over.

We have analyzed the N-N elastic scattering amplitudes resulting from the phase shift analysis of Arndt and Ver West[8)] and have found that the double spin flip helicity amplitude, Φ_2 , can be relatively well approximated by the OPE - δ procedure over a wide range of energies.

The validity of this approach, even at relatively low energies (down to about 100 MeV lab kinetic energy), much below the production threshold (about 280 MeV), is also surprising. Of course for these low energies the absorption mechanism cannot explain the OPE - δ procedure. Neglecting the unitarity violating terms seems to be a more adequate prescription in this case. Partially this can be achieved by unitarization[3)].

Another problem related to the above OPE - δ procedure is the extraction of the πNN form factor. In some publications[9)-12)] a generalization [6)] of the OPE - δ procedure is used together with a form factor in order to fit the experimental data.

It was pointed out to us by Magda Ericson[13)] that in these procedures the effect of short range repulsion has been neglected. Indeed, as we shall see in the subsequent sections, the short range repulsion (or absence of the interaction at short distance) can greatly influence the results which are obtained. What was considered to be the consequence of introducing a form factor comes out as a combined effect of the short range repulsion and of the form factor. Thus the extracted form factors should be reinterpreted along these lines.

Our attempt to explain the OPE - δ procedure in the N-N ϕ_2 amplitude is based on the assumption that pions are effectively not exchanged at short internucleon distances.

2. - PRELIMINARY ANALYSIS OF THE ϕ_2 AMPLITUDE

The double spin flip $\phi_2 = \langle \frac{1}{2}\frac{1}{2}|T(E)|-\frac{1}{2}-\frac{1}{2}\rangle$ helicity amplitude of pp (ϕ_2^{pp}) or np scattering (ϕ_2^{np}) has pion poles in both the t and u channels. In order to simplify the analysis we take a combination of isospin amplitudes which has only a pole in the t channel. In this way we can search for energy independent patterns. Assuming isospin conservation, such a combination is

$$\phi_2 = \frac{1}{6}\left(3\phi_2^1 - \phi_2^0\right) = \frac{1}{3}\left(\phi_2^{np} - 2\phi_2^{pp}\right), \tag{2.1}$$

where ϕ_2^0 and ϕ_2^1 are the isospin zero and isospin one amplitudes, respectively. The c.m. one pion exchange (OPE) amplitude is in this case

$$\phi_2^{OPE} = -\frac{\alpha t}{t-\mu^2}, \tag{2.2}$$

where t is the 4-momentum transfer squared, μ is the pion mass and finally

$$\alpha = \frac{g^2}{16\pi E}. \tag{2.3}$$

In the calculations we take $g^2/4\pi = 14.5$ and E is the total c.m. energy of one of the nucleons.

In Fig. 1 we display the amplitudes calculated from the phase shift analysis of Ref. 8. We have observed that starting from about 425 MeV up to the highest energy analysis available - namely 750 MeV - the amplitudes display almost the same t-dependence. This deviates by approximately a constant value from the OPE formula (2.2). The OPE - δ procedure[1] consists of replacing the numerator by its value at the pole, i.e. by μ^2 :

$$-\frac{\alpha t}{t-\mu^2} \Rightarrow \frac{-\alpha\mu^2}{t-\mu^2}. \tag{2.4}$$

Now the data in Fig. 1 are qualitatively well described by this new formula. For energies lower than 425 MeV we also find a good qualitative description using the OPE - δ procedure - although, as one can see in Fig. 2, the data do not show the same t-dependence as in Fig. 1.

In Sections 3 and 4 we shall present models which give a much better quantitative description of the data. Before comparing our results with the data, we should point out that most of the measurements of the observables used in the phase shift analysis (400 $\div$ 800 MeV), were not performed at small angles. Therefore the results for small angles should be considered as extrapolations and are less reliable than the attached error bars indicate.

3. - THE NO-EXCHANGE MODEL

The OPE - δ procedure is usually explained in terms of s-wave absorption. But, as the OPE - δ procedure is applicable to the ϕ_2 amplitude below the pion production (280 MeV), we have to reject this interpretation. Instead, we shall assume that at short distances, say below some separation distance R, pions are simply not being exchanged. If we let R approach zero, this procedure will be exactly equivalent to the OPE - δ. On the other hand, with a suitable choice of R, we can get a relatively good description of the data of Fig. 1. Before presenting our results a few important remarks about the behaviour of these data should be made.

The isospin combination of eq.(2.1) is such that it eliminates the u channel poles of the exchanged isovector particles (such as π , ρ or δ mesons), but this combination does not eliminate the u channel poles of exchanged isoscalar particles (like ω or ε mesons). However we do not see any structure at backward angles (no u channel interference), indicating that there may be a cancellation mechanism for the isoscalar meson exchanges (or multipion exchanges of the isoscalar type). Within the OPE - δ procedure the data seem to be almost purely OPE, indicating also that the role of isovector meson exchanges different from the pion is relatively unimportant.

On the other hand we know that all the partial waves entering the amplitude are strongly affected by the repulsive core of the N-N interaction. Still, in the real part of the ϕ_2 amplitude, we do not see a direct contribution from the core (like, for instance, the ω -exchange).

A similar situation exists with respect to the Coulomb interaction. The ϕ_2 amplitude arising from photon exchange is very small, almost unnoticeable [14], yet all partial waves entering the ϕ_2 amplitude are affected by the Coulomb interaction. The picture emerging from this discussion is that the ϕ_2 amplitude is mostly OPE providing that the modification induced by the presence of the core is taken into account and that there is no direct contribution from the core itself to the real part of the ϕ_2 amplitude.

A consequence of the existence of the core is that at the core distances, or nearby (where the probability density of the two nucleons is quite small), we do not expect the pions to be exchanged, simply because the two nucleons cannot be found (or can hardly be found) at these distances.

This statement can be analytically expressed in terms of the distorted wave Born approximation (DWBA) for the scattering amplitude ϕ ,

$$\phi_{DWBA} = \phi^{core} - \frac{m^2}{2E} \langle \psi^{(-)}_{core} | V_{res} | \psi^{(+)}_{core} \rangle , \qquad (3.1)$$

where ϕ^{core} is the amplitude resulting from the scattering off the core, V_{res} is the residual interaction outside the core and $\psi^{(\pm)}_{core}$ are the wave functions of the repulsive core itself. According to the previous considerations we do not expect a direct contribution to ϕ_2 from the short range interaction (i.e., $\phi_2^{core} \simeq 0$).

The detailed calculation of (3.1) will be described in Section 4. Here we shall adopt a simpler attitude, namely we shall assume that pions are not exchanged up to a distance R. The formulae of this no-exchange model (NEM) are relatively simple and meaningful; moreover, as we shall see in Section 4, the resulting amplitudes do not differ considerably from the DWBA ones.

Our procedure is the following one. First we Fourier transform the ϕ_2^{OPE} amplitude to obtain a potential in coordinate space. Then we Fourier transform it back to the momentum space, to get the scattering amplitude, but start the interaction from the distance R and go upwards. We use the notation $q = \sqrt{-t}$. Following the procedure and normalizations of Ref. 15 and using eqs. (2.2) and (2.3), we obtain:

$$V_2^{OPE}(r) = -\frac{2}{\pi m^2 r} \int_0^\infty j_0(qr) E \phi_2^{OPE} q^2 dq$$

$$= \frac{g^2\mu^2}{8\pi m^2} \left[\frac{e^{-\mu r}}{r} + \frac{4\pi \delta^{(3)}(\vec{r})}{\mu^2} \right] \qquad (3.2)$$

$$\Phi_2^{NEM}(R,q) = -\frac{m^2}{2E}\int_R^\infty j_0(qr)\,V_2^{OPE}(r)\,r^2 dr$$
$$= \frac{\alpha\mu^2 e^{-\mu R}}{q^2+\mu^2}\left(\cos qR + \frac{\mu}{q}\sin qR\right). \tag{3.3}$$

The last formula will be used in analyzing the data. The effect of a possible form factor will be studied in Section 5.

In Fig. 3 we compare the data with the curves (denoted NEM) resulting from (3.3), where the core radius is R = 1.1 fm. In the figure we also display the curves corresponding to OPE and to the OPE - δ procedure.

Our no-exchange model seems to reproduce the general trend of the experimental amplitudes fairly well, particularly at not too small angles, where we are more confident about the data. The rather large core radius which is needed here has to be considered as an effective one, as we shall see in the next section.

Eq.(3.3) could also be related to other meson exchanges. However the factor exp(- μR) indicates that the contributions of mesons heavier than the pion are greatly suppressed by the existence of the core. This might partly explain the fact that OPE is so dominant for the real part of the ϕ_2 amplitude.

4. - DISTORTED WAVES FROM THE CORE

We shall now consider the OPE amplitude given by eq.(3.1), taking into account the distorsion produced by the repulsive core on the nuclear wave functions. We use the hard sphere wave functions

$$\Psi^{(+)}(k,r,\theta) = \frac{1}{2}\sum_{\ell=0}^{\infty} i^\ell (2\ell+1)\left[h_\ell^{(2)}(kr) + S_\ell h_\ell^{(1)}(kr)\right] P_\ell(\cos\theta)$$

for $r \geq R$ (4.1)

where

$$S_\ell = -h_\ell^{(2)}(kR)/h_\ell^{(1)}(kR), \tag{4.2}$$

R is the radius of the hard sphere, k is the c.m. momentum and $h_\ell^{(1)}$, $h_\ell^{(2)}$ are the spherical Hankel functions.

By inserting (4.1) into eq.(3.1), with $\phi^{core} = 0$ and $V_{res} = V_2^{OPE}$, we obtain, after some algebraic manipulations:

$$\phi_2 = \phi_2^{NEM}(R,q) - \frac{m^2}{2E}\sum_{\ell=0}^{\infty}\frac{(2\ell+1)}{j_\ell^2(kR)+n_\ell^2(kR)}P_\ell(\cos\theta)\cdot$$

$$\cdot\Big[j_\ell^2(kR)\int_R^\infty dr\, r^2 V_2^{OPE} j_\ell^2(kr) + 2n_\ell(kR)j_\ell(kR)\int_R^\infty dr\, r^2 V_2^{OPE} j_\ell(kr)n_\ell(kr) -$$

$$- j_\ell^2(kR)\int_R^\infty dr\, r^2 V_2^{OPE} n_\ell^2(kr)\Big] , \qquad (4.3)$$

where $\phi_2^{NEM}(R,q)$ is the same as in eq.(3.3), j_ℓ and n_ℓ are the spherical Bessel functions and V_2^{OPE} is given by eq.(3.2).

The amplitudes calculated with eq.(4.3) are displayed in Fig. 3 (continuous line), where a core radius R = 0.7 fm has been utilized. The corresponding curves are close to the ones obtained in the simple no-exchange model with R = 1.1 fm. The larger R value needed for the NEM amplitude can be considered as an effective radius which somehow incorporates the effects of the DWBA. Indeed by using the wave functions (4.1) the probability of finding a nucleon close to the core is still very small so that the effective region where no interaction takes place turns out to be more extended than the core radius itself.

Nevertheless the physics behind formula (4.3) is still the same as in the no-exchange model: by a suitable choice of the core radius, the real part of the ϕ_2 amplitude can be well reproduced at different energies by the simple OPE mechanism, except for the shorter distances, where no exchange occurs.

5. - THE EFFECT OF A FORM FACTOR

As we have pointed out in the introduction, the analysis of the ϕ_2 amplitude has often been intended to determine the πNN vertex form factor. The model presented here is not incompatible with the presence of a form factor. But, since the latter is essentially of short range character, we should expect its effects to be greatly diminished by the existence of the core.

Let us consider, as an example, a monopole form factor

$$F(q) = \frac{\Lambda^2-\mu^2}{\Lambda^2+q^2} \qquad (5.1)$$

at each πNN vertex. The OPE contribution to ϕ_2 is then changed from eq.(2.2) to

$$\phi_2^{OPE} = -\frac{\alpha q^2}{q^2+\mu^2}\left(\frac{\Lambda^2-\mu^2}{\Lambda^2+q^2}\right)^2$$

$$= \frac{\alpha\mu^2}{q^2+\mu^2} - \frac{\alpha\mu^2}{q^2+\Lambda^2} - \frac{\alpha\Lambda^2(\Lambda^2-\mu^2)}{(q^2+\Lambda^2)^2} . \tag{5.2}$$

We can now apply to (5.2) the same procedure of Section 3 to get the corresponding NEM amplitude. Eqs.(3.2) and (3.3) will then be replaced by

$$\bar{V}_2^{OPE} = \frac{g^2\mu^2}{8\pi m^2}\left[\frac{e^{-\mu r}}{r} - \frac{e^{-\Lambda r}}{r} - \frac{\Lambda^2(\Lambda^2-\mu^2)}{2\Lambda\mu^2}e^{-\Lambda r}\right] \tag{5.3}$$

and

$$\bar{\Phi}_2(R,\Lambda,q) = \frac{\alpha\mu^2 e^{-\mu R}}{q^2+\mu^2}\left(\cos qR + \frac{\mu}{q}\sin qR\right)$$

$$- \frac{\alpha\mu^2 e^{-\Lambda R}}{q^2+\Lambda^2}\left(\cos qR + \frac{\Lambda}{q}\sin qR\right)$$

$$- \frac{\alpha\Lambda^2(\Lambda^2-\mu^2)e^{-\Lambda R}}{(q^2+\Lambda^2)^2}\left(\cos qR + \frac{\Lambda^2-q^2}{2\Lambda q}\sin qR\right). \tag{5.4}$$

By comparing eqs.(5.4) and (5.2) we can see that the presence of the core suppresses the relative effect of the form factor by an approximate factor $\xi = \exp\left[-(\Lambda - \mu)R\right] \times (1+ \Lambda R/2)/(1+ \mu R)$. For example, with $\mu = 135$ MeV, $R = 0.7$ fm and $\Lambda = 1400$ MeV it is $\xi = 0.026$, which becomes $\xi = 0.12$ with $\Lambda = 900$ MeV. The more extended, effective radius used in Section 3, $R = 1.1$ fm, would reduce the above ξ values to 0.002 and 0.028, respectively. One can compare the resulting curves in Fig. 4.

Hence we deduce that in the framework of our model only a small fraction of what seems to be a vertex function comes from the form factor.

6. - SUMMARY AND CONCLUSIONS

In this paper we have suggested the use of the no-exchange model, which assumes that pions are not being exchanged up to some distance R, in order to explain the OPE - δ procedure.

In the framework of this model one obtains analytical formulae which are simple to use [eqs.(3.3), (5.4)]. A more elaborate version of this model presented in Section 4, is the distorted wave Born approximation, with distorted waves from a hard core. As we have indicated in Section 5 the core effects overshadow the form factor effects. Using the techniques mentioned above we have demonstrated that the real part of the ϕ_2 amplitude above T_{lab}= 400 MeV is represented quite accurately by the OPE contribution distorted by a core effect. Below 400 MeV the OPE contribution also seems to dominate - even at $T_{lab} \cong$ 100 MeV.

It still remains unclear why the real part of the ϕ_2 amplitude is almost purely OPE (distorted by a core effect). As we have indicated in Section 2, above T_{lab}= 400 MeV there is no evidence for isoscalar meson exchanges. The isovector (apart from the pion) contribution also seems to be very small. The nature of the cancellations is not clear. Part of the answer may be the factor exp(- μR), indicated in Section 3, which suppresses the contributions of the heavier meson exchanges.

As was indicated in Section 5, a similar factor exp(- ΛR) seems to be also responsible for the suppression of the form factor effects - making it an almost impossible task to extract the πNN form factor from N-N scattering experiments.

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FIGURE CAPTIONS

Fig. 1 : The normalized ϕ_2/α amplitude of eq.(2.1) from the phase shift analysis of Ref. 4 for energies 425, 500, 600 and 750 MeV as a function of the momentum transfer $q \equiv \sqrt{-t}$.

Fig. 2 : The same as in Fig. 1 but for energies 100 ÷ 425 MeV.

Fig. 3 : Predictions of the normalized ϕ_2/α amplitude of the no-exchange model (NEM) for R = 1.1 fm and the DWBA from a core of radius R = 0.7 fm (continuous line), compared with the phase shift analysis[8] results. The OPE and OPE - δ contributions are also indicated.

Fig. 4 : The normalized ϕ_2/α amplitude of eq.(5.4) as a function of the momentum transfer q, for T_{lab}= 425 MeV. The continuous line is for R = 1.1 fm without a form factor and with Λ = 1.5 GeV (the two curves coincide on the figure). The double-dot-dashed line corresponds to R = 0.7 fm, Λ = 0.9 GeV and the long dashed-dotted line to R = 0.7 fm, $\Lambda = \infty$. The OPE - δ contribution is also indicated (dashed line).

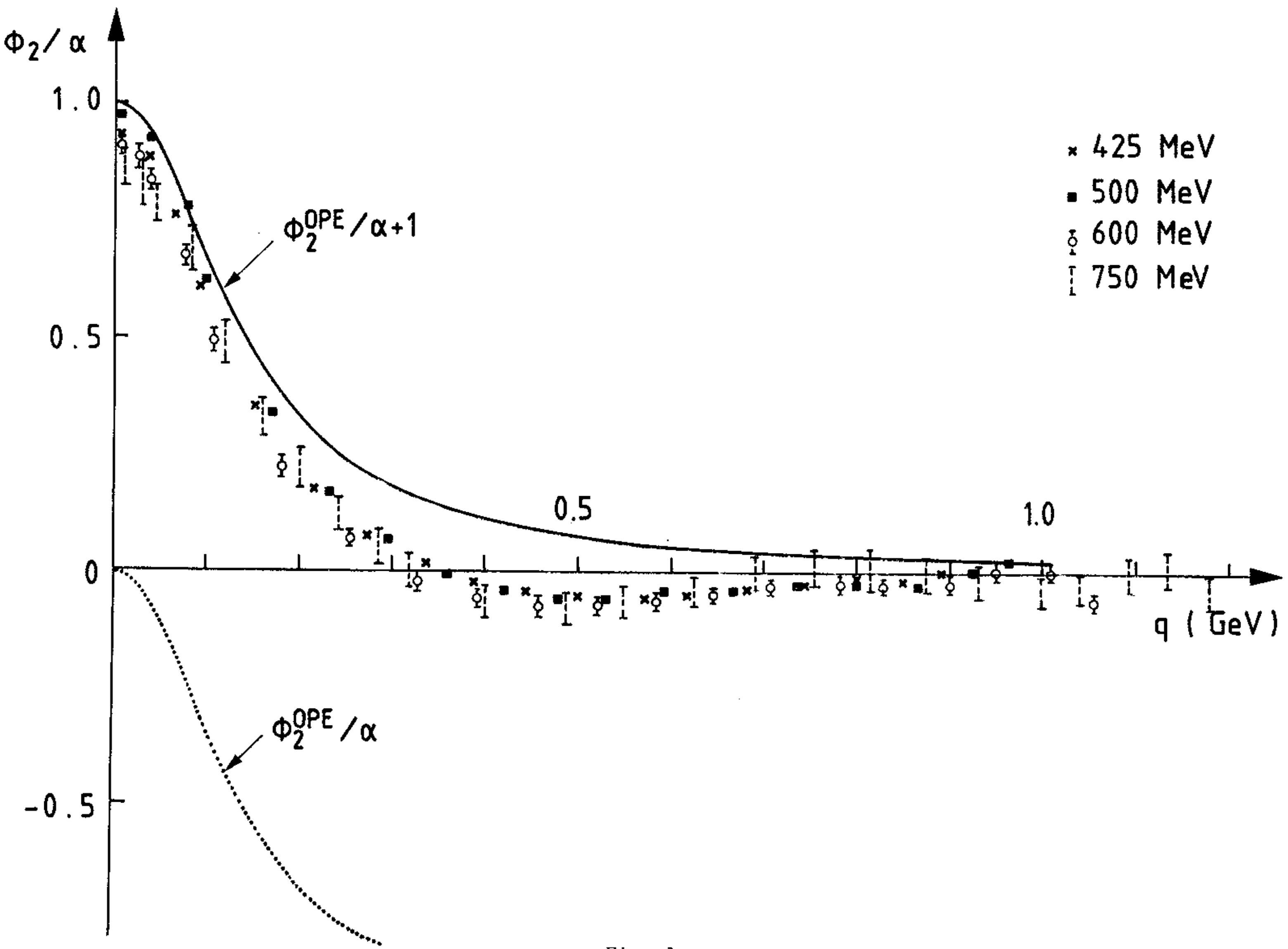

Φ_2/α
1.0
0.5
0
-0.5
$\Phi_2^{OPE}/\alpha+1$
Φ_2^{OPE}/α
× 425 MeV
■ 500 MeV
ǫ 600 MeV
I 750 MeV
0.5
1.0
q (GeV)

Fig. 1

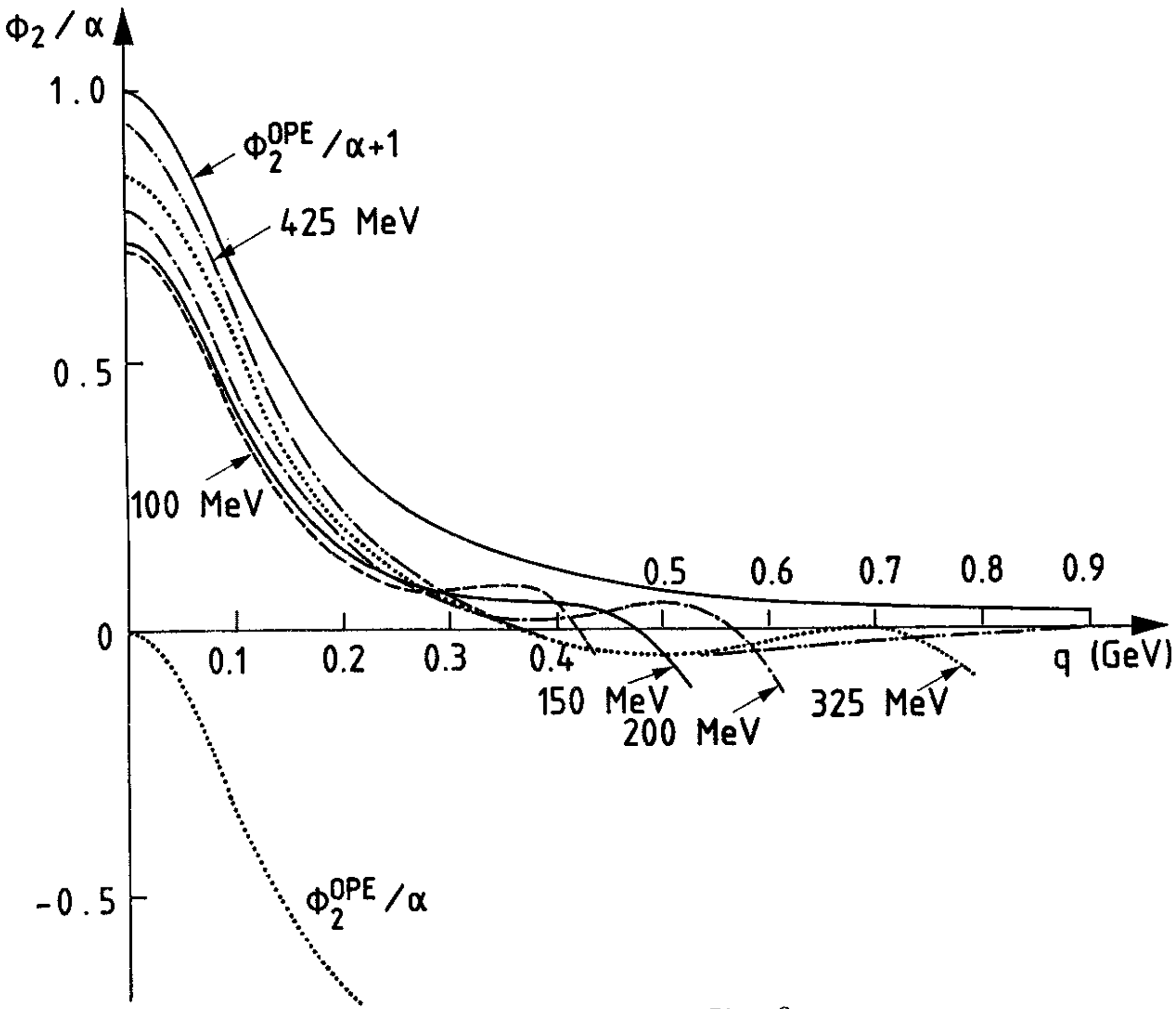
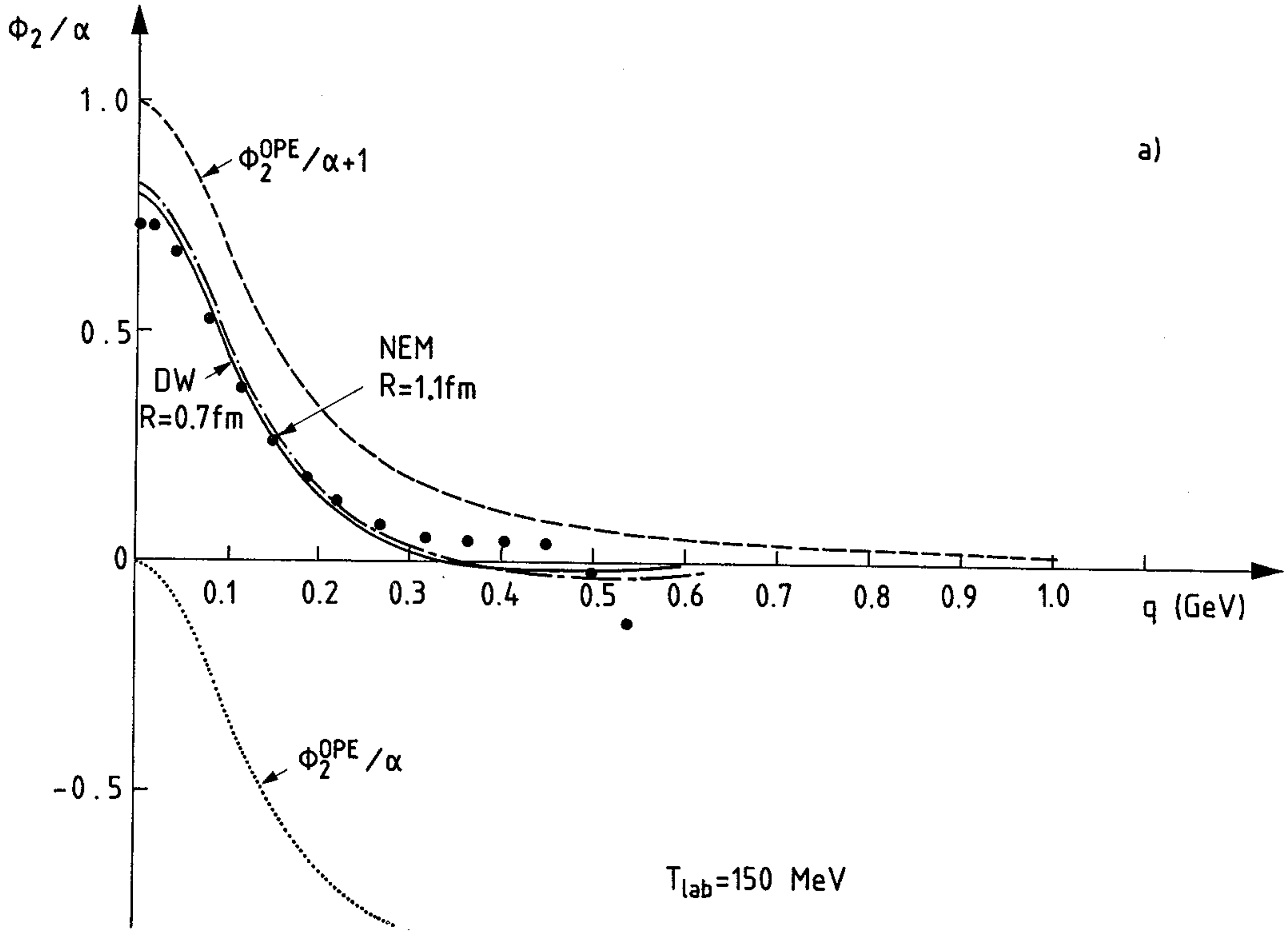
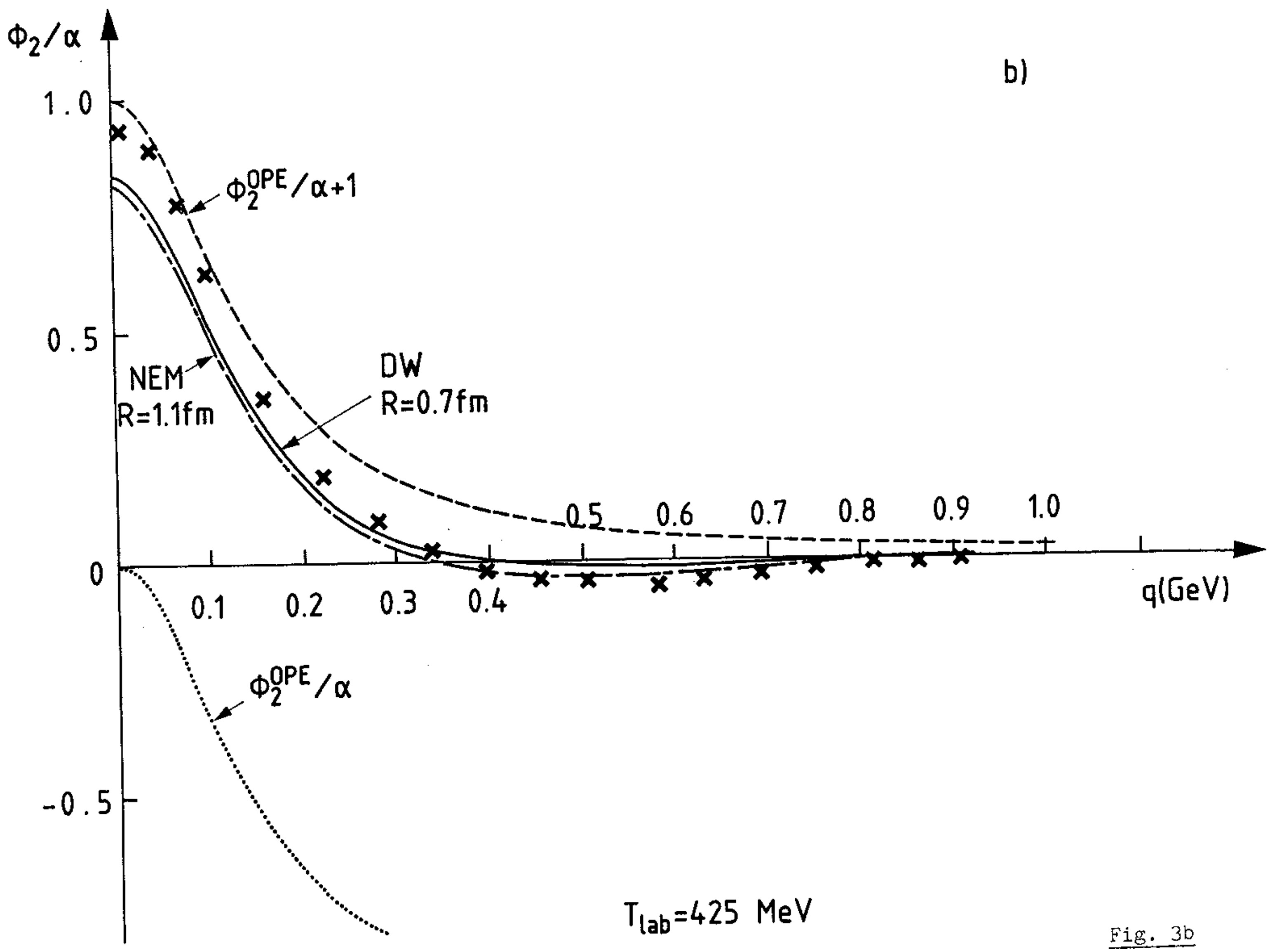

Φ2 / α
1.0
0.5
0
-0.5
Φ2OPE /α+1
425 MeV
100 MeV
150 MeV
200 MeV
325 MeV
Φ2OPE /α
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
q (GeV)

Fig. 2



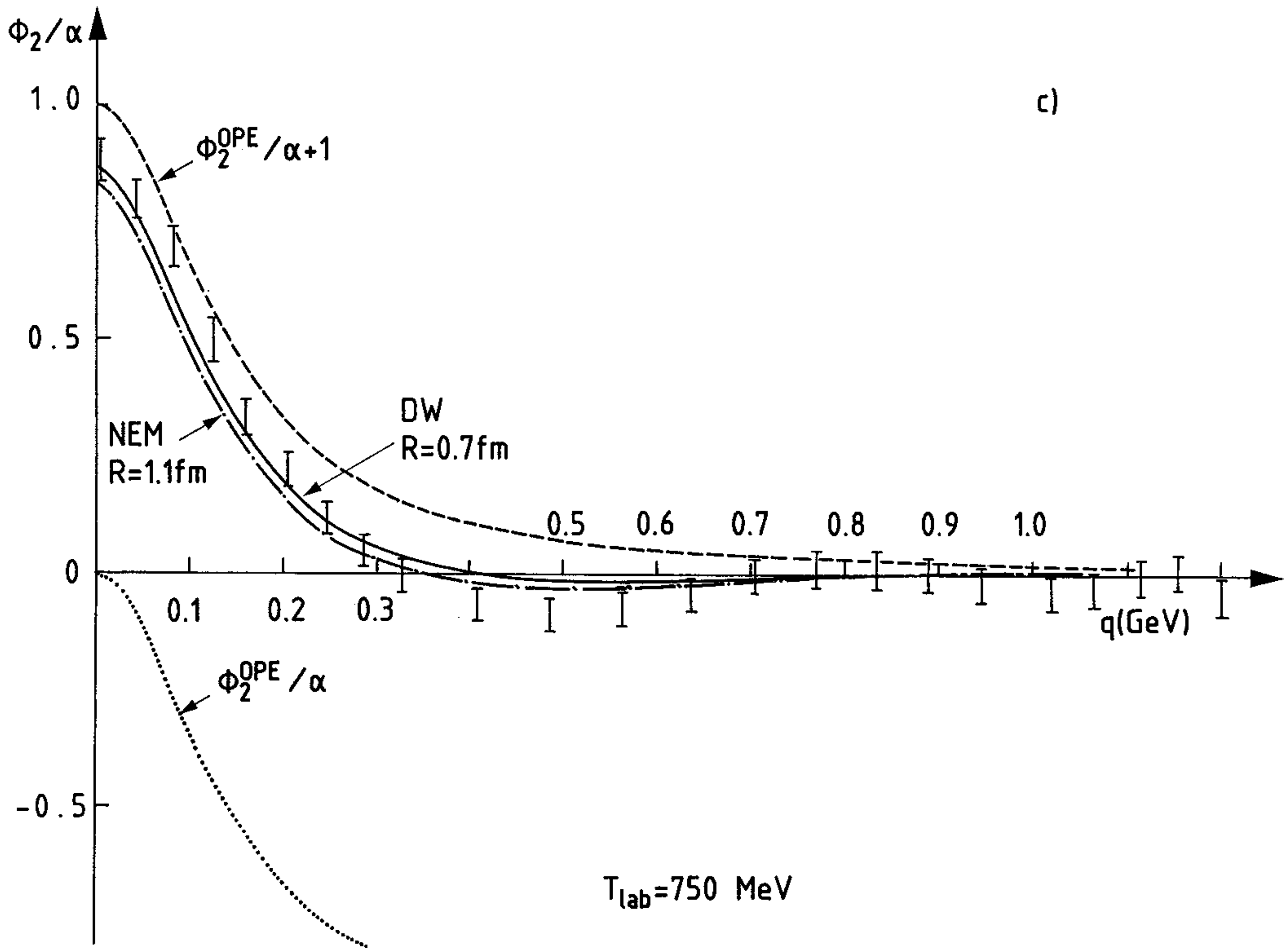
Φ2 / α
1.0
0.5
0
-0.5
Φ2OPE/α+1
a)
NEM
R=1.1fm
DW
R=0.7fm
Φ2OPE/α
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
q (GeV)
Tlab=150 MeV

Fig. 3a



Φ2/α
b)
1.0
0.5
0
-0.5
Φ2OPE/α+1
NEM
R=1.1fm
DW
R=0.7fm
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
q(GeV)
Φ2OPE/α
Tlab=425 MeV

Fig. 3b



Φ2/α
1.0
0.5
0
-0.5
c)
Φ2OPE/α+1
NEM
R=1.1fm
DW
R=0.7fm
0.1
0.2
0.3
0.5
0.6
0.7
0.8
0.9
1.0
q(GeV)
Φ2OPE/α
Tlab=750 MeV

Fig. 3c

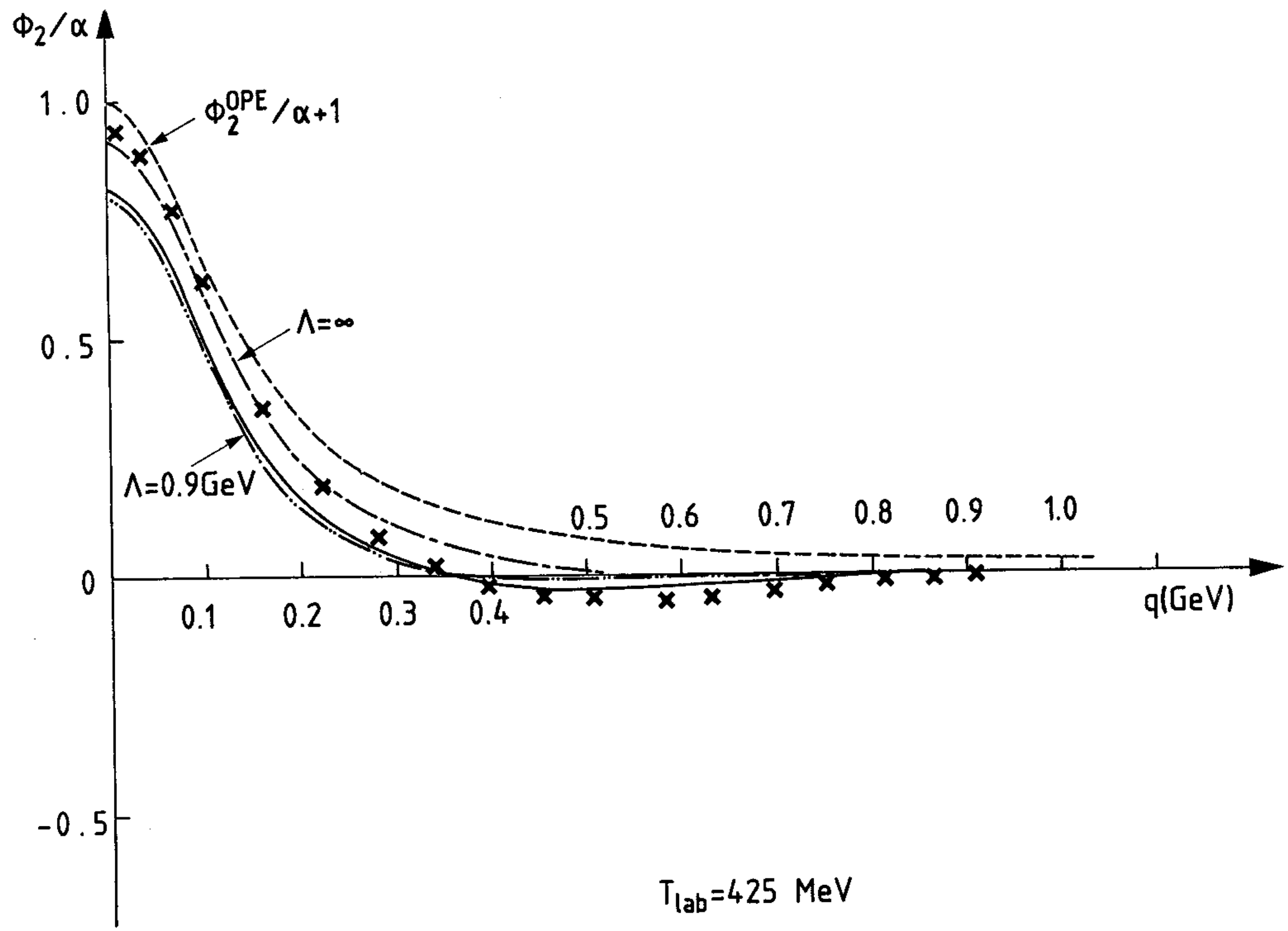

Φ_2/α
1.0
0.5
0
-0.5
$\Phi_2^{OPE}/\alpha+1$
$\Lambda=\infty$
$\Lambda=0.9$GeV
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
q(GeV)
T_{lab}=425 MeV

Fig. 4