



β -beating in the effective model of the LHC using PTC

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Summary

An effective model of the LHC optics has been developed based on measurements of magnetic field, alignment errors and closed orbit. This model utilizes the Polymorphic Tracking Code, with MAD-X as a front-end, to allow the inclusion of harmonics to arbitrary order in thick lattice elements. β -beating calculations have been performed with this model at injection optics and at 3.5 TeV with squeezed optics ($\beta^*=3.5$ m at the interaction point) The model predictions are in good agreement with the measurements performed in the 2010 LHC commissioning run.

1 The effective model: introduction

The high intensity and energy of the LHC proton beams requires accurate control of the transverse beam dynamics in order to guarantee machine protection. This imposes tight tolerances on the optics errors. Geometric and magnetic field errors affect the optics and have to be corrected for safe operation.

An effective model of the optics of the LHC has been built based on measurements of the alignment and magnetic errors. The aim is to have a realistic model including the maximum knowledge available from the machine at present. This model has been performed using MAD-X [1] and the Polymorphic Tracking Code (PTC) [2] which allows treatment of the magnetic errors up to an arbitrary order in the thick elements lattice. After introducing these errors, correction of the beam orbits, tunes, chromaticities and transverse coupling are performed to retrieve the nominal settings of the machine. After all the corrections, the β -beating obtained as a result of the remaining errors is calculated along the machine. The β -beating is defined as the relative variation of the modeled (or measured) β -function with respect to the nominal value, *i.e.*, $\Delta\beta/\beta = (\beta_{Model} - \beta_{Nom})/\beta_{Nom}$. The results of the model at injection optics (450 GeV) and at 3.5 TeV with squeezed optics ($\beta^*=3.5$ m at the interaction point) are presented in this paper, and are compared to the measured β -beating after the correction of local errors performed during the 2010 run [3].

2 Alignment and magnetic field errors

The magnet errors introduced in the effective model come from the simulation tool Windows Interface to Simulation Errors (WISE) [4, 5]. WISE is a generator of geometric and magnetic errors based on measurements of the LHC magnets.

Magnetic field measurements with a low excitation current, the so-called “warm” measurements, are carried out in the industry for all magnets. Once the superconducting magnets are installed at CERN, measurements of a fraction of them are performed under operational conditions, the so-called “cold” measurements. For magnets for which “cold” measurements are not performed, warm-to-cold correlations are introduced in the modeling of the field. In addition to the uncertainties from the warm-to-cold correlation for the fraction of magnets not measured at “cold”, other uncertainties are included in WISE for all magnets, as for example relative and absolute measurement errors or hysteresis and power supply accuracy [5, 6]. With all the available information, WISE produces a number of instances of the most likely LHC machine. 60 different ‘seeds’ are available containing harmonics until 15th order for bending and quadrupole magnets. It is worth noting that the uncertainty from the warm-to-cold correlation for the quadrupolar (b_2) component is not negligible in the case of the quadrupole magnets.

Alignment errors of the magnets are also generated by WISE based on measurements of the mechanical and magnetic axis. Transverse and longitudinal measurements of the assemblies and the magnets in the assemblies are available [5, 7].

The errors introduced in the simulations presented in this paper were generated using the magnetic measurements and the magnets sequence in the machine after the repair performed following the September 2008 incident. The tables corresponding to the magnetic errors at injection and at 3.5 TeV date from 15th May 2011 and the 30th July 2010 respectively, and the tables corresponding to the alignment errors date from 18th February 2010.

3 MAD-X and PTC models comparison

In the modeling of the optics the PTC code has been used as an extension to the MAD-X program¹. In MAD-X only normal quadrupolar and sextupolar errors in the bending magnets can be assigned into the thick elements. All other components (normal and skew) must be assigned to thin elements. PTC is a code dedicated to beam dynamics calculations in the nonlinear regime. Magnetic errors of any arbitrary order (normal and skew) can be assigned to thick elements.

Figure 1 shows a comparison of the β -beating modeled along the machine, starting from the Interaction Point (IP) 3, at injection optics with MAD-X and PTC. Quadrupolar field errors from WISE were introduced in the bending magnets of the thick elements lattice for the LHC Beam 1, together with the correction of such errors with the trim quadrupoles. Both models are in perfect agreement.

¹The PTC code is embedded into the MAD-X code.

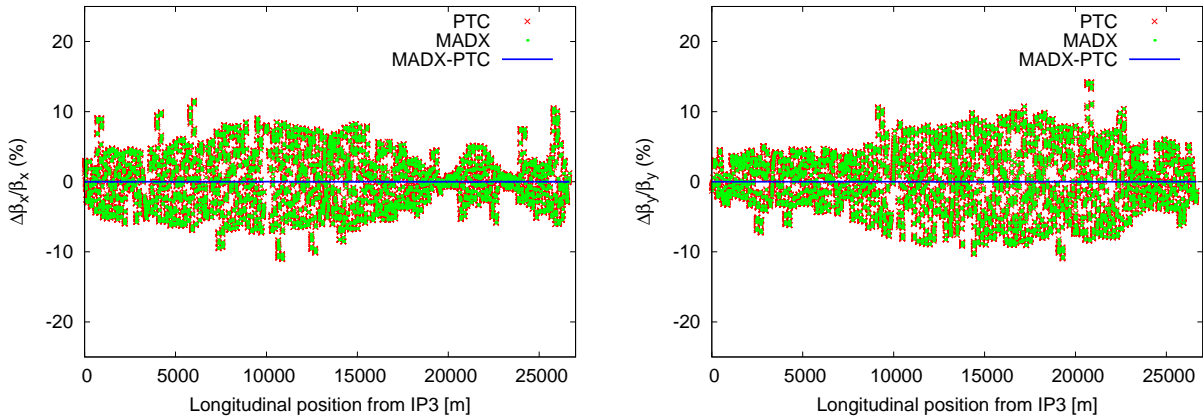


Figure 1: Comparison of the modeled β -beating with MAD-X and PTC when introducing quadrupolar field errors from WISE (seed 1) in the bending magnets as thick elements for the LHC Beam 1 at injection optics in the horizontal (left) and vertical (right) planes. MADX-PTC represents the difference between the predictions of these two models.

4 Thick elements model including different magnet errors

Different models of the LHC optics have been performed with PTC introducing progressively different magnet errors in order to evaluate the effect on the β -beating of those magnetic or alignment errors. It should be noted that when introducing any error, the corresponding correction is also included in the model. Quadrupolar errors are corrected with the trim quadrupoles. Sextupolar, octupolar and decapolar spool pieces are included to correct higher order multipoles. The coupling is corrected with the skew quadrupoles, and the tunes and chromaticities are matched to the nominal values for all the cases. When introducing the alignment errors the orbit is also corrected with the corresponding orbit correctors. The modeling has been performed for the injection optics and at 3.5 TeV for the squeezed optics.

The biggest effect on the β -beating comes from the quadrupolar field errors. The effect on the optics at injection when introducing these errors in the bending magnets, and correcting them, is of the order of 7 or 10%, as can be seen in Fig. 1 and in Table 1. This table shows the peak and standard deviation β -beating along the machine for the different models studied for the LHC Beam 1 corresponding to seed 1 from WISE. The contribution of the quadrupolar errors in the quadrupole magnets is of the same order of magnitude as in the bending magnets (see Table 1), but in the former, as mentioned previously, the uncertainty from the warm-to-cold correlation is non-negligible and the shape of the β -beating along the longitudinal axis changes significantly for the 60 different seeds of the most likely LHC machine created by WISE.

The contribution to the β -beating when introducing all the harmonics (skew and normal components), from 2nd to 15th order, in all bending and quadrupole magnets (and the corresponding correction to the errors), compared with the case where only quadrupolar errors were introduced, is very small. Figure 2 shows the difference in β -beating of these two cases. The maximum difference along the machine is about 1%. It indicates that the feed-down

Table 1: Summary of the peak and standard deviation (std) β -beating and peak dispersion deviation with respect to the nominal value around the machine for the different models: introducing b_2 errors in the bending magnets (MBs), adding b_2 errors in the quadrupole magnets (MQs), introducing all the harmonics in the MBs and MQs, adding to them the alignment errors and correcting the orbit to zero or to the measured one. The results correspond to the LHC Beam 1 seed 1 from WISE, and in two of the models the std and peaks of the averaged values over the 60 seeds are also shown. The results are representative of the LHC Beam 2.

Model	$\Delta\beta_x/\beta_x$ (%)		$\Delta\beta_y/\beta_y$ (%)		ΔD_x (cm)	ΔD_y (cm)
	std	peak	std	peak	peak	peak
Injection optics						
b_2 in MBs	4.0	11.6	4.7	14.2	22.1	0.0
b_2 in MBs & MQs	7.5	-16.8	5.0	19.8	-21.5	0.0
All harmonics	7.4	-16.8	5.0	19.7	-44.1	-26.1
+Misalign, zero orbit (averaged over 60 seeds)	8.1 (6.2)	17.5 (16.9)	5.4 (4.3)	20.3 (14.3)	-41.9 (29.4)	-28.3 (24.2)
+Misalign, meas orbit	8.2	17.4	5.8	21.4	-58.9	-33.3
3.5 TeV 3.5 m β^* optics						
All harmonics	6.9	18.1	3.9	13.2	-35.7	-21.1
+Misalign, zero orbit (averaged over 60 seeds)	6.7 (6.2)	18.4 (16.0)	5.0 (3.2)	15.1 (12.1)	-38.7 (-33.9)	-24.1 (-26.7)

multipoles are negligible as the closed orbit distortion is small.

Going one step further in the modeling of the β -beating, the alignment errors have also been added together with all the magnetic errors. The orbit has been corrected to zero and the corresponding corrections of the magnetic errors have been performed. The resulting β -beating for injection optics is shown in Fig. 3. The effect of introducing the alignment errors compared to the case where only the magnetic errors were introduced is small, but not completely negligible, as can be seen in Fig. 4 and in Table 1. The effect of the alignment errors has also been modeled at 3.5 TeV for the squeezed optics. The difference in β -beating with and without alignment errors for this case is shown in Fig. 5. The effect of the alignment errors after corrections is slightly bigger for the 3.5 TeV squeezed optics case, as expected, due to the stronger sextupoles, but it remains at about 4%.

The impact on the β -beating of the closed orbit has also been studied. For this purpose, the optics has been modeled including all the magnetic and alignment errors as in the previous case, but correcting the orbit to the measured one during the LHC run instead of correcting it to zero. The difference in β -beating between these cases is very small, as shown in Fig. 6. The impact of the closed orbit is almost negligible. The maximum error when correcting to zero instead of to the measured orbit is about 1 or 2%.

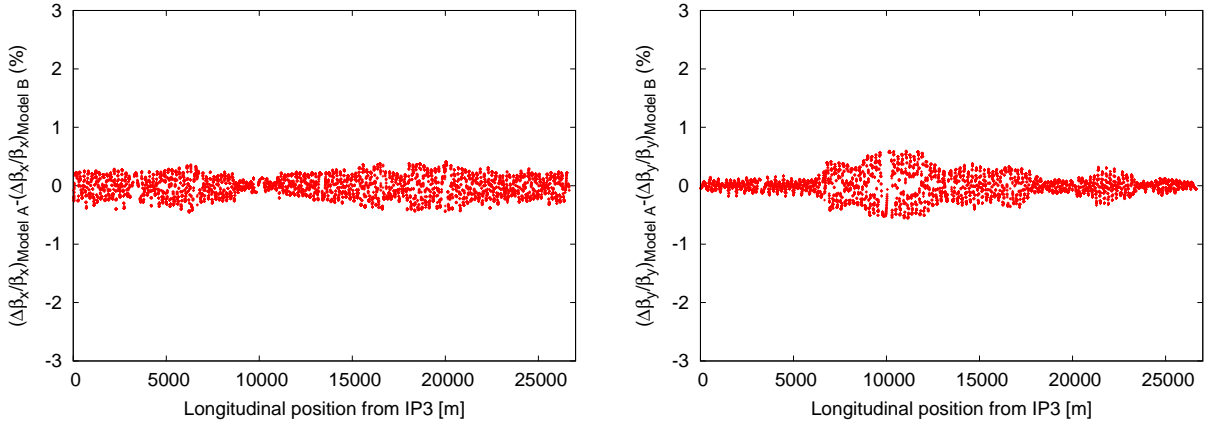


Figure 2: Difference in β -beating between introducing all multipoles and introducing only the quadrupolar errors from WISE (seed 1) in the LHC Beam 1 at injection optics, in the horizontal (left) and vertical (right) planes.

5 Correction of the local optics errors

First optics measurements in 2010 revealed unexpectedly large errors in the beta functions up to 60% in Beam 2 [8, 3]. This is considerably larger than the model predictions shown above. It indicates that there are some magnets whose tolerances are considerably out of the specifications. By applying the segment-by-segment technique [9, 8] it was found that a few magnets were responsible for a significant fraction of the measured β -beating. Namely these magnet types were the warm quadrupoles (MQW) in the collimation sections and the triplet quadrupoles (MQX) in the Interaction Regions (IRs). After the local correction suggested by the segment-by-segment technique the β -beating was reduced to levels between 20% and 30%, closer to the estimates from the magnetic model. Furthermore, later magnetic measurements of MQW-type magnets, confirmed the existence of significant errors in the calibration curves of these magnets [3]. It has not been possible to perform magnetic measurements of MQX-type magnets a posteriori. The calibration errors of these magnets can be regarded as outliers in the distribution of magnetic errors. Global corrections using a large number of quadrupoles were applied to further reduce the β -beating. These global corrections partially compensate the errors estimated from the magnetic measurements which are included in the model. For this reason, the effective model that has been built should not be compared with measurements performed after the global corrections. In the following the LHC model is compared to the optics measurements after the local corrections (to exclude the outliers) but prior to the global corrections.

6 Effective model *vs* measured β -beating and dispersion

The effective model has been studied for all the different instances of magnetic errors generated by WISE (60 seeds) at injection and at 3.5 TeV squeezed to 3.5 m β^* . The 60 seeds

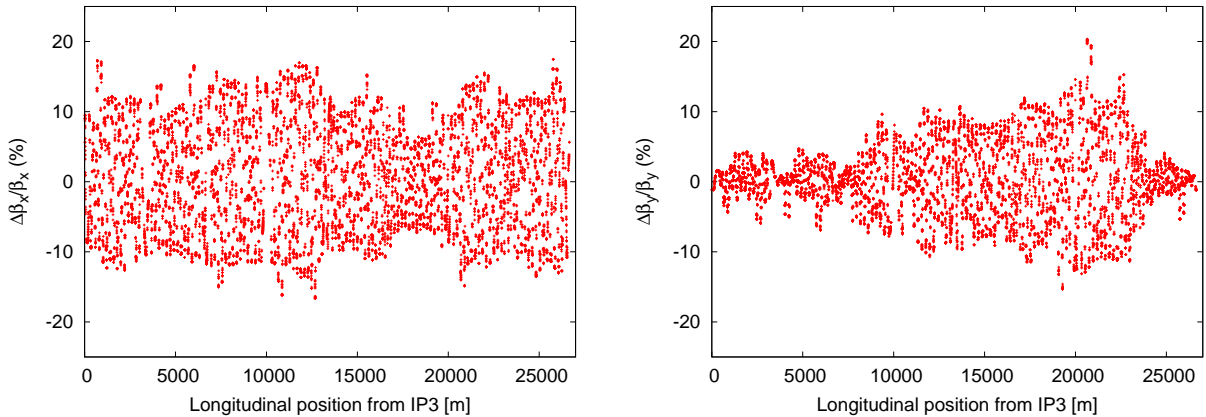


Figure 3: Modeled β -beating when introducing the alignment errors and all the multipoles from WISE (seed 1) in the LHC Beam 1 at injection optics, in the horizontal (left) and vertical (right) planes.

corresponding to the errors at injection include the statistical error generation component for both quadrupolar and higher order errors. In the case of 3.5 TeV, the 60 seeds include the statistical error generation in the quadrupolar errors, but not in the higher order components, for which all the seeds include the same instance of the machine. This does not affect the results as the effect of the higher order multipoles is negligible after the correction of orbit, tune, chromaticity and coupling. The alignment errors have also been added and the orbit has been corrected to zero. For each seed the quadrupolar errors in the bending magnets have been corrected with the trim quadrupoles, the coupling has been corrected and the tunes and chromaticities have been matched to the nominal values. The average over the 60 seeds of the β -beating obtained at each location along the machine are shown in Figs. 7 and 8 for Beam 1 at injection and 3.5 TeV squeezed to 3.5 m β^* respectively. The model is compared to the corresponding measurements of the β -beating at injection and 3.5 TeV after the correction of the local errors [8, 3]. The β -beating along the machine is smaller at 3.5 TeV compared to the case at injection. This is a result of the reduction of the optics errors during the ramp due to the lower persistent current effects in the superconducting magnets and the lower remnant magnetization in the normal conducting ones. It has been observed that the β -beating at injection, measured in the same conditions, fluctuates by 8% between periods of a few months [3], while the variations at 3.5 TeV are about a factor of two smaller.

The averaged β -beating over the 60 seeds as modeled for Beam 2, together with the measurements performed after the correction of the local errors, are shown in Figs. 9 and 10 at injection and 3.5 TeV squeezed to 3.5 m β^* respectively.

The β -beating obtained from the effective model is in good agreement, between the errors, to the measured data after correcting the local errors. From this model, which includes the best knowledge that we can have at present from the machine, one can conclude that the effect on the β -beating of the residual errors after all corrections is of the same order of magnitude as the measured effect. The results indicate that some localized errors may still be corrected at injection optics in some IRs. At 3.5 TeV there is a good agreement between

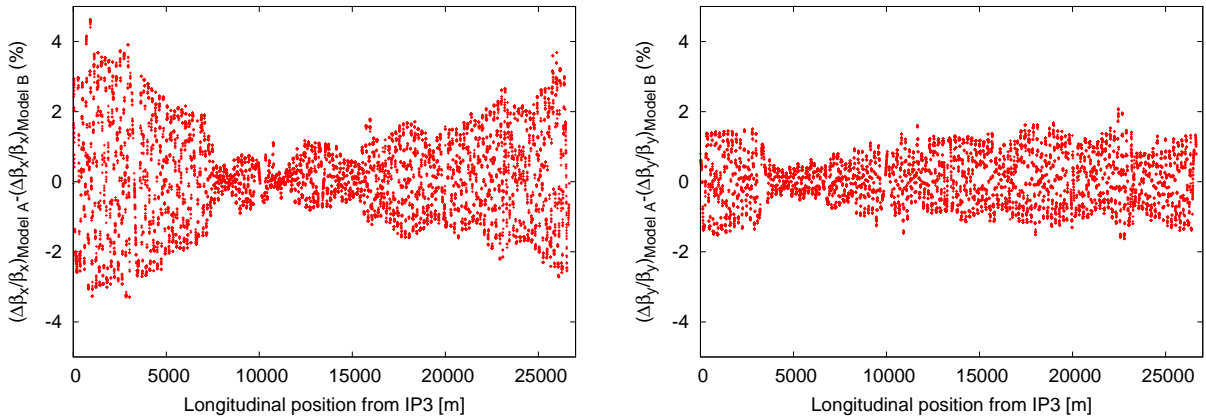


Figure 4: Difference in β -beating between introducing all the magnetic and alignment errors and introducing only the magnetic errors (corresponding to the seed 1 from WISE), in the LHC Beam 1 at injection optics, in the horizontal (left) and vertical (right) planes.

the model and the measurements, except for Beam 2 in the horizontal plane where some errors in IP8 could still remain. In general, this indicates that no significant errors, as for example cable swaps in strong magnets, remain in the machine after correction of main errors at the beginning of the commissioning and after the local error correction was performed in the collimation sections and the IRs.

The dispersion has been calculated around the machine for the different models of the LHC optics performed with PTC introducing progressively different magnet errors. Table 1 shows the peak deviation of the horizontal and vertical modeled dispersions with respect to the nominal values for LHC Beam 1. When only quadrupolar errors are introduced the vertical dispersion remains zero as expected, and when introducing skew components and misalignments the deviation from nominal values increases, but remains smaller than in the horizontal plane. This is in accordance with the measured dispersions during the 2010 run.

The dispersions in the complete model have been obtained for the 60 seeds from WISE, after introducing all the magnetic and alignment errors and their corresponding corrections, and correcting the orbit to zero. The averaged dispersions over the 60 seeds have been calculated around the machine, and the deviations with respect to the nominal values are shown in Figs. 11 and 12 for Beam 1 at injection and 3.5 TeV squeezed to 3.5 m β^* respectively. The model is compared to the corresponding deviation with respect to the nominal values of the measured dispersions during the 2010 run. The deviation of the dispersion at injection optics and at 3.5 TeV squeezed optics to 3.5 m β^* are comparable.

7 Summary and conclusion

An effective model of the LHC optics has been built based on the best knowledge presently available concerning the alignment and magnetic field errors along the machine. These errors are generated by WISE, based on measurements of the LHC magnets. The model has been built with the MAD-X code together with PTC to allow the treatment of the magnetic errors up to an arbitrary order in the thick elements lattice. After introducing the errors, the

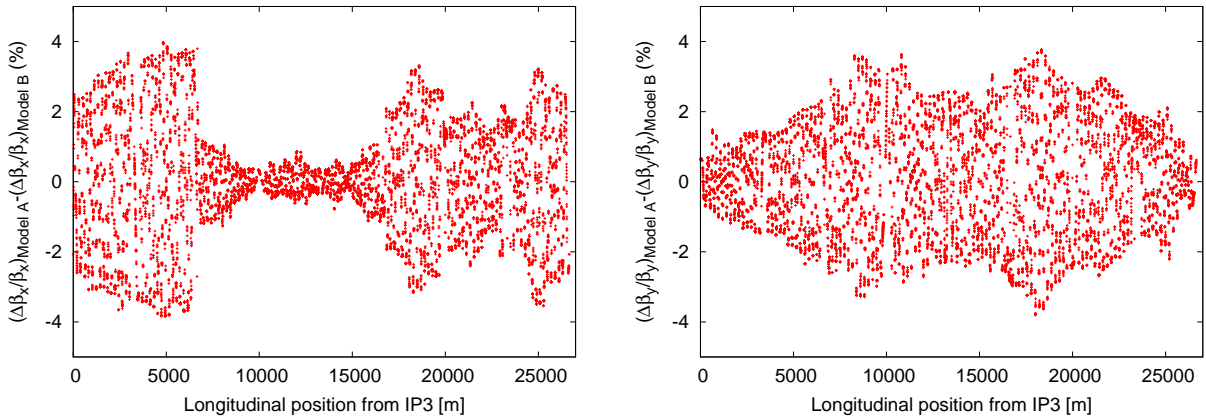


Figure 5: Difference in β -beating between introducing all the magnetic and alignment errors and introducing only the magnetic errors (corresponding to the seed 1 from WISE), in the LHC Beam 1 at 3.5 TeV squeezed to 3.5 m β^* , in the horizontal (left) and vertical (right) planes.

corresponding corrections are performed. The orbit is corrected to zero, quadrupolar errors in the bending magnets are corrected with the trim quadrupoles, the coupling is corrected and the tunes and chromaticities are matched to the nominal values.

The main effect on the β -beating arises from the quadrupolar errors in the bending and quadrupole magnets. There is a non negligible uncertainty of this component in the quadrupoles due to the high warm-to-cold correlation uncertainty for magnets that were not measured under operational conditions. Higher order multipoles have an almost negligible contribution. The feed-down multipoles are very small due to the very small closed orbit errors. A small effect on the β -beating arises when the alignment errors are included and the orbit is corrected to zero, *i.e.*, about 4%, and the effect of the closed orbit is almost negligible, about 1 or 2%.

In order to study the effect of the remaining errors after all the corrections, β -beating calculations have been performed with the model at injection optics and at 3.5 TeV squeezed to 3.5 m β^* . The results have been compared to the measurements of the β -beating performed after the correction of local errors during the 2010 run and show a good agreement, especially at 3.5 TeV, which indicates that no significant localized errors remain in the machine. Some local errors may still be corrected in certain IRs, mainly at injection optics.

The dispersion has also been modeled and compared with the measurements performed during the 2010 run. Larger deviations with respect to the nominal values are observed in the horizontal plane than in the vertical one.

The effective model described in this note is the first model capable of carrying through the high degree of precision of the measured magnetic and alignment errors into the beam optics calculation, prior efforts being inherently limited by the necessity of using a thin lens approximation for high order magnetic terms. Utilizing this high degree of precision it has been possible to quantify the contribution to the β -beating arising from high order magnetic errors, and the magnet misalignments in the presence of small closed orbit distortions. With such prior knowledge it is now possible to conclude that it is safe to use a simpler model

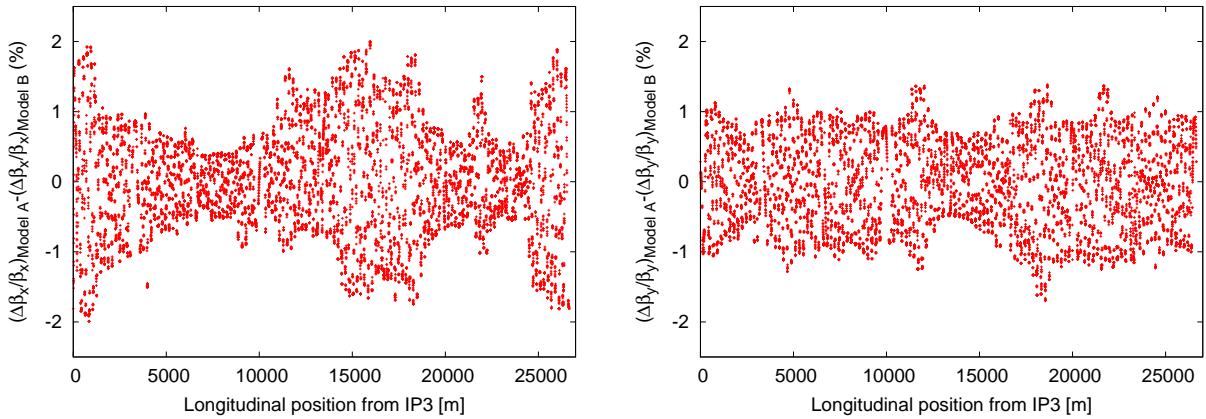


Figure 6: Difference in β -beating between correcting the orbit to the measured one and correcting to zero, for the LHC Beam 1 at injection optics, in the horizontal (left) and vertical (right) planes.

without the need of the PTC code in similar conditions. Nevertheless, under other conditions such as large orbit errors or large off-momentum deviations, the MAD-X and PTC model predictions may present discrepancies.

The effective model is being implemented into the LHC online model for general use² [10].

²Implementation into the LHC online model performed by G. Müller and K. Fuchsberger.

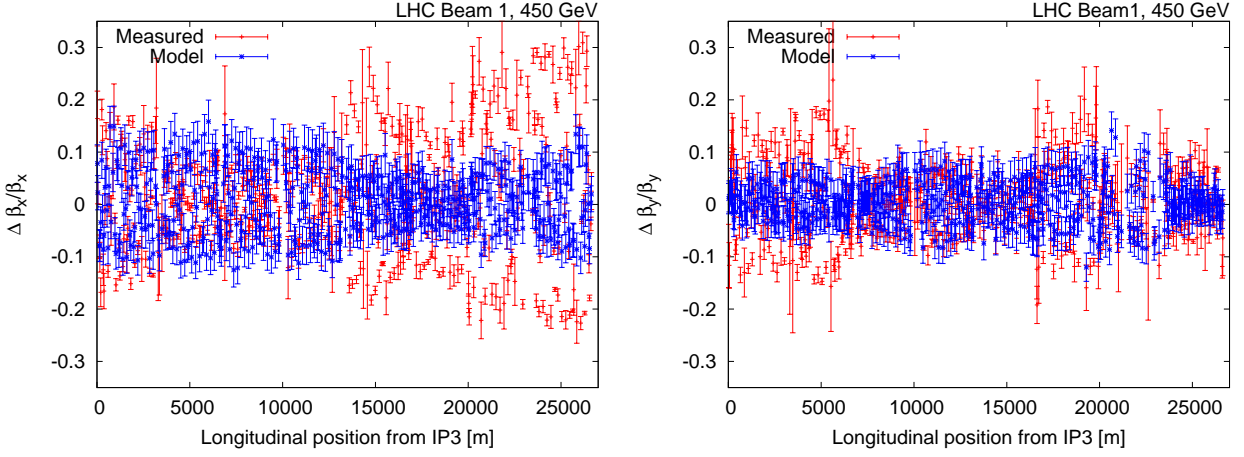


Figure 7: Modeled β -beating (averaged over the 60 seeds) when introducing the alignment errors and all the multipoles from WISE in the LHC Beam 1 at injection optics in the horizontal (left) and vertical (right) planes, together with the measurements performed after the correction of the local errors on 12th August 2010.

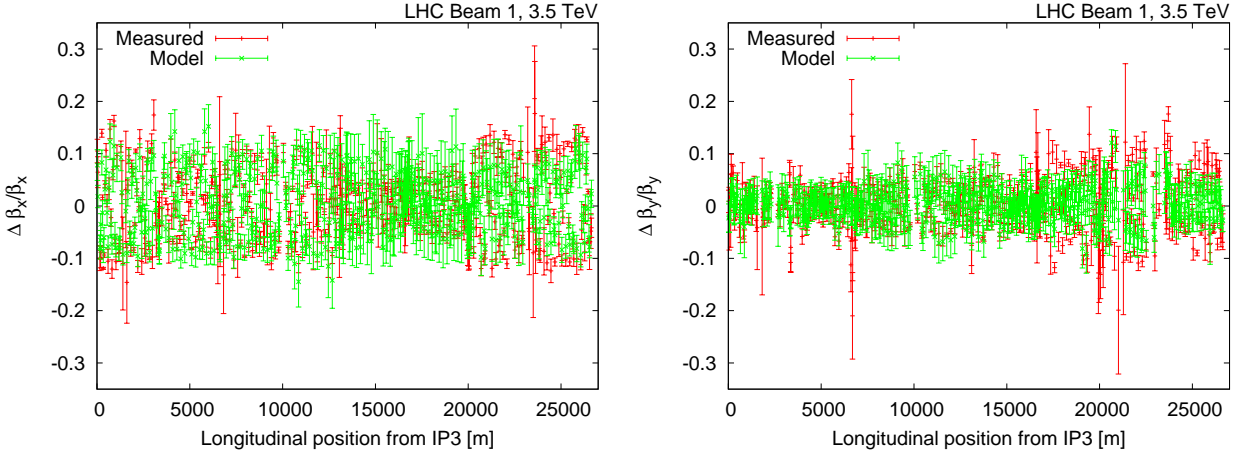


Figure 8: Modeled β -beating (averaged over the 60 seeds) when introducing the alignment errors and all the multipoles from WISE in the LHC Beam 1 at 3.5 TeV squeezed optics to 3.5 m β^* in the horizontal (left) and vertical (right) planes, together with the measurements performed after the correction of the local errors on 4th September 2010.

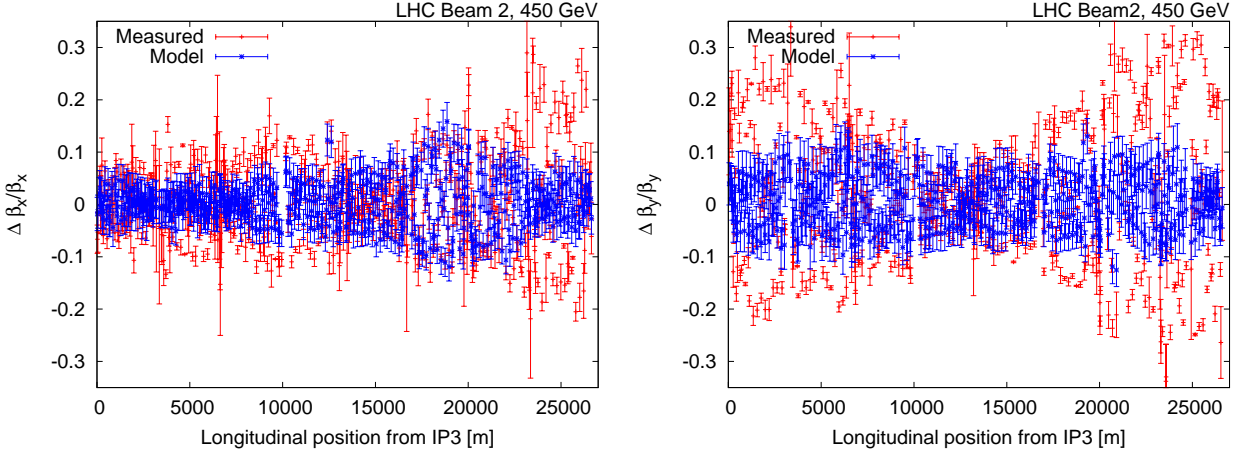


Figure 9: Modeled β -beating (averaged over the 60 seeds) when introducing the alignment errors and all the multipoles from WISE in the LHC Beam 2 at injection optics in the horizontal (left) and vertical (right) planes, together with the measurements performed after the correction of the local errors on 19th August 2010.

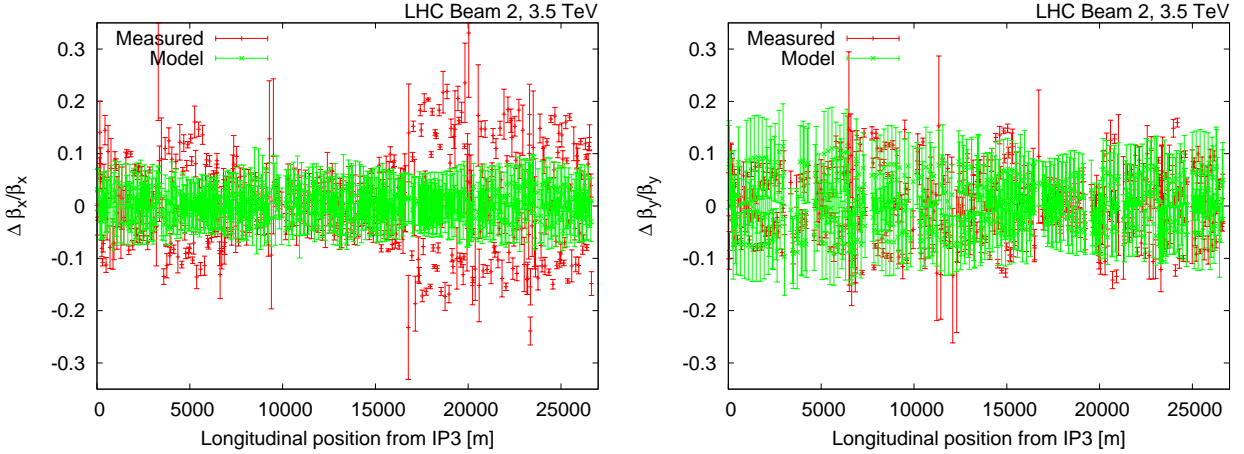


Figure 10: Modeled β -beating (averaged over the 60 seeds) when introducing the alignment errors and all the multipoles from WISE in the LHC Beam 2 at 3.5 TeV squeezed optics to 3.5 m β^* in the horizontal (left) and vertical (right) planes, together with the measurements performed after the correction of the local errors on 4th September 2010.

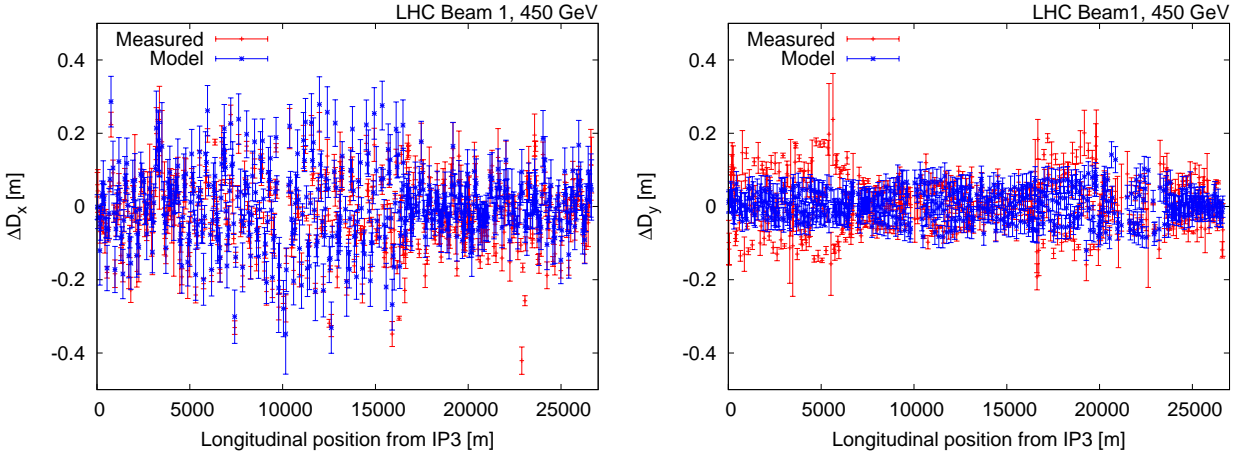


Figure 11: Deviation with respect to the nominal values of the modeled dispersions around the machine (averaged over the 60 seeds) when introducing the alignment errors and all the multipoles from WISE in the LHC Beam 1 at injection optics in the horizontal (left) and vertical (right) planes, together with the measurements performed after the correction of the local errors on 19th June 2010.

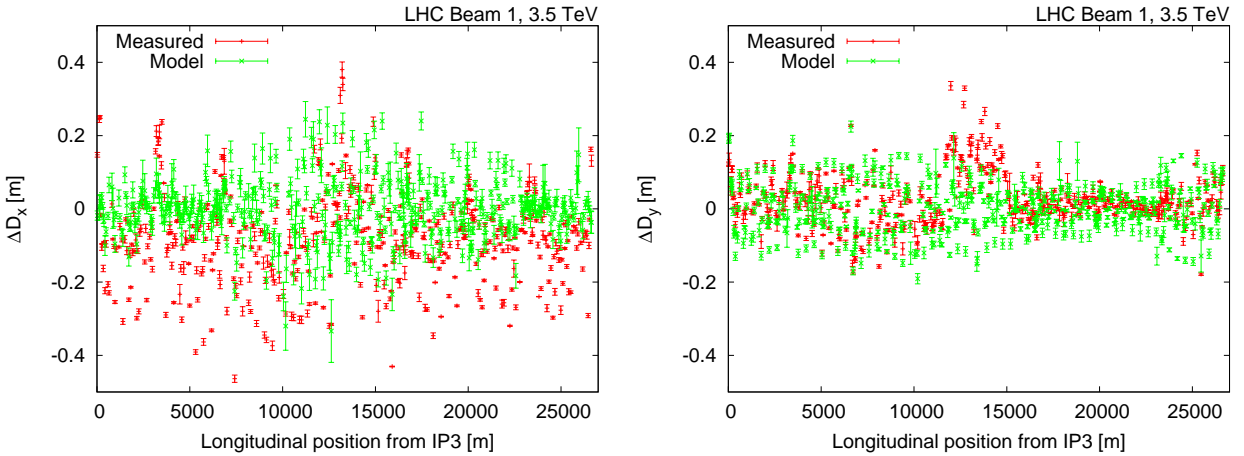


Figure 12: Deviation with respect to the nominal values of the modeled dispersions around the machine (averaged over the 60 seeds) when introducing the alignment errors and all the multipoles from WISE in the LHC Beam 1 at 3.5 TeV squeezed optics to 3.5 m β^* in the horizontal (left) and vertical (right) planes, together with the measurements performed after the correction of the local errors on 6th September 2010.

8 Acknowledgments

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