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OPTIMUM CONDITIONS FOR APPLYING THE "INEFFICIENT COUNTER" METHOD FOR SEPARATING PARTICLES ACCORDING TO THEIR PRIMARY IONIZING POWER

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The "inefficient counter" method has been used repeatedly in experimental research or cosmic radiation, as being the simplest method of rough estimation of the ionizing power of a single charged particle, which at the same time will ensure relatively good accuracy in determining the mean ionizing power for a fairly large group of particles selected by any other criteria. An example of such an application of the "inefficient counter" method can be found in a paper by Alikhanov and Eliseev[1]. However, when the energy of the particles investigated is high, owing to the possibility of using a great number of counters, this method can give the greatest accuracy of measurement of the ionizing power also for a single particle. In order to obtain the required accuracy, it is necessary to use a sufficient number of counters and to take special steps to eliminate systematic errors. The insensitive counter method can be used successfully in the search for quarks with fractional charge and for the detection of rare particles with low ionizing power in a background of a large number of ordinary particles. A set-up with several hundreds of "inefficient counters" can be used to separate particles of different mass in particle beams with a given momentum, according to the relativistic growth of the initial ionizing power.

Since this kind of work requires a very cumbersome and complicated set-up, the question of optimal measuring conditions is particularly important. Actually the accuracy of separating particles according to their ionizing power depends not only on the number of counters in the set-up, but also on the choice of their efficiency.

Let us consider the problem of separating two categories of particles of different ionizing power by the "inefficient counter" method. Let us denote by α and β the mean numbers of primary ionization events in a single counter for particles of the first and second type, respectively. Moreover, we shall assume that the production of even one electron in the counter guarantees that a particle has been recorded. Then the inefficiency for recording these particles will be $\eta_1 = e^{-\alpha}$ and $\eta_2 = e^{-\beta}$. Consequently, the efficiency of recording these particles will be

$$f_1 = 1 - e^{-\alpha} \qquad \text{and} \qquad f_2 = 1 - e^{-\beta}.$$

In each single case of measurement according to the number of operating counters n, we estimate the efficiency for recording a given particle as $f = n/N$, where N is the over-all number of counters in the set-up. The comparison of this estimate with the values f_1 and f_2 is the basis for determining, in the most reliable way, to what category the particle belongs.

The value n for particles that belong, according to their ionizing power, to one of the categories of particles studied, fluctuates according to a binomial law. The variance of these distributions is, respectively:

$$D_1 = Nf_1(1 - f_1) \quad \text{and} \quad D_2 = Nf_2(1 - f_2). \tag{1}$$

When $N \gtrsim 100$ and also when f is not too close to the limiting values 0 or 1, the binomial distribution is well described[2)] by the de Moivre-Laplace asymptotic formula representing a normal distribution with the variance $D = Nf(1 - f)$.

Thus the spread in the number of counters operated decreases as the efficiency of the counters approaches one of the limiting values 0 or 1. However, at the same time there is a lessening of the distance (expressed in the number of counters) between the centres of the two distributions subjected to experimental separation. For this reason there must be some optimum value of the efficiency of the counters, ensuring the very best separation of particles for a given total number of counters in the set-up.

The question of the optimum conditions for separating particles according to their ionizing power, however, permits a different formulation depending on the advisability of taking one or another ratio between so-called errors of the first and second order. Let us examine this question for the case when rare particles with less ionizing power are to be separated accurately from a background of a large number of particles with normal ionizing power. Exactly this formulation of the problem is met with in experiments to search for quarks, or when separating rare antiprotons in a beam of negative relativistic particles of given momentum. In this case, obviously the most undesirable errors will be those imitating rare particles, when particles with normal ionizing power (which we shall later put in Class 1) are in isolated cases recorded as rare particles of Class 2 with less ionizing power. It is clear that such particularly undesirable errors will result from the variance of the empirical value $f = n/N$ for particles of the first type, characterized by the root-mean-square deviation $\sqrt{f_1(1 - f_1)/N}$. The variance of a similar value for the rare particles being separated can lead only to loss of efficiency in the recording of these particles, not to a false result. Therefore the optimum conditions for separating rare particles of Class 2 from the background of a large number of particles of Class 1 will correspond to the minimum ratio of the root-mean-square deviation $\sqrt{Nf_1(1 - f_1)}$ for the resulting number of counters operated n_1, to the difference in the mean numbers of counters operated for particles of the 1st and 2nd class $\Delta = \bar{n}_1 - \bar{n}_2 = N(f_1 - f_2)$. In order to determine the optimum value of counter efficiency f_0, corresponding to the minimum of the relation

$$\frac{\sqrt{D_1}}{\Delta} = \frac{1}{f_1 - f_2}\sqrt{\frac{f_1(1 - f_1)}{N}},$$

it is necessary to equate to zero the derivative

$$\frac{d}{d\alpha}\left(\frac{\sqrt{D_1}}{\Delta}\right) = \frac{d}{d\alpha}\frac{\left[e^{-\alpha}(1 - e^{-\alpha})\right]^{\frac{1}{2}}}{(e^{-\beta} - e^{-\alpha})N^{\frac{1}{2}}} =$$

$$= \frac{1}{N^{\frac{1}{2}}}\left[\frac{e^{\alpha}}{2\sqrt{e^{\alpha} - 1}\left[e^{(\alpha-\beta)} - 1\right]} - \frac{\sqrt{e^{\alpha} - 1}\left(1 - \frac{\beta}{\alpha}\right)e^{(\alpha-\beta)}}{\left[e^{(\alpha-\beta)} - 1\right]^2}\right] \tag{2}$$

Hence, specifying $\beta/\alpha = K$, we obtain the equation for the optimal value of α_0:

$$\frac{(e^{\alpha_0} - 1)e^{-K\alpha_0}}{e^{(1-K)\alpha_0} - 1} = \frac{1}{2(1 - K)},$$

which can be transformed into the following equation for the optimum value of counter efficiency f_0 for recording background particles:

$$(1 - K)\ln(1 - f_0) - \ln\left[1 - 2(1 - K)f_0\right] = 0\,. \tag{3}$$

Consequently the root-mean-square deviation for the empirical estimate of counter efficiency obtained in the case of background particles passing through the set-up will have the value

$$\sigma_0 = \sqrt{\frac{f_0(1 - f_0)}{N}} .$$

Supposing, for instance, that we determine the region of separation of given particles with lower ionizing power, which according to the number of counters operated corresponds to $n \leq \bar{n}_2 = Nf_2$, then the required particles can be selected with a probability of 0.5, and the probability of their being imitated by background particles will be

$$\gamma = \frac{1}{2} \left[1 - \frac{2}{\sqrt{2\pi}} \int_0^H e^{-t^2/2} dt \right] , \qquad H = \frac{\Delta}{\sqrt{D_0}} = \frac{(1 - f_0)^\kappa - (1 - f_0)}{\sqrt{\frac{f_0(1 - f_0)}{N}}} . \tag{4}$$

The relative fraction of wrongly separated particles represents

$$\delta = 2\gamma \frac{I_1}{I_2} ,$$

where I_1 and I_2 are the intensity of the background and separated particles, respectively.

The optimal value of counter efficiency for background particles f_0, according to Eq. (3), is determined by the proportion of primary ionizing power of the particles $K = \beta/\alpha$. If the primary ionizing power of the separated particles is 80% of the primary ionizing power of the background particles*), then the optimum value of counter efficiency $f_0 = 0.889$. Thus for the separation of particles with less ionizing power by the "inefficient counter" method, it is necessary in the optimal case to use counters with a fairly high recording efficiency for the principal particles of the beam. Even higher optimal values of efficiency f_0 correspond to smaller values of K.

*) This is the approximate correlation of the primary ionizing power of antiprotons and pions in a 60 GeV/c beam.

For the case considered $K = 0.8$ when $N = 100$, we obtain a probability of imitation of separated particles by background particles of $\gamma = 2.5\%$. Accordingly, with 100 counters in the set-up, the detection of particles having 20% less primary ionizing power is possible only if the relative fraction in the beam is not less than 1/20. For a set-up with 400 counters, according to Eq. (4), we obtain $\gamma = 5 \times 10^{-5}$. However, when the values taken for the probability γ are too small, the approximation used results in a relatively large underrating of the value γ.

In this case the calculation of γ, determining the accuracy of particle separation, should be carried out on the basis of an incomplete beta function describing the "tail" of the binomial distribution[3]. Using Tables 18 and 19 of Ref. 3, which show the probability f for various limiting values of n and N - n corresponding to the total probability of the "tail" of the binomial distribution, equal to 2.5 and 0.5%, the possibility of separating particles according to their ionizing power can be accurately determined for different values of the total number of counters in the set-up. Table 1 below gives, for a series of values of N, the proportion K of primary ionizing power of the particles; given which it is possible to separate particles with less ionizing power with an accuracy of $\gamma = 2.5$ and 0.5%.

Table 1

γ \ N	25	50	100	220	550
0.025	0 58	0.70	0.78	0.84	0.90
0.005	0.50	0.63	0.73	0.80	0.87

The values of $K = \beta/\alpha$ given in this table correspond to the choice of the optimal values of counter efficiency.

Relation (4) when γ is small leads to an underrating of the result even when the value of N is high. Thus, when $N = 100$ and with the corresponding values of K (Table 1), relation (4), instead of showing $\gamma = 2.5$ and 0.5%, gives 1.5 and 0.2%, and when $N = 550$, -2.0 and 0.3%, respectively.

Comparison with accurate calculations also shows that in spite of the considerable underrating of the value γ when the "tails" of the distributions are calculated according to the de Moivre-Laplace equation, the use of this approximation for determining the optimal counter efficiency leads to correct results for values of $f_0 < 0.95$.

The latter conditions limit the field of applicability of Eq. (3) to $K > 0.67$. When $K = 0.5$, Eq. (3) gives $f_0 = 1.00$, which means that the approximation used is inadmissible.

When the efficiency of the counters is near to unity, even for a large N, the binomial distribution becomes substantially unsymmetrical, and replacing it by a normal distribution leads to some overrating of the optimal value of counter efficiency. Therefore the solutions of Eq. (3) close to the limiting values should be regarded as guiding values for the choice of the counter efficiency corresponding to the optimum measuring conditions, the choice of large values of efficiency being limited to 0.95. Thus for separating particles with lower ionizing power in the whole range of values $K < 0.9$ where this is practically feasible, the optimal values of counter efficiency for background lie in the 0.85 - 0.95 range.

It should be noted that the choice of counter efficiency in the whole of the 0.85 - 0.95 range mentioned, independently of the concrete value $K < 0.9$ and the corresponding value of f_0, has almost no effect on the accuracy of the particle separation. This can be ascertained by determining, according to the accurate tables in Ref. 3, the ratio of the primary ionizing power of particles separated with an accuracy of 2.5 and 0.59% for different values of efficiency f. Thus for $N = 24$ and $Y = 0.5\%$, the value $K = 0.520$ remains invariable, at least to the third figure, in the whole region of counter efficiency from 0.82 to 0.93. For values of efficiency of 0.95, 0.97, and 0.99 the value K is 0.518, 0.507, and 0.46, respectively. From Eq. (3) for $K = 0.520$ it follows that $f_0 = 0.99$. A similar picture is also obtained for other values of N. For $N = 50$ and $\lambda = 0.5\%$ with $f = 0.85$, 0.93, and 0.98, K has a value of 0.633, 0.633, and 0.605, respectively. In accordance with Eq. (3), the optimal value of efficiency when $K = 0.633$ is 0.965. For $N = 100$ and $\gamma = 0.5\%$, $K = 0.730$ for efficiency from 0.83 to 0.92. When $f = 0.96$ and 0.99, K is equal to

0.710 and 0.66, respectively. It should certainly be taken into account that even a small reduction of the limiting value of K for separated particles means a considerable reduction of the accuracy of the measurements. Thus according to Table 1 a reduction of the limiting K by 10% for K from 0.7 to 0.8 means that to re-establish the previous accuracy of measurement, the number of counters in the set-up must be doubled.

When the efficiency of the counters is reduced, a gradual lowering of the limiting value of K is observed, beginning only at $f < 0.80$. A considerable deterioration of the accuracy of measurement occurs when $f < 0.50$. Thus, for a counter efficiency of 0.50 and 0.30, in order to attain the accuracy of the optimal variant it is necessary to multiply the total number of counters in the set-up by 1.5 and 4.

The invariability of the limiting value of K in the region of counter efficiency from 0.80 to 0.95 indicates that the reduction of the width of distribution of the number of counters that have been triggered by background particles in this region is exactly counterbalanced by the reduction of the distance between the centres of the distributions for the separated particles. This is very significant for the practical application of the "inefficient counter" method. In the first place, for all problems of separating particles with less ionizing power independently of the value of the relation $K = \beta/\alpha$, one can use the same counters with an efficiency of 0.85-0.90. In addition, since the accuracy of measurement does not depend on the efficiency of the counters in the corresponding range of values, a spread of several per cent in the efficiency of the counters used is permissible. This considerably facilitates the manufacture and adjustment of a multi-counter system. For this same reason, when the system is operated the requirement for high stability should be imposed only on the mean value of efficiency for the counters of the whole system. The main result of the present work is that high efficiency of the counters corresponds to the optimal conditions for solving the problems considered, which is somewhat unexpected for the low-efficiency counter method.

A lower counter efficiency will correspond to the optimal conditions for separating particles having a greater ionizing power than background particles. Thus, for $K = 2$ from Eq. (3) we find $f_0 = 0.50$. For separating quintuply ionized particles, the optimal value of counter efficiency for singly ionized particles should be 0.25.

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In order to achieve high accuracy in separating rare particles, corresponding to high statistical accuracy of the method with a large number of counters in the set-up, it is necessary in practice to take special measures to eliminate systematic errors. First of all, malfunction of the counters due to technical fautls should be completely excluded. When there is a large number of counters, the task of constant control of their operation certainly involves practical difficulties. In addition, it is of the greatest importance that the chosen inefficiency of the counters (5-10%) be wholly dependent on the absence of a primary ionization event in the counter. Otherwise systematic errors unavoidably occur, which make it impossible to achieve the rated accuracy of particle separation.

Actually, if in each counter a primary ionization event admittedly takes place and only the process of recording it is connected with the chosen inefficiency value, then the measurements will be affected by the usual fluctuations which occur in spectrometric scintillators or gas-discharge proportional counters. The "inefficient counter" method gives a higher resolution for the measurement of the ionizing power of the particles than do the existing spectrometric counters, only if the amplification and the discrimination thresholds of the counters used guarantee the recording of every primary ionization event in the counter. Certainly, when a large enough number of spectrometric counters are used in parallel, high accuracy of separation of the required particles can also be achieved. However, statistical calculations of this accuracy can no longer be carried out sufficiently simply and reliably. Therefore, perhaps the best results can be obtained by combining both methods.

Let us assume, for instance, that after amplitude analysis of some scintillation counters events are separated, which are to be investigated by the "inefficient counter" method. In this case it will be advisable to use a large number of miniature flat spark counters, with power supply controlled by a system of spectrometric scintillation counters with automatic output of data on spark formation.

According to the statistical distribution of the number of counters in which a spark discharge did not take place, it is easy to establish the statistical characteristics of particle selection of the control system of scintillation counters, and in addition to step up by a few orders the accuracy of separating particles with less ionizing power. Such a combined system with a moderate quantity of controlled spark counters will have the same possibilities as a system with a much larger quantity of inefficient counters.

The controlled operation of spark chambers makes it necessary to take steps to reduce the inefficiency of the counters due to the diffusion of electrons on to the electrodes and their adhesion to the atoms of electronegative impurities. Accordingly, first of all it is necessary to reduce to a minimum the delay time of the high-voltage pulse of the counter power supply, and also to reduce the amount of impurities in the gas and to fill with gas ensuring a lower electron diffusion coefficient. For this reason it will be advisable to fill the spark counters with helium instead of the neon gas that is generally used in spark chambers.

Another way of reducing the inefficiency of the counters, arising from the disappearance of electrons, can be based on the use of a Penning mixture and a relatively high d.c. voltage in the counter, sufficient for the efficient formation of metastable atoms of the basic gas[4)].

The absence of systematic errors will be guaranteed by the invariability of the spark counter efficiency for small variations in delay time, amplitude, and length of the power supply high-voltage pulse, and also the value of the d.c. voltage in the counters.

When designing a multi-counter system for measuring the primary ionizing power of particles, one should bear in mind the possibility of achieving a somewhat faster system by using gas-discharge counters with d.c. voltage, which ensures that weak discharge will arise. Such a system will be considerably more complicated, since the radiation accompanying this discharge should be recorded with high efficiency in each counter by an individual photomultiplier.

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