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A SIMPLE APPROACH TO THE DESCRIPTION OF THE THREE-BODY SYSTEM IN MESIC ATOM PHYSICS

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1. A number of calculations have been made by now of various characteristics of mesic molecular processes in the isotopic mixture of hydrogen [viz., for instance, reviews[1)]].

The method of calculation formulated in papers[2,3)] is based on the expansion of the wave function of a three-particle system interacting according to the Coulomb law in terms of the wave functions of the two-centre problem (i.e. the adiabatic concept in the three-body problem). The results show that the two-level approximation of the adiabatic method may be used with approximately 10-30% accuracy in order to calculate the cross-sections of virtually all elastic and inelastic scattering processes of mesic atoms[4)], mesic molecular production[5)], etc., i.e. it is possible to solve the system of two related equations[3,4)]:

$$\left[\frac{d^2}{dR^2} + 2ME - \frac{J(J+1)}{R^2}\right] X_1 = K_{11}X_1 + K_{12}X_2 + 2Q_{12}\frac{dX_2}{dR}\,,$$

$$\left[\frac{d^2}{dR^2} + 2ME - \frac{J(J+1)}{R^2}\right] X_2 = K_{21}X_1 + K_{22}X_2 + 2Q_{21}\frac{dX_1}{dR}\,, \qquad (1)$$

$$M = M_0/m_a, \quad M_0^{-1} = M_a^{-1} + M_b^{-1}, \quad m_a^{-1} = m_\mu^{-1} + M_a^{-1}\,. \qquad (2)$$

Here m_μ and $M_a \geq M_b$ are the masses of the μ meson and a and b nuclei of the hydrogen isotopes respectively, R is the distance between the nuclei, J is the total orbital moment of the three-particle system, and $K_{ij} = K_{ij}(R)$ and $Q_{ij} = Q_{ij}(R)$ are the effective potentials calculated with $\sim 10^{-8}$ accuracy in papers[6)]. All the values in equations (1) are shown in the units (e.z.) of the problem $e = \hbar = m_a = 1$, and, where specifically indicated, the mesic atom units are used (m.a.e.) $e = \hbar = m_\mu = 1$. However, in some cases the accuracy of the two-level approximation is not enough, e.g.:

a) calculation of the energy levels of the μ mesic molecules[2)];

b) calculation of the elastic scattering cross-section of the $p\mu$ mesic atoms in the lower state of the hyperfine structure[4)]

$$p\mu(\uparrow\downarrow) + p \to p\mu(\uparrow\downarrow) + p\,; \qquad (3)$$

c) calculation of the spin flip rate in reaction[4)]

$$d\mu(\uparrow\uparrow) + d \to d\mu(\uparrow\downarrow) + d\,. \qquad (4)$$

In the above cases the system (1) of basic equations must be expanded, for instance in accordance with the scheme proposed (and partly implemented) in papers[2)].

In this paper we propose a simple approach which allows reasonable results to be obtained for all the above-mentioned cases even in the two-level approximation (1).

2. By using the results from papers[2,7,8], it may be shown that for the effective potentials in the system of equations (1) at $R \to \infty$ the following asymptotic relations hold

$$K_{ij}(\infty) = Q_{ij}(\infty) = 0 \ , \tag{5}$$

$$\begin{aligned} (2M)^{-1} K_{11}(\infty) &= -\frac{1}{2} \text{ e.z.} = -\frac{m_a}{2} \text{ m.a.e.} = E_a \ , \\ (2M)^{-1} K_{22}(\infty) &= -\left(\frac{1}{2} + \frac{x}{2M}\right) \text{ e.z.} = -m_a\left(\frac{1}{2} + \frac{x}{2M}\right) \text{ m.a.e.} \ , \end{aligned} \tag{6a}$$

$$x = (M_b - M_a)/(M_b + M_a) \ . \tag{6b}$$

The (6a) value coincides with the energy E_a of the basic state of the isolated atom μM_a but the (6b) value coincides with the energy E_b of the μM_b atom with accuracy only up to terms $\sim M^{-1}$ inclusive. The splitting of the energy levels of the μM_a and μM_b atoms in this approximation equals:

$$(2M)^{-1}\left[K_{22}(\infty) - K_{11}(\infty)\right] = -\frac{x}{2M} \text{ e.z.} = -m_a \frac{x}{2M} \text{ m.a.e.} \tag{7}$$

Furthermore, the M_0 mass (4) in equation (3) does not coincide with the true masses M_a and M_b of the $\mu M_a + M_b$ and $M_a + \mu M_b$ systems respectively:

$$\begin{aligned} M_a^{-1} &= (m\mu + M_a)^{-1} + M_b^{-1} \ , \\ M_b^{-1} &= M_a^{-1} + (m\mu + M_b)^{-1} \ . \end{aligned} \tag{8}$$

In order to eliminate the above-mentioned shortcomings of the conventional approach, we propose below a modification to the two-level approximation which consists of the following:

a) as the effective mass M in equation (3) we use the value

$$\tilde{M} = M_a/m_a \ ; \tag{9}$$

b) as the effective value x we use the value $\tilde{x}$ calculated from the equation

$$-m_a \frac{\tilde{x}}{2\tilde{M}} = E_b - E_a = \frac{m_\mu}{2}\left[\left(1 + \frac{m_\mu}{M_a}\right)^{-1} - \left(1 + \frac{m_\mu}{M_b}\right)^{-1}\right] . \tag{10}$$

It turns out that this formal procedure yields unexpectedly good results when calculating the various characteristics of mesic atom and mesic molecular processes without departing from the framework of the two-level approximation.

Table 1 shows the results of the calculation of the ε_{Jv} energy levels of the μ-mesic molecules of hydrogen isotopes using three methods, 1) the two-level approximation (3), with M and x values corresponding to the standard adiabatic concept (2)

and (6b); 2) the full system of equations identical to (3) including the discrete and continuous spectrum of the two-centre problem[2], and using the same M and x values; 3) The two-level approximation using $\tilde{M}$ and $\tilde{x}$ values determined by means of formulae (9) and (11). It is obvious that in all cases the transition from the (M,x) pair to the $(\tilde{M},\tilde{x})$ pair gives ε_{Jv} values in the two-level approximation which have about the same accuracy as the ε_{Jv} values obtained using the full system of equations for the (M,x) pair.

Table 1

ε_{Jv} energy levels (eV) of the mesic molecules of hydrogen isotopes

	J = 0		J = 1		J = 2	J = 3
	v = 0	v = 1	v = 0	v = 1	v = 0	v = 0
ppμ	247.31[a] 253.55[b] 252.45[c]	-	101.47 106.18 111.16	-	-	-
pdμ	215.68 221.49 220.54	-	91.34 97.90 97.18	-	-	-
ptμ	207.28 213.85 213.55	-	92.21 100.43 98.31	-	-	-
ddμ	322.68 324.99 325.08	33.14 35.66 36.06	224.08 226.29 228.49	0.64 1.96 2.33	83.56 85.67 89.66	-
dtμ	317.03 319.09 318.73	32.20 34.70 34.22	230.10 232.25 232.77	-0.47 0.85 0.58	99.90 102.29 103.51	-
ttμ	361.56 362.89 363.03	81.60 83.68 84.07	287.65 288.96 290.22	43.23 45.02 45.93	170.95 172.26 174.80	46.81 48.14 51.28

The following particle mass values were used in the calculations $m_\mu = 206.769$; $M_p = 1836.152$; $M_d = 3670.481$; $M_t = 5496.918$ and $R_y = 13.60535$ eV [10].

a) Two-level approximation using the adiabatic method[2].

b) Full system of equations using the adiabatic method[2].

c) This paper.

The result obtained for the intrinsic-value problem suggests that the scattering problem solved in the two-level approximation using effective potentials for the $(\tilde{M},\tilde{x})$ pair is significantly closer to reality than the same problem using the same potentials for the (M,x) pair. Figure 1 shows the results of the calculation of the elastic scattering cross-sections (3) for various values of the mass

$\tilde{M}$ ($\tilde{x} = 0$). The results unambiguously point to a critical dependence of the elastic scattering cross-section $p\mu(\uparrow\downarrow) + p$ on the effective mass $\tilde{M}$: when $\tilde{M}$ varies by 10% the cross-section varies by three orders of magnitude.

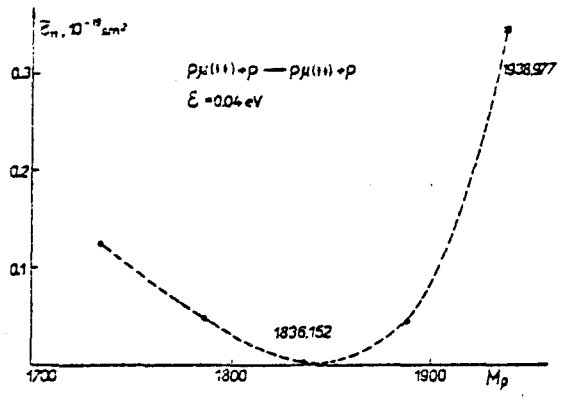

Fig. 1 Elastic scattering cross-section $\bar{\sigma}_{11}$ (3) at a collision energy of $\varepsilon = 0.04$ eV as a function of the effective mass of the proton M_p. The cross-section $\bar{\sigma}_{11} = 1.6 \times 10^{-22}$ cm^2 corresponds to a physical mass of $M_p = 1836.152$.

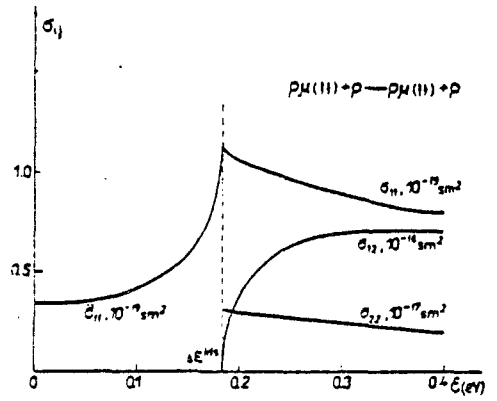

Fig. 2 σ_{ij} cross-sections of (3) and (11) scattering processes of mesic atoms of hydrogen on protons. Splitting of the levels of the hyperfine structure $\Delta E^{hfs} = 0.183$ eV.

Figure 2 shows the results of the calculation of the σ_{11} and σ_{ij} cross-sections for elastic and inelastic scattering of protons on mesic atoms of hydrogen at $\tilde{M} = 5.203$, as calculated from relation (9):

$$(\bar{\sigma}_{11})\ p\mu(\uparrow\downarrow) + p \rightarrow p\mu(\uparrow\downarrow) + p\ , \quad \varepsilon < \Delta E^{hfs}\ ,$$

$$\left.\begin{array}{ll} (\sigma_{11}) & p\mu(\uparrow\downarrow) + p \rightarrow p\mu(\uparrow\downarrow) + p \\ (\sigma_{12}) & p\mu(\uparrow\downarrow) + p \rightarrow p\mu(\uparrow\uparrow) + p \\ (\sigma_{21}) & p\mu(\uparrow\uparrow) + p \rightarrow p\mu(\uparrow\downarrow) + p \\ (\sigma_{22}) & p\mu(\uparrow\uparrow) + p \rightarrow p\mu(\uparrow\uparrow) + p \end{array}\right\} \quad \varepsilon > \Delta E^{hfs}\ . \tag{11}$$

Table 2

Cross-sections and spin flip rate for the pμ + p process at a collision energy $\varepsilon = 0.04$ eV below the reaction threshold ($\Delta E^{hfs} = 0.183$ eV) and $\varepsilon' = 0.05$ eV above the threshold*)

σ_{ij}	M = 4.940	$\tilde{M}$ = 5.203
$\bar{\sigma}_{11}$, 10^{-19} cm^2	0.16×10^{-2}	0.35
σ_{11}, 10^{-19} cm^2	0.18×10^{-1}	0.99
σ_{12}, 10^{-19} cm^2	1.9	5.8
σ_{21}, 10^{-19} cm^2	8.9	27
σ_{22}, 10^{-19} cm^2	6,2	27
λ, 10^{-13} cm^{3}s^{-1}	1,3	3,9
Λ, 10^9 s^{-1}	5,5	17

Tables 2 and 3 show the results of the calculation of the $\bar{\sigma}_{11}$ and σ_{ij} cross-sections at collision energies $\varepsilon = 0.04$ eV and $\varepsilon' = \varepsilon - \Delta E^{hfs} = 0.05$ eV (ΔE^{hfs} is the energy of the hyperfine splitting of the levels of the mesic atoms pμ and dμ) for the pμ + p and dμ + d systems at the M and $\tilde{M}$ values given by formulae (2) and (9) respectively.

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*) The λ and Λ spin flip rates are determined by using the formulae $\lambda = 1/3 \times \sigma_{21}\, v$ and $\Lambda = \lambda N_0$ where v is the relative collision rate, and $N_0 = 4.25 \times 10^{22}$ cm^{-3} is the density of the liquid hydrogen nuclei.

Table 3

Cross-sections and spin flip rates for the dμ + d process at a collision energy of ε = 0.04 eV below the reaction threshold (ΔE^{hfs} = 0.049 eV) and ε′ = 0.05 eV above the threshold*)

σ_{ij}		J = 3/2		J = 1/2	
		M = 9.376	$\tilde{M}$ = 9.632	M = 9.376	$\tilde{M}$ = 9.632
$\bar{\sigma}_{11}$,	10^{-19} cm^2	2.4	2.1	1.5	1.4
σ_{11},	10^{-19} cm^2	2.5	2.3	1.6	1.5
σ_{12},	10^{-19} cm^2	4.3×10^{-2}	3.0×10^{-2}	6.9×10^{-2}	4.9×10^{-2}
σ_{21},	10^{-19} cm^2	8.5×10^{-2}	6.0×10^{-2}	14×10^{-2}	9.6×10^{-2}
σ_{22},	10^{-19} cm^2	1.2	1.2	2.1	1.9
λ,	10^{-13} cm^{3}s^{-1}	2.6×10^{-2}	1.8×10^{-2}	4.2×10^{-2}	2.9×10^{-2}
Λ,	10^{9} s^{-1}	0.11	0.78	0.18	0.12

Table 4

Scattering lengths for mesic atoms of hydrogen isotopes (in units $a_\mu = \hbar^2/m_\mu e^2 = 2.56 \times 10^{-11}$ cm)

Process	a_g		a_u	
	M	$\tilde{M}$	M	$\tilde{M}$
pμ + p	-13.5	-29.4	3.51	3.51
dμ + d	5.34	4.91	2.98	2.95
tμ + t	- 6.72	- 8.93	2.25	2.21

Table 4 shows the scattering lengths for pμ + p → pμ + p reactions with no allowance for the hyperfine structure of the mesic atoms.

This research shows that, by using the $(\tilde{M},\tilde{x})$ pair determined according to formulae (9) and (10), it is possible to obtain binding energies for the three-body system in the two-level approximation which are extremely close to the true values.

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*) The total cross-sections $\bar{\sigma}_{11} = \frac{2}{3}\,\bar{\sigma}_{11}(J = \frac{3}{2}) + \frac{1}{3}\,\bar{\sigma}_{11}(J = \frac{1}{2}) = 1.87$ and the flip rate $\lambda = \frac{1}{3}\,\lambda(J = \frac{3}{2}) + \frac{1}{6}\,\lambda(J = \frac{1}{2})$ are calculated with an allowance for the statistical weights of states with different values of the total momentum J of the three-particle system.

It is most likely that this approach will also give reasonable results for scattering problems (3) and (4). However, a final conclusion on this subject and also a comparison with the experimental results[9] will have to wait until the multi-channel scattering problem has been solved.

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