Four-Jet Production at the Large Hadron Collider at Next-to-Leading Order in QCD

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We present the cross sections for production of up to four jets at the Large Hadron Collider, at next-to-leading order in the QCD coupling. We use the BLACKHAT library in conjunction with SHERPA and a recently developed algorithm for assembling primitive amplitudes into color-dressed amplitudes. We adopt the cuts used by ATLAS in their study of multijet events in pp collisions at $\sqrt{s} = 7$ TeV. We include estimates of nonperturbative corrections and compare to ATLAS data. We store intermediate results in a framework that allows the inexpensive computation of additional results for different choices of scale or parton distributions.

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Pure-jet events are abundant at the Large Hadron Collider (LHC), providing a window onto new strongly interacting physics [1]. The wealth of data being accumulated by the LHC experiments motivates comparisons with precise theoretical predictions from first principles, based on a perturbative expansion in quantum chromodynamics (QCD) within the QCD-improved parton model. The leading order (LO) contribution in the QCD coupling, α_s, does not suffice for quantitatively precise predictions, which require at least next-to-leading-order (NLO) accuracy in the QCD coupling.

The ATLAS [2] and CMS [3] Collaborations have recently measured multijet cross sections in pp collisions at 7 TeV. In this Letter, we provide NLO QCD predictions for the production of up to four jets and compare them to ATLAS data. Our study agrees with the earlier two- and three-jet studies performed by the ATLAS Collaboration [2] using NLOJET++ [4]; the four-jet computation is new.

NLO QCD predictions of jet production at hadron colliders have a 20-year history, going back to the original computations of single-jet inclusive and two-jet production [5,6]. These were followed by results for three-jet production [4,7]. A long-standing bottleneck to obtaining NLO predictions for a larger number of jets at hadron colliders, the evaluation of the one-loop (virtual) corrections, has been broken by on-shell methods [8–10], whose efficiency scales well as the number of external legs increases. Recent years have witnessed calculations with up to five final-state objects [11], among many other new processes [12–14].

We illustrate the virtual contributions to four-jet production in Fig. 1. To evaluate them, we have made a number of significant improvements to the BLACKHAT package [15]. In particular, assembly of the color-summed cross sections for subprocesses from primitive amplitudes [16] has been automated [17], and the recomputation needed upon detection of numerical instabilities has been reduced [18]. The pure-glue contributions dominate the total cross section yet would be the most complex to compute in a traditional Feynman-diagram approach because of their high tensor rank. We include all subprocesses and the full color dependence in QCD in all terms. We treat the five light-flavor quarks as massless and drop the small (percent-level) effects of top quark loops.

We use AMEGIC++ [19], part of SHERPA [20], to evaluate the remaining NLO ingredients: the real-emission amplitudes and the dipole-subtraction terms used to cancel their infrared divergences [21]. AMEGIC++ was cross-checked with the COMIX package [22]. The phase-space integrator exploits QCD antenna structures [23,24].

We have carried out extensive checks, including numerical stability, independence of the phase-space separation parameter α_{dipole} [4], and cancellation of infrared singularities. Our results for two- and three-jet production agree with those obtained by running NLOJET++ [4] to within 1%. (For this comparison we used the k_T jet algorithm [25] and CTEQ6M partons [26] to match the default choices in NLOJET++.) We have compared the virtual matrix elements

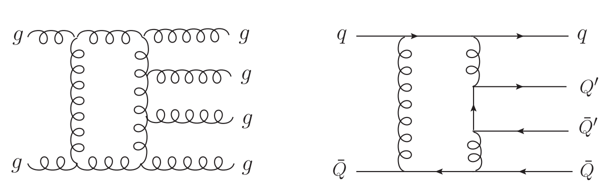

FIG. 1. Sample diagrams for the six-parton one-loop amplitudes for $gg \to gggg$ and $q\bar{Q} \to qQ'\bar{Q}'\bar{Q}$.

for two-, four-, and six-quark processes at selected points in phase space to HELAC-NLO [27]; they agree to 10 digits. In Supplemental Material [28], we provide reference numerical values of the virtual matrix elements at a specific phase-space point.

In the fixed-order perturbative expansion of any observable, it is important to assess whether large logarithms of ratios of physical scales arise in special kinematic regions. Dijet production, in particular, suffers from a well-known instability at NLO [29]. If identical cuts on the transverse momentum p_T of the two jets are used, then soft-gluon radiation is severely restricted when the leading jet is just above the minimum p_T, while the virtual corrections are unaffected. This leads to a large logarithm and a divergence of the NLO corrections at the minimum p_T. Instead of resumming the logarithms [30], we follow ATLAS's approach [2] of imposing asymmetric cuts, with the minimum p_T of the leading jet larger than that for additional jets. Large logarithms are then mitigated at the price of increased scale dependence for the two-jet prediction: By p_T conservation, the lowest p_T bins for the first two jets can be populated only if there is additional real radiation, and the NLO two-jet prediction effectively becomes LO there. The production of three or more jets, and particularly the new NLO prediction for four-jet production, do not suffer from this problem.

In addition to fixed-order parton-level LO and NLO results, we also present results for a parton-shower calculation matched to fixed-order LO matrix elements (ME + PS) [31]. We obtained the latter results by using a RIVET [32] analysis within the SHERPA framework. We also use SHERPA to estimate nonperturbative correction factors which we then apply to our NLO results. These correction factors are obtained by comparing parton-level results, after showering, to fully hadronized predictions including a simulation of the underlying event. We use two different hadronization models: cluster fragmentation as implemented by SHERPA [20] and string fragmentation using the algorithm in PYTHIA 6.4 [33].

We consider the inclusive production of up to four jets in pp collisions at a center-of-mass energy $\sqrt{s} = 7$ TeV. Jets are defined by using the infrared-safe anti-k_T algorithm [34]. We parallel ATLAS in presenting results for jet-size parameters $R = 0.4$ and $R = 0.6$. We order the jets in p_T. We implement the ATLAS cuts from Ref. [2]; we require all jets to have $p_T^{\rm jet} > 60$ GeV and the leading jet to have $p_T^{\rm jet} > 80$ GeV. Observed jets are also required to have rapidity $|y| < 2.8$. We use the MSTW2008 LO and NLO parton distribution functions (PDFs) [35] at the respective orders. We use a five-flavor running $\alpha_s(\mu)$ and the value of $\alpha_s(M_Z)$ supplied with the parton distribution functions.

We present our predictions for the LO, ME + PS, and NLO parton-level inclusive cross sections for two- through four-jet production in Table I. The strong sensitivity of LO cross sections and distributions to the variation of the unphysical renormalization scale μ_R and factorization scale μ_F is significantly reduced at NLO. The wide range of scales probed in distributions requires us to use an event-by-event scale characteristic of the kinematics. We choose $\mu_R = \mu_F \equiv \mu = \hat{H}_T/2$ as our central scale [13,14], where $\hat{H}_T \equiv \sum_i p_T^i$ and the sum runs over all final-state partons i. We use a standard procedure to assess scale dependence, varying the central scale up and down by a factor of 2 to construct scale-dependence bands as in Ref. [11]. The central scale $\mu = \hat{H}_T/2$ is a characteristic measure of the momentum transfers in the event. It is approximately the jet p_T in the two-jet case and rises somewhat in the three- and four-jet cases. Although it was not tuned in any way, for three and four jets it happens to lie near the maximum of the NLO prediction as a function of scale, causing the scale-dependence bands to be largely to the low side of the central value. The lowest value in the band comes from lowering μ to the lower end of its range, $\hat{H}_T/4$. (We have not varied the scale in the ME + PS calculation, as its choice is linked to the tuning of various parameters in the parton shower and hadronization model. Error sets for these parameters are not available.)

In the penultimate column of Table I, we give the nonperturbative underlying event and hadronization (NP) correction factor using the PYTHIA-type string fragmentation model. The cluster fragmentation model gives essentially identical results, within our integration uncertainties, so we do not quote them. We use this factor as an estimate for the NP correction to the NLO cross section as well, shown with

TABLE I. Total cross sections in nanobarns for jet production at the LHC at $\sqrt{s} = 7$ TeV, using the anti-k_T jet algorithm with $R = 0.4$. We compare ATLAS results against LO, ME + PS, and NLO theoretical predictions. The penultimate column gives nonperturbative corrections estimated by using a string fragmentation model. In all cases, numerical-integration uncertainties are given in parentheses. The scale dependence shown with LO and NLO predictions is given as superscripts and subscripts. The three uncertainties shown with the ATLAS data are statistical, jet-energy scale, and detector unfolding; in addition, there is a $\pm 3.4\%$ luminosity uncertainty. The jet-energy scale uncertainties are asymmetric, so they are given as subscripts and superscripts.

No. jets	ATLAS	LO	ME + PS	NLO	NP factor	NLO + NP
≥ 2	$620 \pm 1.3^{+110}_{-66} \pm 24$	$958(1)^{+316}_{-221}$	559(5)	$1193(3)^{+130}_{-135}$	0.95(0.02)	$1130(19)^{+124}_{-129}$
≥ 3	$43 \pm 0.13^{+12}_{-6.2} \pm 1.7$	$93.4(0.1)^{+50.4}_{-30.3}$	39.7(0.9)	$54.5(0.5)^{+2.2}_{-19.9}$	0.92(0.04)	$50.2(2.1)^{+2.0}_{-18.3}$
≥ 4	$4.3 \pm 0.04^{+1.4}_{-0.79} \pm 0.24$	$9.98(0.01)^{+7.40}_{-3.95}$	3.97(0.08)	$5.54(0.12)^{+0.08}_{-2.44}$	0.92(0.05)	$5.11(0.29)^{+0.08}_{-2.32}$

TABLE II. The LO, ME + PS, and NLO predictions for the distribution $d\sigma/dp_{T,4}$ [pb/GeV] in the transverse momentum of the fourth jet, $p_{T,4}$, for $R = 0.4$, compared to ATLAS data. The penultimate column gives the nonperturbative correction factor using the string model. The final column gives the NLO prediction including this factor.

p_T	ATLAS	LO	ME + PS	NLO	NP factor	NLO + NP
60–80	$170 \pm 1.8^{+61}_{-33} \pm 12$	$399(1)^{+295}_{-157}$	157(4)	$219(6)^{+4}_{-100}$	0.92(0.06)	$202(14)^{+4}_{-93}$
80–110	$24 \pm 0.56^{+5}_{-3.8} \pm 2.3$	$57.6(0.1)^{+42}_{-23}$	23.7(0.7)	$32.6(0.8)^{+0.3}_{-12.9}$	0.93(0.05)	$30.3(1.9)^{+0.3}_{-12.0}$
110–160	$2.6 \pm 0.15^{+0.79}_{-0.47} \pm 0.31$	$5.25(0.01)^{+3.9}_{-2.1}$	2.28(0.08)	$3.3(0.1)^{+0.0}_{-0.9}$	0.89(0.06)	$2.9(0.2)^{+0.0}_{-0.9}$
160–210	$0.15 \pm 0.035^{+0.047}_{-0.034} \pm 0.026$	$0.395(0.001)^{+0.29}_{-0.16}$	0.18(0.01)	$0.24(0.01)^{+0.0}_{-0.06}$	0.93(0.08)	$0.22(0.02)^{+0.0}_{-0.06}$

the correction in the last column. (As NLO parton-shower programs are developed beyond the dijet case [36], it will become possible to carry out estimates of nonperturbative corrections in a manner more compatible with NLO calculations.) These nonperturbative corrections are of the order of 10% or less for the production of four or fewer jets. For dijet production, the LO and NLO theory predictions are not in good agreement with the data; as discussed above, this is not surprising given the kinematic constraints as well as the soft-radiation instability. In contrast, for the three- and four-jet cases, both the NLO and ME + PS predictions agree with the data, within the experimental uncertainties, whether or not we account for the small nonperturbative corrections.

Ratios of cross sections typically reduce both theoretical and experimental uncertainties. In particular, we have compared the ratio of four- to three-jet cross sections appearing in Table I to the value obtained by ATLAS:

$$\text{ATLAS}: 0.098 \pm 0.001^{+0.004}_{-0.005} \pm 0.005,$$
$$\text{ME + PS}: 0.100(0.003), \qquad \text{NLO}: 0.102(0.002),$$

where the quoted ATLAS uncertainties are, respectively, statistical, jet-energy scale, and detector unfolding [2]. We display only the statistical integration errors for the theoretical predictions; in the ratio, the (correlated) scale dependence cancels and is not a useful estimate of uncertainty. We have not included the nonperturbative corrections; they also largely cancel in jet ratios. We estimate the residual theoretical uncertainty by comparing ME + PS and NLO results; from here we deduce that the residual theoretical uncertainty is under 5%. This is within our numerical-integration uncertainty and also smaller than the experimental uncertainty.

In Table II, we present the LO, ME + PS, and NLO p_T distribution of the fourth-leading jet, comparing to ATLAS data [2]. The penultimate column gives the nonperturbative correction factor, estimated by using SHERPA, as discussed above. The final column displays the NLO results including this factor. From this table we see that both ME + PS and NLO results are in good agreement with the data, within uncertainties. The estimated nonperturbative corrections are smaller than current experimental uncertainties.

We also consider the $(n + 1)/n$ jet-production ratios $[d\sigma^{n+1}/dp_T]/[d\sigma^n/dp_T]$ as a function of the leading-jet p_T. Figure 2 displays the 3/2 and 4/3 jet-production ratios for $R = 0.6$, comparing the 3/2 ratio with ATLAS data. For the 3/2 ratio, we find very good agreement between NLO theory and the ATLAS data [2], except for the first bin, where the denominator is affected by the kinematic constraint and soft-radiation instability mentioned earlier. The agreement remains good even with increasing leading-jet p_T, where the ratios grow to 0.6 and 0.35 for the 3/2 and 4/3 ratios, respectively. The ME + PS prediction is also in very good agreement with data and consistent with NLO, implying that these processes are under good theoretical control. It will be interesting to compare our theoretical predictions for the 4/3 ratio to future LHC data.

We have estimated the PDF uncertainty by using the 100-element NNPDF 2.1 error sets, the MSTW2008 68% error sets, and the CT10 90% C.L. sets. With MSTW2008, we find one-sigma uncertainties of 1.2% for two-jet production, 1.6% for three-jet production, and 2.5% for four-jet production. The NNPDF 2.1 and MSTW2008 central values agree to well within these values, and the NNPDF 2.1 one-sigma uncertainties are comparable. The CT10 PDF uncertainty estimate is about 25% greater than for MSTW2008. However, the CT10 central value for three-jet production is 5.8% low, outside combined two-sigma errors. At high p_T,

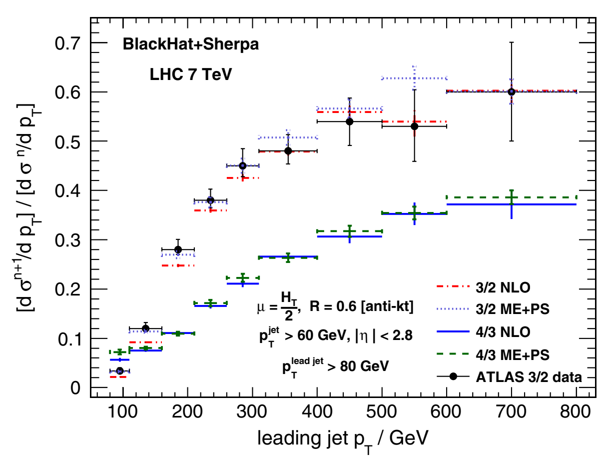

FIG. 2 (color online). A comparison of the 3/2 and 4/3 jet-production ratios to ATLAS data [2] for $R = 0.6$. We show the NLO and ME + PS predictions for these ratios. Vertical bars on the theory predictions represent Monte Carlo statistical uncertainties.

the uncertainty grows somewhat but remains smaller or comparable to our numerical-integration errors.

We have studied the dependence of the jet cross sections on the jet-size parameter R for anti-k_T. LO multijet cross sections always decrease with increasing R, because whenever two partons are merged the event is lost. At NLO, the R dependence is a dynamical question. We find that the NLO three-jet cross section increases with R for our usual range of scale variation. Whether the four-jet cross section increases or decreases with R is sensitive to the choice of scale.

For each event we generate, we record the squared matrix element, the momenta of all partons, and the coefficients of various functions that control the dependence of the final result on the renormalization and factorization scales, as well as on the PDFs. We store this information in ROOT-format n-tuple files [37]. The availability of these intermediate results in a standard format makes it computationally inexpensive to evaluate cross sections and distributions for different scales and PDF error sets. They also offer an easy and reliable way of furnishing our theoretical predictions to experimental collaborations while allowing them to modify cuts or compute additional distributions [38].

In this study of pure-jet processes, we have imposed cuts typical of standard-model measurements at the LHC. The same tools used here can also be used to study backgrounds to new physics signals, such as those arising from colored resonances or higher-dimension effective operators. The improved efficiencies developed in the course of our study should allow us to continue increasing the number of jets accessible to NLO predictions.

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