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# MEASUREMENT OF THERMAL NOISE OF THE PROTON BEAM IN THE STORAGE RING NAP-M*

by

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## Abstract

The measurements of beam noise spectra and noise power with normal and cooled proton beams carried out at NAP-M are described. The characteristics of the noise due to the interaction of particles in the cooled beam are discussed.



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## 1. INTRODUCTION

Due to the finite number of particles the coasting beam moving in the storage ring induces signals on pick-up electrodes. The signal from one particle is a short pulse which repeats with the revolution frequency $w_i$. The spectrum of this signal consists of lines spaced by the revolution frequency $w_i$. The total spectrum from all particles of the beam will consist of lines around the harmonics of the mean revolution frequency. The width of such a peak near the n-th harmonic is determined by the revolution frequency spread $n \cdot \delta w$. The power of such a signal (i.e. integral over the spectrum around given harmonic $n\,w_o$) does not depend on the revolution frequency spread $\delta w$; it is proportional to the total number of particles in the beam N, which is characteristic of Schottky noise. This model gives a good description of the noise spectrum for a beam with low intensity and large momentum spread, when the interaction between particles is negligible.

However, the first measurements of the beam noise carried out at NAP-M[1], have shown that the noise power of the cooled beam is substantially smaller than the level predicted by the usual theory of the beam Schottky noise[2]. The explanation of this phenomenon has been given in a theoretical paper[3].

According to this paper, if the beam density is increased or the momentum spread in the beam is decreased (which means a decrease in the beam temperature) the interaction of beam particles strongly affects the behaviour of the density fluctuations. In a storage ring below transition repulsion of the particles does not allow the development of density fluctuations because the interaction energy of the particles, which create these fluctuations, cannot be substantially higher than the beam temperature. Therefore, when the temperature is decreased below some threshold value, the level of the fluctuations also decreases and the Schottky noise of the beam becomes thermal noise with the power decreasing proportionally to the beam temperature.

Apart from changing the noise power the interaction of the particles leads to a change in the evolution of the fluctuations. The fields induced by fluctuations propogate along the beam in two waves (in the direction and against the beam motion) with a propogation speed depending on the beam intensity. The presence of these waves splits the noise spectrum into two peaks around the harmonic of the revolution frequency.

In this paper we describe the experimental investigation of the thermal noise on the coasting beam carried out in NAP-M[4]. Electron cooling has been used to control the proton beam temperature and to study its thermal noise over a wide range of temperatures.

## 2. INSTRUMENTATION

The noise spectrum measurements have been carried out at the 5th and 8th harmonic of the revolution frequency using an electrostatic pick-up electrode. The block diagram of the 8th harmonic channel is given in Fig. 1. In order to enhance the sensitivity, the pick-up capacity $C_d$ and inductance L making up a resonance circuit, were tuned to the harmonic frequency (17.6 MHz). The Q-value of the circuit is Q = 500, and its resistance at resonance is 40 kΩ. The noise in the system is determined practically by the thermal noise of this circuit and is equal to $10^{-18}$ W in the frequency band $\pm$ 1 kHz (the amplifier noise is 12 db smaller). In the latest scheme a double transformation of the frequency was used. The output signal of the second mixer (synchronous detector) is converted by means of an analog-digital converter (ADC) for input to a computer. The bandwidth of the signal measured by the ADC is equal to 1 kHz, the repetition rate of the measurements by the ADC is 2 kHz; the conversion accuracy is 0.1%.

The analysis of the noise spectrum was carried out by a fast Fourier-transformation program. For this, blocks 512 words long were used containing the results of the measurements made during .25 sec. Using the method of parallel Fourier-analysis, the time to measure the spectrum of width $\Delta F$ at given resolution $\Delta f$ is $\Delta F/\Delta f$ times smaller than the time consumed when the method of consecutive sweeping of the spectrum is applied. This point is especially important when making measurements of non-stationary processes, because it allows to decrease substantially the influence of instabilities of the storage ring guiding field and, correspondingly, of the revolution frequencies on the results.

## 3. RESULTS

The main effect which was found already in the first measurements of the noise signals of the cooled beam was a strong reduction of the signal power[1). Improvement of the instrumentation enables us to measure reliably the extremely weak signals of the cooled beam. Fig. 2 shows the dependence of noise power on the proton current $J_p$ for a normal and a cooled beam (curve a and b respectively). One can see that due to the cooling the noise power decreases approximately by two orders of magnitude, and it does not depend on current $J_p$ in the range of $0.5 \div 10$ μA for a cooled beam. For the normal beam, one can see that the noise power increases linearly with $J_p$.

The voltage on the pick-up electrode and the amplitude of the azimuthal harmonic of the beam density $A_n$ are related by

$$\langle |U_n|^2 \rangle = 2 \left( e \frac{Q}{C_d} \cdot \frac{l_d}{2\pi R_o} \right)^2 \langle |A_n|^2 \rangle$$

Here Q is the Q-value at the input resonance circuit, ld and $C_d$ are length and capacity of the pick-up electrode, $2\pi R_o$ is the orbit circumference, e is the charge of the electron. The brackets <> mean averaging over the beam. This relation was used for the calculation of the harmonic amplitude $A_n$ shown in Fig. 2.

The azimuthal beam density can be written in the form :

$$\rho(\theta,t) = \sum_{n=-\infty}^{\infty} \frac{A_n(t)}{2\pi} \exp(in\theta), \qquad A_n(t) = \sum_{a=1}^{N} e^{in\theta_a(t)} \tag{1}$$

with 'a' labelling the individual particles, N the number of particles in the beam. If the particles do not interact their motion is uncorrelated and it is clear that

$$\langle A_n \rangle = 0, \quad \langle |A_n|^2 \rangle = N.$$

As it was shown in reference 13, particle interaction via surrounding media leads to suppression of density fluctuations according to

$$\langle |A_n|^2 \rangle = \frac{N}{1 + N/N_{th}} \tag{2}$$

where $N_{th}$ is determined by equating the longitudinal coherent tune shift

$$\Omega_n^2 = n^2 \frac{Ne^2 w_s w_s'}{2\pi R_o} \left(\frac{Z_n}{n}\right)$$

to the revolution frequency spread $n\delta w$ around $nw_s$. This yields

$$N_{th} = \frac{2\pi R_o \; \delta w^2}{e^2 w_s w_s'} \left(\frac{n}{Z_n}\right) \tag{3}$$

with $Z_n$ the vacuum chamber impedance, $w_s = w_o(p_s)$ the revolution frequency of reference particle, $w_s' = dw_o/dp$.

Eq. (2) shows that the dispersion of the density fluctuations $\langle |A_n|^2 \rangle$ is proportional to N (Schottky noise of the beam) if the number of particles is small $N \ll N_{th}$. One can see that the measured values of the density fluctuations are in good agreement with Eq. (2) (see fig. 2, curve a).

During cooling, $N_{th}$ can become smaller than N. In this case, the beam noise power does not depend on the number of particles but it is determined by the beam temperature:

$$\langle |A_n|^2 \rangle = N_{th} = \frac{2\pi R_o \; \delta w^2}{e^2 w_s \; w_s'} \left(\frac{n}{Z_n}\right), \qquad N \gg N_{th} \tag{2a}$$

These features of the beam noise can be seen clearly in Fig. 2. The increase of the noise power with proton current for $J_p > 10\ \mu A$ (curve b) can probably be traced to the increase in momentum spread of the proton beam due to intrabeam scattering[6].

In Fig. 3 the noise spectra of the normal (curve a) and the cooled (curve b,c) beam are shown. The most interesting feature in the spectrum of the cooled beam, when current is high, is the presence of two sharp peaks (curve b). As it was pointed out, this feature is related to the propogation of waves of fluctuations in the beam direction and opposite it. In reference 3), it was shown that the distance between these peaks is equal to $2\Omega_n$. When the beam current decreases the separation between these two maxima decreases and, at lower current (curve c, $J_p = .5\ \mu A$ ), they can no longer be resolved by our equipment.

The dependence of coherent tune shift $\Omega_n/n$ on the proton beam current, worked out from the distance between the maxima at the 5th and 8th harmonic of the revolution frequency is shown on Fig. 4. One can see that $\Omega_n/n$ is proportional to $\sqrt{J_p}$ in agreement with Equ. (3).

From the known storage ring parameters[4] an effective impedance of the vacuum chamber $Z_n$ can be calculated

$$\frac{Z}{n} = \left(\frac{\Omega_n}{n w_s}\right)^2 \frac{p_s v_s}{e J_p} \cdot \frac{1}{\eta} \qquad (5)$$

Here $J_p = eNw_s/2\pi$ , $\eta = 1/\gamma^2 - 1/\gamma_t^2$, $\gamma$-relativistic factor, $\gamma_t$ - transition energy over rest energy. The value Z/n calculated from equ. (5), turned out to be 557 ± 50 Ω. This is in good agreement with the impedance of the vacuum chamber with perfectly conducting walls with a chamber radius of b = 3.5 cm and a proton beam radius $a_p = 0.01$ cm, which is equal to 540 Ω.

The data obtained from the measurement of the noise spectrum can also be used for the calculation of the momentum spread in the proton beam. For large momentum spreads ($N_{th} \gg N$) $\Delta p/p$ determines the bandwidth of the noise spectrum:

$$\frac{\Delta p}{p} = \frac{1}{\eta} \left(\frac{\Delta w_n}{n w_s}\right)$$

For instance, the bandwidth of the spectrum of the normal beam, given in Fig. 3a, is equal to 250 Hz at the 8th harmonic of the revolution frequency ($w_s/2\pi = 2.2.10^6$ Hz). This gives for the momentum spread

$$\frac{\Delta p}{p} = 1.8.10^{-4}.$$

In the other case, after cooling ($N_{th} << N$), the width of the spectrum is determined by the beam intensity N (Fig. 3, b + c), and the momentum spread determines the noise power $<|A_n|^2>$ by equ. (2a) which can be written now in the form:

$$\frac{W_n}{W_{no}} = \frac{<|A_n|^2>}{N} = \left(\frac{n\delta w}{\Omega_n}\right)^2$$

where $W_{no}$ is the noisepower of the normal beam with the same intensity (see curve a, Fig. 2). Using the data given in Fig. 2, a,b, and Fig. 4, we get for the cooled beam with a current $J_p < 10\ \mu A$:

$$\frac{\Delta p}{p} = \frac{1}{\eta} \frac{A_n}{n w_s} \sqrt{\frac{W_n}{W_{no}}} \simeq 1.4 . 10^{-6}$$

It is interesting to note that this momentum spread corresponds to an extraordinarily small longitudinal temperature of the beam $T = 1^oK$ in the rest frame of the beam.

The equilibrium value of the momentum spread is determined by the balance between the power of friction and diffusion. When the cooling rates are decreased, the momentum spread and, consequently, the noise power, are increased. In our measurements, the value of the cooling rate was changed by modulating the electron gun voltage with a frequency of f = 5 kHz[1]. The dependence of noise power upon the modulation amplitude is given in Fig. 5. One can see that the noise power is enhanced with increasing amplitude of modulation.

The results of this paper indicate that the specific features of the noise spectra due to interaction of particles should be taken into account when the noise signals from a coasting beam in a storage ring are analyzed, say, for diagnostic purpose.

## Acknowledgements

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## Figure Captions

Fig. 1 Block diagram of the circuit used to measure the beam noise at the 8th harmonic of the revolution frequency.

Fig. 2 Noise power versus proton current a) normal beam, b) cooled beam.

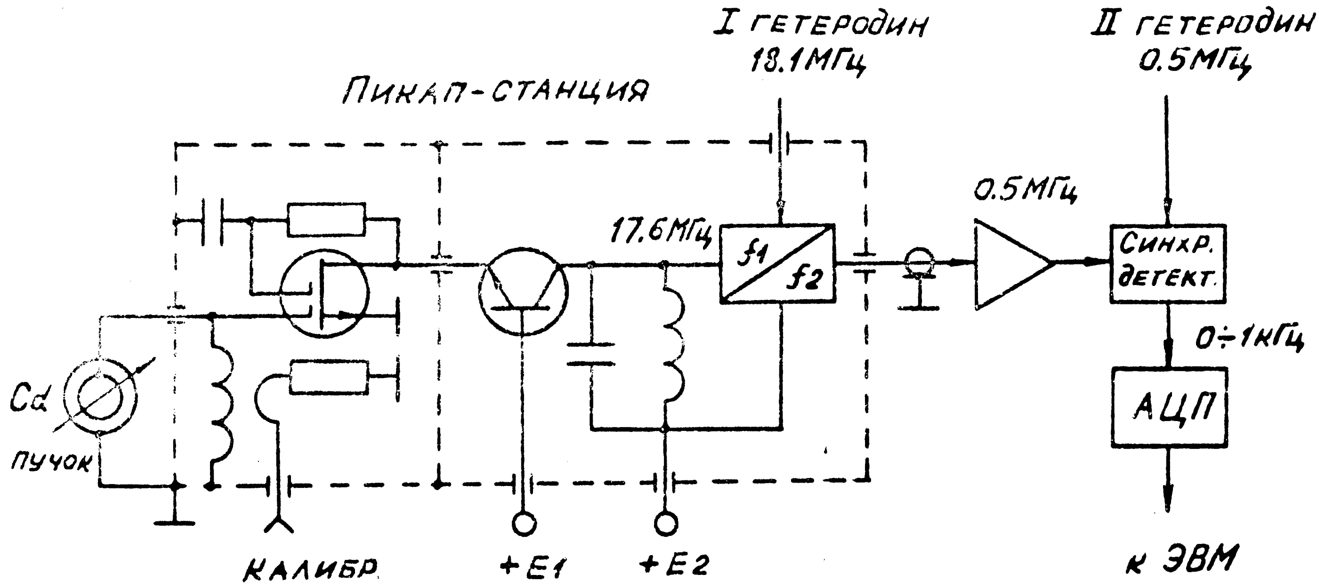
Fig. 3 Noise spectra a) normal beam $J_p$ = 30 μA
b) cooled beam $J_p$ = 30 μA
c) cooled beam $J_p$ = 5 μA

Fig. 4 Longitudinal coherent frequency shift over harmonic number $\Omega n/n$ versus proton current :

+ : measurement at the 8th harmonic of the revolution frequency

x : measurement at the 5th harmonic

Fig. 5 Noise power of the proton beam versus the amplitude of the gun voltage modulation.

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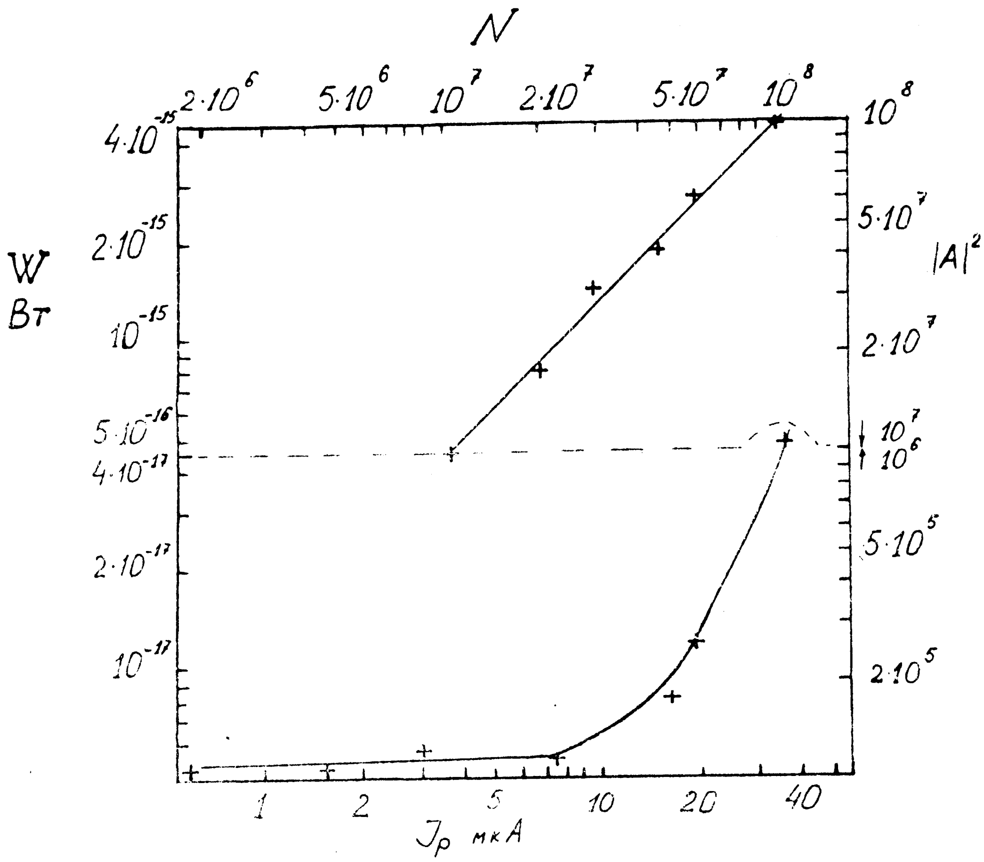
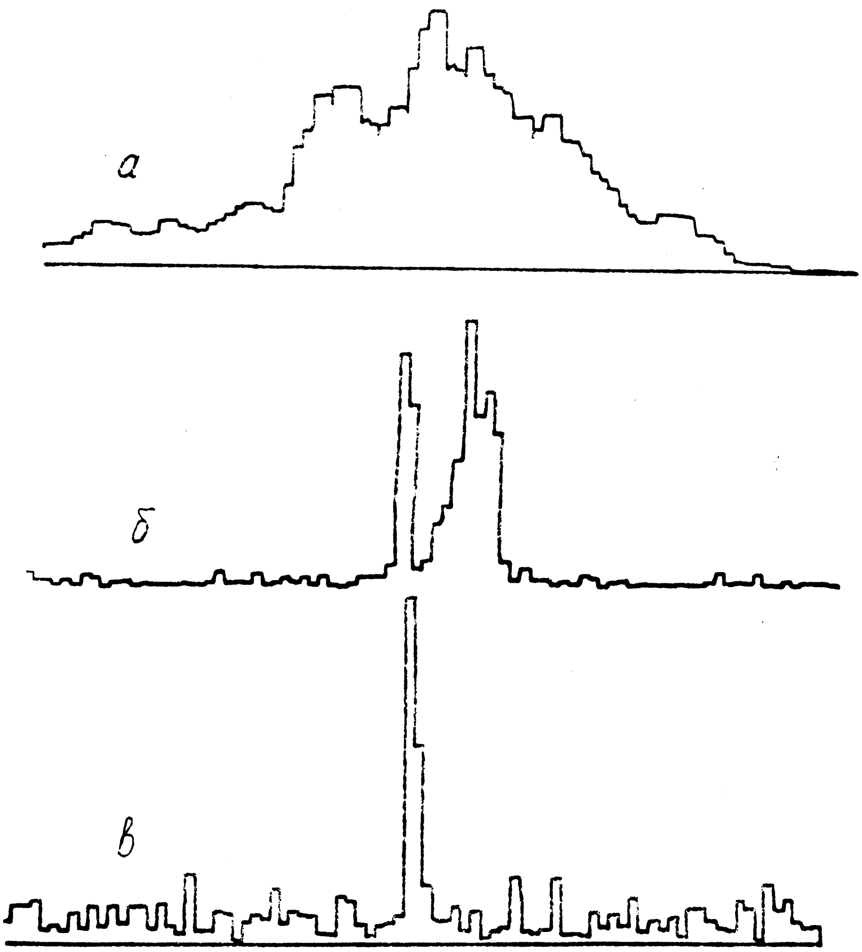



Fig. 2 Noise power versus proton current

a) normal beam
b) cooled beam




<u>Fig. 3</u> Noise spectra a) normal beam $J_p = 30\ \mu A$
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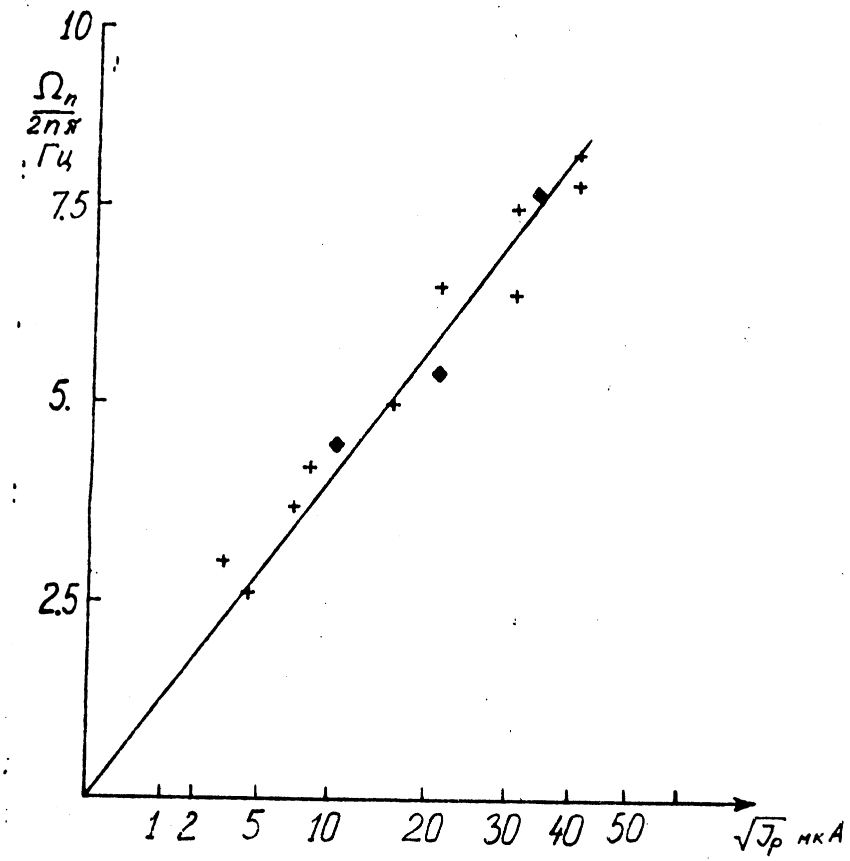


Fig. 4 Longitudinal coherent frequency shift over harmonic number Ωn/n versus proton current:
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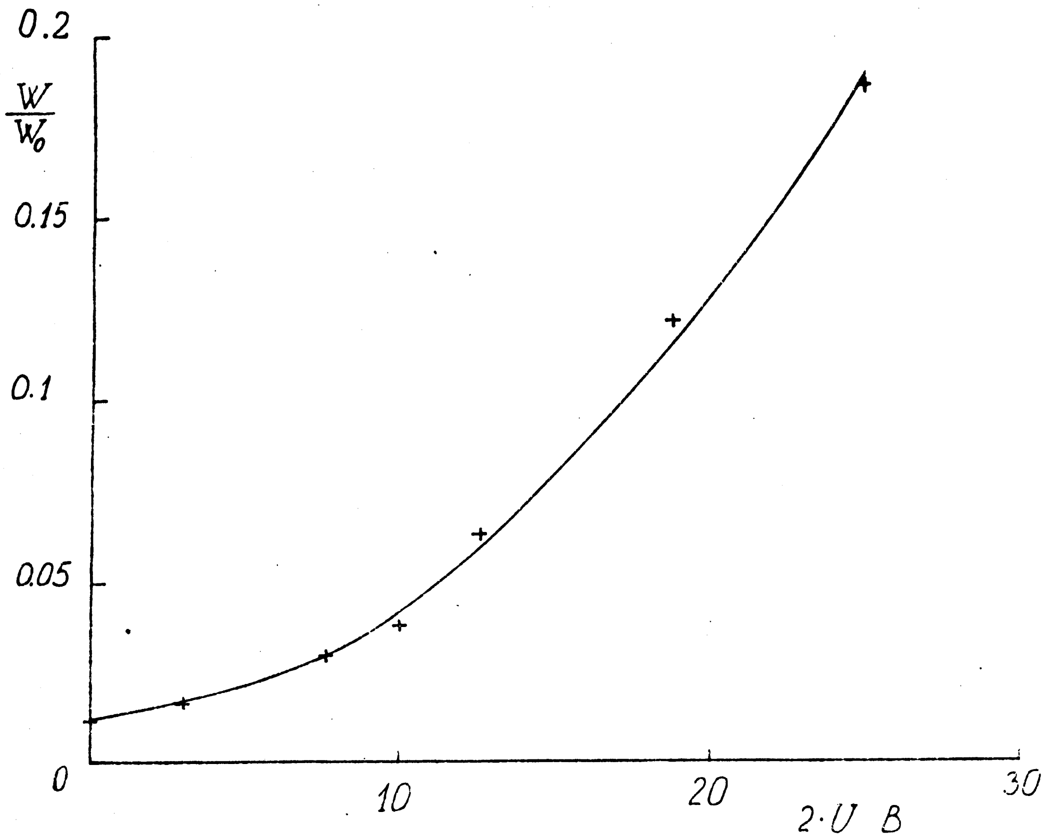


Fig. 5 Noise power of the proton beam versus the amplitude of the gun voltage modulation