ATLAS NOTE

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A combined measurement of the top quark pair production cross-section using dilepton and single-lepton final states

The ATLAS collaboration

Abstract

We present a combined measurement of the top quark pair production cross-section in pp collisions at $\sqrt{s} = 7$ TeV using the results of dilepton and single-lepton analyses with 35 pb^{-1} of data. The result is $\sigma_{t\bar{t}} = 180 \pm 9$ (stat.) $\pm$ 15 (syst.) $\pm$ 6 (lumi.) pb, which is in excellent agreement with the Standard Model prediction.

1 Introduction

The measurement of the top-quark pair-production cross-section, $\sigma_{t\bar{t}}$, is one of the key milestones for the early LHC physics program. A precise measurement of $\sigma_{t\bar{t}}$ allows precision tests of perturbative QCD, where uncertainties on $\sigma_{t\bar{t}}$ are now at the level of 10% [1]. In addition, $t\bar{t}$ production is an important background to the search for the Higgs boson and various searches for physics beyond the Standard Model. New physics may also give rise to additional $t\bar{t}$ production mechanisms or modification of the top quark decay channels.

Within the Standard Model, top quarks are predicted to decay to a W boson and a b-quark nearly 100% of the time, and the decay topologies are determined by the decays of the W bosons. The single-lepton mode, with a branching ratio of 34.4%, and the dilepton mode, with a branching ratio of 6.5% (both including small contributions from taus decaying to electrons and muons), give rise to final states with one or two leptons, missing transverse energy and jets, two of which have b-flavour.

With a data sample corresponding to 2.9 pb^{-1} taken with the Large Hadron Collider at a centre of mass energy of $\sqrt{s} = 7$ TeV, and using both the single-lepton and the dilepton decay channels, ATLAS measured the top quark production cross-section to be $\sigma_{t\bar{t}} = 145 \pm 31^{+42}_{-27}$ pb [2], which is in good agreement with the theoretical prediction $\sigma_{t\bar{t}} = 165^{+11}_{-16}$ pb, assuming a top mass of 172.5 GeV [3].

This note presents a combined measurement of the $t\bar{t}$ production in the single-lepton and dilepton channels using 35 pb^{-1} of data taken with a centre-of-mass energy of $\sqrt{s} = 7$ TeV in 2010. This luminosity estimate has an uncertainty of 3.4% [4]. The measurements in the three dilepton channels ($ee, e\mu$, and $\mu\mu$) are based on simple cut-based analyses of events with at least two jets [5], while the single-lepton channel measurements (e+jets and μ+jets) are based on a multivariate discriminant distribution in $3, 4$, and ≥ 5 jet bins using b-tagging [6]. The results of the five cross-section measurements from each individual channel, as well as the dilepton and single-lepton combinations, are shown in Table 1. Even though the dilepton analysis does not require b-tagging, the combination with the single-lepton channel assumes the branching ratio of $t \to Wb$ is 100%.

The two sets of analyses share some common sources of systematic uncertainty, which are treated consistently in order to form a combination. The likelihood function in each channel is a function of the signal cross-section $\sigma_{t\bar{t}}$, the luminosity $\mathcal{L}$, and several nuisance parameters α_j that parametrize the effect of various sources of systematic uncertainty.

While the three dilepton analyses and the two single-lepton analyses were each combined from the complete likelihood functions, the full five-channel combination was implemented with an approximate method, in which the single-lepton likelihood function was approximated by a multivariate Gaussian with covariance given by the Hessian matrix from MINUIT's HESSE algorithm [7]. The complete dilepton likelihood function was used without approximation. To avoid including constraint terms for systematics common to the dilepton and single-lepton channels more than once, constraint terms corresponding to common systematics were removed from the dilepton likelihood when forming the five-channel combined likelihood.

Each measurement was based on the profile likelihood ratio

$$\lambda(\sigma_{t\bar{t}}) = \frac{L(\sigma_{t\bar{t}}, \hat{\hat{\mathcal{L}}}, \hat{\hat{\alpha}}_j)}{L(\hat{\sigma}_{t\bar{t}}, \hat{\mathcal{L}}, \hat{\alpha}_j)} \tag{1}$$

where $\hat{\sigma}_{t\bar{t}}, \hat{\mathcal{L}}, \hat{\alpha}_j$ denote the maximum likelihood estimate of all the parameters and $\hat{\hat{\mathcal{L}}}$ and $\hat{\hat{\alpha}}_j$ represent the conditional maximum likelihood estimates of $\mathcal{L}$ and α_j holding $\sigma_{t\bar{t}}$ fixed. The best fit value of the cross-section is simply $\hat{\sigma}_{t\bar{t}}$ and the 68% confidence interval is derived from the values of $\sigma_{t\bar{t}}$ which give $-2\ln\lambda(\sigma_{t\bar{t}}) = 1$.

2 The dilepton combined likelihood function

The likelihood functions for each of the dilepton channels have a similar form, with a single Poisson term for the number of observed events with ≥ 2 jets and several Gaussian constraint terms for the nuisance parameters α_j. The combined likelihood L_{ll} is given by the product of the Poisson terms and a product of the constraint terms

$$L_{ll}(\sigma_{t\bar{t}}, \mathcal{L}, \alpha_j) = Gaus(\mathcal{L}_0 | \mathcal{L}, \sigma_{\mathcal{L}}) \prod_{i \in \{ee, \mu\mu, e\mu\}} Pois(N_i^{obs} \,|\, N_{i,tot}^{exp}(\alpha_j)) \prod_{j \in syst} Gaus(0 \,|\, \alpha_j, 1)\,, \qquad (2)$$

where *Pois* is the Poisson distribution, *Gaus* is the Gaussian distribution, $\mathcal{L}_0$ and $\sigma_{\mathcal{L}}$ are the measured luminosity estimate and its error, and the constraint terms on common systematics are only included once. The variation in the expected number of events from the signal and each background process was estimated from dedicated studies of each of the systematic effects. The total number of expected events $N_{i,tot}^{exp}(\alpha_j)$ is then parametrized via piece-wise linear interpolation in the nuisance parameters α_j associated with each source of systematic uncertainty using the RooFit/RooStats software package [8, 9]. The profile likelihood ratios for the individual channels as well as the dilepton combination are shown in Fig. 1. The dominant systematic uncertainties for the dilepton analysis are the jet energy scale, the theoretical uncertainty on the Z cross section, the fake lepton estimates and the parton showering model. Because the dilepton analyses do not use b-tagging, they are not sensitive to the associated systematic uncertainties [5].

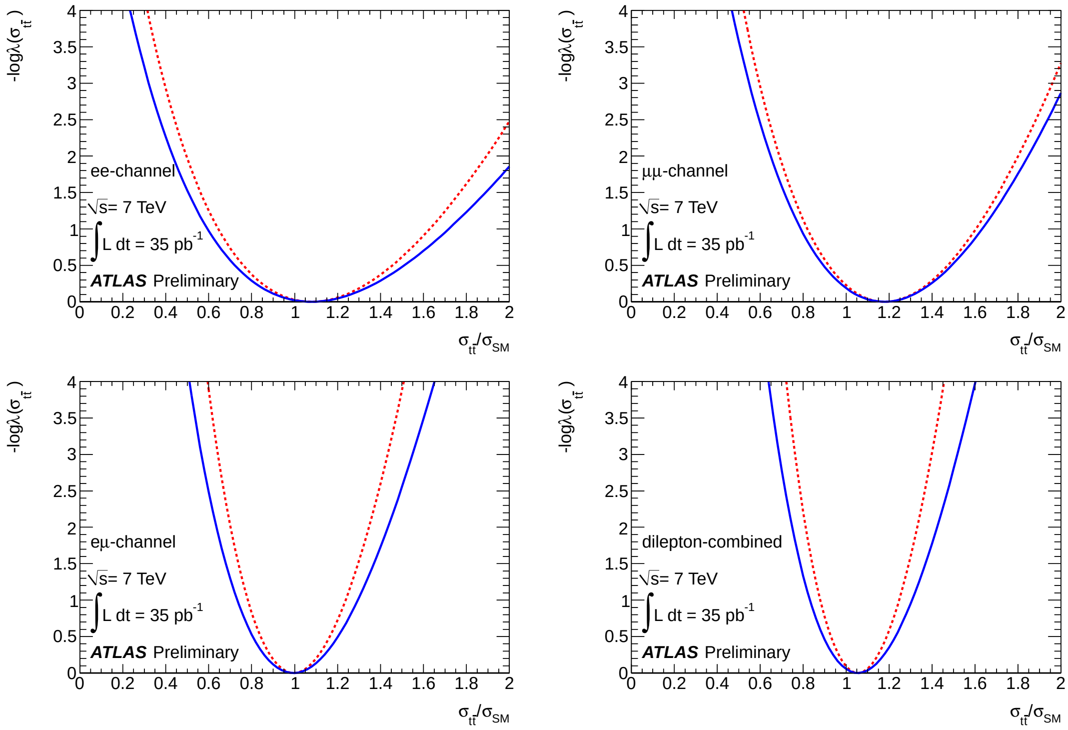

Figure 1: Plots of $-\log\lambda(\sigma_{t\bar{t}})$ as a function of $\sigma_{t\bar{t}}/\sigma_{\rm SM}$ with (blue, solid) and without (red, dashed) systematic uncertainties for the ee (top, left), $\mu\mu$ (top, right), $e\mu$ (bottom, left), and three-channel combined fit (bottom, right).

3 Approximating the single-lepton likelihood function

The likelihood function for the single-lepton channels was formed from the e+jets and μ+jets taking into account common systematics. The single-lepton analysis uses continuous b-tagging: one of the input variables to the likelihood discriminant used in the template fit is the light-flavor probability as obtained from the JetProb b-tagging algorithms [10]. The uncertainties in the b-tagging and mistag rates are included as nuisance parameters in the fit [6]. That likelihood function consists of the parameter of interest $\sigma_{t\bar{t}}$ and 37 nuisance parameters $\vec{\alpha}$, which are together denoted $\vec{\theta} = (\sigma_{t\bar{t}}, \vec{\alpha})$.

Let the maximum likelihood estimator of this two-channel single-lepton combination be denoted $\hat{\vec{\theta}}$. For the purposes of the five-channel combination, the likelihood from the single-lepton channels was approximated with a multivariate Gaussian as the original likelihood was implemented in a different software framework. Figure 2 shows a plot of $-\log\lambda(\sigma_{t\bar{t}})$ as a function of $\sigma_{t\bar{t}}/\sigma_{\rm SM}$, where it can be seen that the likelihood is very symmetric and parabolic, indicating that a multivariate Gaussian is a good approximation to the likelihood function. The covariance matrix comes from the Hessian matrix of the negative-log-likelihood function evaluated at the best fit point,

$$V^{-1}_{ij} = -\frac{\partial^2}{\partial\theta_i \partial\theta_j} \ln L(\vec{\theta}) \Big|_{\hat{\vec{\theta}}} . \quad (3)$$

With the covariance matrix, one can construct the multivariate Gaussian

$$L_{l+\rm jets}(\vec{\theta}) = G(\hat{\vec{\theta}}\,|\,\vec{\theta}, V) = \frac{1}{(2\pi)^{k/2}|V|^{1/2}} \exp\left(-\frac{1}{2}(\hat{\vec{\theta}} - \vec{\theta})^T V^{-1} (\hat{\vec{\theta}} - \vec{\theta})\right) , \quad (4)$$

where $k = 38$ is the dimensionality of the parameter space.

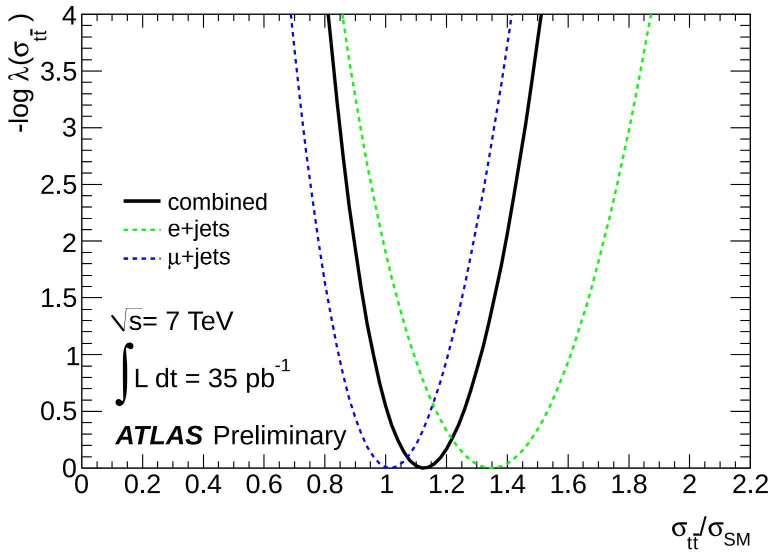

Figure 2: Plot of $-\log\lambda(\sigma_{t\bar{t}})$ as a function of $\sigma_{t\bar{t}}/\sigma_{\rm SM}$ from the full (not approximate) e+jets (green, dashed), μ+jets (blue, dashed), and l+jets combined likelihood (black, solid).

4 The five-channel combined likelihood function

The dilepton and single-lepton channels share several common sources of systematic uncertainty:

- electron energy scale and resolution uncertainties,
- electron identification and trigger efficiency uncertainties,
- muon momentum scale and resolution uncertainties,
- muon identification and trigger efficiency uncertainties,
- dependence of acceptance on Monte Carlo generator as well as initial- and final-state radiation,
- jet energy resolution, jet energy scale, and jet efficiency uncertainties,
- uncertainty due to the effect of pileup,
- cross-section uncertainties for diboson and single top backgrounds.

Because the likelihood function from the single-lepton analysis is approximated by a single multivariate Gaussian, the constraint terms that are common with the dilepton channel must be removed when forming the five-channel combination. Before combining, the dependence of the conditional maximum likelihood estimates $\hat{\hat{\alpha}}_j$ as a function of $\sigma_{t\bar{t}}/\sigma_{\rm SM}$ were compared for the dilepton and single-lepton channels. Those studies did not indicate any unexpected tension in the shared nuisance parameters that would entail combining incompatible results.

The final five-channel likelihood is formed from a product of the approximate single-lepton likelihood $L_{l+\rm jets}$ over the parameter of interest and 37 nuisance parameters (15 of which are shared with the dilepton channels, including a luminosity constraint), the Poisson terms corresponding to the cut-based analyses for the dileptons (which depend on the parameter of interest), and Gaussian constraints for the remaining 21 nuisance parameters that only affect the dilepton channels. In total, there are 59 parameters in the five-channel combined fit.

$$L_{5chan}(\sigma_{t\bar{t}}, \mathcal{L}, \alpha_j) = L_{l+\rm jets}(\sigma_{t\bar{t}}, \mathcal{L}, \alpha_j)\times Gaus(\mathcal{L}_0|\mathcal{L}, \sigma_{\mathcal{L}}) \prod_{i\in\{ee,\mu\mu,e\mu\}} Pois(N_i^{obs}|N_{i,tot}^{exp}) \prod_{j\in ll \text{ only syst}} Gaus(0|\alpha_j, 1)\,. \tag{5}$$

5 Results and Conclusions

The result of fitting this combined model to the observed data gives a $\hat{\sigma}_{t\bar{t}}$ of 180 ± 18 pb, with the 68% confidence interval inferred from the asymptotic properties of the profile likelihood ratio, which is shown in Fig. 3. This interval includes the effect of all systematic and statistical uncertainties, including their correlated effects on the signal and backgrounds in the five channels. The statistical uncertainty is obtained by fixing all the nuisance parameters associated with underlying sources of systematics to their best fit values. The component of the total uncertainty attributed to the effect of systematics is obtained by subtracting in quadrature the statistical contribution from the uncertainty obtained by including all sources of systematics except for the luminosity uncertainty. Finally, the uncertainty attributed to the luminosity uncertainty is obtained by subtracting in quadrature the combined systematic and statistical uncertainty from the total uncertainty, ensuring that the quadratic sum of all three components is consistent with the uncertainty from all contributions. The final results are in good agreement with a simple approximate calculation in which $\sigma_{t\bar{t}}$ is estimated by a weighted sum based on the inverse of the error of the dilepton and single-lepton results and the statistical error is reduced according to the larger sample size.

The dominant systematics in the five-channel combination are from W+jets heavy flavor content (± 8.5 pb), b-tagging (± 7.6 pb), ISR/FSR modeling (± 7.6 pb), jet-energy scale (± 5.3 pb), and jet reconstruction efficiency (± 3.0 pb). The uncertainty from the data-driven estimates of Drell-Yan and fake lepton estimates, which are important systematics in the dilepton channels, have very little effect in the five-channel combined measurement. Furthermore, the fact that the dilepton analysis does not use b-tagging serves to reduce the magnitude of the correlation coefficients between $\sigma_{t\bar{t}}$ and the nuisance parameters related to b-tagging and the heavy flavor content of W+jets, thus reducing the total systematic error slightly.

Figure 4 shows various cross-section measurements from Tevatron and LHC results overlaid on the theoretical predictions as a function of centre-of-mass energy [11]. Figures 5 and 6 show summary plots from the cross-section measurements made by ATLAS with the 2010 dataset [5, 6, 12]. The results show good agreement with the Standard Model predictions.

Channel	$\sigma_{t\bar{t}}$ (pb)
ee	$178\ ^{+67}_{-57}$ (stat.) $^{+37}_{-27}$ (syst.) $^{+9}_{-5}$ (lumi.)
$\mu\mu$	194^{+57}_{-51} (stat.) $^{+20}_{-15}$ (syst.) $^{+12}_{-5}$ (lumi.)
$e\mu$	164 ± 26 (stat.) ± 18 (syst.) $\pm\ ^{7}_{6}$ (lumi.)
di-lepton combined	173 ± 22 (stat.)$^{+18}_{-16}$ (syst.)$^{+8}_{-7}$ (lumi.)
e+jets	223 ± 17 (stat.) ± 27 (syst.) ± 8 (lumi.)
μ+jets	168 ± 12 (stat.)$^{+20}_{-18}$ (syst.) ± 6 (lumi.)
l+jets combined	186 ± 10 (stat.)$^{+21}_{-20}$ (syst.) ± 6 (lumi.)
five-channel combined	180 ± 9 (stat.) ± 15 (syst.) ± 6 (lumi.)

Table 1: Measured values of $\sigma_{t\bar{t}}$ in each of the five individual analyses, the dilepton and single-lepton combinations, and the full five-channel combination.

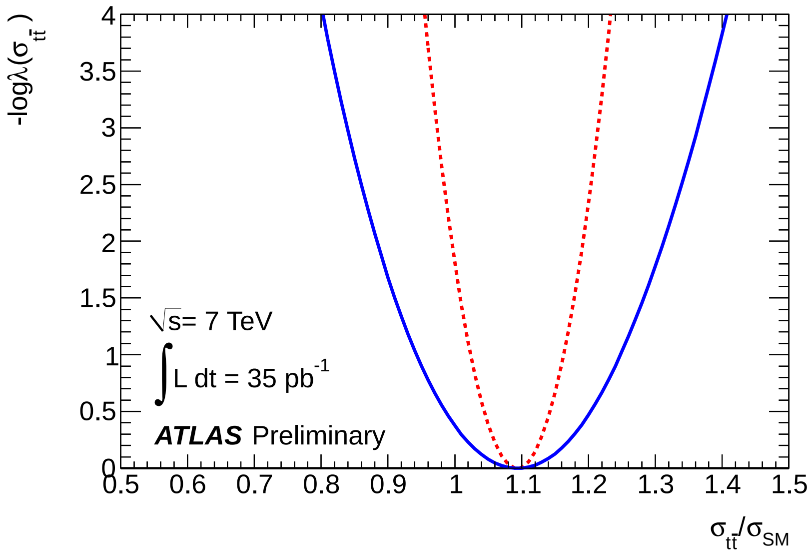

Figure 3: Plots of $-\log \lambda(\sigma_{t\bar{t}})$ as a function of $\sigma_{t\bar{t}}/\sigma_{\mathrm{SM}}$ with (blue, solid) and without (red, dashed) systematics for the five-channel combined fit.

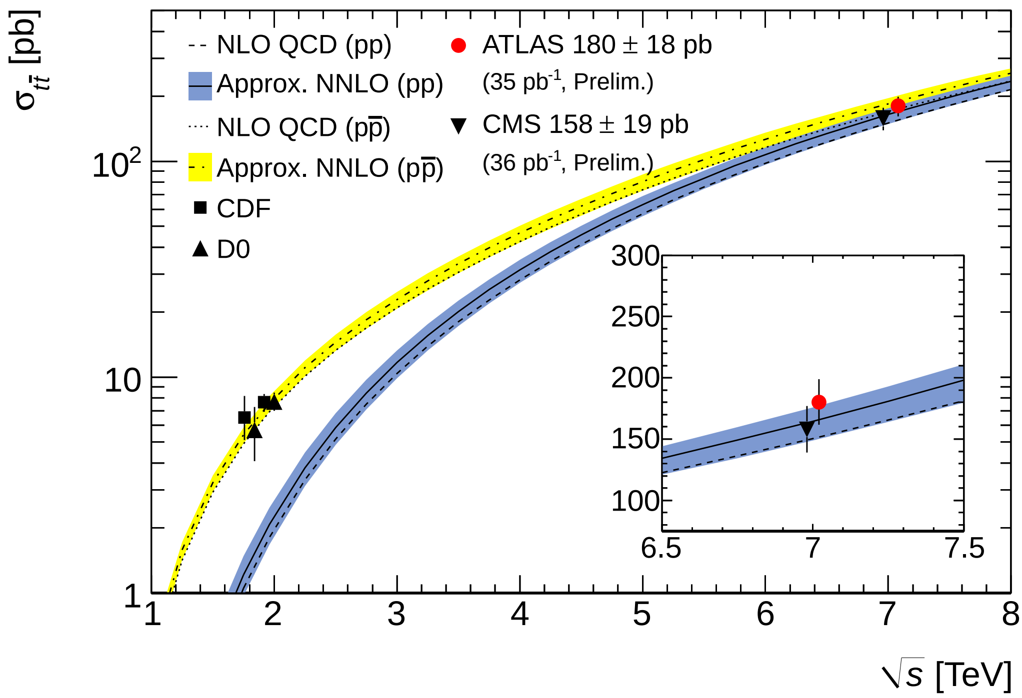

Figure 4: Measurements of $\sigma_{t\bar{t}}$ from ATLAS and CMS in pp collisions, and CDF and D0 in $p\bar{p}$ collisions, compared to theoretical predictions assuming a top mass of 172.5 GeV as a function of $\sqrt{s}$. The present result is indicated by the red circle.

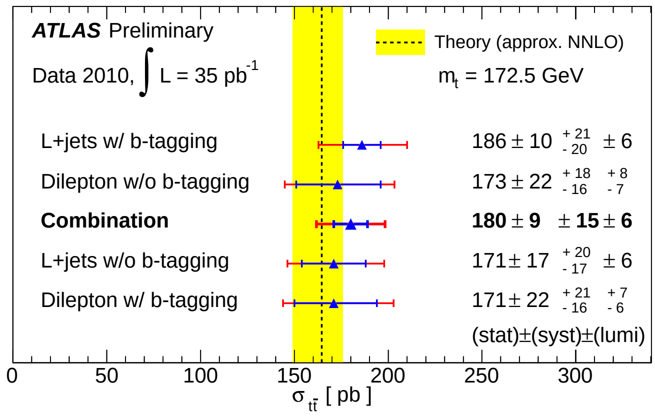

Figure 5: Plots of the measured value of $\sigma_{t\bar{t}}$ in the single-lepton with b-tagging channel, the dilepton without b-tagging channel, and the combination of these two channels, including error bars for both statistical uncertainties only (blue) and with full systematics (red). Results from from auxiliary single-lepton and dilepton measurements are shown as well. The approximate NNLO prediction is shown as a vertical dotted line with its error in yellow.

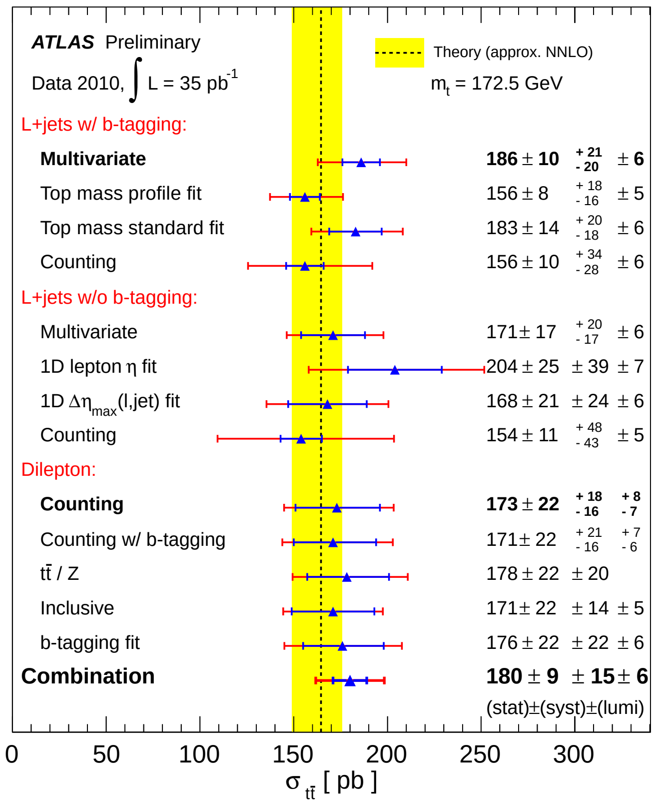

Figure 6: Plots of measured $\sigma_{t\bar{t}}$ using several analyses in each decay channel, including errors bars for both statistical uncertainties only (blue) and all systematics (red). The combined result is based on the L+jets b-tag multivariate and the dilepton counting analyses. The approximate NNLO prediction is shown as a vertical dotted line with its error in yellow.

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