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# Some comments on texture analysis and the use of co-occurrence matrices

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# SOME COMMENTS ON TEXTURE ANALYSIS AND THE USE OF CO-OCCURRENCE MATRICES

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## ABSTRACT

Digital processing techniques used for discrimination or classification of image data are generally based on the grey scale or intensity information. However, there is a growing awareness that the decision making process could be improved by the use of additional information such as texture. Texture is the spatial distribution of the grey values in the image. This report, while commenting on the various available techniques for quantifying texture, is mainly concerned with the technique based on the co-occurrence matrices of the grey scale values. The meaning of various texture parameters based on these matrices and which have been described in the literature are discussed and illustrated by simple examples. Finally an example is given based on real image data.


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## 1. Introduction

In visual pattern recognition we appear to make use of three main properties of an image, grey scale values (i.e. intensity), boundaries (i.e. shape), and the spatial distribution of the grey-scale values (i.e. texture). Whereas most automatic digital processing techniques use just one property, namely the grey scale values, for digital classification purposes (Duda and Hart 1973), this simplification is compensated to a certain extent by using multidimensional data where each dimension corresponds to one of a number of measurement channels. However, there is a growing awareness that if the other two properties (shape and texture) were introduced into the decision making process then classification would be improved. Ideally one would like to calculate some parameters which reflect the texture at each image point and use these as simply other channels of information in the classification process.

Texture is of particular interest because it may be relatively insensitive to temporal variations and changes in the mean or local values of intensity in an image. It is of particular relevance to side-looking radar images where the basic amplitude of the measured return is affected by so many variables that the use of amplitude alone for classifying the image is not very effective. Other types of imagery where texture is of importance are images of biological material, cloud patterns, geological strata, metal surfaces (e.g. grain inclusions in etched surfaces) and images produced by radiography.

In general there are three main problems to be solved before texture can be used for discrimination purposes:-

(a) texture discrimination;

(b) texture description; and

(c) location of boundaries between regions of different texture.

There are no general theories or mathematical rules for solving these three problems and it is largely left to the intuition and pragmatism of the individual user. This report is wholly concerned with (b), the problem of texture description, since if suitable parameters for describing the texture can be obtained the task of discrimination will be much easier.

The various algorithms or techniques for describing texture are usually referred to as first or second order statistics. First order statistics are those that characterise a particular relationship between each picture element and its neighbours from which various features or parameter values can be obtained for each picture element. The parameters are calculated by the use of a local digital operator which is applied to each picture element in turn. The usual parameters derived from first order statistics are average, variance, Laplacian, spread, etc. Second order statistics are generally more complex and expensive to compute and in this case parameters are calculated for local regions of the picture based on all the picture elements within this region. The number of pixels in a local region tends to be larger than the number of neighbours used in calculations based on first order statistics. Examples of second order statistics are Fourier Transforms and the grey-scale dependent relationship used to construct co-occurrence matrices.

This report is mainly concerned with a set of texture parameters suggested by Haralick based on the co-occurrence matrices of the grey-scale values. The main properties of the various suggested texture parameters are illustrated by examples. A few brief comments and examples are also given of the use of power spectra.

## 2. Texture

Texture is an important characteristic when identifying objects or regions in an image since it contains information about the structural arrangement of surfaces and their relationship to the surrounding environment. Texture has been described as that characteristic of an image which leads to descriptions such as coarse, smooth, fine, grainy, speckled, etc. An obvious example of texture in an image is a very regular periodic pattern, such as the furrows in a ploughed field or the lines of trees in an orchard. In practice, however, repetitive patterns are rarely obtained and instead rather vague or varying patterns at random orientations are obtained (i.e. replication of generalised pattern classes rather than a periodic pattern). In addition texture properties very often undergo a slow spatial variation which, while apparent to the eye, is difficult to detect and quantify digitally. Since the pattern may not be identical from place to place and there may be no tendency to periodicity, a quantitative description of texture has to be statistical.

The analytical representation of texture is by the spatial arrangement of the grey scale levels and several methods have been proposed for calculating parameters to characterise the texture at each point in a picture (Rosenfeld and Kak 1976, Lipkin and Rosenfeld 1970). Following the concept of a repetitive pattern, texture is often referred to as a local property since a local region of picture, which is the size of the repetitive pattern, can be processed digitally to calculate the characteristic texture parameter.

In digital processing an algorithm or operator is used to calculate a property of the texture at every point in the image from the local region surrounding each point. The statistics of these local property values provide the parameter which characterises the texture. For example, in a "busy" picture and a "uniform" picture the average values of a parameter characterising contrast (e.g. edge operator) would be high and low respectively.

The types of operators used for characterising texture may conveniently be considered as falling under four main headings (Hawkins 1970):

(a) spatial frequency content;

(b) grey level statistics;

(c) geometrical or shape content; and

(d) higher order measures.

### (a) Spatial Frequency Content

The use of Fourier analysis or autocorrelation would at first sight seem the obvious solution. The "power spectrum" obtained from a Fourier analysis of an image with a periodic pattern will exhibit high power at the characteristic frequency. The autocorrelation function should fall off very rapidly for a "busy" picture and more gradually with a more uniform picture. However, neither technique is generally useful and their use should be restricted to analysing images showing strong periodicity, ideally related to a few spatial frequencies, and a basic texture pattern which extends over moderately large distances. They also suffer from the disadvantage of requiring long computing times. Most of the images we have been asked to classify tend to be composed of many different spatial frequency components which are present in a variety of overlapping and constantly changing texture patterns with random orientations. It should be recognised that, since moderately large areas are required for each calculation, the discrimination resolution of the techniques is relatively poor.

(b) Grey Level Statistics

The computation of relatively simple local grey level statistics such as average, variance, maximum contract, spread etc., which in certain situations may be very effective, is in general rather crude and misleading. This is because the derivation of such statistics are related to average properties of the image and this can smear out the properties of the texture patterns themselves. For example, some of these parameters would have the same values whether the set of intensity values were arranged in a pattern or had a random arrangement. These first order grey level statistics do however have the advantage of being applicable to very small areas and thus can give good spatial resolution for discrimination. The types of images for which these statistics have most success are where the features are intensity dependent rather than pattern dependent, e.g. radiographs, target identification and medical images, and are particularly useful in "restoring" noisy images.

(c) Shape

Local shapes or some geometrical aspect of the pattern can provide parameters which work surprisingly well, particularly for images with sharp contrasts and few effective grey scale levels (e.g. cloud pictures, pictures for the identification of targets and biological material). Normally the characteristic shapes are simply blobs selected by a suitable threshold and parameters such as average size, perimeter length, etc., are estimated. It may be advantageous to study the effect of varying the threshold on the parameter values. For example, smaller texture elements will contain more edge in comparison with total area than larger elements. Alternatively, the use of a specific local shape operator may classify an image. However, this procedure will not be very effective where the characteristic texture cannot be readily revealed by simple thresholding.

(d) High Order Measures

The higher order measures are designed to have properties more closely related to the texture in an image and these measures usually contain directional information. The actual operator may be a simple algorithm such as an edge detector but one can choose to store some property of the operator in the local region as a function of direction (e.g. the number of edges encountered). The idea of using edges is that boundary changes in texture patterns lead to edges. Edges are important in the study of urban areas, targets and medical radiographs. Rosenfeld and Thurston (1971), have suggested the use of the Roberts gradient. This is the sum of the absolute values of the difference between diagonally opposite neighbouring pixels, in pairs of non-overlapping neighbourhoods bounding each particular pixel. Other workers have suggested the use of the Laplacian gradient: Sutton and Hall (1972) extended the idea by making the gradient a function of the distance between pixels and when it was applied to the identification of pulmonary disease in radiographic images, the identification accuracy between normal and abnormal lungs was 80%. A number of higher order measures which represent texture quality are obtained from the statistics of the joint grey level occurrences. These are the subject of discussion in the following sections of the report. In general the higher order measures are complex, expensive in computer time and have moderately poor resolution. Since the texture parameters depend on the relationship of the joint grey level occurrences, they are most useful for classifying images with texture like structures and a wide range of grey levels. Applications can be found in aerial photographs of forests, or geological strata and the images of metallurgical and medical specimens.

A study by Weszka and Rosenfeld (1975) of the classification of terrain in aerial photographs using texture parameters compared three techniques based on Fourier power spectrum, grey level co-occurrences and on statistics of local properties. The Fourier based technique performed the worst and that based on the statistics of grey level differences performed the best. The state of the art is such, however, that none of the measures mentioned above are the complete answer to the general characterisation of texture in an image. The number of published successes is very limited and the successes tend to be applications where some specific region in an image is easily characterised by one particular parameter and where, as often as not, the discrimination is based on intensity, rather than on the texture pattern. One recent new development which will revolutionise the use of texture parameters is parallel array processors which will make the calculation of some spatial parameters almost instantaneous.

One of the basic problems in characterising texture is simply selecting the size of the local region. Obviously for a periodic or replicated pattern the local region should, as far as possible, correspond to the size of the basic pattern. However, in practice, the size of the basic pattern varies and its visual effect may be markedly affected by the surrounding image. One possible solution is to examine the parameter as a function of the size of the local region and then select a value which gives the best discrimination between the various patterns in the data. In practice, the choice of the region size tends to be arbitrary (Thompson 1976). If the region is too large there is a loss of resolution, while if too small the accuracy of the calculated texture parameters will be severely limited.

One of the deficiencies in the use of local regions is a failure to describe macro-texture or grouping. In order to overcome this deficiency, Ehrich (1978) has devised a method based on relational trees which makes use of a local minimum and maximum in the intensity distribution and the nesting of such peaks within the macro-regions. Mitchel et al (1977) had previously proposed that important texture information is contained in the relative frequency of local extremes in intensity (i.e. peaks and troughs).

## 3. Grey Level Co-occurrence Matrices

Co-occurrence matrices contain the statistics of joint grey level occurrences and represent the frequencies with which specified pairs of grey levels occur at some defined separation distance in the picture. Such frequencies can retain directional sensitivity by calculating a separate matrix for different directions.

A mathematical definition may be defined as follows. If the size of image, I, is r by s pixels, the set of resolution cells in an image I may be defined as $A_r \times B_s$ ($|A_r| = r$, $|B_s| = s$), and if the set of grey values that can appear in I is G, then:

$$I : A_r \times B_s \rightarrow G \ ,$$

describes the image.

If we let the vector $\underline{\rho}$, $\left(\frac{P_1}{P_2}\right)$, define the direction associated with a particular co-occurrence matrix, then the co-occurrence matrix for a general integral vector $\underline{\rho}$ is:

$$M_{\underline{\rho}}(i,j) = \sum_{x=|P_1|+1}^{r-|P_1|} \ \sum_{y=|P_2|+1}^{s-|P_2|} \delta^{i}_{I(x,y)} \ \delta^{j}_{I(x+P_1,y+P_2)} \qquad (1)$$

where $\delta^{\alpha}_{\beta} = \begin{cases} 1 & \alpha = \beta \\ 0 & \alpha = \beta \end{cases}$ is Kroneckers' symbol.

In order to reduce the computational effort the ability to examine all directions is generally restricted and the information for vectors directly opposite each other is added together, thus equation (1) becomes:

$$M_{\underline{\rho}}(j,i) = \sum_{x=|P_1|+1}^{r-|P_1|} \ \sum_{y=|P_2|+1}^{s-|P_2|} \delta^{i}_{I(x,y)} \ \delta^{j}_{I(x+P_1,y+P_2)} + \delta^{i}_{I(x,y)} \ \delta^{j}_{I(x-P_1,y-P_2)} \qquad (2)$$

If a given picture has a maximum of k grey levels then the number of occurrences of neighbouring points having various pairs of grey levels can be stored in a $k \times k$ matrix for any specified displacement between the neighbouring points. The (i,j) element of this matrix is the number of times that a point having grey level i occurs at some specified distance in some specified direction for a point having grey level j.

Consider the case for a displacement of one resolution cell in the image. Then, as shown in Fig. 1, each resolution cell is surrounded by eight nearest neighbour resolution cells. Directional sensitivity is examined by calculating matrices for the four directions shown in Fig. 1, i.e. $0^{o}$, $45^{o}$, $90^{o}$ and $135^{o}$. For $0^{o}$ the resolution cell numbers 1 and 5 are the nearest neighbours to resolution cell X. If the intensity of X is i and that of resolution cell 1 is j then one is added to the element P(i,j) in the co-occurrence matrix and if the intensity of cell 5 is k then similarly, one is added to element P(i,k). In order to illustrate the various texture parameters to be mentioned later a series of simple test images are shown in Fig. 3. The ten test images show a wide variety of situations and have been chosen primarily to illustrate the properties of the various parameters rather than illustrate real pictures. The spatial size is 9 × 9 and the range of grey levels is 1 to 9. Two examples of co-occurrence matrices are given in Fig. 4. The first is for the $0^{o}$ direction of test pattern (2), and the second for the $45^{o}$ direction of pattern 10.

It can immediately be seen that the co-occurrence matrices with our present definition are symmetric and that for the smooth picture the high values in the matrix are concentrated near the main diagonals, whereas in the random picture (e.g. (10)), the values are almost equally spread over the entire matrix. With a regular, high contrast pattern high values would be obtained but would tend to spread away from the diagonals. Thus the distributions in the various co-occurrence matrices relates to the underlying texture or pattern.

To allow for differing numbers of combinations of nearest neighbours in the various directions the terms in the matrices are usually normalised by dividing by:

$$0^{o} \quad - \quad 2\,N_y(N_x-1)$$

$$90^{o} \quad - \quad 2\,N_x(N_y-1)$$

$$45^{o} \text{ and } 135^{o} \quad - \quad 2\,(N_x-1)(N_y-1)$$

where $N_x$ and $N_y$ are the number of pixels in the horizontal and vertical directions respectively of the region being examined. Examples are given in Fig. 5 of the normalisation of the co-occurrence matrices given in Fig. 4.

## 4. Data Normalisation

### 4.1 Introduction

One important requirement of texture parameters is that they should be independent of the absolute grey scale and consequently, independent of practical processes such as the degree of development of an image. Analytically, either the quantised images must be invariant under a monotonic grey level transformation or the texture parameters must be invariant.

Normalisation of an image is customary before digitally processing it. Two customary techniques are described in the next two sub-sections. However, even after normalisation of an image the texture parameter can still depend on the local conditions (shadowing, effect of terrain, etc.). Such effects can be removed by first normalising each local region of an image, but two penalties are involved. Small differences in contrast can be over-emphasised which lead to misleading results and computer time can be expensive. Further studies of such aspects need to be performed.

### 4.2 Standard Normal Distribution

This technique transforms data with a mean m and standard deviation $\sigma$ into a standard distribution with mean $\bar{m}$ and standard deviation $\bar{\sigma}$. The transformation of a point x is given by:

$$x^1 = (x - m) \times \frac{\bar{\sigma}}{\sigma} + \bar{m}$$

One could choose $\bar{\sigma}$ and $\bar{m}$ for example to give 32 levels in the range $\bar{m} \pm 3\bar{\sigma}$. The characteristics of the original distribution are obviously best preserved if this has a roughly normal type distribution.

### 4.3 Equal Probability Quantising

An equalisation or equal probability quantising algorithm transforms a given distribution into a flat distribution with equal number of pixels in each level. This is obtained by varying the width of the grey scale steps in the original intensity distribution. This is most easily done by first calculating the frequency histogram of the original distribution. Then if the number of levels in the new distribution is to be, say, 32 the steps in this frequency histogram are found which give as near as possible N/32 (N total number of samples, or pixels) pixels in each successive level. Equal probability quantising provides a near optimal way to reduce the number of grey levels in a picture and yet still retain an accurate representation of the original image (Conners and Harlow 1978). This technique is also often used for plotting picture with a certain number of grey levels. It has the effect, for pictures with a normal type distribution of grey levels, of spreading out the information in the peak of the distribution (where most of the pixels lie) while losing grey scale resolution in the wings of the normal distribution (in which there are few pixels). Equal probability quantising is thus better suited to preserving small changes in grey scale for the majority of the picture elements than the standard normal distribution where such information can be lost, especially if the number of allowed level is much less than the original distribution.

## 5. Texture Parameters from Co-occurrence Matrices

### 5.1 Parameters

A set of statistical parameters has been suggested by Haralick (1974), Haralick and Shanmugan (1973), and Haralick et al (1973), which can be calculated from the co-occurrence matrices. These have been selected to characterise the various distributions in the co-occurrence matrices. In order to understand their physical significance the values of the parameters have been calculated for the nearest neighbours of the 10 test patterns listed in Fig. 3 and are plotted in Fig. 6. The parameters can be made invariant under rotation by calculating the mean over the 4 directions. In Fig. 6 the values for the 4 directions described in Section 3 are plotted.

An explanation of the notation is however first required. All the following quantities can refer to any particular direction and distance used for calculating the co-occurrence matrices.

$P(i,j)$ is the $(i,j)$ entry in the normalised spatial grey scale dependence matrix

$P_x(i)$ is the $i^{th}$ entry in the marginal probability matrix obtained by summing the rows of $P(i,j)$

i.e. $P_x(i) = \sum_{j=1}^{k} P(i,j)$ 1 .....k range of grey scale values

Similarly,

$$P_y(j) = \sum_{i=1}^{k} P(i,j)$$

but because $P(i,j)$ is symmetric, $P_x(i) = P_y(i)$

$\mu_x$, $\sigma_x$ are the mean and standard deviation of the distribution of $P_x$ and

$\mu_x$, $\sigma_y$ are the mean and standard deviation of the distribution of $P_y$.

Because of the symmetry of $P(i,j)$, $\mu_x = \mu_y = \mu$ and $\sigma_x = \sigma_y = \sigma$ and both $\mu_x$ and $\sigma_x$ are average grey level ($\mu$) and standard deviation ($\sigma$) of the considered image.

$$P_{x+y}(n) = \sum_{i=1}^{k} \sum_{\substack{j=1 \\ i+j=n}}^{k} P(i,j) \qquad n = 2,3....., 2k$$

This is the sum of all the spatial grey level dependence frequencies such that the sum of the 2 grey levels i and j is constant. For the co-occurrence matrices it is the sum of lines of entries in a direction of $45^{o}$. The first line is the single entry P(1,1), the second entry is the line P(1,2) + P(1,2), the third entry the line P(3,1) + P(2,2) + P(1,3), etc.

$$P_{x-y}(n) = \sum_{i=1}^{k} \sum_{\substack{j=1 \\ |i-j|=n}}^{k} P(i,j) \qquad n = 0,1....., k-1$$

This is the sum of all spatial grey level dependence frequencies such that the absolute difference between two grey levels i and j is constant. For the co-occurrence matrices, it is the sum of lines of entries at $135^{o}$. The first entry is the line P(1,1) + P(2,2) + P(3,3) + etc., etc.

The 4 distributions $P_x(i)$, $P_y(j)$, $P_{x+y}$ (n1 and $P_{x-y}(n)$) may be more easily understood by referring to Fig. 2 which illustrates the 4 directions which these distributions represent in each co-occurrence matrix. Several of the texture parameters to be described below are calculated from these distributions and are in effect attempts to characterise the inherent pattern of the entries in the co-occurrence matrices.

The 12 parameters to be calculated are described below but their order is not the same as the order of the examples given in Fig. 6. Parameters $f_1$ to $f_5$ are general properties of the co-occurrence matrices while $f_6$ to $f_8$ are properties of the $P_{x+y}(n)$ distributions, $f_9$ to $f_{11}$ of the $P_{x-y}(n)$ distributions, and $f_{12}$ is based on an information measure of correlation.

Angular Second Moment

$$f_1 = \sum_{i=1}^{k} \sum_{j=1}^{k} \{P(k,j)\}^2$$

This is a measure of homogeneity in a picture and since it only depends on the distribution of entries in the co-occurrence matrix and not absolute intensity values it is invariant. Its maximum value is 1.0 for a completely uniform picture (test patterns (7) and (8)) and approaches 0.0 for a random or disordered image (test pattern (10)). This is because the square of the entries in the co-occurrence table are taken (the square of one large value is greater than the sum of squares of many smaller terms). One should note a number of other results. The horizontal rows in test pattern (2) are uniform but the value of $f_1$ only equals 0.5 since alternative rows have different intensity values. For more random patterns such as test patterns (4), (5) and (6) the values of $f_1$ are small. The spread of the $f_1$ values as a function of angle is also relatively small, except for the very regular pattern in test pattern (9). This measure is very effective for separating out highly textured areas from uniform areas (e.g. pattern (8) from (4)), provided there is a reasonable range of grey scale values in the texture. The angular second movement is also a measure of the degree of "busyness" of the texture, for example "blobs" in pattern (4) have a lower value than the very regular texture pattern in pattern (5).

Entropy

$$f_2 = - \sum_{i=1}^{k} \sum_{j=1}^{k} P(i,j) \log P(i,j)$$

This is an invariant measure borrowed from information theory and it is a maximum when all events are equally probable (i.e. maximum disorder) and lowest when there is maximum order. Consequently its extreme behaviour is opposite to that of the angular second moment parameter since the random pattern (10) has the highest value while the uniform patterns (7) and (8) have zero value.

Sum of Squares: Variance

$$f_3 = \sum_{i=1}^{k} \sum_{j=1}^{k} (i - \mu)^2 P(i,j)$$

$\mu$ is the average grey level of the image, i.e.

$$\mu = \sum_{i=1}^{k} \sum_{j=1}^{k} iP(i,j) = \sum_{i=1}^{k} \sum_{j=1}^{k} jP(i,j),$$ because of the symmetry of P(i.j).

Also because of symmetry the i in the $(i-\mu)^2$ term can be replaced by j. As the equation implies it is a variance, with the square of the deviation from the mean grey level $\mu$ weighted by the number of terms P(i,j) in the co-occurrence matrix.

This parameter is a measure of the variability in the image and has the highest values for test patterns (2) and (9) where the intensity values are either two or nine and consequently well away from the mean. Test pattern (6) is similar. The value for $f_3$ is less for test pattern (1) where the majority of the numbers are ones with some nines. For a uniform image the result is zero. This parameter, since it depends on the individual grey scale values, is not invariant.

Since the variance parameter is high for highly contrasting regular patterns it is well suited to separating the linear pattern in (2) and (9) or the regular high contrast pattern (6) from the highly correlated pattern (3), the random pattern (10) and the array of "blobs" in pattern (4).

Correlation

$$f_4 = \frac{\sum_{i=1}^{k} \sum_{j=1}^{k} ij\, P(i,j) - \mu^2}{\sigma^2}$$

The parameter $f_4$ is the ratio between the co-variance and the variance of the grey levels in the image, i.e.

$$f_4 = \frac{\text{co-variance}}{\text{variance}} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{k} (i-\mu_x)(j-\mu_y) P(i,j)}{\sum_{i=1}^{k} \sum_{j=1}^{k} (i-\mu)^2 P(i,j)}$$

since $\mu_x = \mu_y = \mu$, $\sigma_x = \sigma_y = \sigma$

$$f_4 = (\Sigma \Sigma \; i \; j \; P(i,j) + \Sigma \; \Sigma - \mu \; (i+j) \; P(i,j) + \mu^2)/\sigma^2$$

$$\left( \sum_{i=1}^{k} \sum_{j=1}^{k} P(i,j) = 1 \quad \text{and} \quad \sum_{i=1}^{k} \sum_{j=1}^{k} i \; P(i,j) = \mu \right)$$

$$\therefore \quad f_4 = \left( \sum_{i=1}^{k} \sum_{j=1}^{k} ij \; P(ij) - \mu^2 \right) \Big/ \sigma^2$$

This parameter is a measure of the correlation of the grey levels in a particular direction and is highest (0.0) when the co-variance is equal to the variance. Thus it is high for linear features ($\pm$ 1.0), (e.g. test pattern (2)), and low for uniform grey levels (0.0) (e.g. test patterns (7) and (8)). The value of $f_4$ is very high for values at the extreme ends of the diagonals of the co-occurrence matrix. The features need not be uniform, for example test pattern (3), where all the entries are concentrated along one of the diagonals (Fig. 6), gives very high correlation. Hence correlation is high for regular patterns and good contrast. Very often $f_4$ is rather similar to variance, but examples of the difference are test pattern (3), where the contrast varies and hence the variance is not so high, and test pattern (9), where the variance is high but the correlation is low in the $90^o$, $45^o$ and $135^o$ direction. The correlation parameter is not invariant.

The correlation parameter is thus useful for separating out highly correlated patterns (e.g. (2) and (3)) from more random type texture patterns (e.g. (4) and (5)).

Inverse Difference Moment

$$f_5 = \sum_{i=1}^{k} \sum_{j=1}^{k} \frac{1}{1 + (i-j)^2} \; P(i,j)$$

This non-invariant parameter emphasises homogeneous areas with low contrast, i.e. where difference (i-j) is small (uniform test patterns (7) and (8) and almost uniform patterns). Test pattern (2) shows large differences in the various directions. This is because in the horizontal direction each line is uniform and i=j and hence $f_4$ = 1.0, while for the other directions the (i-j) difference is large. The entire picture need not be uniform for $f_5$ to have a high value as shown by pattern (2) in the horizontal direction with alternate lines of intensity values 9 and 2, but the entries in the co-occurrence matrices should be along the diagonals.

Sum Average

$$f_6 = \sum_{i=2}^{2k} i\, P_{x+y}(i)$$

Since $P_{x+y}(i)$ is the normalised frequency of pairs of resolution cells such that the sum of their grey level is i, $f_6$ represents the average value. It is clearly seen from Fig. 2 that this parameter will be high for uniform areas of high intensity (i is large for entries in the bottom right hand corner of co-occurrence matrix). Hence test pattern (8) has a far higher value than test pattern (7), even though both are uniform patterns. All the other test patterns have intermediate values and the parameter is almost independent of direction. It is not invariant.

Sum Variance

$$f_7 = \sum_{i=2}^{2k} (i-f_6)^2\, P_{x+y}(i)$$

$$= \sum_{i=2}^{2k} i^2\, P_{x+y}(i) - \left(\Sigma i P_{x+y}(i)\right)^2$$

The parameter $f_7$ is not invariant and represents the variance of data for varying i. It is highest for alternate rows of very high and low intensity. Hence $f_7$ is very high for test pattern (2) in the $0^o$ direction where there are alternate rows of 2's and 9's, but in the other directions it is zero since the patterns of alternate 2's and 9's are identical for all lines. Test pattern (9) is similar in the $0^o$ direction but is not zero for the other direction, in spite of the fact that the lines of data are identical, since at the nearest neighbour separation the two nearest neighbours have different values. The parameter is thus highly directional sensitive. Typical pattern structures as in test cases (4), (5) and (6) with areas of high intensity and areas of low intensity have relatively high values for the sum variance parameter but little spread compared to patterns with strong directional properties.

Sum Entropy

$$f_8 = \sum_{i=2}^{2k} P_{x+y}(i) \log \{P_{x+y}(i)\}$$

Since entropy, as seen earlier, is a measure of disorder its highest values occur when there are many small random entries in the co-occurrence matrix (e.g. test pattern (10)) and is zero for uniform images (test patterns (7) and (8)). It is an invariant parameter.

A good example of the difference between entropy and sum entropy and the usefulness of statistics based on the $P_{x+y}$ terms is given by test pattern (3). The individual values are set out in Table 1.

TABLE 1

Values of Entropy and Sum Entropy for Test Pattern (3)

| | Entropy | Sum Entropy |
|---|---|---|
| $0^o$ | 2.77 | 2.08 |
| $90^o$ | 2.19 | 0.0 |
| $45^o$ | 2.77 | 0.69 |
| $135^o$ | 2.77 | 0.69 |

The unusual co-occurrence matrices for test pattern (3) are shown in Fig. 7. For the entropy calculation there are a considerable number of separate entries and the resulting values are similar and of medium magnitude. However, the sum entropy is based on distributions of sum of the entries in the rows of values running at $45^o$ (see Fig. 2). While the $0^o$ matrix results in a somewhat similar situation to the entropy calculations, the remaining directions result in $P_{x-y}(i)$ values greater than zero for only either two entries in the $P_{x+y}(n)$ distribution ($135^o$ and $45^o$) or one entry ($90^o$). Hence the sum entropy for only one entry is zero ($90^o$) and for the two entries ($135^o$ and $45^o$) there is a relatively low result. Hence a gradient, giving centres along the main diagonal gives a high value of the sum entropy as may be found for many small areas of uniform but varying grey scale values (e.g. pattern (4) or a certain direction of pattern (3)).

Contrast

$$f_9 = \sum_{i=0}^{k-1} i^2 P_{x-y}(i)$$

This non-invariant parameter, based on the $P_{x-y}(i)$ distribution (see Fig. 2), is particularly large for large variation in intensity (accentuated by the $i^2$ factor) and is zero for uniform images (since i = 0). In test pattern (2) for example, in the $90^o$, $45^o$ and $135^o$ directions all nearest neighbours differ by an intensity difference of 7 and therefore $f_9 = 49$. However, in the $0^o$ direction the intensity difference is zero and hence $f_9 = 0.0$. Hence $f_9$ emphasises high contrast in the local patterns (pattern (6)), and unlike the variance parameter $f_9$ will not have a high value for highly contrasting patterns but will have when the patterns are nearly homogeneous (e.g. pattern (4)).

Difference Variance

If difference average $DIF = \sum_{i=0}^{k-1} i \, P_{x-y}(i)$

$$\text{then} \quad f_{10} = \sum_{i=D}^{k-1} (i-DIF)^2 \, P_{x-y}(i)$$

$$= \sum_{i=0}^{k-1} i^2 \, P_{x-y}(i) - \left( \sum_{i=0}^{k-1} i \, P_{x-y}(i) \right)^2$$

This non-invariant parameter is large for large differences in values of the $P_{x-y}$ distribution and zero for uniform pictures. For example, for test pattern (9) in the $0^o$ direction there are only entries on the diagonal of the matrix and therefore there is only one term of $P_{x-y}$, that for i = 0, and hence the variance is zero. However, for the other direction there are 2 values for the $P_{x-y}$ distribution for i = 0 and i = 7 and therefore the variance is large. Note, however, that for test pattern (2), which contains alternate lines of intensity values of 9's and 2's, there is only one $P_{x-y}$ term for all directions and the variance is always zero. The difference variance is rather like the contrast parameter $f_9$ except that it requires patterns with both a high-low and a low-high contrast to give a high value. An example of the latter is the regular pattern in test pattern (6), whereas for pattern (4) the almost uniform "blobs" lead to a relatively low value of the difference variance (similarly with the random pattern (10) and the highly correlated pattern (3)).

Difference Entropy

$$f_{11} = - \sum_{i=0}^{k-1} P_{x-y}(i) \log (P_{x-y}(i))$$

The difference entry is an invariant parameter rather similar in context to the entropy calculation. The parameter gives high values for random intensity levels and low values for uniform areas. However, the main difference occurs because we are now dealing with the $P_{x-y}$ distribution. Hence, in the following three cases, test pattern (9) ~ $0^o$, test pattern (1) ~ $135^o$ and test pattern (3) ~ $0^o$ (see Fig. 7) there is only one entry in the $P_{x-y}$ distribution and in contrast to the normal entropy values the difference entropy is zero. The difference entropy is large for areas having a variety of contrasting regions.

The three parameters calculated using the $P_{x-y}$ distribution have the interesting feature that certain linear patterns (as for example in test patterns (1), (2), (3) and (9)) are treated as areas of uniform intensity with zero variance contrast and entropy. Similar effects can be obtained with the $P_{x+y}$ distribution.

Information Measure of Correlation

$$\text{If } HXY2 = -\sum_{i=1}^{k}\sum_{j=1}^{k} P_x(i)\, P_y(j) \log \{P_x(i)\, P_y(j)\}$$

$$\text{and } f_2 = -\sum_{i=1}^{k}\sum_{j=1}^{k} P(i,j) \log P(i,j) \quad \text{(Entropy)}$$

$$f_{12} = \left(1 - \exp(-2.0\,(HXY2-f_2)\right)^{\frac{1}{2}}$$

This parameter is very similar to the correlation parameter $f_4$ except that there are no negative values.

5.2 Other Spatial Separations

Up to this point the examples chosen to illustrate the use of texture parameters have been based on the nearest neighbour spatial separation. To illustrate the changes caused by increasing the separation we have selected two parameters, namely the Inverse Difference Moment and the Sum Variance. These have been calculated for a spatial separation of two units (second nearest neighbour) and the results are shown in Fig. 8. Comparing Figs. 6 and 8 it can be seen that the uniform and random patterns, as expected, exhibit little change with spatial separation. It has also been observed that the results for the parameters, variance, entropy, angular second moment and correlation for such patterns also do not depend strongly on the spatial separation chosen. This is because they are rather general properties of an image. However, linear and regular patterns can give very different answers for the remaining parameters.

The linear test patterns (2) and (9) give very different results for the examples given in Figs. 8 and 6 for the two different spatial separations. For example with the Inverse Difference Moment parameter, pattern (2) has the value 1.0 for all angular directions for the second nearest neighbour separation. This is because each value in the image has a similar value for its second neighbour while for the nearest neighbour separation in Fig. 6 this was only true for the $0^{o}$ direction. With pattern (9) the opposite effect is seen. With the Sum Variance parameter something similar occurs and for pattern (2) the value is high for all directions since for the second nearest neighbour there are only two entries, (2,2) and (9,9), in the co-occurrence matrices and thus two widely separated $P_{x+y}$ terms. However, for the nearest neighbour separation, except for the $0^{o}$ direction there are two entries, (9,2) and (2,9) but only one $P_{x+y}$ term. Highly correlated patterns as in (3) also show large differences for these two parameters.

With normal texture patterns such dramatic effects may not be expected because the patterns are not so regular but differences may occur and hence it is important to try out a range of spatial separations. The texture parameters when calculated for a range of spatial separations often show a characteristic curve (as can be seen in the later examples given in Figs. 13 and 14) which rises from a small value for a spatial separation of one unit to some maximum value and eventually reaches a saturation value independent of spatial separation. A simple explanation of this behaviour can be given in terms of the relative size of the spatial separation and the texture pattern. For separations small compared to that of the texture pattern the image behaves as if it were of uniform intensity. As the spatial separation increases the values of the various parameters rise and reach a maximum when the spatial separation is equal to the average distance between the pattern and the background or the distance between two distinct types of patterns. It is at this spatial separation that the variation of the texture pattern with angle is usually greatest. At the other extreme, where the spatial ratio is large compared to the basic texture pattern, the grey level relationship essentially becomes random and so the texture parameter becomes independent of the choice of spatial separation. However, if the pattern is sufficiently regular instead of a saturation value one should see a periodic pattern in the parameter as a function of the spatial separation.

### 5.3 Power Spectra of Test Texture Patterns

A real example will be considered in the next section, but, first, let us consider the power spectra for two reasonably realistic test patterns, namely patterns (4) and (6). In order to derive the Fourier transform the basic $9 \times 9$ pattern has been repeated to give an array of size $64 \times 64$. A two dimensional Fourier transform has been calculated and a power spectrum obtained by summing the amplitudes in radial zones in the frequency plane about zero frequency. The power spectra have been normalised by dividing by the number of entries in each zone and multiplying by the area of the zone. Thus for a random picture the power spectrum would take the form of a line passing through the origin. The results for test patterns (4) and (6) are shown in Fig. 9. The co-ordinates along the x-axis are given in terms of both frequency (number of cycles per unit distance) and wavelength (in terms of the number of unit distances, where one unit is the basic picture cell size).

The regular test pattern (6) produces, not surprisingly, a sharp peak. The peak frequency of 0.24 (wavelength = 4.2), calls for some explanation. In the test picture the pattern is repeated in both the x and y directions with a frequency of 0.17 (wavelength = 6). As a consequence, in the two-dimensional frequency plane the resultant amplitude peaks occur in a direction at $45^{0}$ to the main axes at a radial frequency distance of $.17x(2)^{\frac{1}{2}} = .24$. This result demonstrates the need to examine the distribution in the two-dimensional frequency plane as well as the power spectrum in order to understand and interpret the periodicities in various directions.

For the slightly more random pattern of "blobs" in test pattern (4) the power spectrum (Fig. 9) is composed of a number of peaks corresponding to wavelengths along the original image axes of on average of about 2.8 and 4 and multiples of these.

## 6. A Practical Example

### 6.1 Introduction

The data used to provide a real example has been obtained by the American remote sensing satellite Landsat. The satellite images the earth with a multi-spectral scanner which has a nominal ground resolution of 80 m and produces digital data which can be transformed into images. We have chosen four test areas, each of size 128 × 128 pixels. For this particular data the image had been resampled to give a pixel size of 50 × 50 $m^2$. Two of the areas correspond to urban areas in Manchester and two to rural areas just outside Manchester. Only data from one of the multi-spectral scanner wavebands, the visible red band (wavelength 0.6 to 0.7 μm), has been considered. Pictures of one of the urban and one of the rural areas are shown in Fig. 10. The light areas represent a high radiation intensity reflected from a ground element and dark areas a low intensity.

These two test areas have no apparent recognisable characteristic texture patterns and they cannot be distinguished on intensity alone (by density slicing) since the intensity distribution for the two types of areas overlaps considerably. This can be seen from the statistical information for the two areas reproduced in Table 2, and the frequency histogram of the intensity values given later in Fig. 16. Thus these two radically different test areas, like features in so many other pictures, are very difficult to separate.

TABLE 2

Properties of Urban and Rural Test Sites

| | Range of Intensities | Mean | Standard Deviation |
|---|---|---|---|
| Urban | 21 - 143 | 47 | 10.5 |
| Rural | 9 - 110 | 34 | 9.5 |

### 6.2 Power Spectra

It was anticipated that the poor spatial resolution of the image would lead to only small differences between the two areas in terms of texture and this was, to some extent, verified when the power spectra for two of the areas were examined (Fig. 11). The overall shape of the two spectra are rather similar and there is roughly a 2:1 power ratio. A broad peak occurs in both spectra at a frequency of about 0.1, equivalent to a wavelength of 10 units (unit cell size equals 50 m on the ground, hence wavelength equals 500 m). The peak is more pronounced for the rural area. A noticeable feature of these spectra is the considerable amount of energy at all frequencies and in this respect real images differ appreciably from the previous test patterns with their very regular patterns and very pronounced peaks in their power spectra. This supports the comments made in Section 2, that for most pictures the texture patterns are not regular enough to produce a very characteristic power spectrum.

## 6.3 Texture Parameters for the Entire Test Areas

When the texture parameters described earlier were calculated from the co-occurrence matrices for the urban and rural test areas a very noticeable difference was obtained. The data was normalised before the calculation of the texture parameters. This was done with the equal probability quantisation technique and the original range of between 9 and 182 was reduced to 32 levels. The intensity levels for the 32 new intensity values were not obtained for the urban and rural test areas separately but were calculated for the total image area from which the test area was extracted. Otherwise any contrast differences between the two areas may have been lost.

The resultant texture parameters exhibited very large differences between the urban and rural test areas, in particular for the parameters involving variance (variance, sum and difference variance) and entropy (entropy, sum and difference entropy) and for the sum average, contrast and to a lesser extent the angular second moment parameters. These differences, for the variance, contrast, entropy and sum average parameters respectively, are shown in Figs. 12 to 15. The parameter values are plotted in each case as a function of the spatial separation of the nearest neighbours used to calculate the co-occurrence matrices. The value plotted is the mean value over the four directions $0^{o}$, $90^{o}$, $45^{o}$ and $135^{o}$ and the range is indicated by the error bars. The range is negligible for the Sum Average and Variance parameters.

The entropy and contrast parameters show the characteristic shape as a function of spatial separation mentioned in Section 5.3, while the variance and sum average parameters appear independent of spatial separation. This is to be expected when one is calculating variance and sum average properties over a large area in which there is no very characteristic regular pattern, since these are average properties. The entropy and contrast parameters are much more dependent upon the degree of disorder and the local contrast, and hence the average "blob" size in the picture. Thus the variation of the latter two parameters with the neighbour separation indicates a tendency to form texture "blobs" in the pictures. It is believed that the difference in the entropy and contrast parameters for the two kinds of test areas does reflect a real difference in texture between the two areas while the difference for the sum average parameter and to a lesser extent the variance parameter is simply a measure of the different mean intensity for the two areas.

## 6.4 Texture Parameter for Each Point in the Test Areas

In the last section it was demonstrated that various texture parameters could distinguish between the rural and urban areas when calculated for the relatively large, entire test area. Such a procedure in practice would give very poor spatial resolution. Hence a compromise must be made between statistics and resolution. However, in order to characterise the texture the size of the local area analysed must be at least twice the separation of the characteristic pattern.

An an example the variance parameter has been calculated at every point in the two test areas using both a 3 by 3 and a 7 by 7 local region to calculate the co-occurrence matrices, for nearest neighbour spatial separation. The average value over all directions is stored at each location in an output file, which has the appearance of a grey scale image. The effect of both a Gaussian normalisation and equal probability quantisation has also been tried. The computing times (see next section) became so large for these runs that only a very limited number of calculations could be carried out.

In order to show the consequences of normalisation, both the frequency histogram of the grey scale values of the original intensity values of the whole picture (700 points by 600 lines) from which the two test areas were extracted and frequency histograms for two of the test areas are shown in Fig. 16. The considerable overlap of the intensity values for the urban and rural test areas is clearly seen in Figs. 16(b) and 16(c). In order to reduce the storage requirements the calculations were performed for only 16 levels. The results for the urban and rural test areas are shown in Fig. 17 for the two kinds of normalisation. For the equal probability quantisation the data for the complete picture was used to define the grey scale boundary levels for the new 16 normalised levels. The results shown in Fig. 17(a) and 17(b) give quite different distributions for the urban and rural areas. For the Gaussian normalisation the mean and standard deviation for the whole picture was used to define the input parameters and for the output a mean of 8 with a standard deviation of ± 5 was used. The results are shown in Fig. 17(c) and 17(d) and do not give as good a separation as the previous method.

The results of the calculations of the variance parameter are given in the table below in the form of the mean and standard deviation of the output image and are reproduced as frequency histograms of the variance values (×5 scale) in Figs. 18 and 19.

TABLE 3

Mean and Standard Deviation of the Variance Parameter Calculated at Each Point in the Test Area

| Test Area | Using a 3 × 3 Local Region | | Using a 7 × 7 Local Region | |
|---|---|---|---|---|
| | Equal Probability Quantisation | Gaussian Normalisation | Equal Probability Quantisation | Gaussian Normalisation |
| Urban | 25 ± 28 | 19 ± 25 | 51 ± 33 | 21 ± 18 |
| Rural | 12 ± 16 | 16 ± 17 | 36 ± 33 | 29 ± 20 |

The results (see Fig. 18) when using the local area of 3 × 3 are such that very poor discrimination would be obtained in separating individual points into urban and rural. However for the local area of 7 × 7 (see Fig. 19) there is an observable separation of individual points when the equal probability quantisation is used.

For example, consider the application of a discrimination level of 20 to the variance parameter, then the majority of points below 20 would be from the urban area and the majority of points above 20 from the rural area. A different region size or spatial separation or another texture parameter could possibly improve on this result but such a test has not been carried out.

It seems reasonable that the better performance of the equal probability quantisation compared to the Gaussian normalisation is due to the former preserving better the smaller grey scale differences in the peak of the original picture histogram whereas the latter reduces the contrast across the complete range of grey scale values. Thus the equal probability quantisation is able to preserve the grey scale texture information which occurs most frequently and thus it separates more effectively the real texture differences.

In general when the local area is of the order of the same size as the spatial separation or small (e.g. less than 5 × 5 pixels), there will only be a few entries in the co-occurrence matrices and it is much simpler computationally and probably just as meaningful to calculate some simple grey level statistics.

An alternative approach to calculating the texture parameter at every point in the image is to divide the image into a set of non-overlapping small square sub-images. Unfortunately, although it considerably reduces the computer time, it does produce a much coarser resolution than the original image. Haralik (1975) has also suggested a technique, whereby the co-occurrence matrix can be calculated for a defined arrangement of spatial separations around each point, thus introducing some ideas of context into the analysis.

## 7. Computing Times

The central processor time (CPU) to calculate for four test areas, each 128 × 128 pixels, all 12 texture parameters with a nearest neighbour separation, was only eight seconds on an IBM 3033, including equal probable quantising to 32 levels and graph plotting time. The ten test patterns described earlier, each 9 × 9 pixels, took just 3.5 second CPU time.

However, when the calculation is performed for a local region centred on each pixel in turn the computing time becomes prohibitive. For example, to calculate just the variance parameter for a local region size of 3 × 3 and a nearest neighbour separation for an image of size 64 × 64 pixels toook 160 seconds CPU time on the IBM 3033. While on a PDP11/60 a similar calculation for an area of 128 × 128 took just over one hour.

The present program is very generalised and could be optimised to give reductions in computing times of factors of 2 or 3. Even so, in order to make use of such texture parameters in large images very large amounts of computing time need to be available. Such calculations are obviously very well suited to array processors or dedicated hard-wired devices. Such specialised devices are now appearing on the market and within a few years the calculation of the texture parameters will become commonplace.

## 8. Conclusions

The automated classification of images has mainly relied primarily on differences in the grey scale or intensity value of the various classes. It is now recognised that the ability to characterise the texture within images would also help to improve the classification process.

This report has been mainly concerned with the problem of texture description. Although it emphasises the experience we have gained with the use of a set of texture parameters suggested by Haralick based on co-occurrence matrices, four main techniques for quantifying texture were briefly considered, namely:

(a) spatial frequency content;

(b) grey level statistics;

(c) geometrical or shape content; and

(d) higher order measures.

In our experience, and demonstrated in this report, the use of Fourier analysis has not proved very effective in characterising texture. This is because natural texture patterns are generally insufficiently regular over large enough distances for spatial frequency analysis to provide useful discrimination. In addition, the effective spatial resolution is relatively poor and long computing times are required. However, the method can be very effective in discriminating between areas with regular texture or with very characteristic frequencies.

The most widely used technique is the computation of simple grey level statistics such as average, variance, spread, etc. It is computationally very easy and the use of small local areas results in very good spatial resolution. The statistics are not however very effective at characterising texture differences since they are more related to the average properties and are very strongly dependent on local contrast and edges. Hence they have proved most effective in analysing radiographic images (both industrial and medical) and images of biological specimens where such properties predominate.

The use of parameters which characterise local shapes are again not particularly effective at characterising texture associated with a range of grey scale values. However, if a single intensity discrimination level can break the image into a two level picture, "blobs" and background, then the method is very effective. This approach has had most success with images of clouds or biological material and radar images since these show sharp contrasts and involve few effective grey levels

Fortunately the higher order measures are designed to have properties very much more related to the texture patterns than the last two techniques. The calculation of these measures, unfortunately, generally leads to long computing times, especially if spatial resolution is to be preserved. For very small local areas it may be much easier to calculate some simple grey level statistics. The directional sensitivity of these measures can be very important if there are strong linear characteristics in the patterns. Also when there are a range of grey levels in the texture like patterns these high order measures are the most effective. The two techniques generally used are either based on some form of edge detection such as the number of edges per unit area or the calculation of parameters based on the statistics of the joint grey level occurrences. The use of the edge detection technique has been used very effectively in radiographic images and the parameters based on the co-occurrence matrices in aerial photographs and with geological specimens.

More than ten parameters have been suggested by Haralick based on the co-occurrence matrices. These matrices are calculated for a number of directions and represent the frequencies with which specified pairs of grey levels occur at some defined distance in the image. The various parameters were used to characterise different distributions in the co-occurrence matrices (for example, whether the data is distributed along a diagonal or is uniformly distributed over the image). A series of simple test images was used to illustrate the properties of the various parameters and the main observations are listed below. The first group of parameters are based on the general statistics of the co-occurrence matrices.

<u>Angular Second Moment</u> - measures degree of uniformity and is invariant. Thus it will separate out patterns of varying degrees of complexity ("busyness") such as cloud pictures, micrograph and medical specimens and homogeneous areas from patterned areas.

<u>Entropy</u> - measures amount of disorder or randomness and is invariant. It is very similar to the reverse of the Angular Second Moment.

<u>Variance</u> - measure of spread of values and is non-invariant. Unlike entropy it is not necessarily high for random intensity values but high for rather regular high contrast, e.g. linear features and highly contrasting patterns such as in radar images, radiographs, drainage patterns and urban areas in aerial photography.

<u>Correlation</u> - measure of correlation in the pattern and is high for linear patterns or gradients with good contrast. It is non-invariant.

<u>Inverse Difference Measure</u> - emphasises homogeneous areas or patterns with low contrast. It is non-invariant. It is rather like the Angular Second Moment parameter but much more directional sensitive.

The following parameters are based on the distribution of entries occurring in rows in direction of $45^{o}$ in the co-occurrence matrices.

<u>Sum Average</u> - emphasises near homogeneous areas of high intensity and is non-invariant. Very useful for separating out uniform areas of different intensity, as occurs in radiographs.

<u>Sum Variance</u> - emphasises patterns with areas of high and low intensity. It is non-invariant and very directionally dependent and thus will separate out linear patterns from "blob" type patterns.

<u>Sum Entropy</u> - while emphasising random patterns as does entropy, it also emphasises regular gradients, and areas with a range of grey scale values but with small uniform areas, as for example in radiographs. It is an invariant parameter.

The next three parameters are based on the distribution of entries occurring in rows in directions of $135^{o}$ in the co-occurrence matrices.

Contrast - emphasises large regular differences in contrast between nearest neighbours (i.e. in the local patterns), and unlike variance is not high for sets of homogeneous patterns with the different homogeneous patterns having a high contrast with the background areas. It is well suited to separating highly contrasting patterns from more uniform patterns.

Difference Variance - rather like contrast but to have a very high value a pattern structure is needed with contrast both high-low and low-high in the pattern. It is a non-invariant parameter.

Difference Entropy - high for random pictures as normal entropy and in particular for regular patterns with a gradient giving a range of contrasts. It is an invariant parameter.

Hence for texture patterns the Angular Second Moment and Variance are good for characterising "busyness". Contrast is very effective for high contrast in the pattern, the Difference Variance and Difference Entropy are very effective for picking out patterns containing a wide variety of contrasts while the Sum Variance and Correlation are very responsive to linear features or texture patterns running in particular directions.

In general one parameter is insufficient to characterise several types of texture and therefore it is recommended that a variety of possible parameters are first calculated over a test region. Then one can select one or more parameters which give the best separation for the various types of areas selected.

The choice of the size of the spatial separation distance to use for the calculation of the various parameters is very important. With too small a distance the local areas may seem uniform and the basic texture patterns missed, while if the distance is too large the grey level relationship will become essentially random. There will also be a loss of resolution if the distance is too large. In theory the distance should be as long as the basic patterns. In general the texture parameters exhibit a characteristic shape as a function of distance, which gradually rises to some maximum and then saturates. The optimum separation would appear to be somewhere near the value which gives the maximum parameter value, when the parameters are most susceptible to directional effects and the form of the texture.

In order to be able to compare results from image to image and also to have a standard size of co-occurrence matrix, the texture parameters must be independent of absolute grey scale. This is accomplished by normalising the data to a fixed number of grey levels. The most effective technique is the method of equal probability quantisation.

The main function of the use of the texture parameters is as a means of discrimination or classification in an image. With a practical example it has been demonstrated that when calculated for large areas (128 × 128) corresponding to two separate types of ground features, very good discrimination was obtained. However, in practice much greater resolution is required and it is necessary to calculate the texture parameter in a local region around each individual pixel. This requires a much greater computational effort and it is necessary to reduce the number of parameters to be calculated and give careful attention to the spatial separation and size of the local region to be used. With the practical example it was found that by using a nearest neighbour separation in a 7 × 7 local area around each pixel good discrimination could be obtained for the majority of pixels in the two types of areas used. This was in spite of the fact that the intensity distribution for the two types of areas overlapped each other.

Computation times on a PDP11/60 for the above calculation for an area of 128 × 128 pixels took about one hour. No attempt has been made to optimise a very general program and this time could be reduced by possibly a factor of two. Even so, computing times are extensive. These types of calculation would be well suited to array processors, which are just beginning to be available.

Thus the use of texture parameters based on co-occurrence matrices are a very valuable aid in the discrimination or classification of images which have some type of texture like structure and a range of grey scale values.


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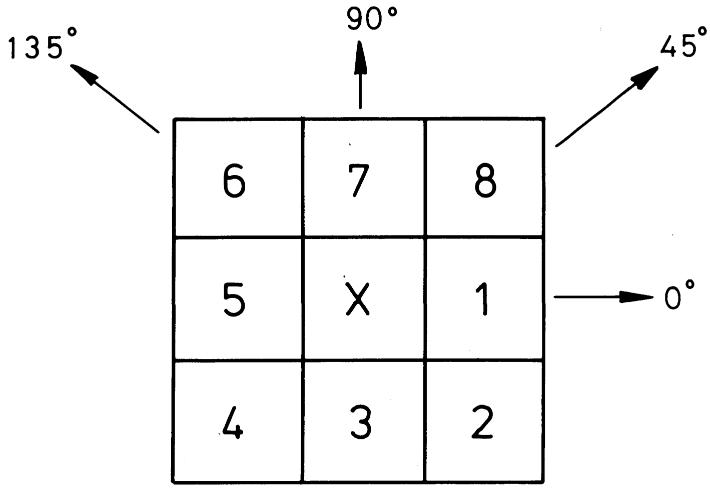



FIG 1. NEAREST NEIGHBOUR RESOLUTION CELLS AND THE CORRESPONDING DIRECTIONS

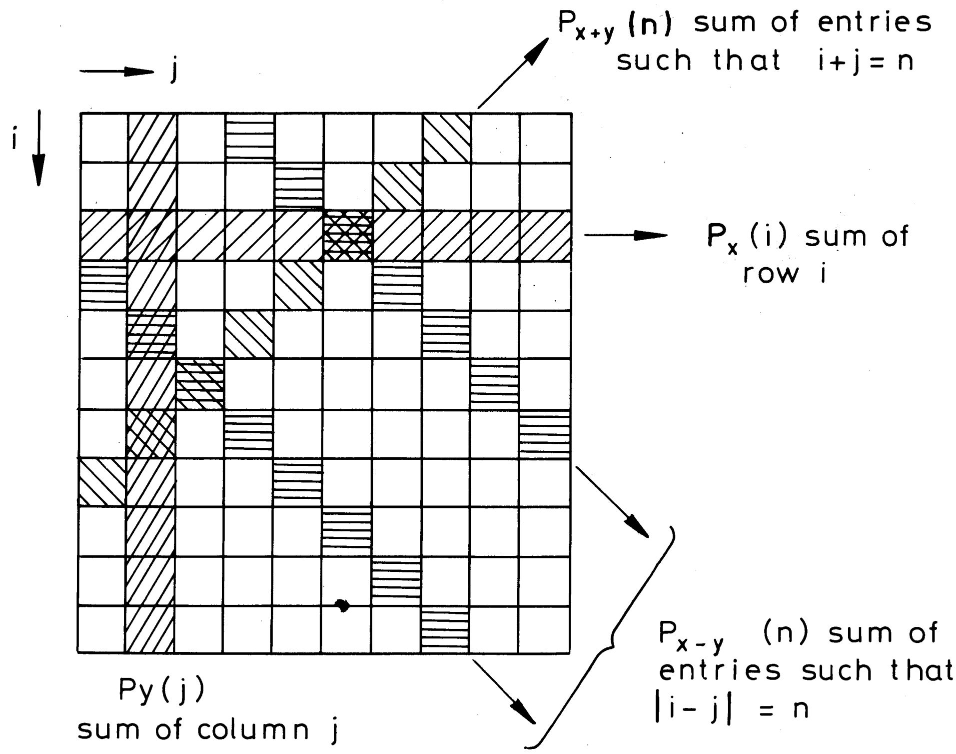



FIG. 2 VARIOUS DIRECTIONAL DISTRIBUTIONS CALCULATED FROM THE CO-OCCURRENCE MATRICES

Fig 3 Test Patterns

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 9 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 9 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 9 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 9 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 9 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 |

(1) Picture with diagonal linear feature

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

(2) Linear pattern,lines one element wide

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

(3) Highly correlated pattern

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8 | 7 | 7 | 2 | 2 | 8 | 7 | 2 | 1 |
| 7 | 8 | 7 | 2 | 7 | 7 | 8 | 7 | 2 |
| 2 | 2 | 2 | 2 | 2 | 7 | 7 | 2 | 1 |
| 2 | 2 | 7 | 2 | 1 | 2 | 2 | 2 | 2 |
| 7 | 7 | 8 | 7 | 2 | 2 | 2 | 7 | 7 |
| 7 | 8 | 7 | 7 | 2 | 2 | 7 | 8 | 7 |
| 1 | 2 | 2 | 2 | 1 | 2 | 8 | 7 | 7 |
| 2 | 2 | 7 | 7 | 7 | 2 | 2 | 2 | 2 |
| 2 | 8 | 8 | 7 | 7 | 2 | 2 | 2 | 2 |

(4) Array of "blobs" of high intensity in low intensity background

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 9 | 9 | 1 | 2 | 9 | 9 | 1 |
| 2 | 1 | 9 | 9 | 2 | 1 | 9 | 9 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 1 | 2 | 9 | 9 | 1 | 2 | 9 | 9 | 1 |
| 2 | 1 | 9 | 9 | 2 | 1 | 9 | 9 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 1 | 2 | 9 | 9 | 1 | 2 | 9 | 9 | 1 |

(5) Regular, low intensity, pattern

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 9 | 9 | 1 | 1 | 1 | 9 | 9 | 9 |
| 9 | 1 | 9 | 1 | 9 | 1 | 9 | 1 | 9 |
| 9 | 9 | 9 | 1 | 1 | 1 | 9 | 9 | 9 |
| 1 | 1 | 1 | 9 | 9 | 9 | 1 | 1 | 1 |
| 1 | 9 | 1 | 9 | 1 | 9 | 1 | 9 | 1 |
| 1 | 1 | 1 | 9 | 9 | 9 | 1 | 1 | 1 |
| 9 | 9 | 9 | 1 | 1 | 1 | 9 | 9 | 9 |
| 9 | 1 | 9 | 1 | 9 | 1 | 9 | 1 | 9 |
| 9 | 9 | 9 | 1 | 1 | 1 | 9 | 9 | 9 |

(6) Regular, highly contrasting pattern

Fig 3 Test patterns cont.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

(7) Uniform picture, low intensity

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |

(8) Uniform picture, high intensity

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

(9) Linear pattern, lines two elements wide

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 4 | 2 | 2 | 8 | 3 | 7 | 4 |
| 6 | 1 | 1 | 5 | 9 | 2 | 8 | 8 | 7 |
| 5 | 4 | 1 | 1 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 4 | 1 | 1 | 8 | 8 | 9 | 8 |
| 5 | 3 | 1 | 2 | 1 | 1 | 7 | 2 | 5 |
| 6 | 2 | 9 | 3 | 2 | 1 | 1 | 2 | 4 |
| 7 | 6 | 2 | 5 | 3 | 2 | 1 | 1 | 5 |
| 9 | 8 | 6 | 4 | 6 | 4 | 3 | 1 | 1 |
| 5 | 4 | 8 | 6 | 4 | 2 | 1 | 8 | 6 |

(10) Random picture

Fig 4 Examples of unnormalised co-occurrence matrices

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 80 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 64 |

(a) Test pattern (2) $0^o$ direction

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 7 | 3 | 5 | 5 | 2 | 1 | 3 | 1 |
| 7 | 0 | 3 | 0 | 4 | 1 | 1 | 1 | 2 |
| 3 | 3 | 0 | 2 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 2 | 4 | 0 | 1 | 0 | 2 | 0 |
| 5 | 4 | 0 | 0 | 2 | 1 | 0 | 1 | 0 |
| 2 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 2 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 4 | 1 |
| 3 | 1 | 0 | 2 | 1 | 1 | 4 | 2 | 1 |
| 1 | 2 | 0 | 0 | 0 | 2 | 1 | 1 | 2 |

(b) Test pattern (10) $45^o$ direction

Fig. 5 Examples of Normalised co-occurrence matrices

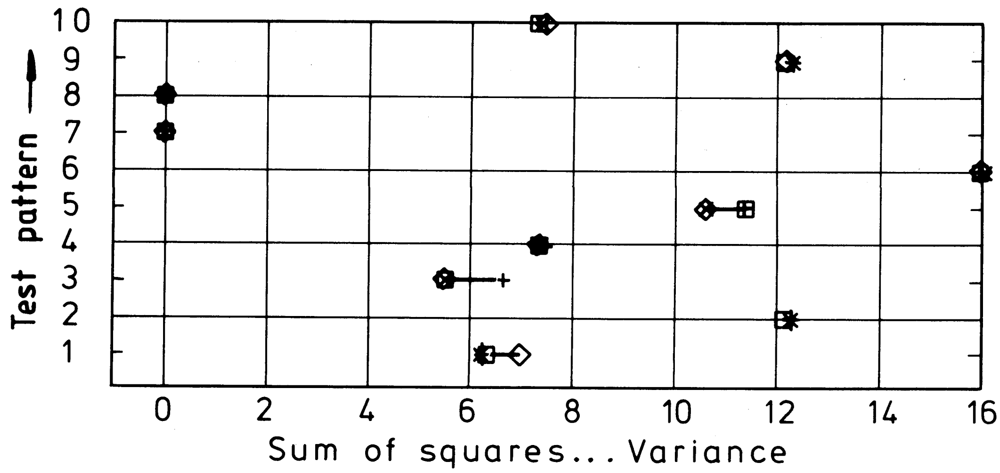
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0.56 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(a) Test pattern (2) $0^{o}$ direction

Fig. 5 Examples of Normalised co-occurrence matrices

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.03 | 0.05 | 0.02 | 0.04 | 0.04 | 0.02 | 0.01 | 0.02 | 0.01 |
| 0.05 | 0 | 0.02 | 0 | 0.03 | 0.01 | 0.01 | 0.01 | 0.02 |
| 0.02 | 0.02 | 0 | 0.02 | 0 | 0.01 | 0 | 0 | 0 |
| 0.04 | 0 | 0.02 | 0.03 | 0 | 0.01 | 0 | 0.02 | 0 |
| 0.04 | 0.03 | 0 | 0 | 0.02 | 0.01 | 0 | 0.01 | 0 |
| 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0 | 0.01 | 0.02 |
| 0.01 | 0.01 | 0 | 0 | 0 | 0 | 0 | 0.03 | 0.01 |
| 0.02 | 0.01 | 0 | 0.02 | 0.01 | 0.01 | 0.03 | 0.02 | 0.01 |
| 0.01 | 0.02 | 0 | 0 | 0 | 0.02 | 0.01 | 0.01 | 0.02 |

b) Test pattern (10) $45^{o}$ direction



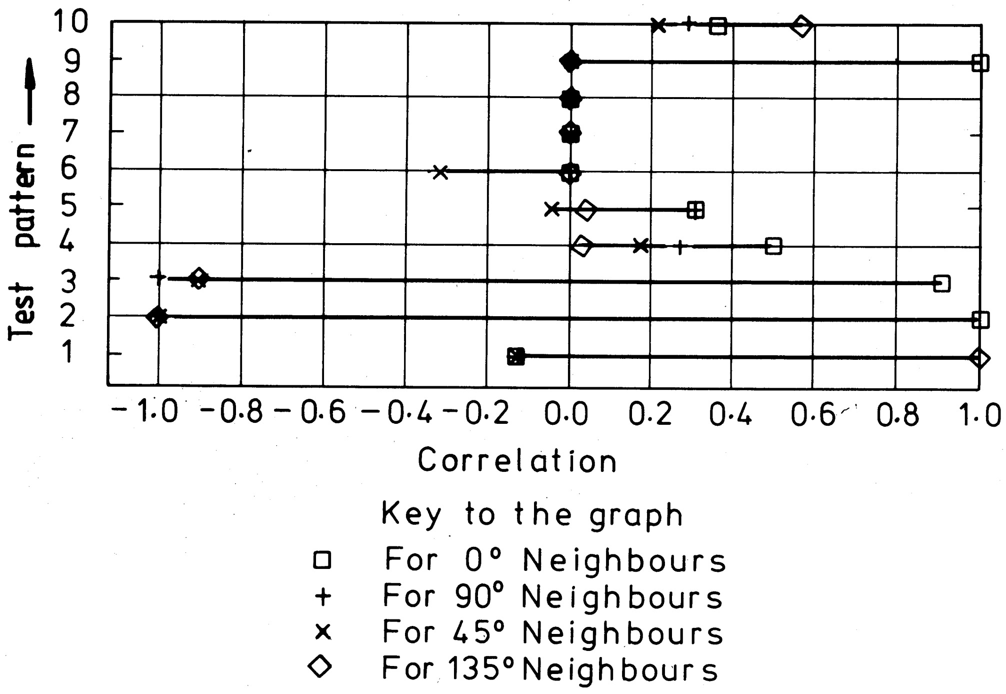
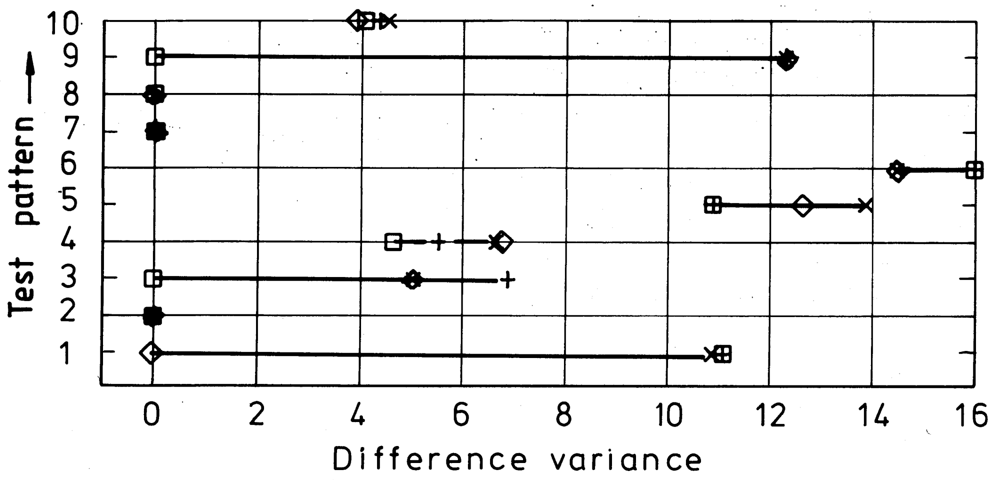
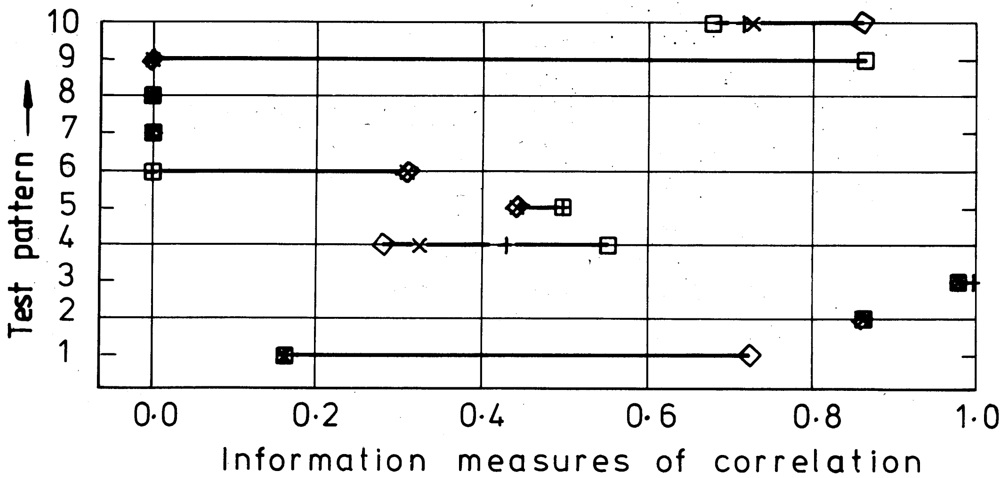




FIG. 6 CALCULATION OF NEAREST NEIGHBOUR TEXTURE PARAMETERS FOR A SERIES OF TEST PATTERNS

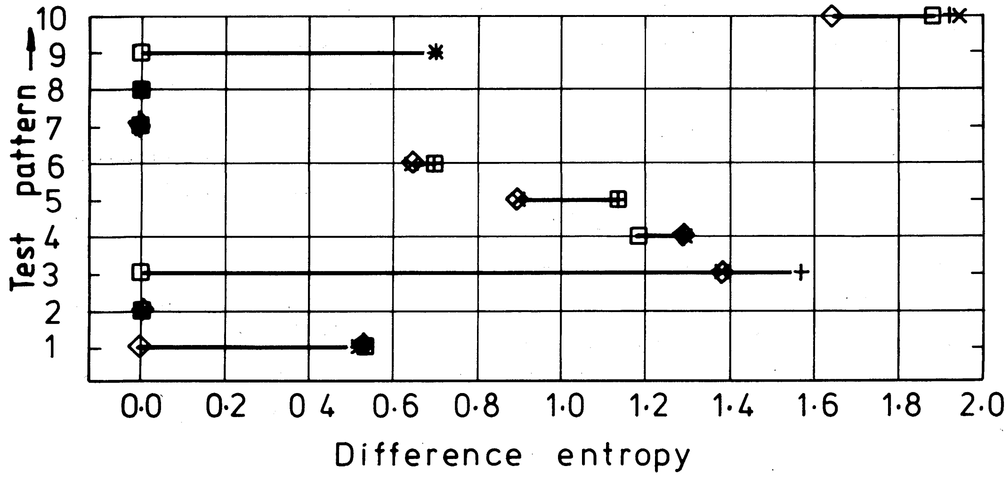


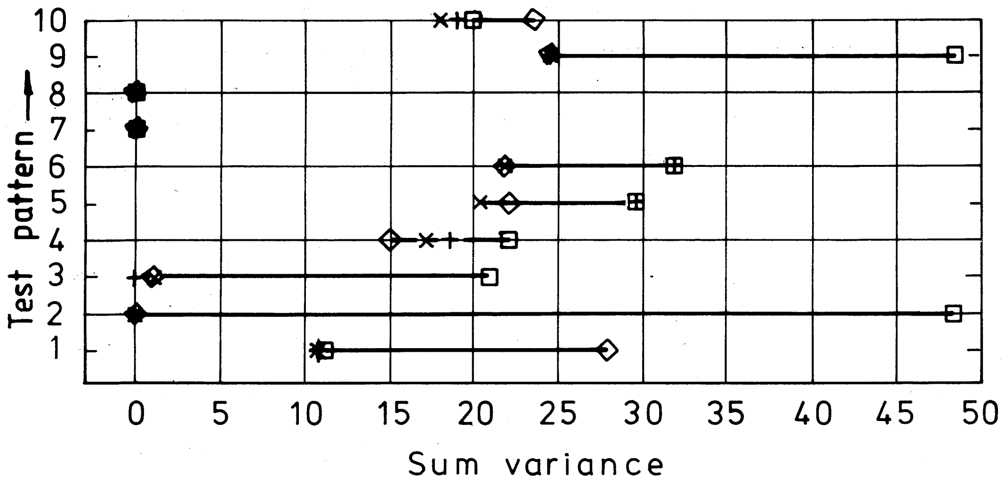




Key to the graph

□ For 0° Neighbours
+ For 90° Neighbours
× For 45° Neighbours
◇ For 135° Neighbours

FIG. 6 cont. CALCULATION OF NEAREST NEIGHBOUR TEXTURE PARAMETERS FOR A SERIES OF TEST PATTERNS

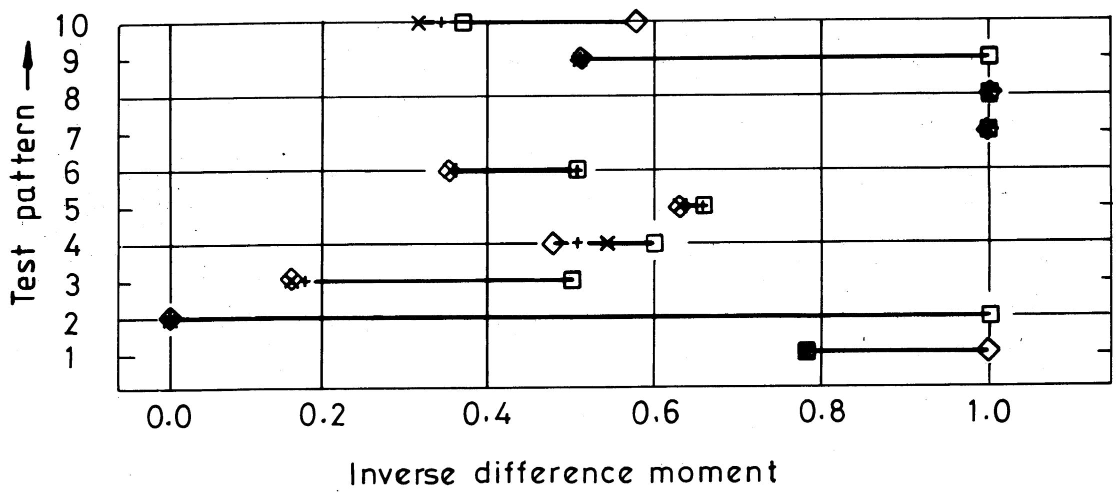


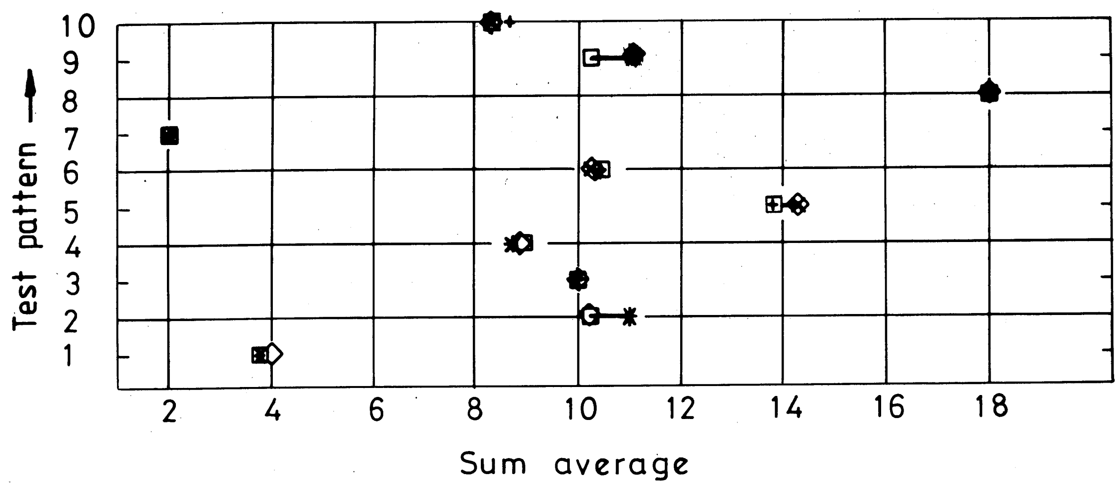




Key to the graph

□ For 0° Neighbours
\+ For 90° Neighbours
× For 45° Neighbours
◇ For 135° Neighbours

FIG. 6 cont. CALCULATION OF NEAREST NEIGHBOUR TEXTURE PARAMETERS FOR A SERIES OF TEST PATTERNS




Key to the graph

□ For 0° Neighbours

+ For 90° Neighbours

× For 45° Neighbours

◇ For 135° Neighbours

FIG.6. cont. CALCULATION OF NEAREST NEIGHBOUR TEXTURE PARAMETERS FOR A SERIES OF TEST PATTERNS

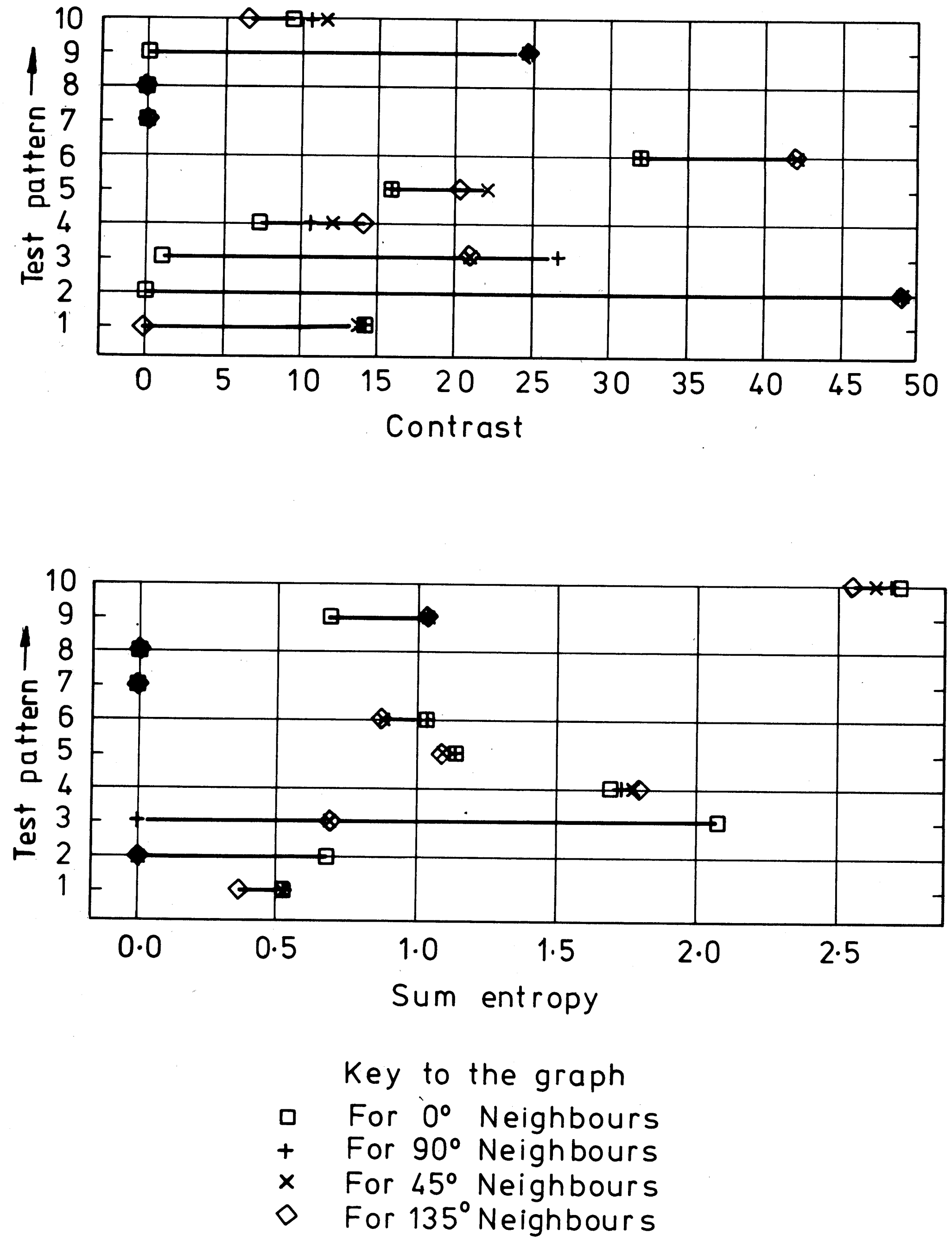
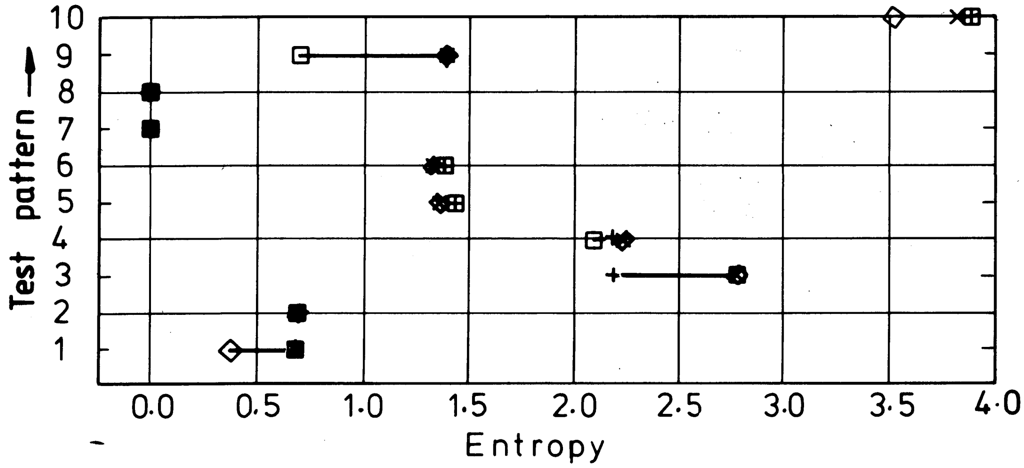
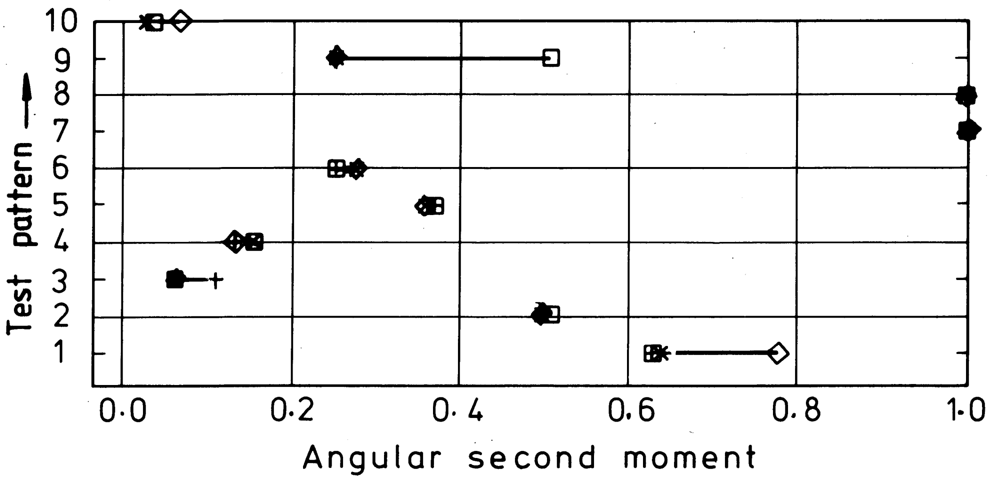



FIG 6 cont. CALCULATION OF NEAREST NEIGHBOUR TEXTURE PARAMETERS FOR A SERIES OF TEST PATTERNS




Key to the graph

□ For 0° Neighbours

+ For 90° Neighbours

× For 45° Neighbours

◇ For 135° Neighbours

FIG 6 cont. CALCULATION OF NEAREST NEIGHBOUR TEXTURE PARAMETERS FOR A SERIES OF TEST PATTERNS

Fig. 7 Normalised co-occurrence matrices of test pattern (3)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.06 | 0 | 0.06 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0.06 | 0 | 0.06 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.06 | 0 | 0.06 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0.06 | 0 | 0.06 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.06 | 0 | 0.06 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0.06 | 0 | 0.06 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.06 | 0 | 0.06 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.06 | 0 |

a $0^o$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0.1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0.1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

b) $90^o$

Fig. 7 Cont'd

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.06 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.06 | 0 | 0.06 |
| 0 | 0 | 0 | 0 | 0 | 0.06 | 0 | 0.06 | 0 |
| 0 | 0 | 0 | 0 | 0.06 | 0 | 0.06 | 0 | 0 |
| 0 | 0 | 0 | 0.06 | 0 | 0.06 | 0 | 0 | 0 |
| 0 | 0 | 0.06 | 0 | 0.06 | 0 | 0 | 0 | 0 |
| 0 | 0.06 | 0 | 0.06 | 0 | 0 | 0 | 0 | 0 |
| 0.06 | 0 | 0.06 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0.06 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

c) $135^o$ and $45^o$

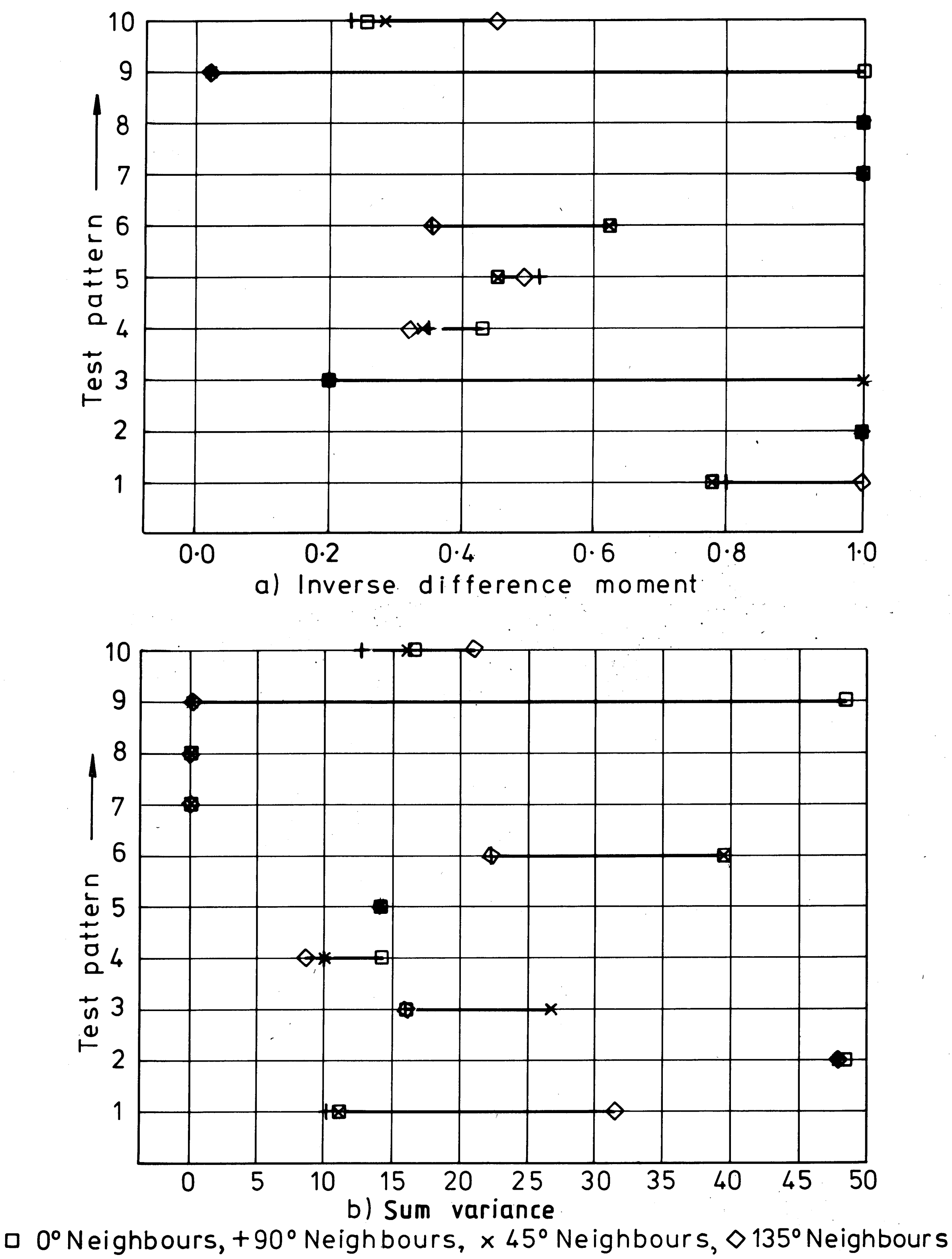
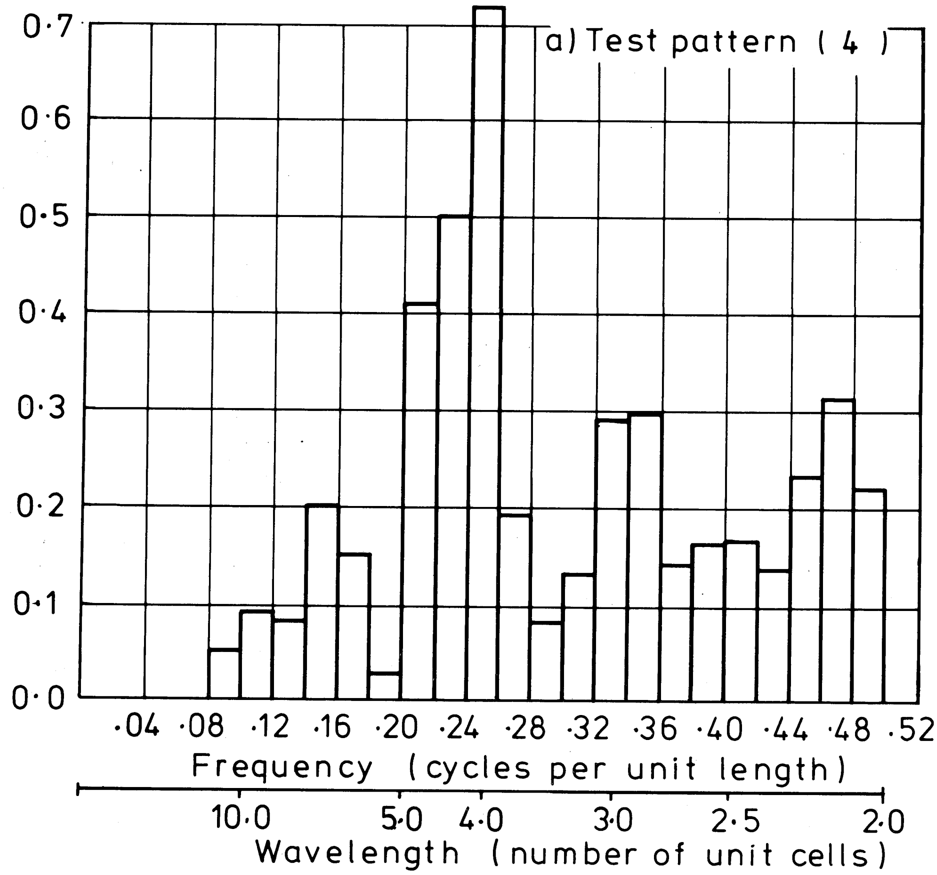



FIG. 8 CALCULATION OF SOME SECOND NEAREST NEIGHBOUR TEXTURE PARAMETERS FOR A SERIES OF TEST PATTERNS



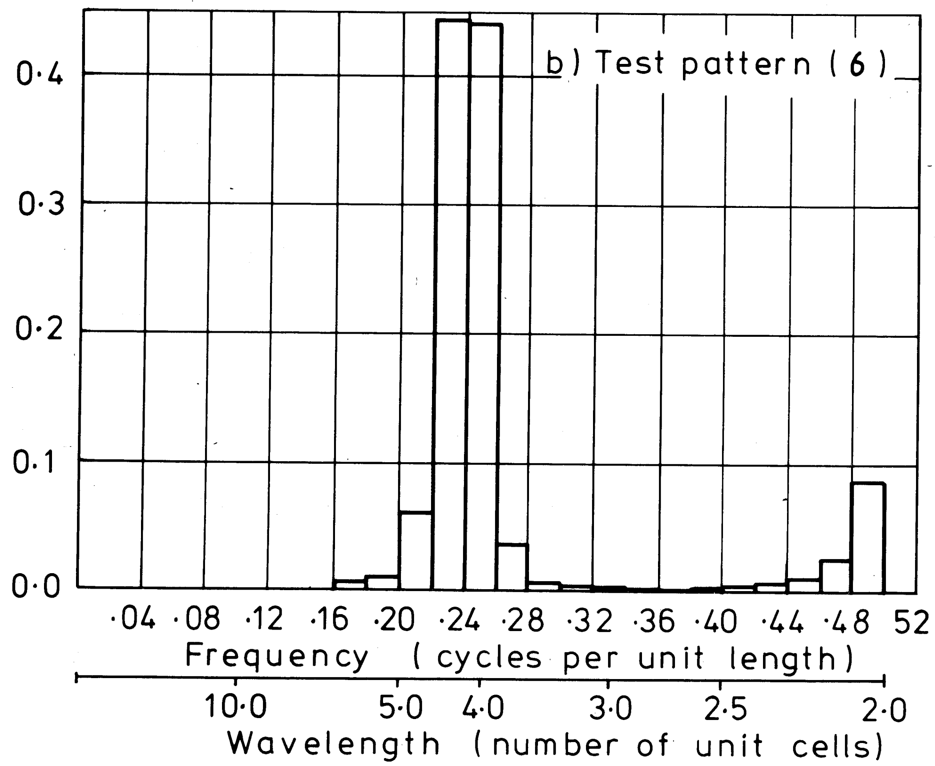




FIG.9. POWER SPECTRA OF TEST PATTERNS ( 4 ) AND ( 6 ).
(BASIC ARRAY REPEATED TO GIVE ARRAY OF SIZE 64 x 64)

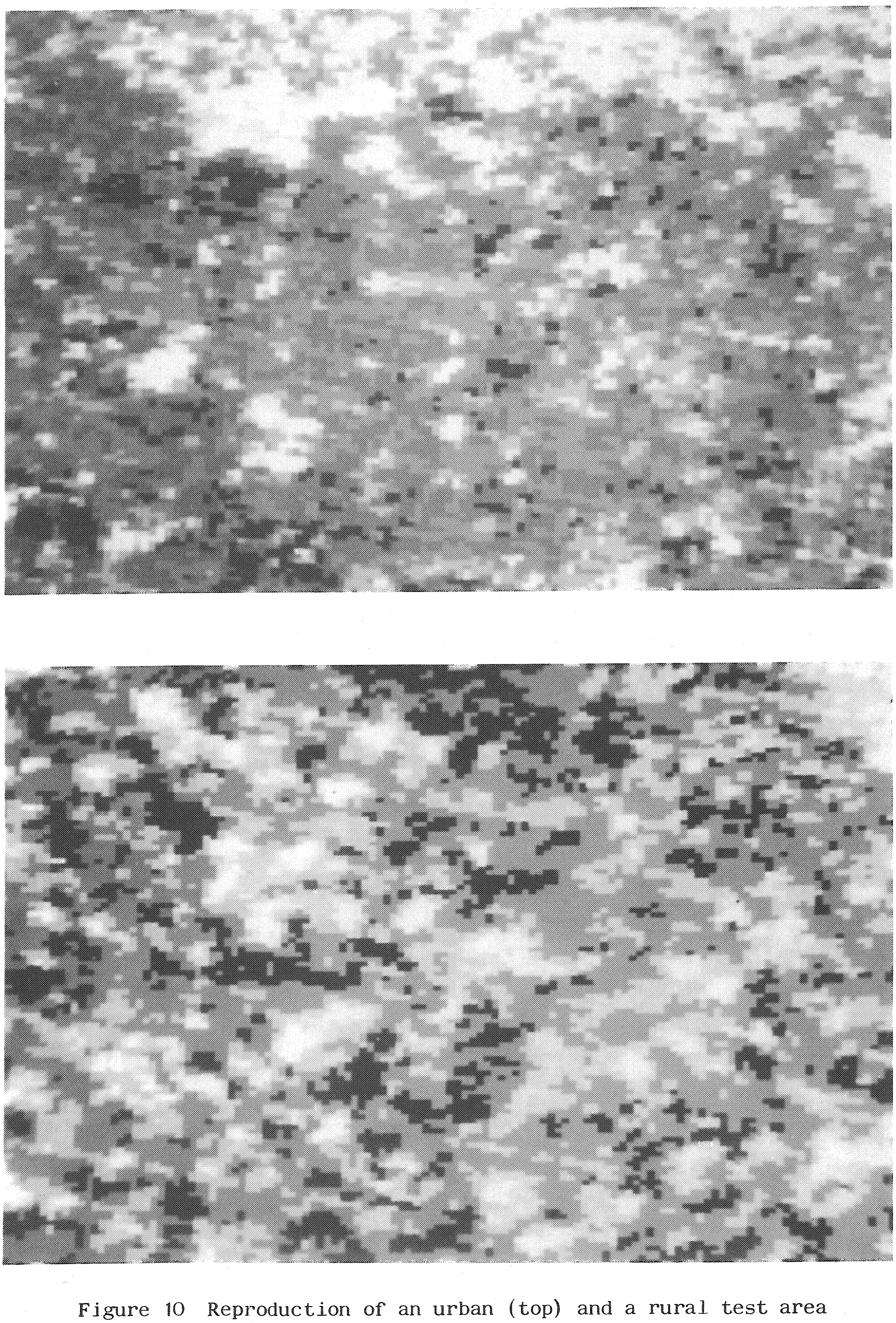


Figure 10 Reproduction of an urban (top) and a rural test area from Landsat satellite data

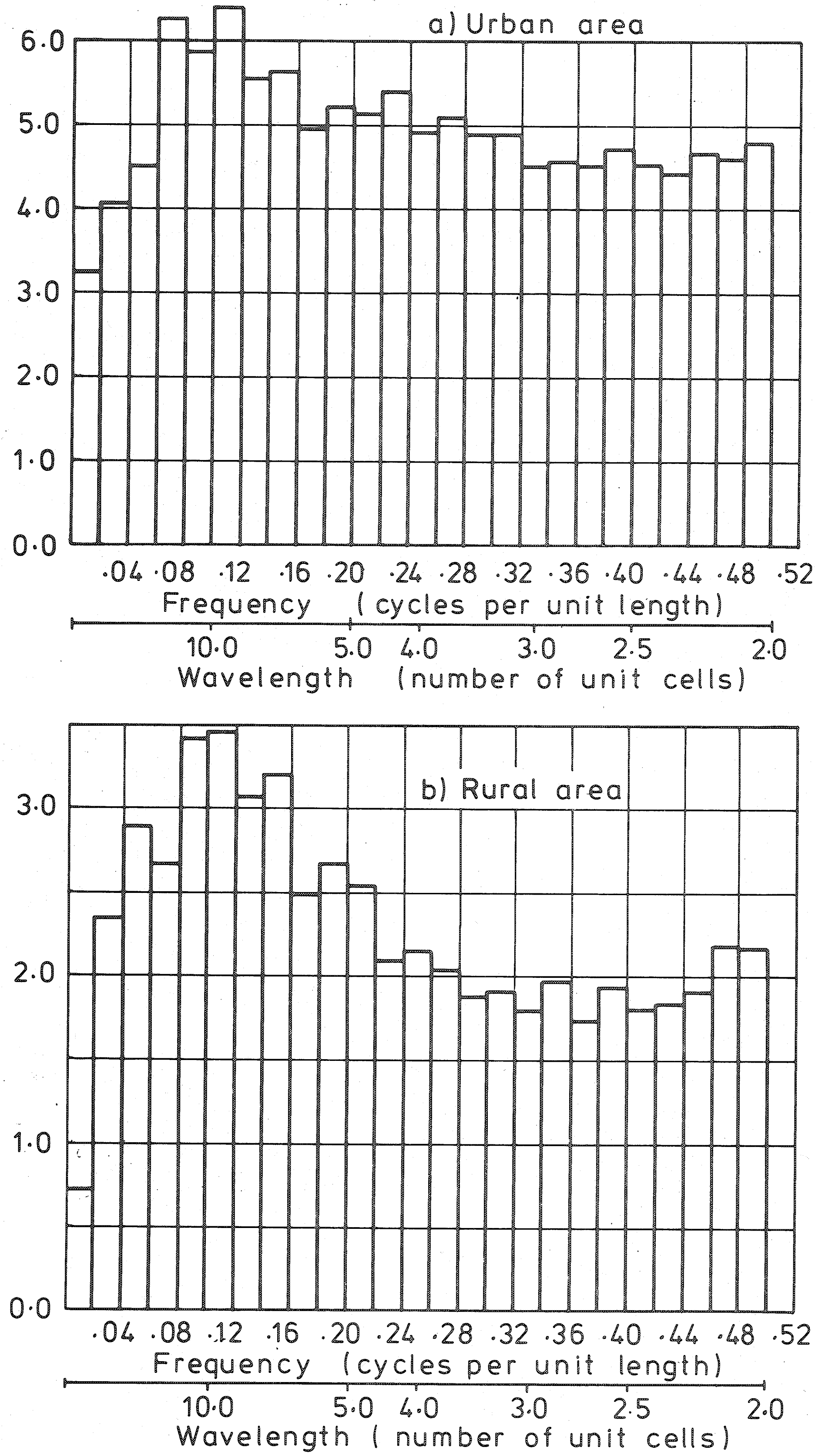



FIG. 11 POWER SPECTRA OF URBAN AND RURAL TEST AREAS FROM A LANDSAT SATELLITE IMAGE

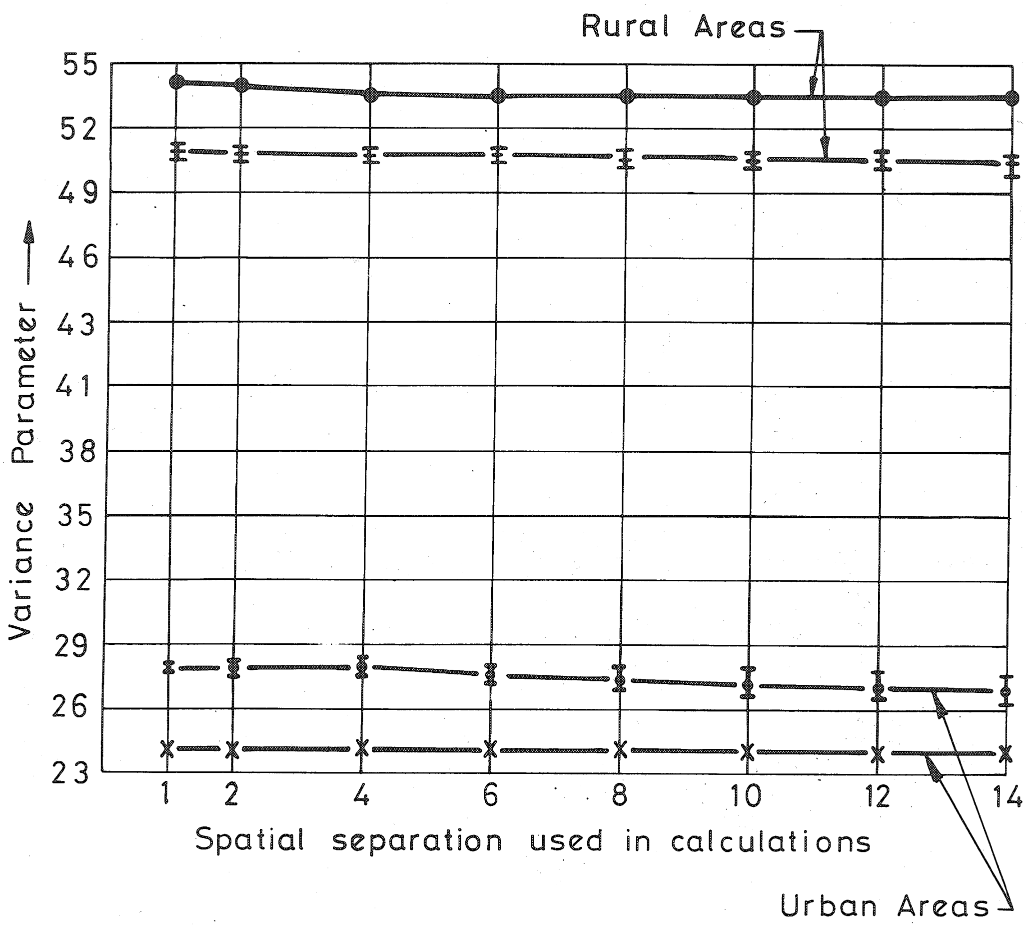



FIG. 12. VARIANCE PARAMETER FOR THE URBAN AND RURAL TEST AREAS

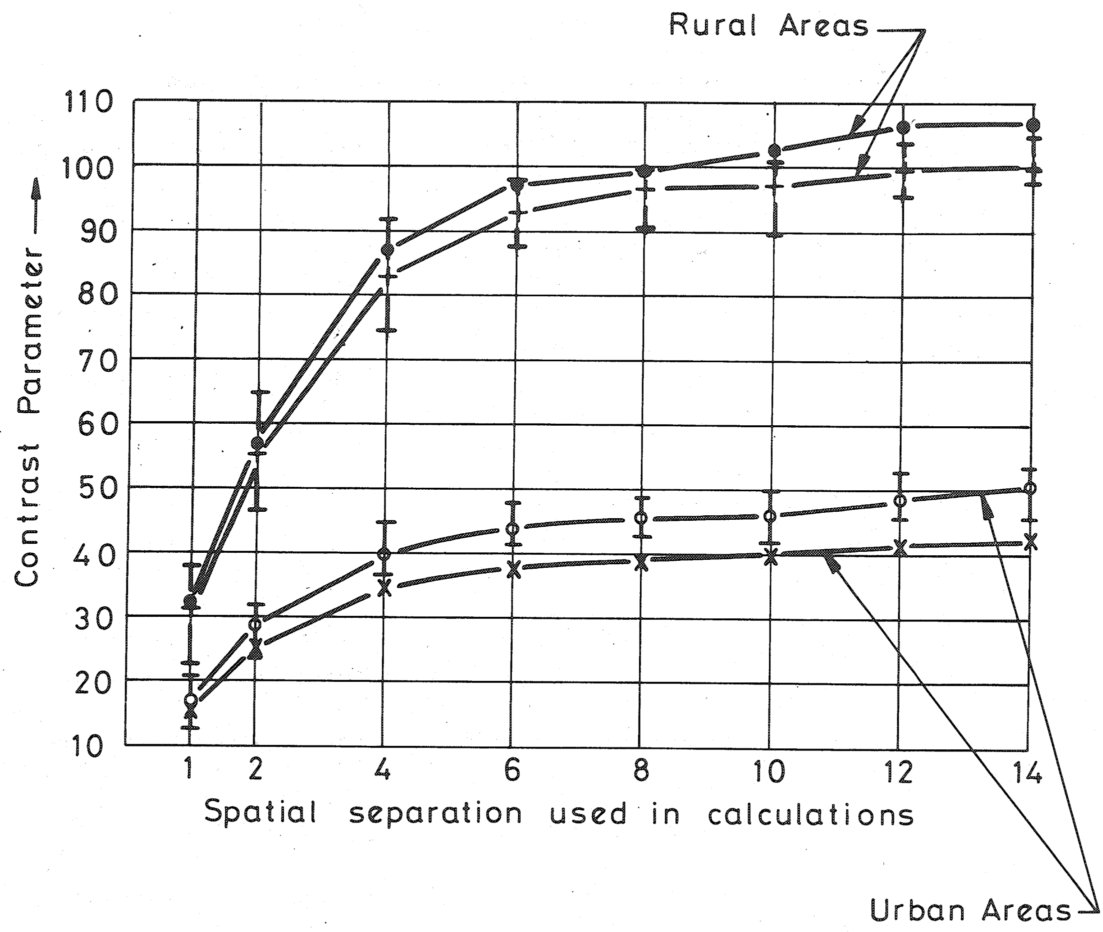



FIG. 13. CONTRAST PARAMETER FOR THE URBAN AND RURAL TEST AREAS

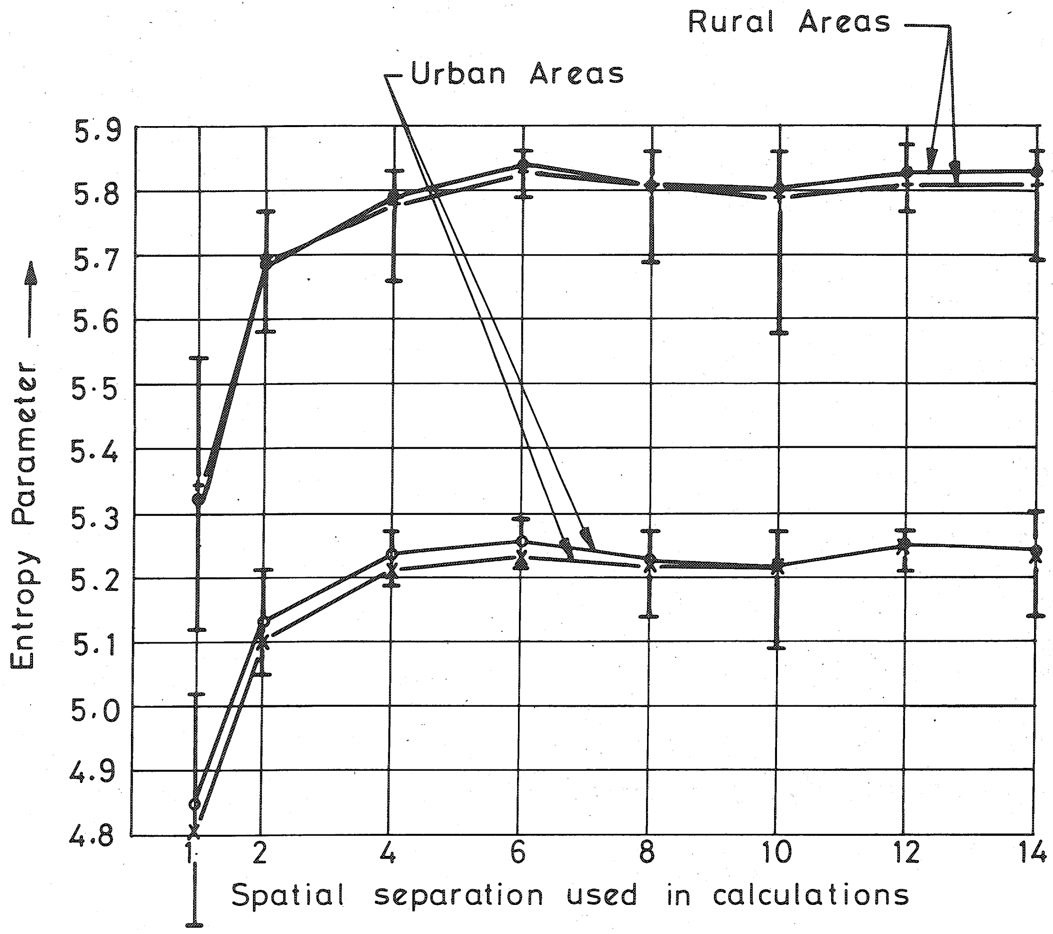



FIG. 14. ENTROPY PARAMETER FOR THE URBAN AND RURAL TEST AREAS

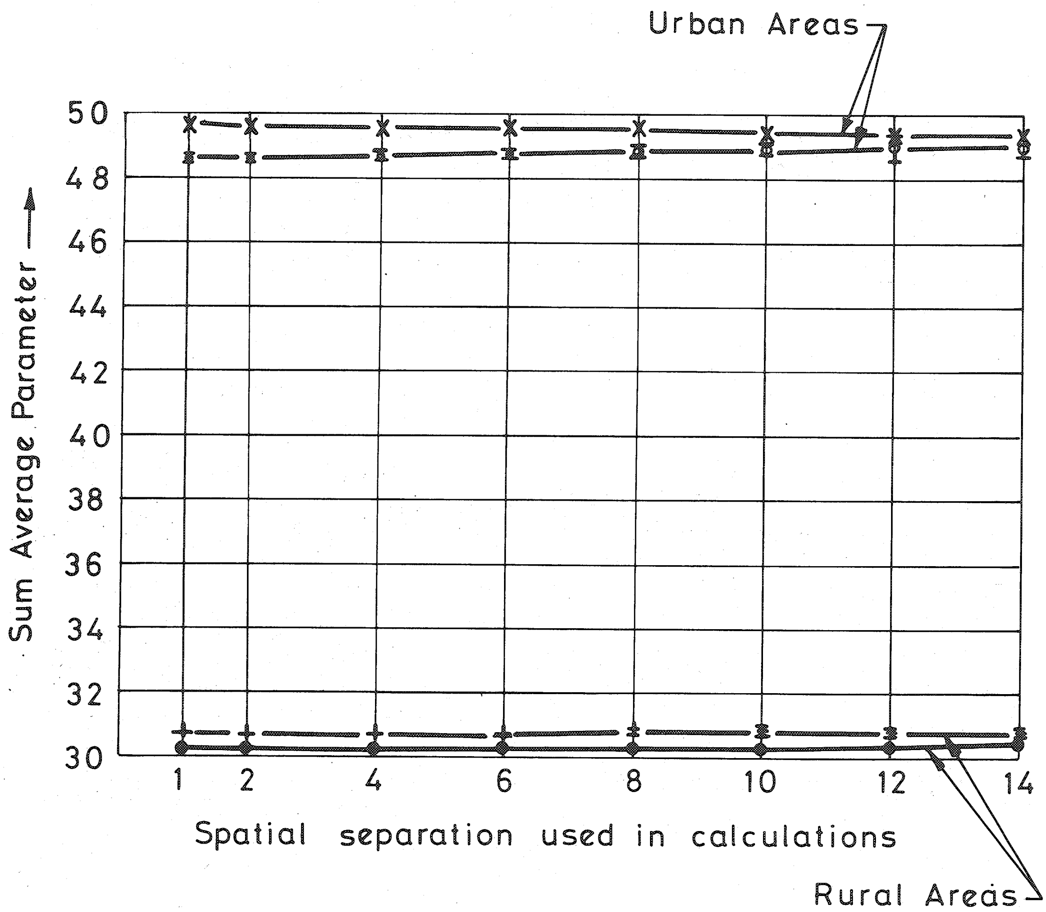



FIG. 15. SUM AVERAGE PARAMETER FOR THE URBAN AND RURAL TEST AREAS

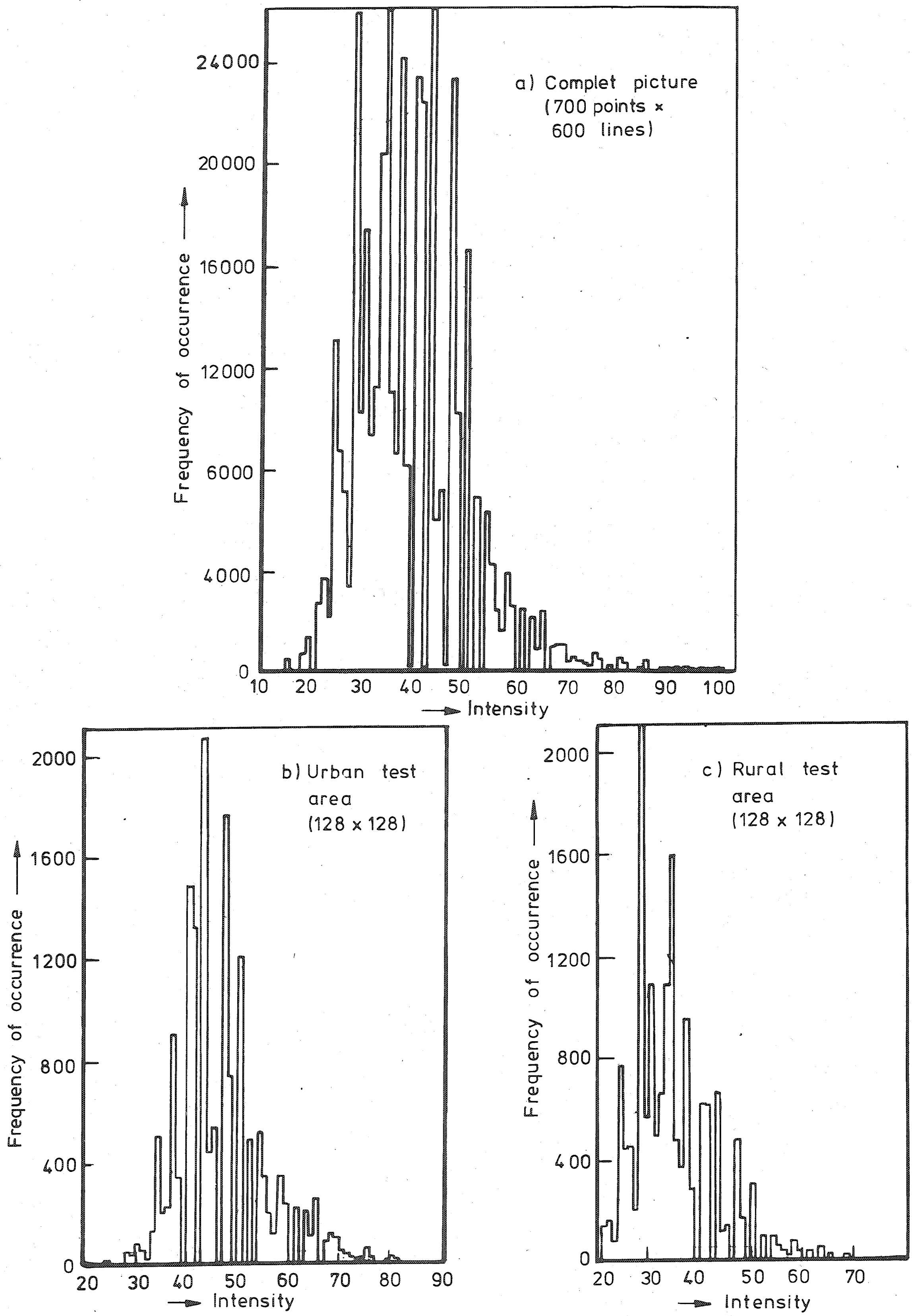



FIG. 16 HISTOGRAM OF ORIGINAL DATA

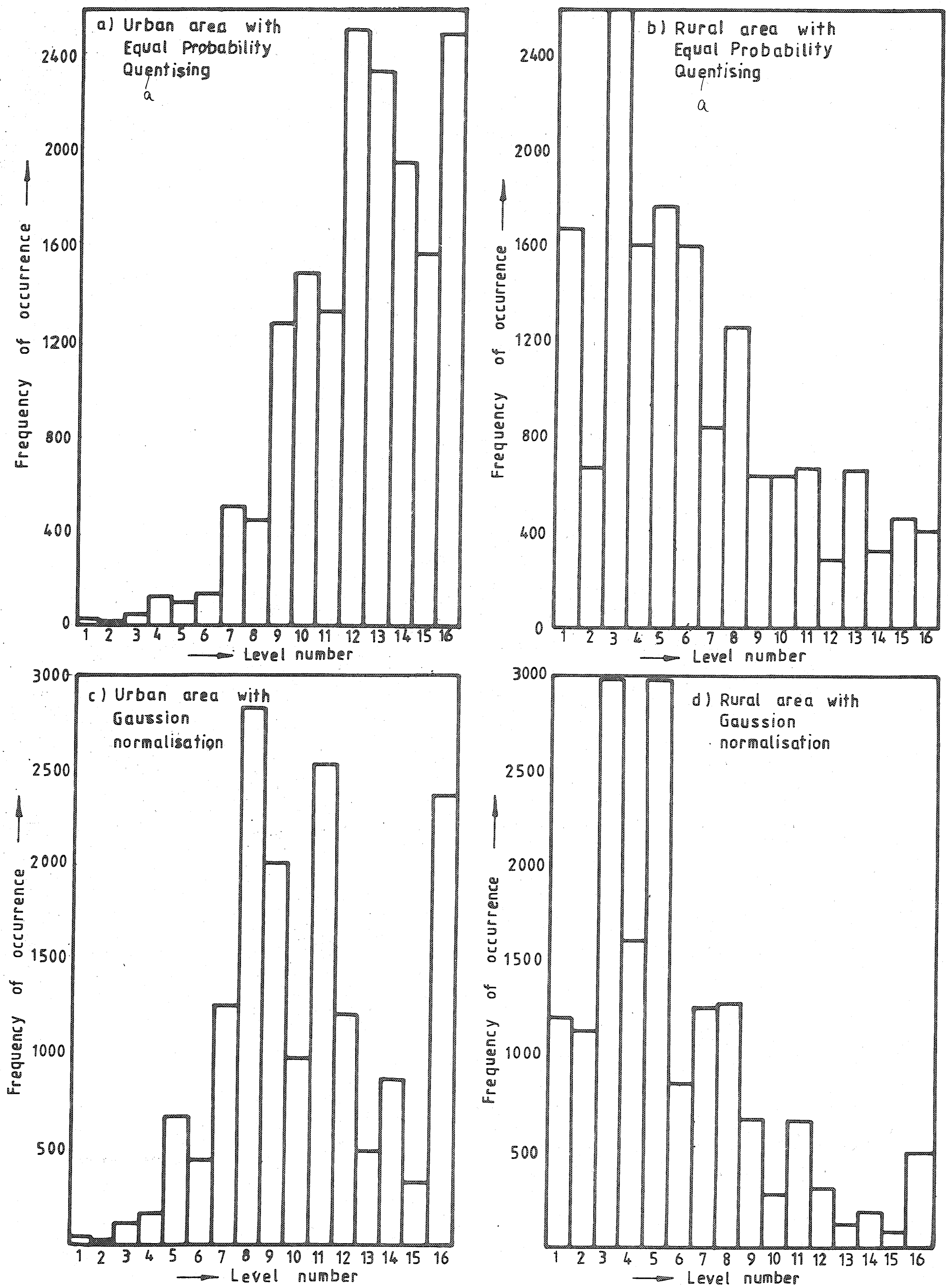



FIG. 17. HISTOGRAMS OF TEST AREAS AFTER NORMALISATION TO 16 LEVELS

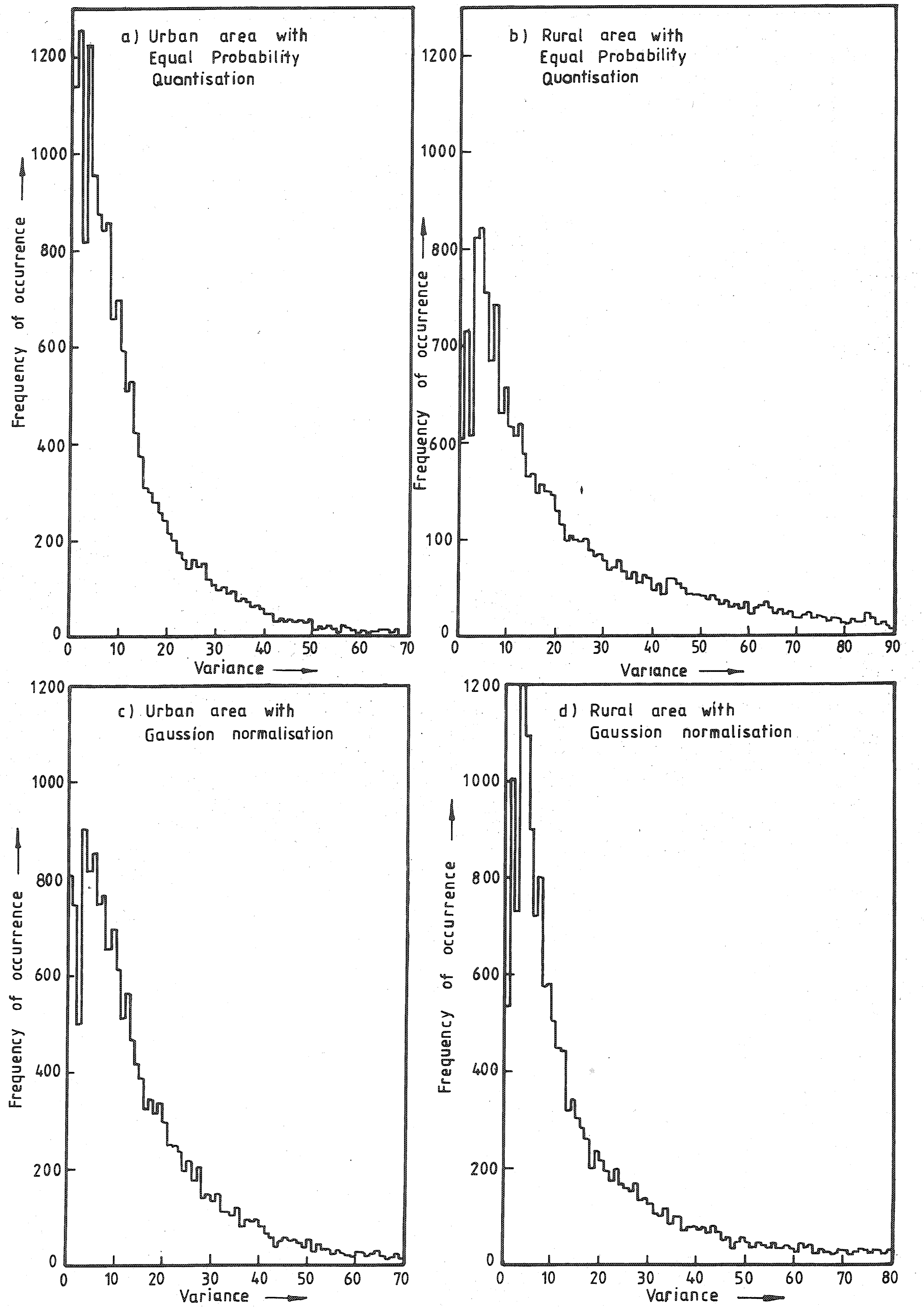



FIG. 18 NEAREST NEIGHBOUR VARIANCE PERAMETER CALCULATED WITH A 3×3 SIZED MASK AT EACH POINT IN THE TEST AREAS

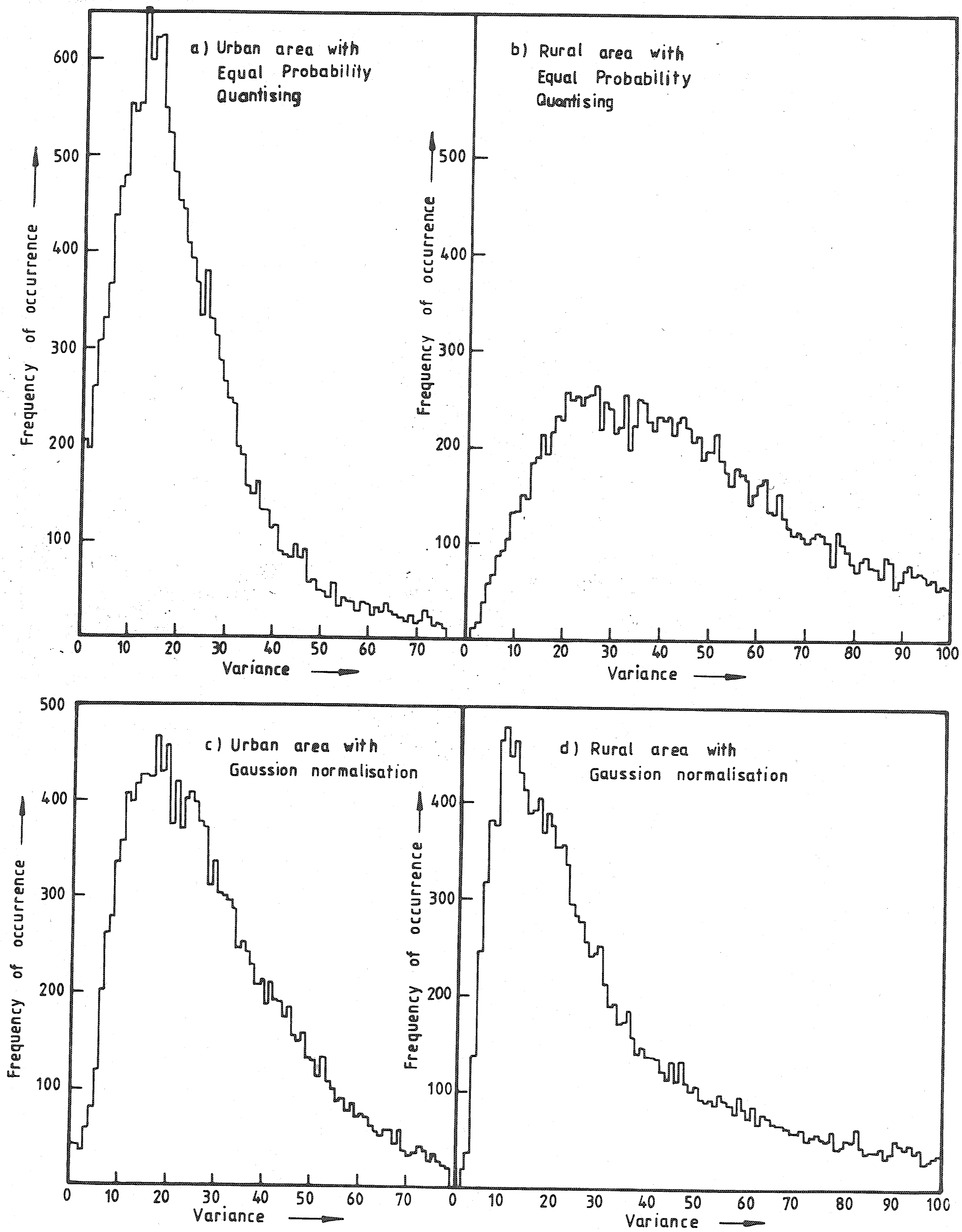



FIG. 19 NEAREST NEIGHBOUR VARIANCE PARAMETER CALCULATED WITH A 7×7 SIZED MASK AT EACH POINT IN THE TEST AREAS