EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH

CERN-ISR-TH/70-47
Closed distribution

THE COUPLING IMPEDANCE OF RESONANT CROSS-SECTION VARIATIONS (SECOND APPROXIMATION)

by

B. Zotter

Geneva, 25th September, 1970

1. Introduction

The resonant behaviour of cavities formed by pairs of opposing cross-section variations of a tube has been described in refs. (1) and (2). The coupling impedance has been defined there in terms of the coefficient of the n-th space harmonic of the longitudinal electric field in the particle beam, which can be found by solution of an (infinite) matrix equation ("n" is the mode number of the perturbation on the beam). Besides direct numerical solution of the matrix equation on a computer, a matrix transformation can be applied that allows the derivation of approximate expressions for the resonant frequencies, coupling impedances, and quality factors ("first approximation"). While this first approximation yields good results for reasonably large cavities, it becomes quite inaccurate for very short or shallow cavities. Furthermore, the effect of finite tube wall resistivity cancels in this approximation.

This report remedies the situation by giving formulae that are valid for all values of geometric ratios. Instead of depending on a single parameter, the correction factors in this "second approximation" depend on three geometric ratios : the "circumference factor" $\alpha = g/2\pi R$, the ratio of outer to inner radius $\lambda = d/b$, and the "aspect ratio" $\varepsilon = \pi b/g$. These factors have been evaluated for a wide range of parameters, and are given in a number of tables and curves. Furthermore, the computer program "SECAPP" evaluates the four factors for any combination of parameters in a very short time.

While the first approximation was based on the fact that replacement of some Besselfunctions by their small argument approximations made the kernel of the matrix equation diagonal, the second approximation retains the exact Besselfunction expressions but limits the matrix to a single element. It thus allows only the calculation of the lowest (axial) resonant mode, but describes also the influence of a finite tubewall resistivity (limited to this point, it was already mentioned in ref. (2)). It yields results in much better agreement with those of the matrix inversion program, as will be shown on hand of a few examples.

Another application of the second approximation is the calculation of resonances of bellows. While the original analysis covers only cavities spaced equally along a tube, bellows usually consist of a group of closely spaced small cavities with long smooth tubes on both sides (periodic with the circumference or superperiod of the machine). The problem can be approximated by considering a single cavitiy with the overall length of the bellows, in which only the lowest axial mode is allowed. All higher modes have radial electric field components and would be strongly damped by the presence of the radial walls of the actual bellows. The field equations may be solved either by field- or impedance-matching. The last method is actually less restrictive [3], but the first one coincides with the second approximation described here. The second approximation thus can also be regarded as an exact solution to an approximate problem (at least when the tube-wall is perfectly conducting), and can give valuable checks on the more complicated matrix inversion routines.

The sidewall resistivity has not been included in the expressions directly, but can be approximately accounted for by increasing the resistivity of the outer cavity wall. From the first order theory, we further know that higher order resonances have decreasing coupling impedances ($\omega_{res}^{-3/2}$), but are multiplied by a 2 due to the non-uniformity of the electric field in the longitudinal direction. On the other hand, the quality factor increases as $\omega_{res}^{1/2}$. In general, we will be safe if we take for the coupling impedance twice the value calculated for the lowest mode. Incidentally, this covers also the results found for bellows by impedance matching where the impedance rises to about twice the value of the lowest mode for some geometries.

2. The Second Approximation

First we assume that only the outer cavity wall has finite conductivity. The matrix equation for the field coefficients in the beam region then is found from ref (2), eq. (4.11) (with R replaced by R')

$$X = \alpha N(U - \alpha R' N^{+} I N)^{-1} R' N Y \qquad (2.1)$$

where

$\alpha = \frac{g}{2\pi R}$ is the circumference factor

U the unit matrix

Y a column vector with a single non-vanishing element

$$Y_n = \frac{a^2}{\chi b^2} \frac{I_1(\chi a)}{I_o(\chi a)} \approx \frac{a^2}{2b^2} \qquad (2.2)$$

I is a diagonal matrix with the elements

$$I_{mm} = \left(x \frac{I_1}{I_o}\right)_{\chi_m b} \qquad (-\infty < m < +\infty) \qquad (2.3)$$

R' a diagonal matrix with the elements

$$R'_{ss} = \left(x \frac{xF_1 - \bar{\eta}\, G_o}{xF_1 - \bar{\eta}\, G_1}\right)_{\Gamma_s b} \qquad (0 \leq s < \infty) \qquad (2.4)$$

with

$$\left.\begin{aligned} F_i(x) &= Y_o(\lambda x)\, J_i(x) - J_o(\lambda x)\, Y_i(x) \\ G_i(x) &= Y_1(\lambda x)\, J_i(x) - J_1(\lambda x)\, Y_i(x) \end{aligned}\right\} \qquad (2.5)$$

$$\lambda = d/b \qquad (\lambda = b/d) \qquad (2.6)$$

and $\bar{\eta} = (1 + i)\eta = (1 + i\,(\frac{\omega b}{c})^2 \;\frac{\delta}{2b} \qquad (2.7)$

(δ is the skindepth of the outer cavity wall)

N is an infinite matrix with the elements

$$N_{ms} = \frac{\pi\alpha m}{(\pi\alpha m)^2 - (\frac{\pi s}{2})^2} \begin{cases} \sin \pi\alpha m & s \text{ even} \\ -i\cos \pi\alpha m & s \text{ odd} \end{cases} \qquad (2.8)$$

with the same limits on m and s as above.

N^{+} is the Hermitian conjugate of N, thus

$$N^{+}_{sm} = N^{*}_{ms} \qquad (2.9)$$

The radial propagation constants are

$$\Gamma_s^{\,2} = \left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi s}{g}\right)^2 \qquad (2.10)$$

$$\left.\begin{aligned} \chi_m^{\,2} &= \left(\frac{\omega}{c}\right)^2 - \left(\frac{m}{R}\right)^2 \\ \chi &\equiv \chi_n = \frac{n}{\gamma R} \end{aligned}\right\} \qquad (2.11)$$

In these formulae we have assumed that the mode number n of the perturbation is integer. This assumption will be discussed further in Section 6.

For the second approximation we restrict the subscript s to the single value s = 0. The solution of the matrix equation (2.1) for the n-th component of X then is simply

$$X_n = \frac{N_{no}\, R'_{oo}\, N^{+}_{on}}{1 - R'_{oo}\, W_{oo}}\, Y_n \qquad (2.12)$$

where

$$W_{oo} = \sum_{m=-\infty}^{\infty} N^{+}_{om}\, I_{mm}\, N_{mo}$$

$$\approx \left(\frac{J_1}{xJ_o}\right)_{\frac{\omega b}{c}} + 2\sum_{m=1}^{\infty}\left(\frac{J_1}{xJ_o}\right)_{\chi mb}\left(\frac{\sin \pi\alpha m}{\pi\alpha m}\right)^2 \qquad (2.13)$$

With the relation[1]

$$\frac{Z}{n} = -i\,\frac{2 Z_o}{\left(\frac{\omega a}{c}\right)^2}\, X_n \qquad (2.14)$$

we can calculate the coupling impedance as a function of frequency, resp. $x = \frac{\omega b}{c}$

$$\frac{Z}{n} = -i\,\frac{2 Z_o}{\pi x^4}\,\frac{\beta^3 b^2}{Rg}\,\sin^2\left(\frac{gx}{2\beta b}\right)\frac{R'_{oo}(x)}{1 - \alpha W_{oo}(x) R'_{oo}(x)} \qquad (2.15)$$

The computer program BELZ 1 has been written to evaluate this expression for various geometric parameters. We find that Z/n makes circles in the impedance plane which pass through the origin and are symmetric to the real axis. The maximum absolute value of Z/n thus is entirely real. The program evaluates these "resonances" by finding the zeros of the imaginary part of Z/n.

3. Analytic Expressions

For the first approximation we replace the Besselfunctions in the expression for I_{mm} by the small argument approximations, and thus find $I_{mm} \approx \frac{1}{2}$. The sum for W_{oo} (eq. 2.12) then can be evaluated,[2] and yields $W_{oo} \approx \frac{1}{2\alpha}$. Actually W_{oo} can deviate considerably from this value, and we express **it in the form**

$$W_{oo} = \frac{1}{2\alpha}(1 + \xi_1) \qquad (3.1)$$

where ξ_1 is a function of all three geometric parameters and the frequency, and can be evaluated numerically with Eq. (2.13).

For metal walls at not too high frequencies we further have $\eta = \left(\frac{\omega b}{c}\right)^2 \frac{\delta}{2b} << 1$, and we may approximate the quantity R'_{oo} by

$$R'_{oo} \simeq \left[x \frac{F_o}{F_1} + (1 + i) \frac{4\eta}{\lambda(\pi x F_1)^2} \right]_{\frac{\omega b}{c}} \qquad (3.2)$$

under the assumption that F_1 is not too small (the zeros of F_1 are essentially antiresonances, i.e., frequencies where the coupling impedance vanishes. At resonances the values of F_1 turn out to be finite).

Eq. (2.11) then yields

$$X_n = \frac{\alpha a^2}{2b^2} \frac{\left(\frac{xF_o}{F_1}\right)_{\frac{\omega b}{c}} \left(\frac{\sin \pi\alpha n}{\pi\alpha n}\right)^2}{1 - \frac{1 + \xi_1}{2}\left(x\frac{F_o}{F_1} + \frac{4i\eta}{\lambda(\pi x F_1)^2}\right)_{\frac{\omega b}{c}}} \qquad (3.3)$$

when we neglect the small additional real part of R'_{oo} in the denominator, and the total additional part (proportional η) in the numerator. We have to retain the imaginary part in the denominator however small it is, as it yields the limitation of X_n to finite values at resonance, (the real part only shifts the resonance frequency slightly).

The resonance condition is then

$$\left(x \frac{F_o}{F_1}\right)_{\frac{\omega b}{c}} = \frac{2}{1 + \xi_1(\frac{\omega b}{c})} \tag{3.4}$$

and we call

$$x_1 = (\frac{\omega b}{c})_{01} \tag{3.5}$$

the lowest solution of this transcendental equation (since ξ_1 is also a function of $\frac{\omega b}{c}$, the sum (2.13) has to be evaluated for each frequency in order to determine ξ_1 from (3.1)).

The real part of the denominator vanishes at resonance, and thus X_n becomes entirely imaginary, resp. Z/n entirely real. The value of the coupling impedance is then found to be

$$\left(\frac{Z}{n}\right)_{01} = 4\pi \, Z_o \frac{F_1^2 \, (x_1)}{x_1^4 \, (1 + \xi_1)^2} \frac{\beta^3 b^2 d}{Rg\delta} \sin^2 \left.\frac{\omega g}{2\beta c}\right|_{01} \tag{3.6}$$

The indices 0 and 1 refer to the number of zero-crossings of the field in the cavity - zero in axial and one in the radial direction for the lowest resonant cavity mode.

We can rewrite Eqs. (3.5) and (3.6) in a more practical form, after eliminating the skindepth that is still a function of frequency

$$\left.\begin{aligned} \left(\frac{\omega}{c}\right)_{01} &= \frac{j_1}{d} \, X_1' \, (\varepsilon,\lambda) \\ \left(\frac{Z}{n}\right)_{01} &= C_1 \, \Phi_1' \, (\varepsilon,\lambda) \, \beta^3 \sigma^{\frac{1}{2}} \, \frac{(2d)^{5/2}}{Rg} \, \sin^2 \left.\frac{\omega g}{2\beta c}\right|_{01} \end{aligned}\right\} \tag{3.7}$$

where $j_1 = 2.405$ is the first zero of the zero-order Besselfunction, and

$$C_1 = \frac{Z_o^{3/2}}{2\pi\, j_1^{7/2}\, J_1^2(j_1)} = 200.42 \quad \Omega^{3/2} \text{ is a constant} \qquad (3.8)$$

The functions X_1' and Φ_1' are chosen in such a manner that they tend to unity for large values of λ, corresponding to a closed cavity (vanishing tubeholes in the sidewalls). Eqs. (3.7) then tend towards known expressions by simply leaving off the correction factors.

Actually, the functions depend on all three geometric ratios, but for small vlaues of the "aspect ratio" ε (not too short cavities) they are practically independent of the circumference factor α (when α is not too large). The functions are defined by

$$X_1'(\varepsilon,\lambda) = \frac{\lambda x_1}{j_1}$$

$$\Phi_1'(\varepsilon,\lambda) = \frac{\pi^2\, j_1^{7/2}\, J_1^2(j_1)\, F_1^2(x_1)}{\lambda^{3/2}\, x_1^{7/2}\,(1+\xi_1)^2}$$

They are listed in the tables, and shown in Figures 1 and 2 as functions of $\bar{\lambda}(= 1/\lambda)$ with ε as parameter (for $\alpha = 0.1$). Their dependence on α is shown for one particular value of $\lambda(\bar{\lambda} = 0.5)$ in Figures 5A and 5B. The first approximation is also shown in Figures 1 and 2 for comparison.

From Figure 1 we see that X_1' increases initially with $\bar{\lambda}$, and for small values of ε the rise is much more rapid than predicted by the first approximation. The curves reach a maximum depending on ε, and decrease again toward unity for $\bar{\lambda} = 1$, while the first approximation keeps rising and yields completely incorrect values for $\bar{\lambda} \gtrsim 0.7$. The resonant frequencies are thus determined by the cavity radius d for small values of $\bar{\lambda}$, tend towards the frequencies of a slotmode for higher values of $\bar{\lambda}$ (determined by d-b), and are determined by the cavity radius again when $\bar{\lambda}$ comes close to unity ($d \approx b$).

The function Φ_1' deviates less drastically from the first approximation.

The coupling impedance, however, may still be quite different from the value found by the second approximation due to the incorrect transit-time factor. For values of $\varkappa$ close to unity, the first approximation yields consistently too low values for the correction factor ϕ_1.

4. Tube Walls of Finite Conductivity

The matrix equation for the field coefficient vector in the case of finite tube wall resistivity was given in ref.(2) in Eq. (6.8). The terms $(U - \eta I_{nn})^{-1}$ on either side of the RHS can be approximated by unity as long as $\chi b << 2.4$, (or $n << 2.4 \frac{\gamma R}{b}$) which is usually well fulfilled for particles with $\gamma >> 1$. We may then use Eq. (6.9) or

$$\left.\begin{aligned} X &= \alpha N \left[U - \alpha S N^{+} I \, (U - \bar{\eta} I)^{-1} \, N \right]^{-1} S N^{+} Y \\ \text{where} \quad S &= R' - \bar{\eta} \, (2U - U^{o}) \end{aligned}\right\} \qquad (4.1)$$

when we also include the resistivity of the outer cavity wall.

In the second order approximation we get as before

$$\left.\begin{aligned} X_n &= \alpha \frac{N S_{oo} \, N^{+}_{on}}{1 - \alpha S_{oo} W'_{oo}} \, Y_n \\ \text{where} \quad W'_{oo} &= \sum_{m=-\infty}^{+\infty} \frac{N^{+}_{om} \, I_{mm} \, N_{mo}}{1 - \bar{\eta} \, I_{mm}} \end{aligned}\right\} \qquad (4.2)$$

Since the resonance of the lowest mode is always below the cut-off frequency of the tube - although it may come close to it for very small cavities - the values of I_{mm} stay limited. For small enough η we may thus write (as $N^{+}_{om} = N_{mo}$)

$$\begin{aligned} W'_{oo} &\approx \sum_{m=-\infty}^{+\infty} I_{mm} \, N^{2}_{mo} + \bar{\eta} \sum_{m=-\infty}^{+\infty} I^{2}_{mm} \, N^{2}_{mo} \\ &= \frac{1}{2\alpha} (1 + \xi_1) + \frac{\bar{\eta}}{4\alpha} (1 + \xi_2) \end{aligned} \qquad (4.3)$$

The n-th field coefficient in the beam region thus becomes to first order in η

$$X_n = \frac{\alpha a^2}{2b^2} \frac{\left(\frac{xF_o}{F_1}\right)_{\frac{\omega b}{c}} \left(\frac{\sin \pi\alpha n}{\pi\alpha n}\right)^2}{1 - \frac{1+\xi_1}{2}\left[x\frac{F_o}{F_1} + i\eta\left(\frac{4}{\lambda(\pi x F_1)^2} + \frac{xF_o}{2F_1}\frac{1+\xi_2}{1+\xi_1} - 1\right)\right]_{\frac{\omega b}{c}}} \tag{4.4}$$

Resonance is still approximated by condition (3.4), and we find the coupling impedance

$$\left(\frac{Z}{n}\right)_{01} = 4\pi Z_o \frac{F_1^2(x_1)}{x_1^4\left[(1+\xi_1)^2 + \xi\lambda\left(\frac{\pi x_1 F_1}{2}\right)^2\right]} \frac{\beta^3 b^2 d}{Rg\delta} \sin^2 \frac{\omega g}{2\beta c}\bigg|_{01} \tag{4.5}$$

with $\xi = \xi_2 - \xi_1 (2 + \xi_1)$

The quantity ξ stays always larger than zero. The effect of the finite tube wall conductivity thus is a reduction of the coupling impedance as expected. Including the possibility of different resistivities of the outer cavity wall and the tube wall we find

$$\left(\frac{Z}{n}\right)_{01} = \frac{\beta^3 C_1 \Phi_1'(\varepsilon,\lambda)}{\rho_c^{\frac{1}{2}} + \rho_\tau^{\frac{1}{2}}\Sigma_1'(\varepsilon,\lambda)} \frac{(2d)^{5/2}}{Rg} \sin^2 \frac{\omega g}{2\beta c}\bigg|_{01} \tag{4.6}$$

where

$$\Sigma_1'(\varepsilon,\lambda) = \lambda\xi\left(\frac{\pi x_1 F_1(x_1)}{2(1+\xi_1)}\right)^2 \tag{4.7}$$

In first approximation, the correction due to the finite tube wall resistivity vanishes or $\Sigma_1 = 0$. Σ_1' depends somewhat more strongly on α than the other correction functions, but for small values of ε and α it is again practically independent of α. It is shown in Figure 3 as function of ε and $\bar{\lambda}$, and in Figure 5C as function of α. For $\bar{\lambda} \to 1$, Σ_1' tends towards $1/\alpha$ as expected for a smooth wall.

The sidewall resistivity can be taken into account approximately by increasing the resistivity of the outer cavity wall in the area ratio

$$\frac{d^2 - b^2}{gd} = \frac{\varepsilon}{\pi} \frac{1 - \lambda\!\!\!^{-}{}^2}{\lambda\!\!\!^{-}} \tag{4.8}$$

For the case of bellows with N convolutions in the length g, we have to multiply this ratio by N. In **general** we find for equal conductivities of all cavity walls

$$\rho_c = \rho \left(1 + \frac{N\varepsilon}{\pi} \frac{1 - \lambda\!\!\!^{-}{}^2}{\lambda\!\!\!^{-}}\right) \tag{4.9}$$

5. The Quality Factor

We found the resonances of the lowest mode by determining the frequency for which the real part of the denominator of Eq. (3.3), resp. Eq. (4.4) vanishes. By looking for the frequencies where the real and imaginary part of the denominator are equal, we find the so-called "45°**-points**" which determine the bandwidth, and hence the Q-factor of the resonance. If we call the value of $x = \frac{\omega b}{c}$ at resonance x_o, at one of the 45° points $x_{\frac{1}{2}}$, we find

$$\left.\begin{aligned} Q &= \frac{x_o}{2\Delta x} \\ \text{where} \qquad \Delta x &= \left| x_o - x_{\frac{1}{2}} \right| \end{aligned}\right\} \tag{5.1}$$

The condition on the frequency of the 45° points becomes from Eq. (4.4)

$$\left(x \frac{F_o}{F_1} - \frac{2}{1 + \xi_1}\right)_{x_{\frac{1}{2}}} = \eta \left(\frac{4}{\lambda(\pi x F_1)^2} + \frac{xF_o}{2F_1} \frac{1 + \xi_2}{1 + \xi_1}\right)_{x_{\frac{1}{2}}} \tag{5.2}$$

We assume that $x_{\frac{1}{2}}$ is quite close to x_o (high Q resonance), and develop the LHS into a Taylor series, including only the first two terms. In the RHS, we replace $x_{\frac{1}{2}}$ directly by x_o as the expression is multiplied by the small quantity η. Taking Eq. (3.4) into account, we find then with Eq. (5.22) from ref.(2)

$$Q = \frac{d}{\delta} \frac{1 - \left(\frac{\pi x F_1}{2}\right)^2 \left(1 - \frac{2\xi_1'}{x(1+\xi_1)^2}\right)}{1 + \lambda\xi \left(\frac{\pi x F_1}{2(1+\xi_1)}\right)^2} \tag{5.3}$$

where ξ_1' is the derivative of ξ_1 with respect to x calculated at x_o. Taking the possibility of different resistivities of cavity and tube wall into account, we can write this equation as

$$Q = D_1 \frac{\Psi_1'(\varepsilon,\lambda)}{1 + \Sigma_1'(\varepsilon,\lambda) . \sqrt{\frac{\rho_\tau}{\rho_c}}} \sqrt{2d\sigma_o} \tag{5.4}$$

where $D_1 = \frac{1}{2}\sqrt{j_1 Z_o} = 15.055\ \Omega^{\frac{1}{2}}$

and $$\Psi_1'(\varepsilon,\lambda) = \sqrt{\frac{\lambda x}{j_1}} \left[1 - \left(\frac{\pi x F_1}{2}\right)^2 \left(1 - \frac{2\xi_1'}{x(1+\xi_1)^2}\right)\right] \tag{5.5}$$

As ξ_1' is usually positive, the Q-factor found from the second approximation is thus usually higher than that from the first one. It is reduced, however, by the tube wall resistivity in the same ratio as the coupling impedance. The correction factor $\Psi_1'(\varepsilon,\lambda)$ is shown in Figure 4 and 5D.

6. Non-integer mode numbers

In general the resonant frequencies do not correspond to integral mode numbers, and this invalidates to a certain degree the assumptions made in the beginning. We can avoid this problem by calculating with non-integral mode numbers from the start. We only have to replace m by $m + n - <n>$ in Eqs. (2.8) and (2.10) ($<n>$ is the integral part of n that is only subtracted to keep the same numbering of the modes). This leads to a somewhat more complicated expression for W_{oo} (resp. W'_{oo}), as the sum over negative terms is no longer equal to the sum of positive terms, and becomes dependent on the beam velocity β. This method has been incorporated into the program BELZ 2, and its main result is the splitting of higher resonant modes into positive and negative ones. It has little influence on the lowest mode as long as the period is much larger than the tube radius. For very short periods, the lowest resonant frequency can then become larger than the cut-off frequency of the tube.

On the other hand, the use of non-integer mode numbers for a circular machine is somewhat questionable, as the phaseshift around the circumference must be 2π (or a multiple thereof). We can keep the mode number integer by adjusting any one of the geometrical parameters, which are somewhat arbitrary anyhow for the idealized geometry. As the required changes are usually quite small, we actually ignore them when evaluating the correction functions with the computer program SECAPP. The frequency of the lowest resonance is then always below the cut-off frequency of the tube.

TABLE I

Approximate formulae for the lowest resonance of periodic cylindrical cavities on a tube

1. Resonant Frequency

1st. Approximation $$f_{01}^{(1)} = \frac{B_1}{d} X_1(\lambda)$$

2nd. " $$f_{01}^{(2)} = \frac{B_1}{d} X_1'(\alpha,\varepsilon,\lambda)$$

with $$B_1 = \frac{cj_1}{2\pi} = 114.74 \text{ MHz}$$

2. Coupling Impedance

1st. Approximation $$\left(\frac{Z}{n}\right)_{01}^{(1)} = C_1 \beta^3 \sigma_c^{\frac{1}{2}} \frac{(2d)^{5/2}}{Rg} \Phi_1(\lambda) \sin^2 \frac{\pi g f_{01}^{(1)}}{\beta c}$$

2nd. Approximation $$\left(\frac{Z}{n}\right)_{01}^{(2)} = C_1 \beta^3 \sigma_c^{\frac{1}{2}} \frac{(2d)^{5/2}}{Rg} \frac{\Phi_1'(\alpha,\varepsilon,\lambda)}{1 + \left(\frac{\sigma_c}{\sigma_\tau}\right)^{\frac{1}{2}} \Sigma_1'(\alpha,\varepsilon,\lambda)} \sin^2 \frac{\pi g f_{01}^{(2)}}{\beta c}$$

with $$C_1 = \frac{Z_o^{3/2}}{2\pi j_1^{7/2} J_1^2(j_1)} = 200.42\ \Omega^{3/2}$$

3. Quality Factor

1st. Approximation $$Q_{01}^{(1)} = D_1 (2d\sigma_c)^{\frac{1}{2}} \Psi_1(\lambda)$$

2nd. Approximation $$Q_{01}^{(2)} = D_1 (2d\sigma_c)^{\frac{1}{2}} \frac{\Psi_1'(\alpha,\varepsilon,\lambda)}{1 + \left(\frac{\sigma c}{\sigma \tau}\right)^{\frac{1}{2}} \Phi_1'(\alpha,\varepsilon,\lambda)}$$

with $$D_1 = \frac{(j_1 Z_o)^{\frac{1}{2}}}{2} = 15.055\ \Omega^{\frac{1}{2}}$$

The Correction Functions X_1, Φ_1, Ψ_1 are shown in Figures 1 to 3 (and listed in ref.2), and the correction functions X_1', Φ_1', Ψ_1', Σ_1' are shown in Figures 1 to 5 and listed in Table II as functions of $\alpha = {}^{g}/2\pi R$, $\varepsilon = {}^{\pi b}/g$, and $\lambda = {}^{b}/d$.

7. Numerical Examples

In this section we compare the results of the first and second approximation, the direct formula for a single cavity harmonic (BELZ program), and the matrix inversion routine CHIMP with various numbers of cavity harmonics. We consider the three cases of a very shallow, a very short, and a normal cavity. The approximate formulae are collected in Table I.

Cavity 1 (envelope of small bellows in the ISR):

$b = .08$, $d = .09$, $g = .1$, $2\pi R = 1.0$ m, $\gamma = 30$,
$\rho_c = 5.10^{-6}$ Ωm (including sidewall resistivity of about 10 convolutions), $\rho_\tau = 0$ or 10^{-6} Ωm

Geometrical parameters : $\alpha = .1$, $\varepsilon = 2.51$, $\lambda = .889$

	f(GHz)	Z/n(kΩ)		Q	
		$\rho_\tau = 0$	$\rho_\tau = 10^{-6}$	$\rho_\tau = 0$	$\rho_\tau = 10^{-6}$
Computer results					
CHIMP (31 harmonics)	1.3765	1194	869	5099	3708
" (3 ")	1.3760	1271	930	5034	3688
" (1 ")	1.3754	1430	1058	4935	3658
BELZ (non-integer n)	1.3750	1431	-	-	-
" (integer n)	1.3749	1431	-	-	-
Second approximation					
$X'_1 = 1.0785$, $\Phi'_1 = .01879$, $\Psi'_1 = 1.543$, $\Sigma'_1 = .7851$	1.3751	1427	1056	4408	3262
First approximation					
$X_1 = 1.871$, $\Phi_1 = .00782$, $\Psi_1 = .2936$	2.385	216	(216)	838	(838)

For this very shallow cavity the first approximation gives quite wrong values, while the second approximation agrees very well with the computer results for one cavity harmonic. The impedance and Q-factor results are less than 20% above the best computer values with 31 harmonics.

<u>Cavity 2</u> (single convolution of large bellows for inflector magnet):

$b = .12$, $d = .17$, $g = .02$, $2\pi R = 1.0$ m, $\gamma = 30$

$\rho_c = 5.10^{-6}$, $\rho_\tau = 0$ or 10^{-6} Ωm (sidewalls included in ρ_c).

Geometrical parameters : $\alpha = .02$, $\varepsilon = 18.85$, $\lambda = .0705$

	f(MHz)	Z/n(kΩ)		Q	
		$\rho_\tau = 0$	$\rho_\tau = 10^{-6}$	$\rho_\tau = 0$	$\rho_\tau = 10^{-6}$
Computer results					
CHIMP (31 harmonics)	932.4	5971	2004	16840	5677
BELZ	932.1	5996	-	-	-
Second approximation					
$X_1' = 1.3772$, $\Phi_1' = .08396$,					
$\Psi_1' = 3.792$, $\Sigma_1' = 3.650$	929.5	5980	2267	14867	5657
First approximation					
$X_1 = 1.244$, $\Phi_1 = .106$,					
$\Psi_1 = .603$	839.7	6163	(6163)	2365	(2365)

For this very short cavity, the first approximation yields reasonable values for f and Z/n when $\rho_\tau = 0$, but is incorrect for the Q-factor and when $\rho_\tau = 10^{-6}$. The second approximation yields acceptable values in all cases.

<u>Cavity 3</u> ("normal cavity")

$b = .1, \quad d = .25, \quad g = .25, \; R = .4 \text{ m} \; ; \; \gamma = 30; \rho_c = \rho_\tau = 10^{-6} \; \Omega\text{m}$

Geometrical parameters : $\alpha = .09947$, $\varepsilon = 1.2566$, ƛ = .4.

	F(MHz)	$^{Z}/_{n}$(kΩ)	Q
Computer results:			
CHIMP (13 harmonics)	481.1	151.8	9965
Second approximation :			
$X_1' = 1.0317$, $\Phi_1' = .5526$			
$\Psi_1' = .9763$, $\Sigma_1' = .01398$	473.5	172.6	10250
First approximation :			
$X_1 = 1.0273$, $\Phi_1 = .5604$			
$\Psi_1 = .9231$	471.5	176.7	9826

For this large cavity the second approximation is only slightly better than the first one, which already yields acceptable values.

8. <u>Conclusions</u>

Improved approximate formulae for the resonant frequency, coupling impedance, and quality factor of the lowest resonant mode of cavities arranged periodically on a tube have been derived and are shown in Table I. The finite conductivity of the outer cavity-wall and the tube-wall have been included in the analysis, while the finite conductivity of the cavity sidewalls can be taken into account approximately by increasing the resistivity of the outer cavity wall in accordance with Eq. (4.9).

The required four correction factors have been defined and evaluated numerically for a wide range of the three parameters α,ε, and λ. The results are given in Table II, and are shown graphically in Figures 1 to 5. Where applicable, also the correction factors of the first approximation are shown in the figures, which appear to become quite inaccurate for short and/or shallow cavities (especially for the resonant frequency). The second approximation has no such limitations of the geometric parameters, and yields values in excellent agreement with the computer results for a single cavity harmonic. Inclusion of more cavity harmonics shows that the actual coupling impedance is actually somewhat lower, while the resonant frequency changes very little. Also the results for finite tube-wll conductivity are found to agree reasonably well with the computer results, while they gave no contribution at all in the first approximation. The various results are compared for a number of odd-shaped cavities in Section 7.

References

1. E. Keil, B. Zotter : "The Coupling Impedance of Corrugated Vacuum Chambers for Cyclic Particle Accelerators", Part I, CERN-ISR-TH/70-30.

2. E. Keil, B. Zotter : Idem, Part II, CERN-ISR-TH/70-33.

3. E. Keil, B. Zotter : "Bellows Resonances in the ISR", BEIC Document 48.

List of Figures

TABLE II

ALPHA = .500

CORRECTION FUNCTION FOR RESONANT FREQUENCY X1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00049	1.00185	1.00377	1.00575	1.00729	1.00797	1.00756	1.00608	1.00378
.20	1.00059	1.00224	1.00456	1.00705	1.00919	1.01036	1.01033	1.00894	1.00634
.50	1.00105	1.00399	1.00827	1.01318	1.01769	1.02100	1.02205	1.02051	1.01579
1.00	1.00193	1.00741	1.01560	1.02526	1.03469	1.04195	1.04507	1.04235	1.03116
2.00	1.00363	1.01396	1.02970	1.04868	1.06784	1.08228	1.08616	1.07422	1.04455
5.00	1.00631	1.02423	1.05162	1.08484	1.11701	1.13596	1.12953	1.09733	1.05051
10.00	1.00760	1.02913	1.06204	1.10182	1.13902	1.15749	1.14392	1.10354	1.05192
20.00	1.00830	1.03180	1.06774	1.11113	1.15101	1.16871	1.15095	1.10649	1.05257
50.00	1.00874	1.03348	1.07132	1.11700	1.15859	1.17563	1.15516	1.10825	1.05296
100.00	1.00889	1.03405	1.07251	1.11896	1.16116	1.17794	1.15660	1.10883	1.05310

CORRECTION FUNCTION FOR COUPLING IMPEDANCE PHI1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.96799	.88079	.75247	.60062	.44044	.29032	.16382	.07109	.01687
.20	.96723	.87872	.75025	.59766	.43825	.28882	.16296	.07076	.01681
.50	.96378	.87071	.73936	.58616	.42893	.28219	.15950	.06942	.01657
1.00	.95818	.85579	.71793	.56398	.41031	.26986	.15292	.06702	.01619
2.00	.94566	.82727	.67814	.52362	.37675	.24778	.14207	.06372	.01587
5.00	.92713	.78324	.62037	.46604	.33186	.22140	.13165	.06147	.01573
10.00	.91853	.76316	.59414	.44075	.31357	.21168	.12836	.06089	.01570
20.00	.91354	.75230	.58035	.42745	.30386	.20676	.12681	.06061	.01568
50.00	.91157	.74555	.57181	.41916	.29774	.20378	.12593	.06044	.01567
100.00	.91009	.74332	.56938	.41673	.29576	.20286	.12559	.06039	.01567

ALPHA = .200

CORRECTION FUNCTION FOR RESONANT FREQUENCY X1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00049	1.00187	1.00377	1.00575	1.00729	1.00797	1.00761	1.00611	1.00379
.20	1.00059	1.00224	1.00459	1.00705	1.00919	1.01036	1.01033	1.00894	1.00634
.50	1.00105	1.00399	1.00827	1.01318	1.01775	1.02100	1.02205	1.02051	1.01584
1.00	1.00194	1.00741	1.01560	1.02526	1.03469	1.04203	1.04534	1.04309	1.03351
2.00	1.00371	1.01427	1.03046	1.05023	1.07077	1.08764	1.09553	1.08899	1.06197
5.00	1.00749	1.02900	1.06291	1.10704	1.15627	1.19757	1.20525	1.16014	1.08243
10.00	1.01037	1.04015	1.08786	1.15178	1.22501	1.27905	1.26013	1.17872	1.08552
20.00	1.01226	1.04752	1.10441	1.18157	1.26978	1.32441	1.28145	1.18478	1.08655
50.00	1.01353	1.05243	1.11542	1.20157	1.29962	1.35113	1.29232	1.18786	1.08710
100.00	1.01398	1.05413	1.11933	1.20867	1.31014	1.36004	1.29573	1.18883	1.08727

CORRECTION FUNCTION FOR COUPLING IMPEDANCE PHI1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.96809	.88040	.75254	.60009	.44049	.29035	.16375	.07107	.01687
.20	.96733	.87880	.74994	.59772	.43829	.28885	.16297	.07076	.01681
.50	.96385	.87076	.73940	.58620	.42868	.28221	.15951	.06943	.01657
1.00	.95746	.85593	.71807	.56413	.41050	.26989	.15287	.06695	.01613
2.00	.94494	.82597	.67602	.52100	.37392	.24501	.13972	.06228	.01547
5.00	.92000	.76374	.59225	.43304	.29955	.19477	.11563	.05591	.01502
10.00	.90002	.71922	.53345	.37375	.25033	.16470	.10552	.05443	.01496
20.00	.88804	.69097	.49694	.33803	.22244	.14986	.10186	.05394	.01493
50.00	.87819	.67267	.47490	.31600	.20517	.14195	.10007	.05370	.01492
100.00	.87491	.66673	.46621	.30810	.19932	.13936	.09953	.05363	.01492

ALPHA = .100

CORRECTION FUNCTION FOR RESONANT FREQUENCY X1

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00048	1.00193	1.00366	1.00583	1.00729	1.00822	1.00740	1.00611	1.00384
.20	1.00059	1.00218	1.00453	1.00714	1.00925	1.01020	1.01038	1.00882	1.00637
.50	1.00103	1.00390	1.00844	1.01301	1.01756	1.02075	1.02189	1.02063	1.01579
1.00	1.00191	1.00735	1.01538	1.02542	1.03518	1.04187	1.04534	1.04328	1.03350
2.00	1.00374	1.01424	1.03057	1.05023	1.07040	1.08739	1.09521	1.08905	1.06276
5.00	1.00749	1.02900	1.06312	1.10770	1.15749	1.20087	1.21372	1.17469	1.09277
10.00	1.01059	1.04106	1.09003	1.15733	1.23871	1.31039	1.30324	1.20693	1.09734
20.00	1.01302	1.05066	1.11217	1.19912	1.30721	1.39352	1.33905	1.21468	1.09845
50.00	1.01489	1.05780	1.12823	1.22916	1.35809	1.44103	1.35227	1.21758	1.09892
100.00	1.01556	1.06026	1.13430	1.24091	1.37570	1.45554	1.35568	1.21844	1.09907

CORRECTION FUNCTION FOR COUPLING IMPEDANCE PHI1

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.96970	.87848	.75409	.59945	.44049	.28967	.16409	.07107	.01685
.20	.96733	.88072	.75071	.59709	.43801	.28930	.16289	.07087	.01680
.50	.96550	.87368	.73709	.58745	.42950	.28286	.15976	.06932	.01658
1.00	.96006	.85793	.72118	.56290	.40841	.27030	.15287	.06680	.01613
2.00	.94113	.82703	.67445	.52100	.37537	.24557	.14015	.06227	.01545
5.00	.92215	.76575	.59078	.43052	.29798	.19295	.11385	.05463	.01480
10.00	.90077	.71809	.53324	.36755	.24067	.15436	.09848	.05224	.01471
20.00	.88929	.67852	.48093	.31462	.20058	.12995	.09237	.05155	.01469
50.00	.85369	.65149	.45792	.29036	.17352	.11852	.09044	.05148	.01467
100.00	.84410	.65771	.44263	.27349	.16714	.11551	.09050	.05138	.01466

ALPHA = .050

CORRECTION FUNCTION FOR RESONANT FREQUENCY X1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00049	1.00187	1.00377	1.00575	1.00729	1.00797	1.00761	1.00611	1.00379
.20	1.00059	1.00224	1.00459	1.00705	1.00919	1.01036	1.01033	1.00894	1.00634
.50	1.00105	1.00399	1.00827	1.01318	1.01775	1.02100	1.02205	1.02051	1.01584
1.00	1.00194	1.00741	1.01560	1.02526	1.03469	1.04203	1.04534	1.04309	1.03351
2.00	1.00371	1.01427	1.03046	1.05023	1.07077	1.08764	1.09553	1.08905	1.06276
5.00	1.00751	1.02906	1.06307	1.10737	1.15725	1.20054	1.21372	1.17595	1.09640
10.00	1.01060	1.04112	1.09036	1.15765	1.23908	1.31402	1.31635	1.22013	1.10330
20.00	1.01315	1.05112	1.11342	1.20173	1.31626	1.42420	1.37156	1.23054	1.10455
50.00	1.01533	1.05967	1.13333	1.24050	1.38524	1.50222	1.38672	1.23325	1.10494
100.00	1.01619	1.06303	1.14114	1.25560	1.41142	1.52350	1.38983	1.23389	1.10504

CORRECTION FUNCTION FOR COUPLING IMPEDANCE PHI1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.96809	.88040	.75254	.60009	.44049	.29035	.16375	.07107	.01687
.20	.96733	.87880	.74994	.59772	.43829	.28885	.16297	.07076	.01681
.50	.96385	.87076	.73941	.58620	.42868	.28221	.15951	.06943	.01657
1.00	.95746	.85593	.71808	.56413	.41050	.26989	.15287	.06695	.01613
2.00	.94494	.82597	.67602	.52100	.37392	.24501	.13973	.06227	.01545
5.00	.91965	.76319	.59164	.43282	.29882	.19357	.11400	.05465	.01472
10.00	.89863	.71598	.52834	.36637	.24131	.15307	.09618	.05126	.01458
20.00	.88114	.67706	.47824	.31563	.19602	.12212	.08798	.05051	.01456
50.00	.86759	.64673	.43837	.27589	.16225	.10382	.08585	.05032	.01455
100.00	.85843	.63627	.42291	.26210	.15136	.09918	.08549	.05026	.01455

ALPHA = .020

CORRECTION FUNCTION FOR RESONANT FREQUENCY X1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00049	1.00187	1.00377	1.00575	1.00729	1.00797	1.00761	1.00611	1.00379
.20	1.00059	1.00224	1.00459	1.00705	1.00919	1.01036	1.01033	1.00894	1.00634
.50	1.00105	1.00399	1.00827	1.01318	1.01775	1.02100	1.02205	1.02051	1.01584
1.00	1.00194	1.00741	1.01560	1.02526	1.03469	1.04203	1.04534	1.04309	1.03351
2.00	1.00371	1.01427	1.03046	1.05023	1.07077	1.08764	1.09553	1.08905	1.06276
5.00	1.00751	1.02906	1.06307	1.10737	1.15725	1.20054	1.21372	1.17599	1.09681
10.00	1.01060	1.04112	1.09036	1.15765	1.23908	1.31402	1.31715	1.22390	1.10654
20.00	1.01315	1.05112	1.11347	1.20159	1.31675	1.42964	1.38855	1.24011	1.10824
50.00	1.01543	1.06047	1.13441	1.24336	1.39466	1.54593	1.40968	1.24298	1.10860
100.00	1.01645	1.06410	1.14385	1.26213	1.43026	1.58007	1.41215	1.24342	1.10866

CORRECTION FUNCTION FOR COUPLING IMPEDANCE PHI1

EPS \ LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.96809	.88040	.75254	.60029	.44049	.29035	.16375	.07187	.01687
.20	.96733	.87880	.74994	.59772	.43829	.28885	.16297	.07076	.01681
.50	.96385	.87076	.73941	.58620	.42868	.28221	.15951	.06943	.01657
1.00	.95746	.85593	.71808	.56413	.41050	.26989	.15287	.06695	.01613
2.00	.94494	.82597	.67602	.52100	.37392	.24501	.13973	.06227	.01545
5.00	.91965	.76319	.59164	.43282	.29882	.19357	.11400	.05463	.01472
10.00	.89863	.71598	.52834	.36637	.24131	.15312	.09608	.05098	.01452
20.00	.88207	.67789	.47771	.31569	.19591	.12060	.08565	.04985	.01448
50.00	.86718	.64613	.43650	.27294	.15822	.09468	.08281	.04953	.01447
100.00	.85948	.62897	.41811	.25619	.14352	.08814	.08247	.04960	.01447

ALPHA = .010

CORRECTION FUNCTION FOR RESONANT FREQUENCY X1

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00093	1.00347	1.00703	1.01073	1.01353	1.01465	1.01358	1.01042	1.00581
.20	1.00098	1.00368	1.00752	1.01150	1.01463	1.01605	1.01529	1.01227	1.00764
.50	1.00130	1.00495	1.01017	1.01603	1.02111	1.02430	1.02477	1.02208	1.01632
1.00	1.00208	1.00796	1.01668	1.02689	1.03665	1.04393	1.04683	1.04392	1.03373
2.00	1.00379	1.01457	1.03106	1.05113	1.07187	1.08879	1.09639	1.08948	1.06285
5.00	1.00754	1.02919	1.06334	1.10778	1.15774	1.20120	1.21415	1.17611	1.09683
10.00	1.01061	1.04118	1.09052	1.15790	1.23944	1.31435	1.31731	1.22397	1.10707
20.00	1.01315	1.05115	1.11353	1.20197	1.31699	1.42981	1.39015	1.24266	1.10944
50.00	1.01543	1.06010	1.13447	1.24344	1.39503	1.55550	1.41767	1.24628	1.10983
100.00	1.01647	1.06419	1.14412	1.26295	1.43344	1.60498	1.41999	1.24665	1.10988

CORRECTION FUNCTION FOR COUPLING IMPEDANCE PHI1

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.96476	.87377	.74319	.59078	.43352	.28615	.16197	.07057	.01682
.20	.96418	.87262	.74135	.58926	.43231	.28537	.16146	.07037	.01677
.50	.96214	.86642	.73379	.58062	.42501	.28032	.15866	.06925	.01655
1.00	.95673	.85328	.71503	.56114	.40839	.26881	.15243	.06684	.01612
2.00	.94465	.82412	.67432	.51954	.37292	.24436	.13951	.06223	.01545
5.00	.91896	.76262	.59080	.43235	.29868	.19324	.11391	.05463	.01472
10.00	.89908	.71575	.52755	.36603	.24105	.15309	.09609	.05097	.01451
20.00	.88244	.67780	.47796	.31536	.19569	.12066	.08540	.04965	.01446
50.00	.86533	.64428	.43600	.27317	.15833	.09267	.08171	.04940	.01445
100.00	.86238	.63004	.41770	.25444	.14174	.08345	.08148	.04938	.01445

ALPHA = .500

CORRECTION FUNCTION FOR TUBEWALL RESISTIVITY SIG

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.00013	.00099	.00313	.00676	.01165	.01716	.02233	.02621	.02862
.20	.00013	.00100	.00317	.00688	.01195	.01785	.02379	.02935	.03661
.50	.00015	.00118	.00381	.00849	.01542	.02448	.03566	.05012	.07763
1.00	.00021	.00162	.00532	.01228	.02340	.03975	.06353	.10362	.21571
2.00	.00032	.00254	.00862	.02110	.04415	.08554	.16086	.30131	.56085
5.00	.00067	.00529	.01841	.04751	.10693	.22092	.40861	.64419	.85917
10.00	.00093	.00733	.02554	.06634	.15052	.30830	.54339	.78658	.95000
20.00	.00110	.00866	.03016	.07847	.17840	.36290	.62255	.86380	.99581
50.00	.00122	.00956	.03331	.08675	.19747	.40006	.67498	.91316	1.02418
100.00	.00126	.00988	.03445	.08975	.20431	.41334	.69339	.93024	1.03387

CORRECTION FUNCTION FOR QUALITY FACTOR PSI

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00000	.99726	.98500	.95639	.90992	.85419	.80965	.80487	.86667
.20	1.00005	.99748	.98551	.95729	.91120	.85565	.81135	.80766	.87415
.50	1.00029	.99852	.98805	.96194	.91795	.86430	.82241	.82542	.91206
1.00	1.00076	1.00055	.99312	.97138	.93216	.88283	.84764	.87068	1.03760
2.00	1.00165	1.00446	1.00299	.99042	.96310	.92978	.92800	1.03547	1.34934
5.00	1.00307	1.01076	1.01948	1.02482	1.02594	1.03956	1.11873	1.31929	1.61808
10.00	1.00375	1.01381	1.02767	1.04267	1.06038	1.10160	1.21704	1.43423	1.69882
20.00	1.00412	1.01547	1.03214	1.05257	1.08004	1.13758	1.27243	1.49516	1.73903
50.00	1.00435	1.01651	1.03492	1.05878	1.09262	1.16094	1.30810	1.53346	1.76373
100.00	1.00443	1.01686	1.03585	1.06087	1.09693	1.16903	1.32048	1.54659	1.77213

ALPHA = .200

CORRECTION FUNCTION FOR TUBEWALL RESISTIVITY SIG

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.00013	.00099	.00314	.00677	.01167	.01719	.02237	.02626	.02868
.20	.00013	.00100	.00318	.00689	.01198	.01788	.02384	.02940	.03666
.50	.00015	.00118	.00381	.00850	.01543	.02451	.03568	.05008	.07644
1.00	.00021	.00162	.00530	.01221	.02320	.03913	.06150	.09593	.17894
2.00	.00029	.00230	.00770	.01857	.03791	.07098	.12876	.24465	.62462
5.00	.00042	.00329	.01147	.03012	.07217	.17722	.47475	1.23739	2.53422
10.00	.00051	.00405	.01457	.04121	.11443	.35925	1.10766	2.32793	3.40197
20.00	.00062	.00495	.01806	.05306	.15917	.55313	1.62908	2.94623	3.77995
50.00	.00073	.00580	.02132	.06356	.19797	.72066	2.01269	3.34218	4.00282
100.00	.00077	.00615	.02259	.06768	.21346	.78786	2.15603	3.48227	4.07913

CORRECTION FUNCTION FOR QUALITY FACTOR PSI

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00000	.99727	.98500	.95639	.90992	.85420	.80972	.80493	.86672
.20	1.00005	.99748	.98553	.95729	.91120	.85566	.81137	.80769	.87419
.50	1.00029	.99852	.98805	.96195	.91801	.86431	.82242	.82538	.91103
1.00	1.00076	1.00055	.99312	.97135	.93204	.88248	.84626	.86457	1.00547
2.00	1.00169	1.00458	1.00320	.99046	.96184	.92356	.90773	.99157	1.41034
5.00	1.00365	1.01308	1.02463	1.03343	1.03611	1.05033	1.20142	1.83735	3.11475
10.00	1.00514	1.01939	1.04078	1.06781	1.10566	1.21816	1.70611	2.76298	3.88188
20.00	1.00611	1.02352	1.05140	1.09126	1.15816	1.37037	2.11016	3.28223	4.21347
50.00	1.00676	1.02622	1.05828	1.10667	1.19496	1.49006	2.40045	3.61059	4.40757
100.00	1.00699	1.02714	1.06064	1.11193	1.20803	1.53595	2.50739	3.72579	4.47373

ALPHA = .100

CORRECTION FUNCTION FOR TUBEWALL RESISTIVITY SIG

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.00013	.00099	.00314	.00677	.01167	.01718	.02238	.02627	.02868
.20	.00013	.00101	.00318	.00689	.01198	.01789	.02384	.02940	.03666
.50	.00015	.00119	.00380	.00851	.01544	.02452	.03568	.05009	.07642
1.00	.00021	.00162	.00532	.01220	.02316	.03914	.06150	.09597	.17888
2.00	.00029	.00230	.00769	.01857	.03797	.07100	.12861	.24291	.58243
5.00	.00041	.00323	.01116	.02901	.06841	.16191	.41196	1.16358	3.59559
10.00	.00041	.00326	.01161	.03177	.08512	.27896	1.21770	3.74712	6.74628
20.00	.00039	.00305	.01111	.03241	.10424	.51292	2.60293	5.85134	8.14054
50.00	.00040	.00322	.01223	.03756	.13285	.82762	3.76109	7.12140	8.85971
100.00	.00042	.00348	.01287	.03958	.14873	.97760	4.20437	7.56250	9.09672

CORRECTION FUNCTION FOR QUALITY FACTOR PSI

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00000	.99730	.98492	.95646	.90992	.85441	.80953	.80493	.86680
.20	1.00005	.99744	.98549	.95736	.91126	.85552	.81142	.80756	.87424
.50	1.00029	.99846	.98816	.96182	.91785	.86410	.82228	.82552	.91095
1.00	1.00075	1.00052	.99297	.97148	.93245	.88234	.84626	.86481	1.00540
2.00	1.00170	1.00457	1.00328	.99046	.96154	.92334	.90730	.99016	1.37350
5.00	1.00365	1.01306	1.02467	1.03346	1.03504	1.04266	1.15775	1.78088	4.01514
10.00	1.00524	1.01974	1.04125	1.06786	1.10061	1.18862	1.82107	3.97452	6.68866
20.00	1.00648	1.02490	1.05419	1.09530	1.15864	1.40300	2.92596	5.74251	7.85478
50.00	1.00743	1.02867	1.06340	1.11509	1.20606	1.64454	3.83336	6.79776	8.45095
100.00	1.00776	1.02996	1.06674	1.12223	1.22366	1.75193	4.17539	7.16142	8.54639

ALPHA = .050

CORRECTION FUNCTION FOR TUBEWALL RESISTIVITY SIG

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.00013	.00099	.00314	.00677	.01167	.01719	.02237	.02627	.02868
.20	.00013	.00100	.00318	.00689	.01198	.01789	.02384	.02940	.03666
.50	.00015	.00118	.00381	.00850	.01543	.02451	.03568	.05008	.07644
1.00	.00021	.00162	.00530	.01221	.02320	.03913	.06150	.09593	.17889
2.00	.00029	.00230	.00770	.01857	.03791	.07097	.12865	.24291	.58191
5.00	.00041	.00322	.01117	.02911	.06848	.16181	.40829	1.09836	3.55466
10.00	.00041	.00321	.01135	.03106	.08211	.25452	1.07735	4.44790	11.26129
20.00	.00033	.00258	.00924	.02634	.07872	.40008	3.55031	10.42477	16.13347
50.00	.00024	.00193	.00708	.02179	.08100	.83391	6.76762	14.29802	18.38573
100.00	.00023	.00184	.00686	.02191	.08903	1.11830	7.98239	15.52153	19.04168

CORRECTION FUNCTION FOR QUALITY FACTOR PSI

EPS LAM	.10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00000	.99727	.98500	.95639	.90992	.85420	.80972	.80493	.86673
.20	1.00005	.99748	.98553	.95729	.91120	.85566	.81137	.80769	.87419
.50	1.00029	.99852	.98805	.96195	.91801	.86431	.82242	.82538	.91103
1.00	1.00076	1.00055	.99312	.97135	.93204	.88248	.84626	.86457	1.00542
2.00	1.00169	1.00458	1.00320	.99046	.96184	.92356	.90764	.99016	1.37305
5.00	1.00366	1.01310	1.02464	1.03323	1.03486	1.04226	1.15485	1.72662	3.96546
10.00	1.00525	1.01977	1.04139	1.06780	1.09921	1.17491	1.72007	4.55320	10.10103
20.00	1.00654	1.02506	1.05440	1.09460	1.15188	1.35182	3.70320	9.44610	13.80331
50.00	1.00764	1.02946	1.06501	1.11653	1.19951	1.71135	6.23346	12.55522	15.48189
100.00	1.00807	1.03117	1.06909	1.12496	1.21917	1.92344	7.17653	13.52642	15.96648

ALPHA = .020

CORRECTION FUNCTION FOR TUBEWALL RESISTIVITY SIG

EPS	LAM .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.00013	.00099	.00314	.00677	.01167	.01719	.02237	.02627	.02868
.20	.00013	.00100	.00318	.00689	.01198	.01789	.02384	.02940	.03666
.50	.00015	.00118	.00381	.00850	.01543	.02451	.03568	.05008	.07644
1.00	.00021	.00162	.00530	.01221	.02320	.03913	.06150	.09593	.17889
2.00	.00029	.00230	.00770	.01857	.03791	.07097	.12865	.24291	.58191
5.00	.00041	.00322	.01117	.02911	.06848	.16181	.40829	1.09754	3.37620
10.00	.00041	.00321	.01135	.03106	.08210	.25402	1.04527	4.08464	14.85865
20.00	.00033	.00256	.00914	.02589	.07597	.35002	3.65918	17.30543	35.04027
50.00	.00019	.00151	.00542	.01581	.05248	.68827	13.94391	33.95810	45.95182
100.00	.00013	.00101	.00369	.01132	.04415	1.26371	18.50391	38.70732	48.52785

CORRECTION FUNCTION FOR QUALITY FACTOR PSI

EPS	LAM .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00000	.99727	.98500	.95639	.90992	.85420	.80972	.80493	.86673
.20	1.00005	.99748	.98553	.95729	.91120	.85566	.81137	.80769	.87419
.50	1.00029	.99852	.98805	.96195	.91801	.86431	.82242	.82538	.91103
1.00	1.00076	1.00055	.99312	.97135	.93204	.88248	.84626	.86457	1.00542
2.00	1.00169	1.00458	1.00320	.99046	.96184	.92356	.90764	.99016	1.37305
5.00	1.00366	1.01310	1.02464	1.03323	1.03486	1.04226	1.15485	1.72601	3.82367
10.00	1.00525	1.01977	1.04139	1.06780	1.09920	1.17456	1.69556	4.25688	12.25809
20.00	1.00654	1.02506	1.05441	1.09453	1.15072	1.32146	3.79736	14.47074	24.63463
50.00	1.00768	1.02960	1.06518	1.11602	1.19117	1.65114	11.62627	26.42166	30.82706
100.00	1.00819	1.03160	1.06980	1.12508	1.20929	2.09168	14.99633	29.72013	32.25189

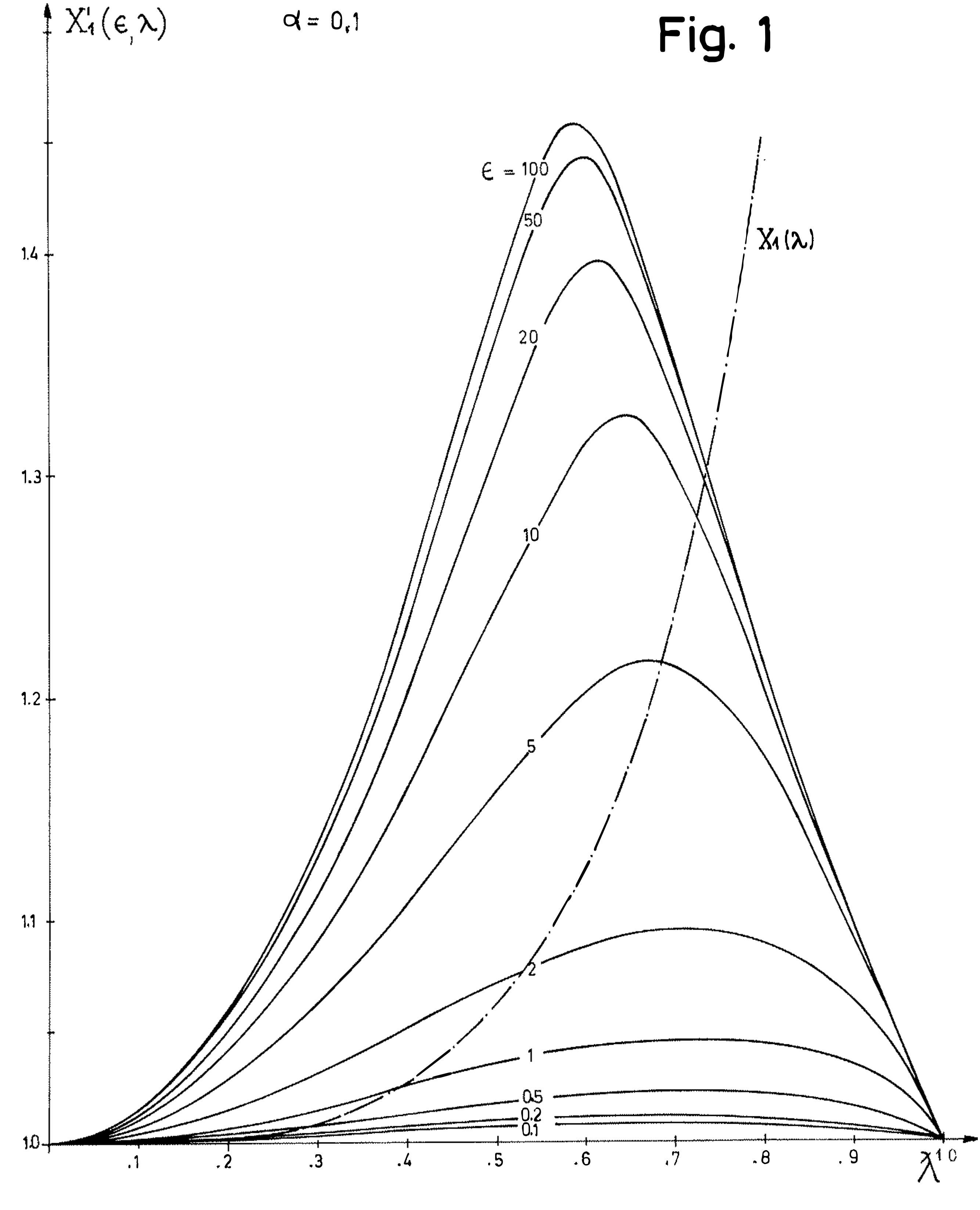
ALPHA = .010

CORRECTION FUNCTION FOR TUBEWALL RESISTIVITY SIG

EPS	LAM .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	.00025	.00193	.00610	.01327	.02309	.03431	.04488	.05252	.05558
.20	.00025	.00192	.00608	.01323	.02306	.03435	.04511	.05325	.05828
.50	.00025	.00192	.00612	.01348	.02394	.03667	.05038	.06469	.08683
1.00	.00027	.00209	.00677	.01536	.02856	.04673	.07052	.10459	.18488
2.00	.00032	.00254	.00847	.02025	.04089	.07542	.13428	.24878	.58667
5.00	.00042	.00329	.01142	.02971	.06972	.16405	.41221	1.10325	3.38254
10.00	.00041	.00324	.01144	.03129	.08263	.25545	1.04948	4.06943	13.98249
20.00	.00033	.00257	.00917	.02600	.07615	.35066	3.45333	18.41210	53.25826
50.00	.00019	.00150	.00538	.01565	.05065	.56157	21.48351	61.35056	88.79092
100.00	.00012	.00090	.00322	.00947	.03294	1.27781	34.40729	75.75423	96.82728

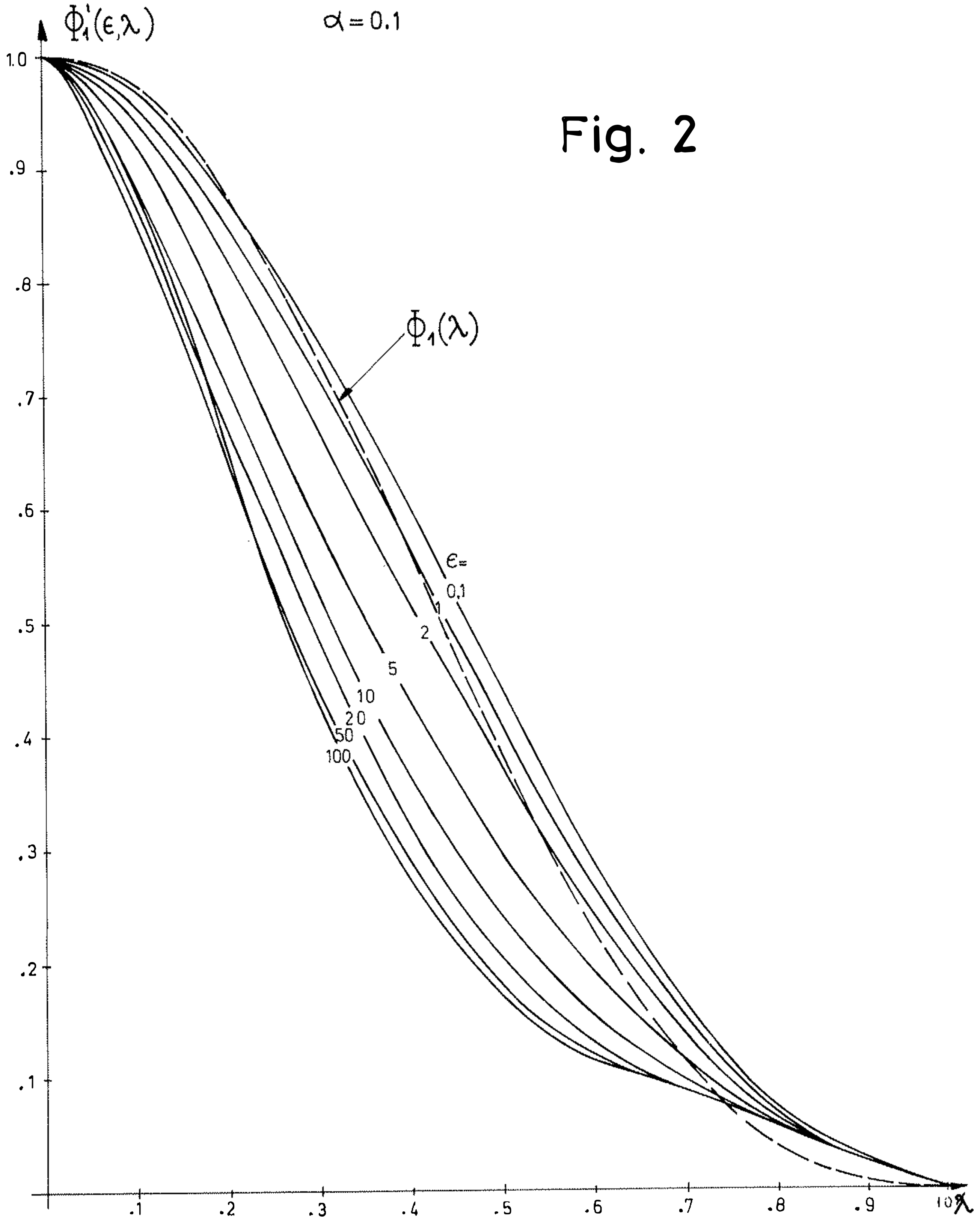
CORRECTION FUNCTION FOR QUALITY FACTOR PSI

EPS	LAM .10	.20	.30	.40	.50	.60	.70	.80	.90
.10	1.00024	.99834	.98781	.96191	.91868	.86649	.82589	.82555	.89062
.20	1.00027	.99846	.98812	.96240	.91933	.86712	.82647	.82632	.89333
.50	1.00043	.99918	.98982	.96550	.92367	.87220	.83250	.83654	.92012
1.00	1.00084	1.00094	.99414	.97343	.93542	.88720	.85221	.87107	1.01063
2.00	1.00173	1.00479	1.00374	.99155	.96367	.92631	.91130	.99458	1.37720
5.00	1.00368	1.01317	1.02485	1.03366	1.03561	1.04373	1.15754	1.73055	3.82913
10.00	1.00525	1.01980	1.04150	1.06801	1.09963	1.17547	1.69860	4.24568	11.67086
20.00	1.00654	1.02508	1.05444	1.09460	1.15094	1.32183	3.64109	15.03237	31.24883
50.00	1.00769	1.02962	1.06519	1.11599	1.19037	1.56833	16.71202	41.72154	45.69242
100.00	1.00820	1.03162	1.06982	1.12485	1.20545	2.12710	25.45105	49.94129	48.77667



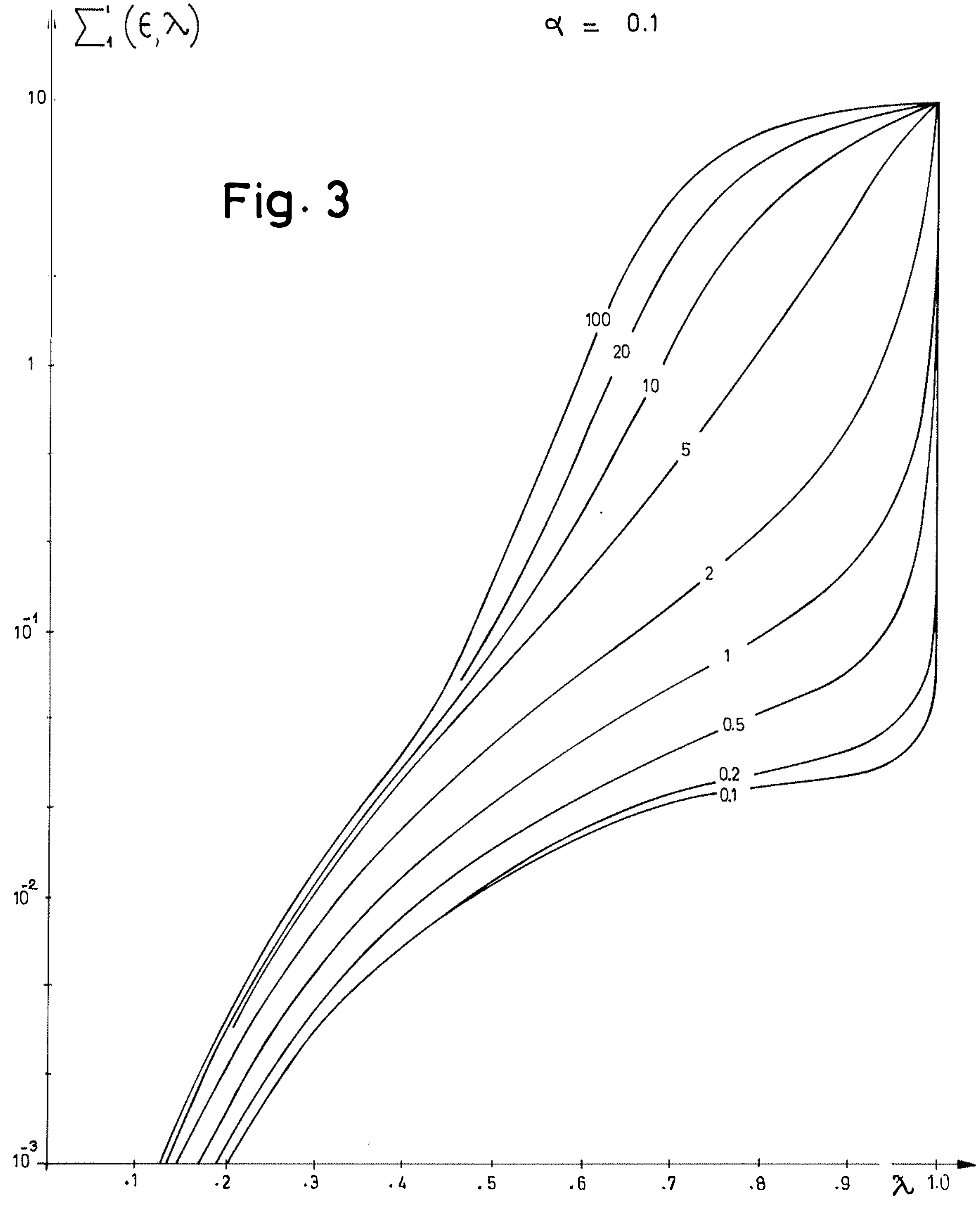
X'1(ε, λ)
α = 0,1
ε = 100
50
20
10
5
2
1
0.5
0.2
0.1
X1(λ)
1.4
1.3
1.2
1.1
1.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
λ

Fig. 1

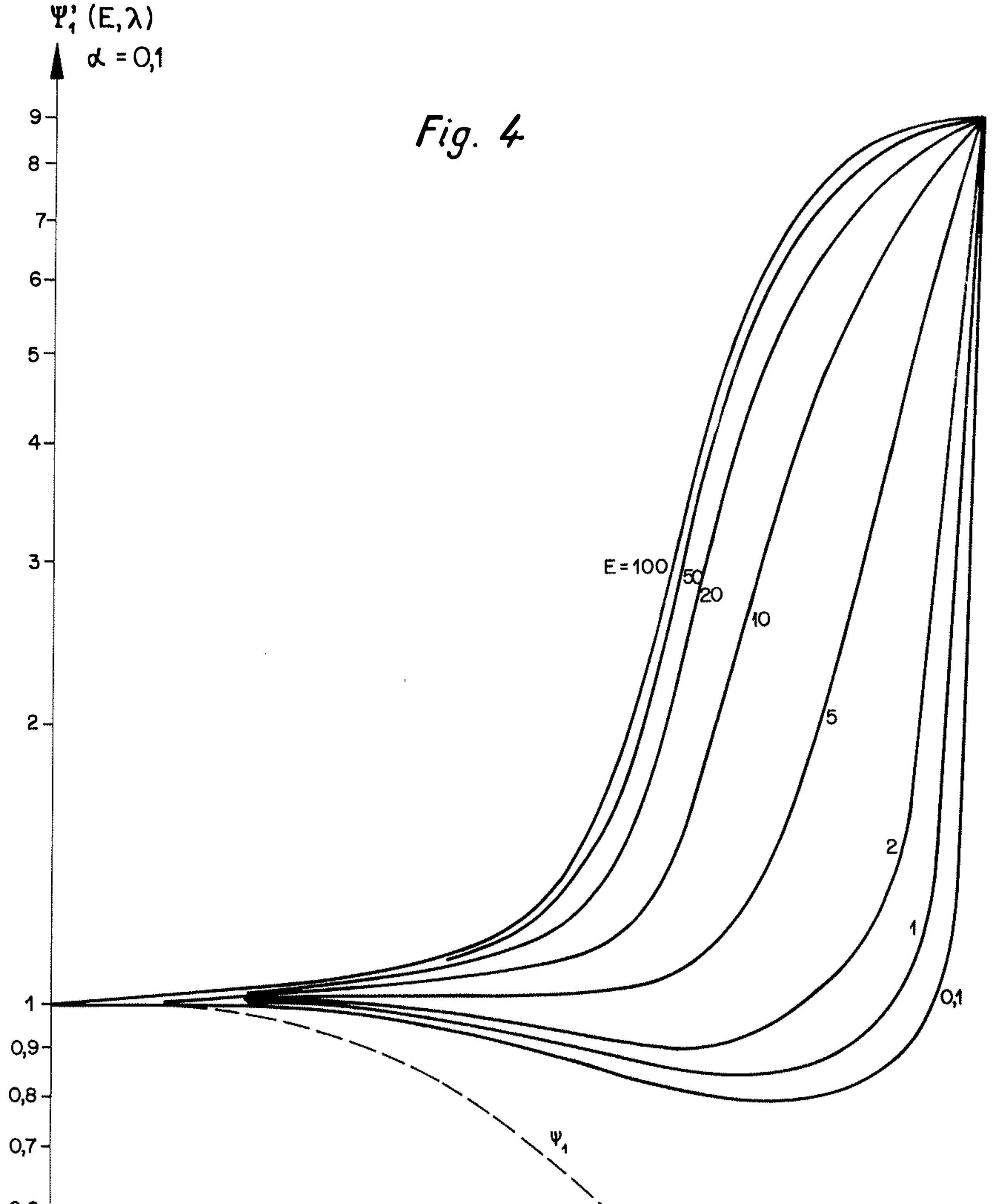


$\Phi_1'(\epsilon,\lambda)$
$\alpha = 0.1$
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
$\Phi_1(\lambda)$
ϵ=
0,1
1
2
5
10
20
50
100
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
λ

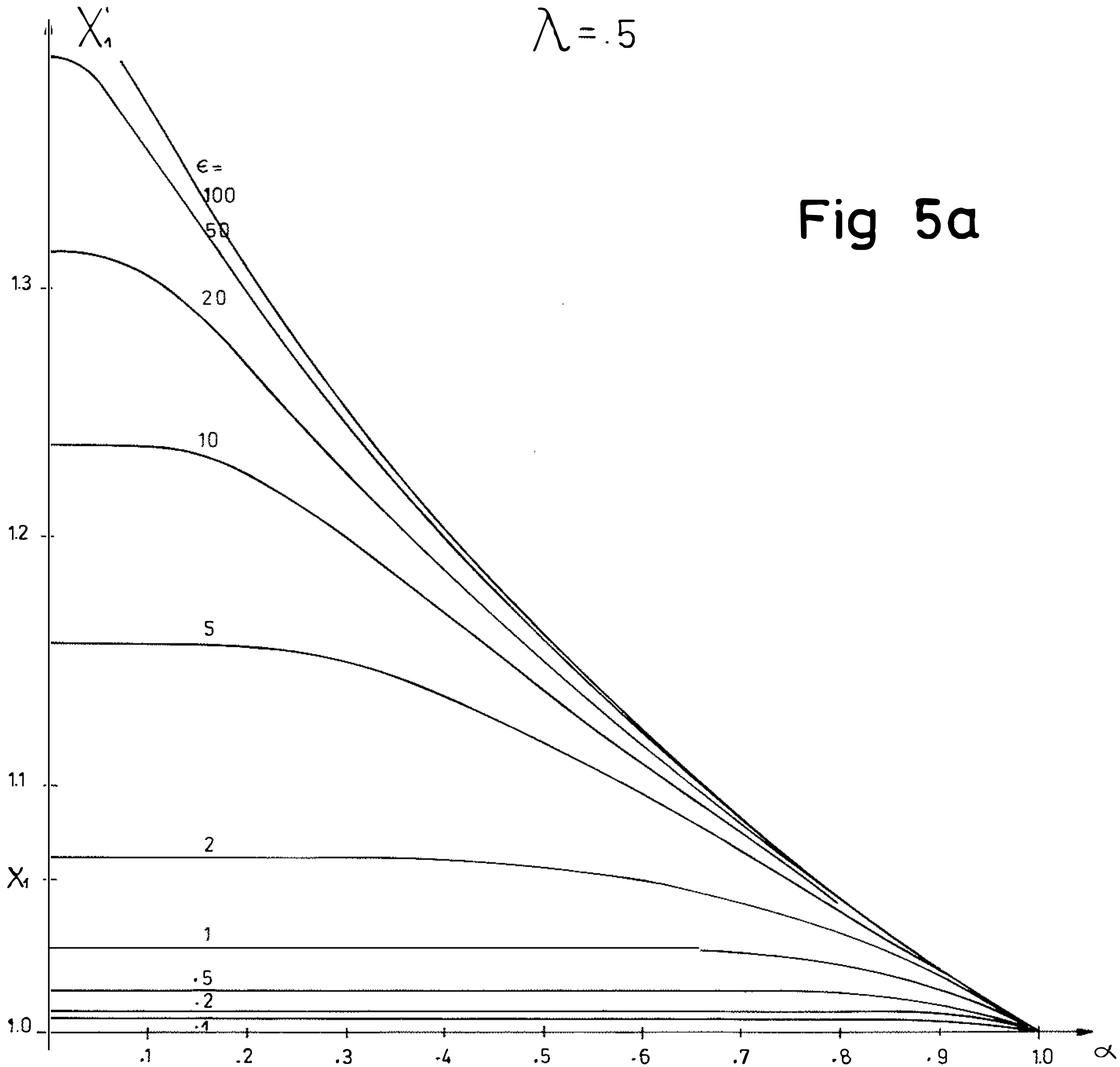
Fig. 2



$\sum_1^1(\epsilon,\lambda)$
$\alpha = 0.1$
Fig. 3
10
1
10^{-1}
10^{-2}
10^{-3}
100
20
10
5
2
1
0.5
0.2
0.1
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
λ



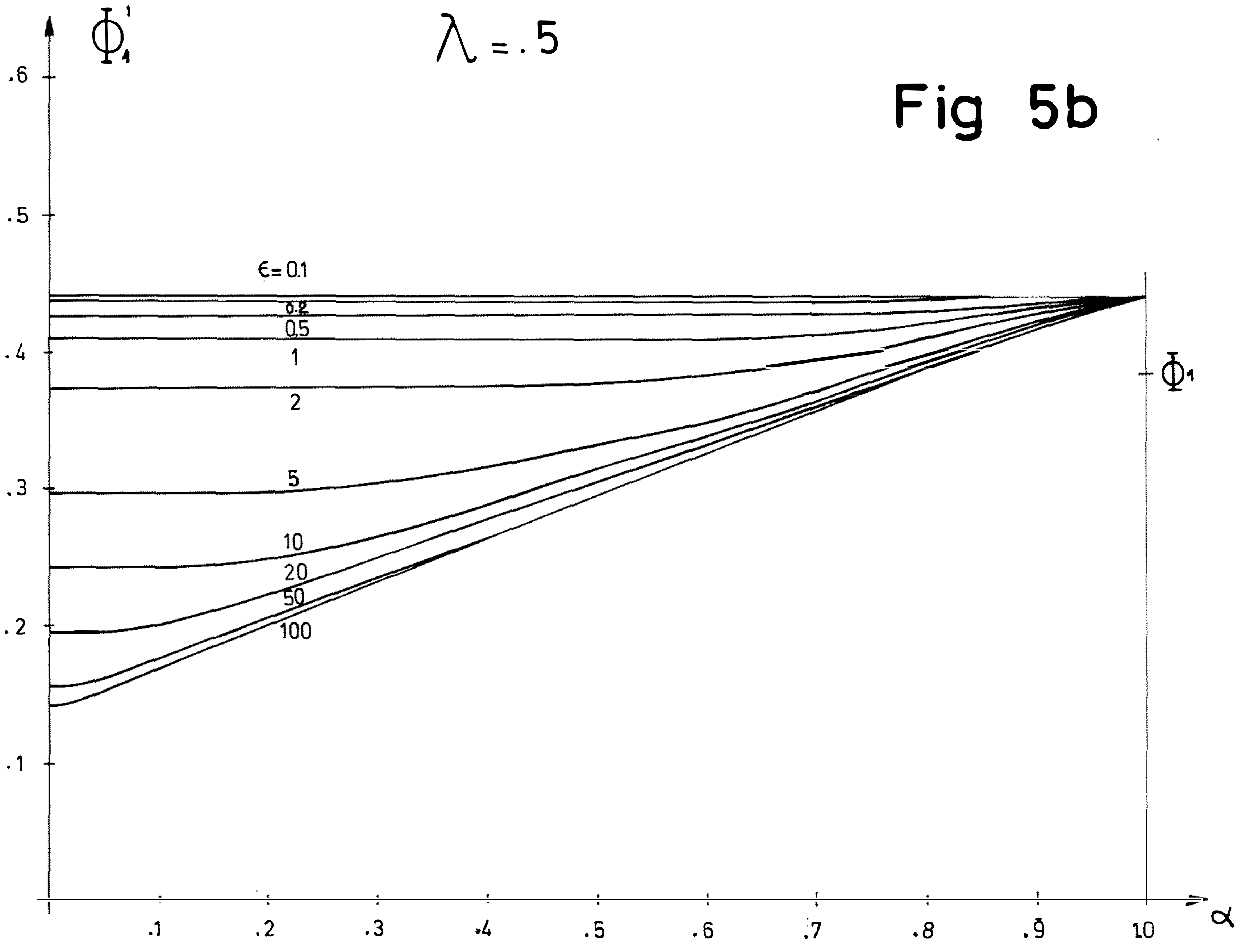
Ψ'₁ (E,λ)
α = 0,1
Fig. 4
9
8
7
6
5
4
3
2
1
0,9
0,8
0,7
0,6
0,5
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
λ
E = 100
50
20
10
5
2
1
0,1
ψ₁



X_1'
$\lambda = .5$
$\epsilon =$
100
50
20
10
5
2
1
.5
.2
.1
1.3
1.2
1.1
X_1
1.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
α

Fig 5a

Fig 5b



Φ_1'
$\lambda = .5$
$\epsilon = 0.1$
0.2
0.5
1
2
5
10
20
50
100
Φ_1
.6
.5
.4
.3
.2
.1
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
α

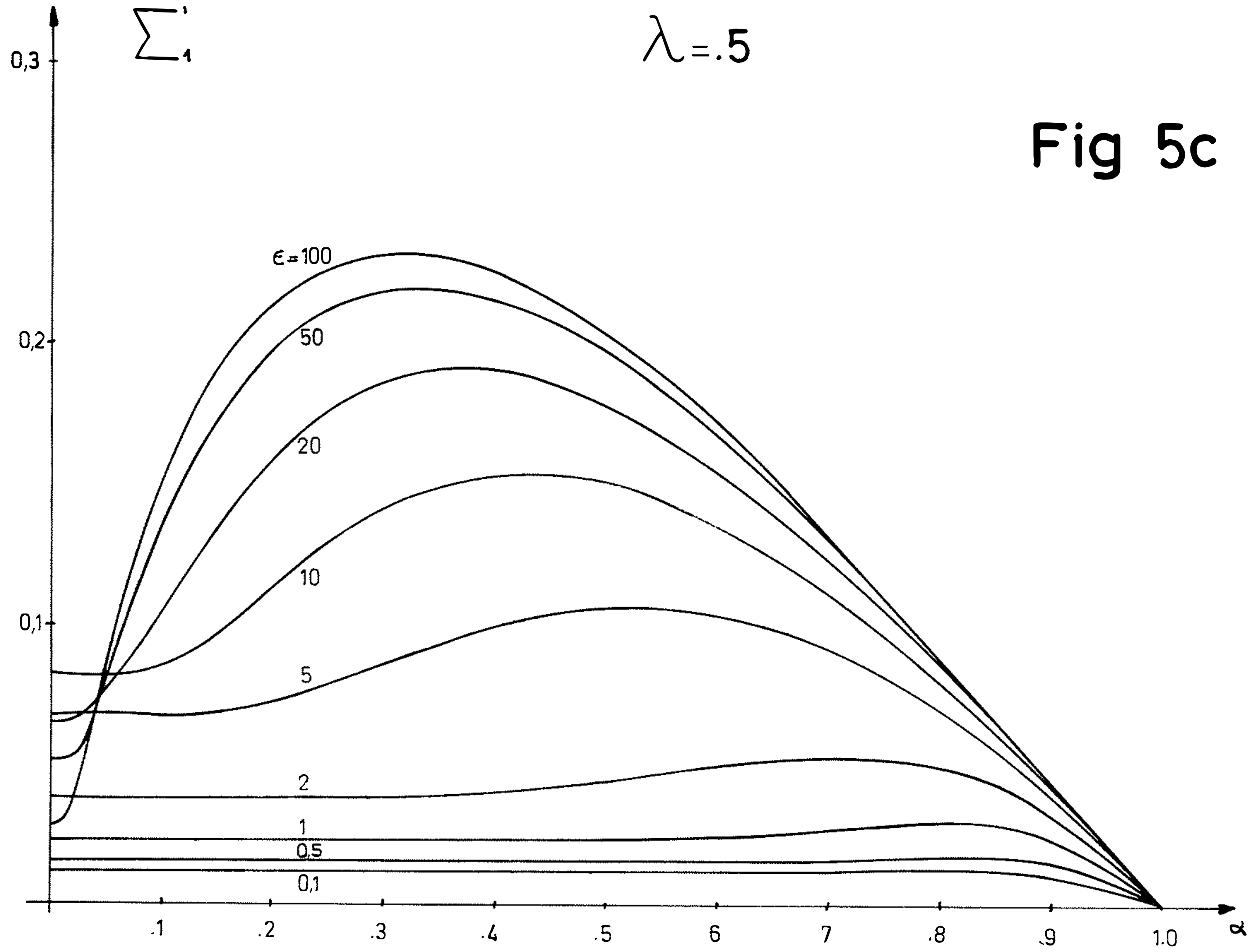


Fig 5c