INSTITUTE OF HIGH-ENERGY PHYSICS, SERPUKHOV
Report IFVE OP 71-74

ON TRANSPORTING POLARIZED BEAMS OF CHARGED PARTICLES

V.I. Voronov, V.P. Kartashev, R.A. Rzaev

Serpukhov 1971

Translated at CERN by R. Luther
(Original: Russian)
Not revised by the Translation Service

(CERN Trans. 76-1)

Geneva
August 1976

1. Introduction

It is very interesting to study the spin dependence of proton interactions at high energies. There are various theoretical predictions concerning the effects of polarization and spin behaviour in this energy region. Not only a polarized target but also polarized proton beams are needed in order to perform experiments to study spin dependence. Beams of this type are planned at the (200-500) GeV accelerator at Batavia /1/.

There are three ways of producing high-energy polarized protons: 1) acceleration of directly polarized protons /2,3/; 2) polarization during elastic scattering /1/; 3) polarized protons from hyperon decay /1/. Without going into detail on each method of obtaining polarized protons, we shall examine in this paper the problems of transporting such beams.

2. Description of particle polarization

The layout of the experiment to produce a polarized beam of particles and capture them in an optics system is shown in fig. 1 (see, for instance / /). The angle θ denotes the direction of particle emission in the nuclear reaction and has arbitrary values. Since the polarization vector $\vec{P}_o$ is perpendicular to the plane of the reaction in proton-proton interactions, for the components p_x, p_y and p_z at the input to the optics system the following relations apply:

$$p_x = -P_0 \frac{y \cos\alpha . \cos\theta}{\sqrt{y^2\cos^2\alpha + \ell^2\sin^2(\theta+\alpha)}},$$

$$p_y = P_0 \frac{\ell \sin(\theta+\alpha)}{\sqrt{y^2\cos^2\alpha + \ell^2\sin^2(\theta+\alpha)}}, \qquad (1)$$

$$p_z = -P_0 \frac{y \cos\alpha . \sin\theta}{\sqrt{y^2\cos^2\alpha + \ell^2\sin^2(\theta+\alpha)}}$$

In order to determine the polarization states of relativistic protons in an external magnetic field, we shall use the quasi-conventional spin motion equation which has the following form in the laboratory frame /5/:

$$\frac{d\vec{P}}{dt} = \frac{e}{mc\gamma}\left[\vec{P}\times\left(\vec{B} + \frac{g-2}{2}\left(\vec{B}_{\parallel} + \gamma\vec{B}_{\perp}\right)\right)\right], \qquad (2)$$

where $\vec{B}_{\parallel}$ and $\vec{B}_{\perp}$ are the magnetic field's components parallel and perpendicular to the direction of the momentum, $\gamma = (1 - v^2/c^2)^{-1/2}$, $g = 5.6$. As can be seen from (2), $P_x^2 + P_y^2 + P_z^2 = \text{const}$, and the polarization vector P precesses around $\vec{B}_{\perp}$ and $\vec{B}_{\parallel}$ at rates:

$$\omega_{\perp} = \frac{eB_{\perp}}{mc\gamma}\left(1 + \frac{g-2}{2}\gamma\right), \qquad \omega_{\parallel} = \frac{g}{2}\,\frac{eB_{\parallel}}{mc\gamma}.$$

Let us find the solution to equation (2) in the field of a quadrupole lens with a gradient G ($B_x = G\cdot y$, $B_y = G\cdot x$). We shall divide the lens into n parts of length z so that variations in the particle's x and y coordinates may be ignored within the limits of each division. Then (2) takes the form of a system of differential equations with constant coefficients for the components of the polarization vector, and an exact solution can be found for each section:

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \frac{1}{k^2}(a^2 + b^2\cos kz) & \frac{ab}{k^2}(1-\cos kz) & -\frac{b}{k}\sin kz \\ \frac{ab}{k^2}(1-\cos kz) & \frac{1}{k^2}(b^2 + a^2\cos kz) & \frac{a}{k}\sin kz \\ \frac{b}{k}\sin kz & -\frac{a}{k}\sin kz & \cos kz \end{pmatrix} \begin{pmatrix} P_x^0 \\ P_y^0 \\ P_z^0 \end{pmatrix} \qquad (3)$$

where

$$k^2 = a^2 + b^2, \quad a = \frac{eGy}{mvc\gamma}\left(1 + \frac{g-2}{2}\gamma\right), \quad b = \frac{eGx}{mvc\gamma}\left(1 + \frac{g-2}{2}\gamma\right),$$

x and y are the particle shifts at the beginning of section z in the horizontal and vertical planes respectively.

In order to obtain the vector conversion formula for a defocussing lens, a must be replaced by -a and b by -b in (3).

This method for describing particle polarization in a quadrupole lens is easier to use and, unlike the method given in paper /4/ in which the depolarization of a non-relativistic particle beam was considered, it does not involve the selection of quadrants for the polarization vector's conversion matrix.

In a homogeneous magnetic field B directed, for instance, along the y axis, the spin precesses according to the law:

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} \cos KL & 0 & -\sin KL \\ 0 & 1 & 0 \\ \sin KL & 0 & \cos KL \end{pmatrix} \begin{pmatrix} P_x^0 \\ P_y^0 \\ P_z^0 \end{pmatrix} . \qquad (4)$$

Here $K = \frac{eB}{mvc\gamma}\left(1 + \frac{g-2}{2}\gamma\right)$ and L is the length of the magnet.

As is known /6/, during motion in a magnetic field the momentum and spin vectors rotate about $\vec{B}$ with the same angular velocity, i.e., the particle's polarization remains at a constant angle to the direction of motion. However, if the particle has an anomalous magnetic moment (such as the proton), its spin precesses in the field at a velocity which is different from the rate of change of direction of the momentum's vector. If the magnetic field is perpendicular to the particle's momentum, the angle of precession of the spin about the field is:

$$\beta = \frac{eBL}{mvc\gamma} \left(1 + \frac{g-2}{2}\gamma\right),$$

and the angle between the spin and momentum vectors is related to the angle of deflection a of the beam in the magnet by the relation

$$\delta = \frac{g-2}{2}\gamma a . \tag{5}$$

By virtue of precession (5) it is possible to alter the orientation of the proton spin in relation to the direction of motion. For instance, if polarization scattering in an accelerator occurs in the vertical plane, the polarization vector of the scattered protons, which always lies perpendicular to the scattering plane, will be in the horizontal plane. By placing a bending magnet in the accelerator's extraction channel and setting the angle of deflection in the horizontal plane **a in such a way that** the condition $\delta = \pi/2$ is fulfilled, it is possible to convert transversely polarized beams into longitudinally polarized beams, and the polarization vector can be either parallel or antiparallel to the protons' velocity in the beam. This method for producing longitudinally polarized protons was first used in an extracted beam at the synchro-cyclotron /7/. (It should be pointed out that, if neutrons are present in the beam as well as protons, they also become longitudinally polarized because their precession angle differs only slightly from the protons' precession angle).

By using expressions (1), (3)-(5) and also the known formulae for converting trajectories in quadrupole lenses and bending magnets, we find the conversion for the particles' polarization vector. By averaging over the number of particles, we can determine the behaviour of beam polarization in the magnet-optics channel.

3. Discussions of the results

In order to design a system for transporting polarized beams in a channel consisting of quadrupole lenses and bending magnets with a homogeneous field and separated by free gaps, a program was written (see Annex) using the usual matrices for converting the trajectories in the M_{ij} optic elements (see, for instance, /8/) and the system parameters found by means of the channel design programs /9,10/.

Beam depolarization in the lenses is determined by the field value, i.e. it depends on the gradient and trajectory of the particles. Therefore, when the optics circuit of a channel is designed, the beam has to be made as small as possible at the points where quadrupole lenses are situated in order to minimize depolarization. If the beam is emitted from a stigmatic point source and identical phase ellipses occur in the horizontal and vertical planes at the channel input, and provided that the channel's transmission is conserved, the arrangement of the lenses in the channel (FD or DF) will not have a significant effect on beam depolarization. Depolarization values which are identical to an accuracy of ± 0.15 are thus obtained.

In order to conserve polarization completely if a beam enters the channel with different phase characteristics in each plane, the lenses and their mode of operation must be selected so that the space dimensions of the beam in the lenses are kept to a minimum.

Tables I and II show the results of calculating beam polarization in two possible systems for transporting protons of momentum P = 70 GeV/c at an angle $\theta = \pi/2$.

Matrix (3) was obtained when the lateral displacement of the particles within each section was constant. This condition is fulfilled very accurately at n values equal to a few units since the trajectories are nearly paraxial at high energies. Whereas the difference in the values of the $\langle P_y \rangle$ components at n = 1 and n = 5 is 1-2%, it does not exceed 0.05% at n = 5 and n = 20 (the averaged values of the $\langle P_x \rangle$ and $\langle P_z \rangle$ components go to zero because of the symmetry of the fields in the quadrupole lenses).

In the first case the maximum depolarization is 4% and is partially compensated along the channel. In the second case it increases and reaches ~ 9% at the channel output.

It therefore follows that, when a channel is built to carry polarized particles, it must be optimized not only with respect to the usual parameters but also in terms of beam polarization.

The tables also illustrate the example of a monochromatic beam converted from transverse to longitudinal polarization with a magnetic field selected in accordance with the requirements $\delta = \frac{\pi}{2}$.

In conclusion, the authors wish to thank V.I. Kotov, S.B. Nurushev and A.V. Samojlov for their useful comments and advice on the paper.

Bibliography

1. National Accelerator Laboratory, 1969, Summer Study Report, v.1.
2. Yu. A. Plis, L.M. Soroko. JINR Preprint 9-5012, Dubna, 1970.
3. Ya. S. Derbenev et al. ZhETF, 60, 1216 (1971).
4. D. Werren. Nucl. Instruments & Methods 58, 84, 1968.
5. M. Froissart, R. Stora. Nuclear Instrum. & Methods., 7, 297. 1960.
6. V.B. Berestetskij, E.M. Lifshchits, L.P. Pitaevskij, Relyativistskaya kvantovaya mekhanika, Part 1, Moscow, 1968.
7. M.G. Meshcheryakov, S.B. Nurushev et al., AE, 14, 38, 1963.
8. S. Penner. Rev. Scient. Instrum. 32, 150, 1961.
9. J.W. Gardner, D. Whiteside, NIRL /M/ 21, 1961.
10. I. Aleksandrov, V. Kartashev, A. Samojlov, IHEP Preprint 67-21, Serpukhov, 1967.

The manuscript was received by the publishing group on 6 September 1971.

Appendix

Fig. 2 shows a block diagram of the program for calculating polarization. The program uses the following initial data:

1. Number of elements in the channel, calculated momentum, beam's initial polarization (P_o), angle θ ;

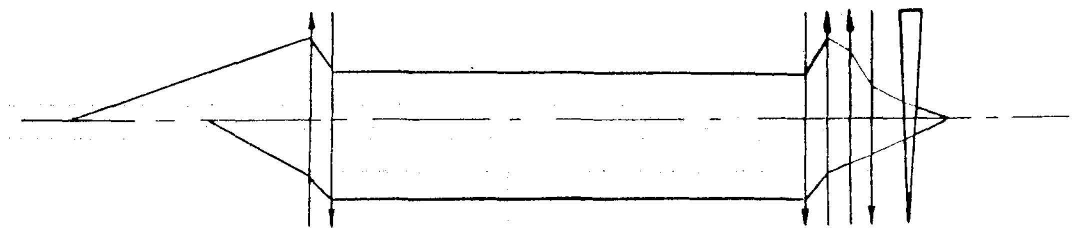
2. Description of elements in accordance with the following table

Type of element		1-Code	2	3	4-number of divisions
Free gap		1	Length in horizontal plane (m)	Length in vertical plane (m)	1
Lens	F	2	Gradient (gauss/cm)	Effective length (m)	n
	D	3	Gradient (gauss/cm)	Effective length (m)	n
Magnet		4	Field (gauss)	Effective length (m)	1

3. Phase volume, momentum range Δp and number of particles in the sample.

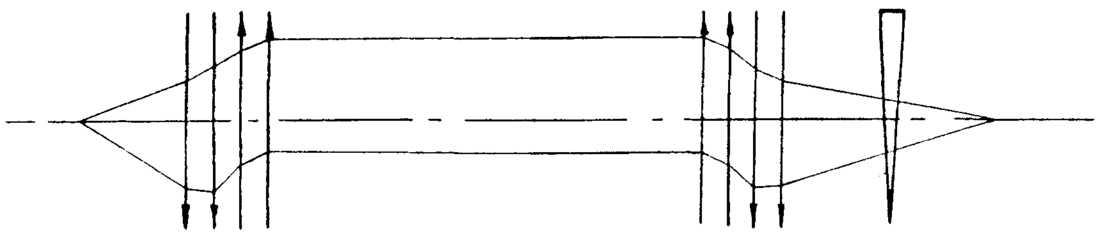
<u>Table I</u>

Parameters of the optics system after each element and the polarization value (in the diagram horizontal rays are above the axis and vertical rays below).



Channel parameters				Polarization		
1	2	3	4	$\langle P_x \rangle$	$\langle P_y \rangle$	$\langle P_z \rangle$
1	22.3960	10.2020	1	0.0000	1.0000	-0.0000
2	3443.2450	1.5400	10	-0.0001	0.9772	-0.0026
1	1.9000	1.9000	1	-0.0001	0.9772	-0.0026
3	3421.6600	1.5400	10	-0.0001	0.9956	0.0011
1	54.6000	54.6000	1	-0.0001	0.9956	0.0011
3	3339.7580	1.5400	10	-0.0001	0.9606	0.0021
1	1.9000	1.9000	1	-0.0001	0.9606	0.0021
2	3152.6920	1.5400	10	-0.0001	0.9865	0.0016
1	1.9000	1.9000	1	-0.0001	0.9865	0.0016
2	3152.6920	1.5400	10	-0.0000	0.9957	0.0012
1	1.9000	1.9000	1	-0.0000	0.9957	0.0012
3	3520.0000	1.5400	10	-0.0001	0.9863	0.0014
1	3.5000	3.5000	1	-0.0001	0.9863	0.0014
4	6654.3300	4.1000	1	-0.0001	-0.0074	-0.9863
1	3.5000	3.5000	1	-0.0001	-0.0074	-0.9863

Table II



Channel parameters				Polarization		
1	2	3	4	⟨P_x⟩	⟨P_y⟩	⟨P_z⟩
1	10.0000	10.0000	1	0,0000	1.0000	-0.0000
3	1155.5610	2.0900	10	-0.0001	0.9778	0.0023
1	2.5000	2.5000	1	-0.0001	0.9778	0.0023
3	1155.5610	2.0900	10	-0.0004	0.9141	0.0039
1	2.5000	2.5000	1	-0.0004	0.9141	0.0039
2	786.8240	2.0900	10	-0.0000	0.9534	0.0033
1	2.5000	2.5000	1	-0.0000	0.9534	0.0033
2	786.8240	2.0900	10	0.0003	0.9778	0.0020
1	50.0000	50.0000	1	0.0003	0.9778	0.0020
2	786.8240	2.0900	10	0.0005	0.9905	0.0006
1	2.5000	2.5000	1	0.0005	0.9905	0.0006
2	786.8240	2.0900	10	0.0007	0.9874	-0.0009
1	2.5000	2.5000	1	0.0007	0.9874	-0.0009
3	1155.5610	2.0900	10	0.0005	0.9805	0.0011
1	2.5000	2.5000	1	0.0005	0.9805	0.0011
3	1155.5610	2.0900	10	0.0004	0.9175	0.0028
1	5.0000	5.0000	1	0.0004	0.9175	0.0028
4	6654.3300	4.1000	1	0.0004	-0.0054	-0.9175
1	5.0000	5.0000	1	0.0004	-0.0054	-0.9175

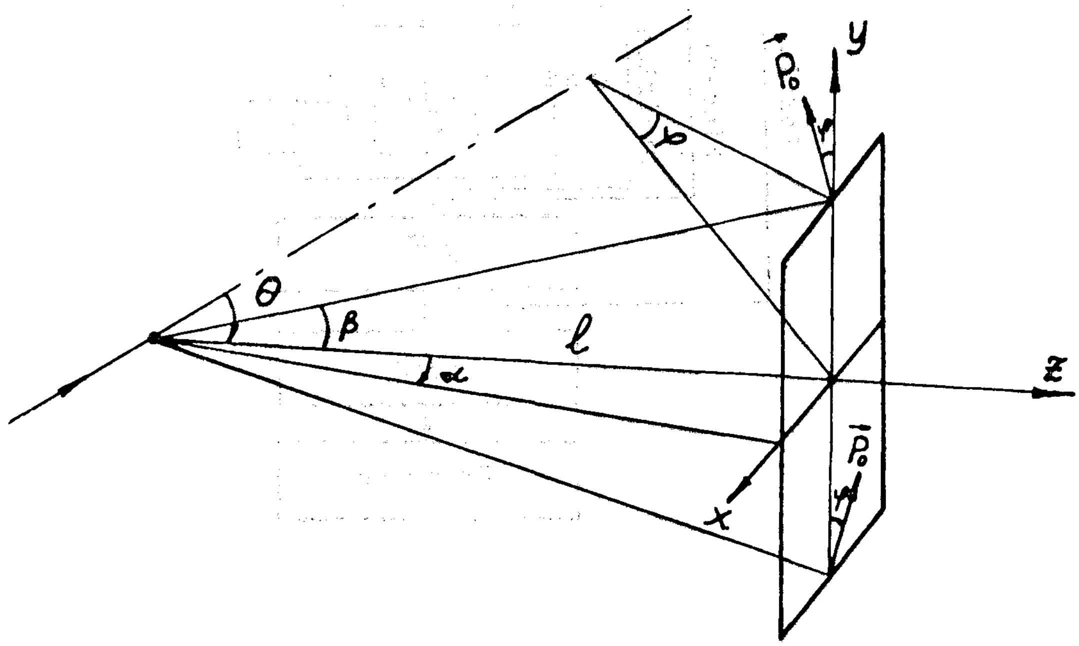


Fig. 1 Experimental conditions for producing and capturing a polarized beam. ℓ - distance from the sources to the lens input, α and β are the angles of capture in the horizontal and vertical planes respectively.

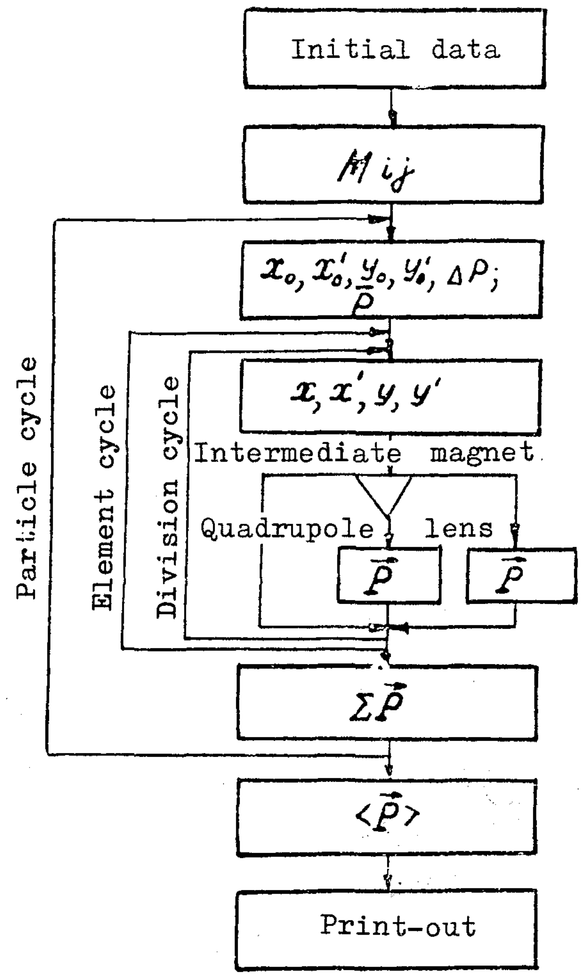


Fig. 2 Block diagram of the program