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REFLECTIONS ON INSTABILITIES STIMULATED BY RECENT OBSERVATIONS ON ADONE

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1. Observations on Adone

On a recent visit to CERN, C. Pellegrini reported on the following observations on Adone.

Over the past months a radial transverse instability was observed which has a threshold of 150 μA/bunch at the injection energy E_i = 300 MeV. The threshold current depends on the parameter as follows:

$$I_{th} \sim L(\Delta Q)\ E^{1/2}$$

where L is the bunch length, E the electron energy and ΔQ the tune spread in the beam. All these dependences were experimentally checked. The threshold does not depend on the number of bunches in the machine, and is independent of the Q value. This suggests that its mechanism is not a long memory effect like the resistive wall instability. Instead, it seems to be due to one single bunch interacting with itself only during a single revolution.

Since the separating electrodes which occupy 80-90% of the circumference were the suspected source of the instability they were all removed during December 1968, and according to very preliminary results, in early January 1969, the threshold of an instability was now 1.5 to 2 mA without any stabilisation.

The new threshold still seems to be due to a radial instability of a single bunch which does not depend on the Q value. There is also a vertical instability with a slightly higher threshold.

2. Physical Model

Pellegrini proposed the following physical model. The particles in the front of the bunch leave wake fields in the vacuum chamber, separating electrodes and other pieces of equipment. These fields influence the particles in the rear of the bunch. In fact, each particle is influenced by the fields of all the preceding particles. The model supposes that the wake fields persist during the passage of the bunch, but that they decay between bunches.

Because of synchrotron oscillations, particles are continuously interchanged between the front and the rear of a bunch. Pellegrini's extensive general theory analyzes the stability of bunches under the combined influence of the near-wake fields and synchrotron oscillations.

We may make an approximate analysis and obtain a rough threshold, by noting that the theory for this instability fits into the general framework of transverse coherent instabilities. We need to consider the real and imaginary part of the Q shift, $Re(\Delta Q)$ and $Im(\Delta Q)$, due to the wake fields. Due to the time delay between the front and rear particles of a bunch and because the coherent transverse oscillation has frequency $Q\omega_0$ the force seen by rear particles is shifted in phase (from that of front particles) by an angle φ, where

$$\varphi \approx QL/R \ . \tag{1}$$

Consequently $Im(\Delta Q)$ becomes approximately:

$$Im(\Delta Q) = \varphi \ Re(\Delta Q) \tag{2}$$

Since the bunch length is much smaller than the betatron wavelength, we have $Im(\Delta Q) < Re(\Delta Q)$. Under these circumstances, the threshold for the coherent instability is essentially determined by the value of $Re(\Delta Q)$.

The sign of $Im(\Delta Q)$ is actually such as to give damping of the centre-of-mass bunch motion. However, modes in which the bunch distorts are unstable, and have an $Im(\Delta Q)$ of the same order of magnitude as given by Eq. (2). Moreover, oscillations of the wake fields (resonant modes on the separating electrodes) will lead to instability of the centre-of-mass mode, with the possibility of even larger values of φ than in Eq. (2).

3. Stability considerations for Adone

We start from formula (4.15) in the theory of the transverse resistive wall instability for bunched beams[1]. This gives the eigenvalue λ_m of the m-th mode for M equally populated bunches:

$$\frac{\lambda_m}{N} = \frac{e^2 U}{L} + \frac{8\pi^{1/2}}{3} \left(\frac{2\pi R}{L} \right)^{1/2} \frac{e^2 W}{2\pi R \omega_o^{1/2}} + \frac{e^2 W}{2\pi R \omega_o^{1/2}} M^{1/2} \, G\left(2\pi, \frac{m + Q}{M}\right) . \tag{3}$$

In this formula, use has been made of (3.23) and (3.24) and the imaginary part of the self-field terms has been neglected. The quantity W is defined as

$$W = (4c\beta^2/b^3) \ (4 \ \pi \ \sigma)^{-1/2} . \tag{4}$$

The relation between the eigenvalue λ_m and the Q shift is

$$Q^2 - Q_o^2 = -\lambda_m/m_o \ \gamma\omega_o^2 . \tag{5}$$

According to Ref. 1), U is given by their formula (3.3):

$$U = -(2/\gamma^2) \ (a^{-2} - b^{-2}) . \tag{6}$$

In the absence of a proper calculation of the effect of separating electrodes on bunched relativistic electron beams it appears to be a reasonable precaution not to rely on the cancellation of electric and magnetic image effects on the separating electrodes. Thus Eq. (6) may be replaced by a formula which has a self-field term and a term due to electrostatic images alone (eeping magnetic images alone, would give a U of the same absolute value):

$$U \approx - 2(a^{-2} \gamma^{-2} - b^{-2}) \approx 2b^{-2} \qquad (7)$$

Table I

Adone Parameter List

Machine radius	R	17 m
Q value	Q	3.25
Harmonic number	M	3
Chamber radius	b	5 cm
Horizontal beam radius	a	0.1 cm
Bunch length	L	1 m
Injection energy/rest energy	γ	600
Conductivity of stainless steel	σ	10^{16} s^{-1}
Angular frequency	ω_o	$2 \cdot 10^7$ s^{-1}
$6 \cdot 10^{10}$ electrons/bunch correspond to a current of		100 mA

The above formulae and the parameters shown in Table I are sufficient for an evaluation of the instability in Adone. We find:

$$U = 0.08 \text{ cm}^{-2} \qquad (8)$$

$$W = 2.7 \text{ cm}^{-2} \text{ s}^{-1/2} \qquad (9)$$

$$\text{ImG} < 2 \qquad (10)$$

We have neglected the real part of G, because its contribution is much smaller than that of the U term. We may use the following relations between Eq. (5) and the real and imaginary part of the Q shift:

$$Re(\Delta Q) + iIm(\Delta Q) = \frac{1}{2Q_o} (Q^2 - Q_o^2) \, . \tag{11}$$

Finally we obtain (from resistive wall theory, but with U modified as in Eq. (7)):

$$Re(\Delta Q) = - 1.5 \cdot 10^{-13} N \tag{12}$$

$$Im(\Delta Q) = - 2.5 \cdot 10^{-17} N \tag{13}$$

The Q spread δQ in the beam due to the octupole component of the magnetic guide field - without the excitation of separate stabilizing octupoles - was experimentally found to be:

$$\delta Q = 10^{-3} \, a^2 \, . \tag{14}$$

A comparison between the Q spread δQ and the Q shift (Eq. (12)) allows us to employ the stability criterion

$$\Delta Q \leq \delta Q \tag{15}$$

and to calculate a threshold population N_U and a threshold current I_U at which Landau damping ceases to be effective:

$$N_U = 6 \cdot 10^7 \qquad I_U = 100 \; \mu A \, . \tag{16}$$

In electron storage rings with a radiation damping time τ_s a bunch with a current above the threshold (Eq. (16)) is not necessarily unstable. As long as the growth time

$$\tau_g = 1/\omega_o \, Im(\Delta Q) \tag{17}$$

is larger than the synchrotron damping time τ_s the motion of the bunch is in fact stable. This yields a new set of thresholds which we call N_V and I_V:

$$N_V = 2 \cdot 10^9 \qquad I_V = 3 \text{ m A} . \tag{18}$$

Observation in Adone gives a threshold of about 150 μA which is in good agreement with our estimate of 100 μA, obtained from the real Q-shift arising from non-cancelling image effects (Eq. 7 and Eq. 12). However, the imaginary part, as evaluated from the resistive wall theory (Eq. 13), must have been underestimated by a large factor. The model described above, however, removes this difficulty since it yields values of Im(ΔQ) which are very much larger than in resistive wall theory. We find from Eqs (1) and (2):

$$\text{Im}(\Delta Q) = - 3 \cdot 10^{-12} N . \tag{19}$$

Thus, at a current about three times threshold, namely 300 μA, we find:

$$\tau_g = 8 \text{ ms} . \tag{20}$$

This figure is in rather good agreement with the experimental observations. Resonant modes would, as remarked above, make τ_g even smaller.

4. Consequences for the CERN Intersecting Storage Rings

The same physical model may be employed to estimate related phenomena in the ISR. We present two alternative ways of making approximate calculations for the ISR. The first method consists in taking the U_c for cylindrical geometry from the transverse resistive wall paper for coasting beams[2], given there as formula (3.17a):

$$U_c = - e^2 N \left[1 - (a/b)^2\right] / 2\pi \, Q \, \omega_o \gamma^3 m_o \, R \, a^2 \, , \qquad (21)$$

and to remove from this formula the cancellation of electric and magnetic image terms by the factor γ^2. Remembering that $Re(\Delta Q) = U_c/\omega_o$ we find:

$$Re(\Delta Q) \approx \frac{N r_o R}{2\pi Q \, \gamma \, b^2} \left(\frac{M_c L_c}{2\pi R} \right) . \qquad (22)$$

The last bracket is just the fraction of the ISR circumference occupied by clearing electrodes.

The second method is based on the paper by Laslett[3] on clearing electrodes. He works in rectangular geometry and obtains in the long wavelength limit for a terminating impedance which is not too badly matched to the electrode impedance (his formulae (11a) and (12a) combined):

$$Re(\Delta Q) = \frac{2 \, N r_o \, R}{Q \, \gamma \, h \, \varpi} \left(\frac{M_c L_c}{2\pi R} \right) , \qquad (23)$$

where we have also inserted the circumference factor. We may compare Eq. (22) and Eq. (23) by putting $h = \varpi = 2b$. In this case the two formulae agree apart from a factor π ; we shall employ (the more conservative) Eq. (23) for subsequent estimates.

Table II

ISR Parameter List

Machine radius	R	150 m
Q value	Q	8.75
Chamber width	w	15 cm
Chamber height	h	5 cm
Proton energy/rest energy	γ	30
Angular frequency	ω_o	$2 \cdot 10^6\ s^{-1}$
Number of clearing electrodes	M_c	264
Length of clearing electrodes	L_c	0.25 m

$4 \cdot 10^{14}$ protons correspond to a current of 20 A

With the ISR parameters given in Table II, we find for the real part of the Q shift:

$$\mathrm{Re}(\Delta Q) = 1.5 \cdot 10^{-17} N \ . \qquad (24)$$

At the design intensity of the ISR - $N = 4 \cdot 10^{14}$ - the Q shift becomes

$$\mathrm{Re}(\Delta Q) = 0.006 \ . \qquad (25)$$

It should be easy to provide the Q spread, required to stabilize the motion, by the poleface windings in the ISR.

The effect of the clearing electrodes is - despite the small circumference factor ($M_c L_c/2\pi R = 0.07$) - stronger than the resistive wall effect which was calculated to require a Q spread $\delta Q = 0.0036$ at the same current[4,5]. Finally, we have neglected resonance effects by using the limiting value (12a) in Laslett's paper[3]. When the wavelength of the perturbation of the beam is of the same order of magnitude as the length of the clearing electrodes, resonance effects become important (they must be calculated using the full expressions) and could require larger Q spreads for stability, but Zotter[6] has suggested that the clearing electrodes can be designed with sufficient resistance that the broadened resonances are innocuous.

6. General remarks

There are several methods available, in the design of machines, for controlling coherent transverse instabilities. Limiting our attention to relativistic electron storage rings, we list them in the order of increasing safety.

i) One takes $\mathrm{Re}(\Delta Q)$ and $\mathrm{Im}(\Delta Q)$ from Ref. 1) or 2) and provides enough Q spread in the beam to achieve Landau damping (against $|\mathrm{Re}(\Delta Q)| + \mathrm{Im}(\Delta Q)$) at the design intensity. This method is sensitive to errors in the calculation of $\mathrm{Re}(\Delta Q)$ or $\mathrm{Im}(\Delta Q)$. Again, it has been shown that some structures, like clearing electrodes[3], do not show the $1/\gamma^2$ cancellation in $\mathrm{Re}(\Delta Q)$ and can have large $\mathrm{Im}(\Delta Q)$, so that this method is very dangerous.

ii) One takes $\mathrm{Re}(\Delta Q)$ and $\mathrm{Im}(\Delta Q)$ from the resistive wall calculations[1] and obtains stability by ensuring that the radiation damping rate is larger than the growth rate, i.e. the threshold is given by Eq. (17). Because of the strong influence which structures other than smooth walls are known to have on the numerical value of $\mathrm{Im}(\Delta Q)$ this procedure appears to be dangerous.

iii) One provides a Q spread in the beam which is bigger than that $\mathrm{Re}(\Delta Q)$ which is calculated without the $1/\gamma^2$ cancellation of electric and magnetic terms. Since generally $\mathrm{Im}(\Delta Q) < \mathrm{Re}(\Delta Q)$ this method is safe, except in resonant situations. Unfortunately, the Q spreads required to fulfill this condition are rather large.

Adone was designed following method (ii), and the observed large growth rate now forces either (1) a structure modification which will reduce Im(ΔQ) and make (ii) effective; or (2) a structure modification which will reduce Re(ΔQ) so that (i) is effective; or (3) addition of Q spread so as to achieve (iii). We believe that the required Q spread across the beam to have situation (iii) (at the design intensity of 100 mA it is approximately 0.01) could be achieved, but appreciate that it presents problems. Alternatively, separating electrodes might be designed so as to preserve the $1/\gamma^2$ cancellation of magnetic and electric forces and thus accomplish situation (ii). Electrodes having an Im(ΔQ) sufficiently small for method (ii) to be effective seem hard to imagine, but there is the possibility of operation without separating electrodes. More detailed comments on these possibilities are given below.

Large Q spread: To obtain high luminosity the transverse beam dimensions must be made large (by external fields); and in this case, the Q spread is easier to accomplish than it would be for beams of natural size. Presumably the Q variation can be confined to the region of the beam, by suitable magnetic pole-face windings, and thus not degrade the injection process.

Cancellation of electric and magnetic forces: It is not clear to what extent this is possible: In the case of long wavelength perturbation Ruggiero[7] has shown that - independent of the number or position of the termination - separating electrodes do not preserve the $1/\gamma^2$ cancellation. For bunches short compared to the electrode, the calculations have yet to be done; perhaps matched terminations at the electrode ends would be advantageous. Quantitative gains are possible by reducing the number of electrically separate structures, but this is limited by the dynamic requirements of a few crossing points.

Operation without clearing electrodes: Since it has already been shown that positron beams can be held in Adone without clearing electrodes, it should be possible to also maintain electrons provided the positron current is kept somewhat larger than the electron current. This mode of operation has - with externally increased beam dimensions - the expectation of high luminosity, but the disadvantage of large interaction volumes.

Beam crossing could be accomplished by purely magnetic methods (as in the two-vacuum chamber Desy design). Clearing of the (now-separated) electron beam would seem to require electrodes, but these - purely clearing - electrodes could be electrically continuous and thus not a source of large instability terms. The requisite magnetic configuration (namely, the number and strength of the non-linear elements) has not yet been ascertained, and might prove impractical for Adone.

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8. References

1) E.D. Courant and A.M. Sessler, Rev.Sci.Instr. 37, 1579 (1966).

2) L.J. Laslett, V.K. Neil and A.M. Sessler, Rev.Sci.Instr. 36, 436 (1965).

3) L.J. Laslett, Proc.Int.Symp. Electron and Positron Storage Rings, Saclay, p. IV-5-1 (1966).

4) E. Keil and A.M. Sessler, ISR-TH/67-39 (1967).

5) B. Zotter, ISR-TH/68-51 (1968).

6) B. Zotter, private communication.

7) R. Ruggiero, private communication.