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MICROWAVE INSTABILITY CRITERIA FOR BUNCHED PROTON BEAMS

by

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Abstract

Stability criteria for bunched proton beams are derived for various stages of the stacking process. During RF-handling, the stability is least when the acceleration stops after the RF-bucket has been shrunk tightly around the bunch. When the RF-voltage is switched off suddenly, the bunch will become unstable during filamentation. However, this leads only to an initial reduction of phase-space density, and is stopped by "shielding" after sufficient protons have been stacked. Adiabatic de-bunching could avoid this blow-up, but requires an even smaller coupling impedance than during RF-handling.

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1. INTRODUCTION

A few years ago[1] experimental observations at the CERN proton synchrotron led to the realization that a longitudinal instability can occur in bunched beams at very high frequencies, with wave lengths small compared to the bunch length. The growth-rates are correspondingly high and the thresholds are given by the coasting beam stability criterion, but with local values for the current and momentum spread. This yields usually much more stringent conditions than those obtained with the average values. Because of the frequency range involved, one often refers to this phenomenon as "microwave instability".

Similar effects occur in electron storage rings, and are considered one of the causes of bunch-lengthening. Several attempts to develop a theoretical understanding [2][3][4] have met with only limited success, and have been followed by a more empirical "scaling law"[5] based on measurements in SPEAR and other electron machines. All these theories have two or more adjustable parameters which are used to predict the high-frequency behaviour of the coupling impedance. The "local" stability criterion gives also a good approximation to these data, and, as it is the most simple, we shall apply it to the various stages necessary for stacking in proton storage rings.

2. LOCAL STABILITY CRITERION

For longitudinal bunch oscillations with wavelengths short compared to the bunch-length, one can use the coasting beam stability criterion with local variables

$$\left|\frac{Z}{n}\right| < F \frac{E_o}{e} \frac{|\eta|}{\gamma I_p} \left(\frac{\delta p}{m_o c}\right)^2 \tag{1}$$

where F is a formfactor of order unity, depending on the momentum distribution of the particles, and the ratio of real to imaginary part of the coupling impedance, $E_o = m_o c^2$ is the particle rest energy.

$\eta = \frac{p}{\Omega}\frac{d\Omega}{dp} = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$ is the revolution frequency dispersion

δp is the full momentum spread at half height (local) and

I_p is the peak current given by

$$I_p = G \frac{h}{m} \frac{2\pi}{\Delta\phi} I_o \tag{2}$$

where G is another form factor of order unity, depending on the longitudinal distribution of particles

h is the harmonic number

m is the number of bunches

$\Delta\phi$ is the bunch length in RF radians

and I_o is the average current.

It is obvious that for a bunched beam I_p is larger than I_o, and for the same momentum spread the threshold value of Z/n is thus smaller than for coasting beams, at least for the same high frequencies. For coasting beams, however, the criterion (1) has to be applied also at low frequencies, where the impedance may be larger due to the wall resistivity contribution.

For bunches with parabolic distributions, F = 4/5 and G = 3/2[6]. Furthermore, the spread at the bottom is $\Delta p = 4\delta p/3$. Combining Eqs (1) et (2) with these values we obtain

$$\left|\frac{Z}{n}\right| < \frac{3}{20\pi}\,\frac{m}{h}\,\frac{E_o}{e}\,\frac{|n|\,\Delta\phi}{\gamma I_o}\left(\frac{\Delta p}{m_o c}\right)^2 \tag{3}$$

3. SMALL BUNCHES IN LARGE BUCKETS

For small bunches in large buckets the shape in phase-space is approximately elliptical, and the bunch-area in $\Delta\beta\gamma$, RF-radians is

$$A = \frac{\pi}{4}\,\Delta\phi\,\frac{\Delta p}{m_o c} \tag{4}$$

Solving for $\frac{\Delta p}{m_o c}$, and substituting into Eq (3) yields the stability criterion

$$\left|\frac{Z}{n}\right| < \frac{12}{5\pi^3}\,\frac{m}{h}\,\frac{E_o}{e}\,\frac{|n|}{\gamma I_o}\,\frac{A^2}{\Delta\phi} \tag{5}$$

In this form, the criterion is useful at injection, when the bunch is determined externally.

After capture in RF-buckets, the bunch length and the momentum spread are related by

$$\frac{\Delta p}{m_o c} = \frac{\beta\gamma f_s}{|n|\,f_{RF}}\,\Delta\phi \tag{6}$$

where $$f_s = f_{RF}\left(\frac{eV|\eta|\cos\phi_s}{2\pi\beta^2\gamma h E_o}\right)^{1/2} \tag{7}$$

is the synchrotron frequency for the RF voltage V and stable phase angle ϕ_s. $f_{RF} = hf_o$ is the RF frequency.

Combining Eqs. (4), (6) and (7) we obtain for the bunch length

$$\Delta\phi = \left(\frac{32h|\eta|A^2E_o}{\pi\gamma eV\cos\phi_s}\right)^{1/4} \tag{8}$$

which may be smaller than the value at injection. Substitution of Eq. (8) into Eq. (5) yields [7]

$$\left|\frac{Z}{n}\right| < \frac{6m}{5\pi^3h}\left(\frac{E_o}{e}\cdot\frac{|\eta|A^2}{\gamma I_o}\right)^{3/4}\left(\frac{\pi}{2}\frac{V\cos\phi_s}{hI_o}\right)^{1/4} \tag{9}$$

In this form, the criterion can be used during matching and acceleration, as long as the RF voltage is not reduced to shrink the buckets around the bunches.

4. BUNCHES IN TIGHT BUCKETS

During the acceleration into the stack, the RF-voltage is usually reduced until the buckets are filled.

Then Eq. (4) is no longer valid, but we can now use expressions from bucket theory[8] to get

$$A = \frac{8\alpha(\Gamma)}{\sqrt{2}Y(\Gamma)}\frac{\Delta p}{m_oc} \tag{10}$$

where $\Gamma = \sin\phi_s$. (11)

$\alpha(\Gamma)$ and $Y(\Gamma)$ are tabulated functions describing the ratio of moving to stationary bucket area and height. Alternately, Eq. (10) can be written

$$\frac{\Delta p}{m_oc} = \frac{A}{2\pi}\zeta(\Gamma) \tag{12}$$

where $$\zeta(\Gamma) = \frac{\pi Y(\Gamma)}{2\sqrt{2}\alpha(\Gamma)} \tag{13}$$

is 2π times the ratio of bucket height to bucket area. Substitution of these expressions into Eq. (3) yields after expansion with A^2

$$\left|\frac{Z}{n}\right| < \frac{3}{160}\,\frac{m}{h}\,\frac{E_o}{e}\,\frac{|\eta| A^2}{\gamma I_o}\; R\,(\Gamma) \tag{14}$$

where

$$R(\Gamma) = \frac{\Delta\phi(\Gamma)\,Y^2(\Gamma)}{4\pi\alpha^2(\Gamma)} = \frac{2}{\pi^3}\Delta\phi(\Gamma)\,\zeta^2(\Gamma) \tag{15}$$

has been defined such that R (O) = 1.
For $\Gamma>0$, $R(\Gamma)>1$, e.g. for $\phi_s = 30^o$, R (½) = 1.617 (see fig. 1). The stability criterion Eq. (14) thus becomes most critical when $\Gamma = 0$, i.e. at the end of the acceleration.

In all the criteria, the common factor

$$\frac{|\eta| A^2}{\gamma I_o} \tag{16}$$

appears. The factor $|\eta|/\gamma$ shows that stability can become critical when the energy is close to transition. $|\eta|/\gamma$ has its maximum value at $\sqrt{3}\gamma_{tr}$, where it becomes $2/(3\sqrt{3}\gamma_{tr}^3)$, and decreases rather slowly above that value.

The factor A^2/I_o shows that the threshold can be increased by increasing the bunch area, even if the current increases proportionally [9].

5. DEBUNCHING

If the RF voltage is switched off suddenly at the end of the acceleration, as is done customarily in the ISR, the bunches will start to filament. As the filaments become longer, their local momentum spread and current decreases continuously, and thus also the threshold given in Eq. (1), which is proportional to the square of the spread divided by the current. Eventually the filaments will thus become unstable, and the momentum spread will not decrease further. Continuing filamentation will fill the phase-space between filaments, and the result is a coasting beam with the same momentum spread as the original bunches, corresponding to a loss in longitudinal density. However, experiments in the ISR show that this effect is reduced after the first few pulses due to "shielding" by the particles already stacked.

On the other hand, if the RF voltage is reduced to zero adiabatically, filamentation will be avoided. However, the momentum spread will shrink by another factor of $\pi/2$, and hence the stability criterion becomes

$$\left|\frac{Z}{n}\right| < \frac{3}{40\pi^2} \frac{m}{h} \frac{E_o}{e} \frac{|\eta| A^2}{\gamma I_o} \tag{17}$$

which is even more stringent than Eq. (14). Furthermore, impedances at lower frequencies have to be taken into account as the bun length increases. Nevertheless, if the coupling impedance can be kept below the threshold given by Eq. (17), it should be possible to debunch without loss of longitudinal density. On the other hand, if the coupling impedance is too high, the resulting instability will again blow up the momentum spread. This seems to be the case in the ISR, where adiabatic debunching has not shown any advantage over sudden voltage reduction and has been abandoned.

6. CONCLUSIONS

The stability criterion for high-frequency oscillations of bunched proton beams is given in several forms applicable to different stages of a stacking cycle. It appears that stability of the bunched beams is most critical after the RF voltage has been reduced to shrink the buckets around the bunches, in particular at the end of acceleration when the stable phase angle is reduced to zero. The thresholds are even lower during adiabatic debunching, and the beam will always become unstable during filamentation if the RF voltage is switched off suddenly.

The thresholds can be increased by injecting at an energy near to or above $\sqrt{3}$ times transition or by increasing the injected bunch area. Nevertheless, the microwave stability remains generally one of the most stringent requirements on the impedance seen by the beam.

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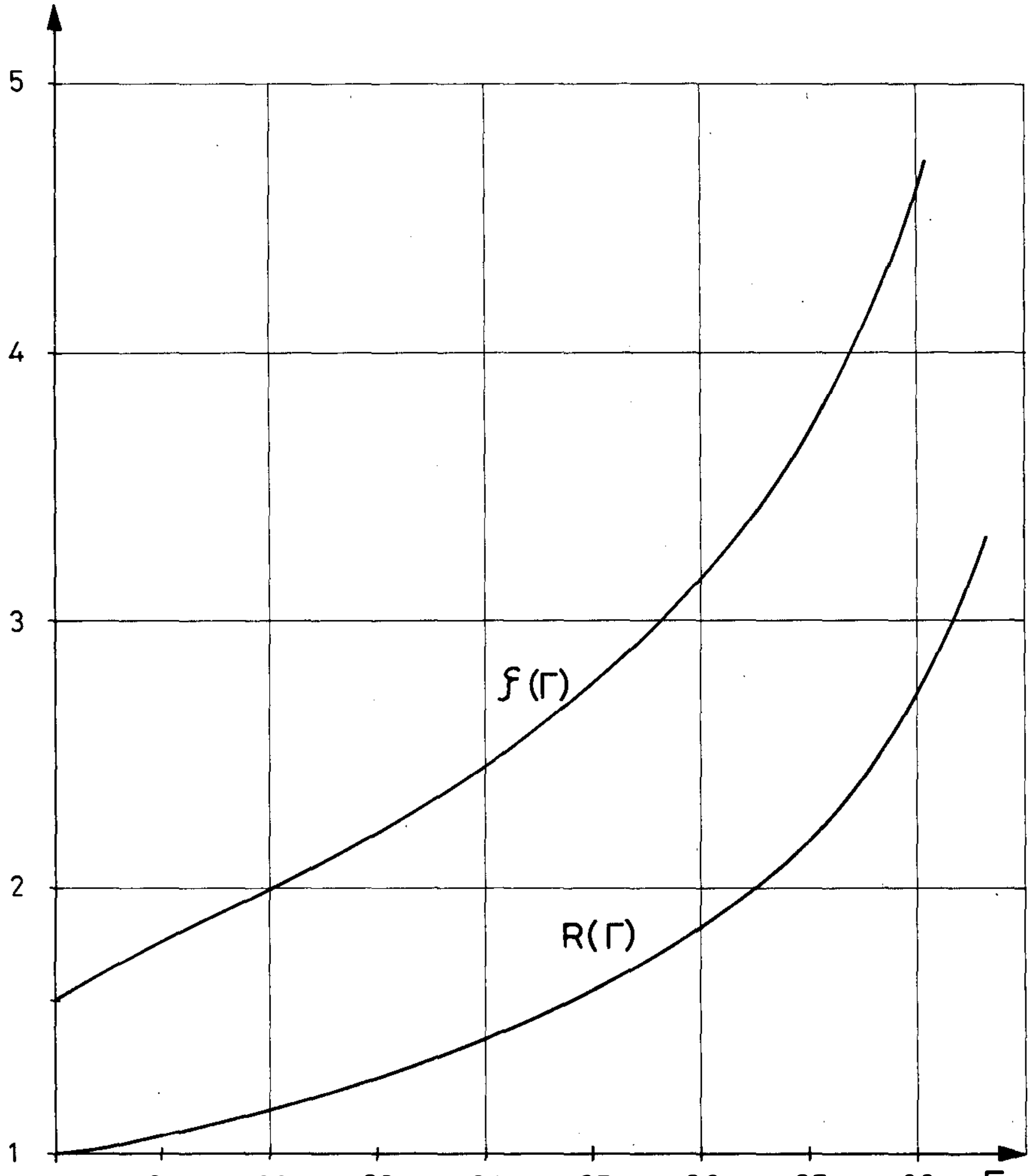

Fig. 1: Normalized bucket functions $R(\Gamma)$ and $f(\Gamma)$