

Study of the effect of shell stabilization of the collective isovector valence-shell excitations along the N=80 isotonic chain

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It is proposed to initiate an experimental program to study the quadrupole-collective isovector valence-shell excitations the so-called mixed-symmetry states (MSSs) of unstable nuclei from the N = 80 isotonic chain. The main aim of this program is to investigate the microscopic mechanism which leads to a concentration or a fragmentation of the MSSs, an effect dubbed *shell stabilization* of MSSs. This will be achieved by identification of MSSs of the unstable nuclei ¹⁴⁰Nd and ¹⁴²Sm. The MSSs of these nuclei will be identified experimentally by measuring their relative populations with respect to the population of the the first 2⁺ states in inverse kinematics Coulomb excitation (CE) reactions on light targets. As a first step of this program we apply for a beam time for the radioactive ¹⁴⁰Nd and ¹⁴²Sm beams at beam energy of 2.85 MeV/u. These beams will be used to determine the absolute $B(E2; 2_1^+ \rightarrow 0_1^+)$ values for ¹⁴⁰Nd and ¹⁴²Sm in Coulomb excitation reaction on a ⁴⁸Ti target.

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I. INTRODUCTION – NUCLEAR STATES WITH PROTON-NEUTRON MIXED SYMMETRY

Atomic nuclei are examples of mesoscopic two-fluid quantum systems. The physics of these systems is determined by three main properties: the many-body aspect, the quantum nature, and the two-fluid character. Nuclear phenomena that reflect these three properties are collectivity, shell structure, and the isospin degrees of freedom. The Quadrupole-collective isovector valence-shell excitations, so-called mixed-symmetry states (MSSs) [1, 2],



represent a unique quantum laboratory in which the balance and interplay between the nuclear collectivity, the shell structures, and the isospin degree of freedom can be studied. A special type of MSSs, the 1^+ scissors mode, was first discovered in nuclei in electron scattering experiments at the TU Darmstadt [3] and subsequently found or suggested to exist in Bose-Einstein condensates [4] and metallic clusters [5]. In this respect, the impact of a deeper understanding of the structure of these states is beyond the field of nuclear structure physics.

States with proton-neutron mixed symmetry have been defined [1] in the framework of the interacting boson model with proton-neutron degree of freedom (IBM-2). The structure and the characteristics of these states are determined by the effective p-n correlations in the valence shell of collective nuclei. Their excitation energies are directly related to the proton-neutron symmetry energy in the valence shell. This fact is obvious in the IBM where the excitation energies of MSSs determine the parameters of the Majorana interaction to which p-n symmetric states are insensitive [1, 2].

The concept of proton-neutron mixed symmetry is formalized by the F-spin quantum number [6], which is the isospin analogue for bosons. Within this concept the fully symmetric states have $F = F_{max} = (N_\pi + N_\nu)/2$ ($N_{\pi,\nu}$ denote the proton/neutron boson numbers), while MSSs are those states with $F = F_{max} - 1$ [6]. In other words, the F-spin quantum number counts the number of proton and neutron pairs which are in phase in the quantum state. The IBM-2 states with maximum F-spin quantum number are called Full Symmetry States (FSSs). The F-spin is an approximate quantum number for low-lying collective states of heavy nuclei. The lowest states in a given nucleus are those formed by the FSSs. The M1 transitions between these states are forbidden and indeed they are observed to be small on an absolute, single-particle scale. With little modifications due to symmetry restrictions, the MSSs in the IBM-2 repeat the multiplet structure observed for the FSSs albeit at higher energy and with different decay properties. The energy difference between the FS and MS states with the same phonon number is determined by the size of the Majorana interaction in the IBM-2 [1]. The most distinct feature of MSSs (those with $F = F_{max} - 1$) is the existence of allowed F-vector ($\Delta F = 1$) M1 transitions to FSSs. This is of importance because the M1 transitions are forbidden between FSSs and can, thus, very well serve as a unique signature for MSSs.

The fundamental MSS in weakly collective vibrational nuclei, is the one-quadrupole phonon $2_{1,ms}^+$ state [1] which is the lowest-energy isovector quadrupole excitation in the valence shell. Its close relation to the 2_1^+ state is evident in the Q-phonon scheme for MSSs [7], where the wave functions of the one-quadrupole phonon excitations are well approximated by the expressions:

$$\begin{aligned} |2_1^+\rangle &\propto [Q_\pi + Q_\nu]|0_1^+\rangle \equiv Q_S|0_1^+\rangle & F = F_{max} \\ |2_{1,ms}^+\rangle &\propto [(Q_\pi/N_\pi) - (Q_\nu/N_\nu)]|0_1^+\rangle \equiv Q_m|0_1^+\rangle & F = F_{max} - 1 \end{aligned}$$

where $Q_{\pi,\nu}$ denote the proton and neutron quadrupole operators and $|0_1^+\rangle$ is the ground state of a collective even-even nucleus. Within the framework of this model the following signature for one-phonon MSSs in vibrational nuclei can be expected:

- The one-phonon $2_{1,ms}^+$ state should be the lowest-lying MSS.
- The $2_{1,ms}^+$ state should decay to the 2_1^+ by a strong M1 transition with an absolute matrix element of about $1 \mu_N$.

- Since the $2_{1,ms}^+$ state is a one-phonon excitation it should have collective E2 matrix elements to the ground state for both, protons and neutrons, however, with opposite signs, which might lead to partial cancellation in the total $\langle 0^+ || E2 || 2_{1,ms}^+ \rangle$ matrix element. Thus, a small-to-weakly-collective (\leq a few W.u.) E2 transition from the $2_{1,ms}^+$ state to the ground state can be expected.
- All MSSs must be expected to be very short lived, typically a few hundred femtoseconds or less, because of the strong M1 matrix elements and typical transition energies to the 2_1^+ of ≈ 1 MeV in vibrational nuclei.

From the above fingerprints it is obvious that the MSSs can be identified experimentally by their unique decay to the low-lying FSSs [1, 7]. This however, comprises a major experimental challenge because it requires full spectroscopic information, i.e. the spin-parities of these highly excited non-yrast states, their lifetimes and the branching and multipole mixing ratios of their γ -decay have to be determined. For more detailed insight in the structure of these states, information on their magnetic moments is also necessary. Until recently obtaining all this information was possible for a hand-full of stable nuclei only. **No MSSs have ever been solidly identified in unstable nuclei on the basis of large absolute M1 transition rates.**

II. PHYSICS CASE - SHELL STABILIZATION PHENOMENON IN THE MASS $A \approx 130$ REGION

Despite their important role for understanding the effective proton-neutron interaction in the valence shell the experimental information on MSSs is relatively scarce. Available information on MSSs of vibrational nuclei has recently been summarized in a review article [7]. Due to the experimental challenges, MSSs have been observed so far in stable nuclei only. The best examples are found in the mass $A \approx 90$ regions [7–15]. Until recently only a few cases have been known in mass $A \approx 130$ -140 region [16–20]. In the last few years, however a tremendous progress on the study of MSSs in mass $A \approx 130$ region has been made. Using Coulomb excitation reactions MSSs have been identified in several low-abundant stable nuclei, namely ^{138}Ce [21], ^{136}Ce [22], ^{134}Xe [23] and ^{132}Xe [24]. This new experimental information reveals several interesting physics phenomena. It has been shown [21] that in contrast to the neighboring even-even isotone ^{136}Ba , the $2_{1,ms}^+$ state in ^{138}Ce is strongly mixed with a nearby 2^+ fully symmetric state with a mixing matrix element of $V_{mix} = 44(3)$ keV, first measured directly for a MSS. This experimental observation led to the hypothesis that the microscopic structure can have a dramatic influence on the properties of these states. In fact, for weak-collective vibrational nuclei the single-particle structure of the wave functions can be the most important factor for preserving or fragmenting the MSSs. The observed mixing in ^{138}Ce is attributed to the lack of **shell stabilization** at the proton $1g_{7/2}$ sub-shell closure. The evolution of the MSSs from ^{136}Ba to ^{138}Ce shows for the first time that the strength concentration of collective-isovector excitations in the valence shell reflects the underlying single-particle structure. This hypothesis was partially confirmed experimentally by observing a fragmented one-phonon MS state in ^{136}Ce [22] and a single isolated MSS in ^{134}Xe [23]. These experimental facts have inspired an extended microscopic calculations of the structures of the MSSs of the N=80 isotones [25] within the framework of the quasiparticle-phonon model (QPM) [26]. The QPM calculations demonstrate that a

TABLE I: Properties of the low-lying 2^+ states of ^{140}Nd predicted by the microscopic QPM [28]. The calculations were performed using the same parameters as in Ref. [25].

| $E_{\text{level}}(\text{keV})$ | | J^π | $B(M1; J^\pi \rightarrow 2_1^+)$ $(\mu_N^2)^b$ | $B(E2; J^\pi \rightarrow 2_1^+)$ W.u. ^{b, c} | $B(E2; J^\pi \rightarrow 0_1^+)$ W.u. ^{b, c} |
|--------------------------------|-------------------|----------------------------------|---|--|--|
| Exp. ^a | QPM. ^b | | | | |
| 773 | 742 | 2_1^+ | | | 17 |
| 1490 | 1506 | 2_2^+ | 0.006 | 24 | 0.076 |
| 2140 ^d | 2209 | 2_3^+ | 0.03 | 6.2 | 3.5 |
| – | 2259 ^e | $2_4^+ \equiv 2_{1,\text{ms}}^+$ | 0.23 | 0.85 | 4.3 |
| – | 2399 ^e | 2_5^+ | 0.11 | 3.6 | 0.39 |

^aExperimentally observed level. From NNDC.

^bQPM calculations.

^c1 W.u. = $43.2 e^2\text{fm}^4$.

^dThis assignment is uncertain.

^eThere is no experimentally known level which can be assigned to this QPM state.

simultaneous description of the properties of the MSSs of the stable nuclei from the $N = 80$ isotonic chain, including the fragmentation of the one-phonon MSS in ^{138}Ce , can be achieved by slight increase of the energy of the $\pi 2d_{5/2}$ orbital with respect to the $\pi 1g_{7/2}$ orbital. This correction weakens the effect of pairing, showing that the shell stabilization is a genuine shell effect [25]. Furthermore the evolution of the MSSs in the $N = 80$ isotonic chain allows the relative proton-neutron interaction to be derived [23], for the first time from the properties of both symmetric and mixed-symmetry states in a simple semi-empirical way as suggested in Ref. [27].

Besides the insight in the nature of the MSSs the new physics effects, observed in the $N = 80$ isotonic chain, also rise some new questions. The generic nature of the shell stabilization mechanism is not apparent. There are only three stable nuclei in the $N = 80$ isotonic chain ^{138}Ce , ^{136}Ba and ^{134}Xe , which cover the transition from proton $g_{7/2}$ subshell closure to the proton $d_{5/2}$ subshell closure. Obviously, if we want to investigate the shell stabilization of MSSs we have to enter the domain of unstable nuclei – ^{140}Nd , ^{142}Sm , ^{144}Gd , ^{146}Dy , ^{148}Er and ^{150}Yb . The identification of the one-phonon $2_{1,\text{ms}}^+$ states in these nuclei should make transparent the shell stabilization of the MSSs (or the lack of it) in the proton $d_{5/2}$ (^{140}Nd , ^{142}Sm , ^{144}Gd , ^{146}Dy) and the proton $d_{3/2}$ (^{148}Er and ^{150}Yb) subshell closures. The most feasible experimental cases for such studies are ^{140}Nd and ^{142}Sm . The proton structure of these nuclei is dominated by the $d_{5/2}$ orbital. Assuming that the shell stabilization mechanism is correct, a single well pronounced MSSs can be expected in these nuclei. However, the QPM calculations for ^{140}Nd , which were performed using the same parameters as in Ref. [25], reveal a more complicated picture [28]. These results are summarized in Table I. The QPM calculations predict that the MSS of ^{140}Nd is fragmented between the fourth and the fifth 2^+ states of this nucleus. The decay of the former state carries the major part of the $M1$ strength. This situation is quite similar to the one observed in ^{138}Ce and questions the simple picture of the shell stabilization [21]. On the other hand, the available experimental information on the 2^+ states of ^{140}Nd does not allow a connection between the experimental and the calculated 2^+ states to be made (see Table I). Furthermore, the semi-empirical fit of the evolution of the MSSs in the $N=80$ isotones predicts that the MSS of ^{140}Nd should appear at 2493 keV, a value which deviates from the QPM predictions. All these facts

clearly demonstrate the inconsistency in our understanding of the microscopic structure of MSSs which can be summarized in the following questions:

- Is the effect of shell stabilization of the MSSs of such a general nature that can be expected to be present in ^{140}Nd and ^{142}Sm in the same way as observed in the stable N=80 isotones [21]?
- Is the microscopic description of the MSSs of the stable N=80 isotones obtained in the framework of the QPM [25] also applicable to the MSS of ^{140}Nd and ^{142}Sm ?
- Are there any other microscopic mechanisms which can explain consistently the evolution of the MSSs in the N=80 isotones, including the properties of the MSS of ^{140}Nd and ^{142}Sm ?

The only clear approach to these questions goes through the identification of the MSSs of ^{140}Nd and ^{142}Sm which is possible in inverse kinematics CE reactions on light targets. The use of light target like carbon is imposed by the nature of the MSSs. As we already discussed the one-phonon MSSs are connected with a strong $M1$ transition to the first 2^+ state and with a weakly collective (\approx few W.u.) $E2$ to the ground state. The $M1$ transitions are strongly suppressed in CE reactions. Therefore, the one-phonon MSSs are populated only through one-step CE excitations from the ground state due to the comparatively large $E2$ excitation strengths. As a result the CE reactions on light targets, *i.e.* CE reactions for which the multi-step excitations are suppressed, serve as a tool for selective/predominant population of the one-phonon MSSs in vibrational nuclei with respect to the population of the other off-yrast 2^+ states.

III. EXPERIMENTAL DETAILS

The MSSs of the radioactive ^{140}Nd and ^{142}Sm nuclei are highly excited ($E_x \approx 2.4$ MeV), off-yrast and short-lived 2^+ states. These states can be populated and their lifetimes can be measured in inverse kinematics Coulomb excitation reaction on light targets, an experimental technique which we successfully employed for identification of MSSs in stable nuclei [21, 23, 24]. However, these kind of experiments require beam energies of the order of 80-85% of the respective Coulomb barrier. These energies are still not accessible at REX-ISOLDE facility but will be within the capabilities of upgraded system HIE-ISOLDE. To prove that these kind of experiments will be feasible at HIE-ISOLDE we use our data on ^{138}Ce [21] obtained with Gammasphere (photopeak efficiency of $\approx 10\%$) [29]. The necessary statistics to perform full spectroscopy of the low-spin states of ^{138}Ce was collected for about 14 hours at a beam intensity of 6×10^9 pps. Using this information the experimental data on ^{138}Ce can be scaled with respect to the beam intensity and the running time. The results from such scalings are shown in Fig. 1. Since the MSSs of ^{140}Nd and ^{142}Sm have characteristics which are similar (or even better in terms of $E2$ excitation strengths) to the MSS of ^{138}Ce , it can be expected that they can be identified if the nuclei of interest can be produced as radioactive beams with intensities of about $10^5 - 10^6$ pps.

The purpose of the present proposal is to initiate an experimental program at REX-ISOLDE which is focused on the investigation of the MSSs in the heavy N= 80 isotones. However, before the actual experiments aiming identifications of MSSs of ^{140}Nd and ^{142}Sm , which apparently have to be foreseen for HIE-ISOLDE, there are two important steps to be undertaken:

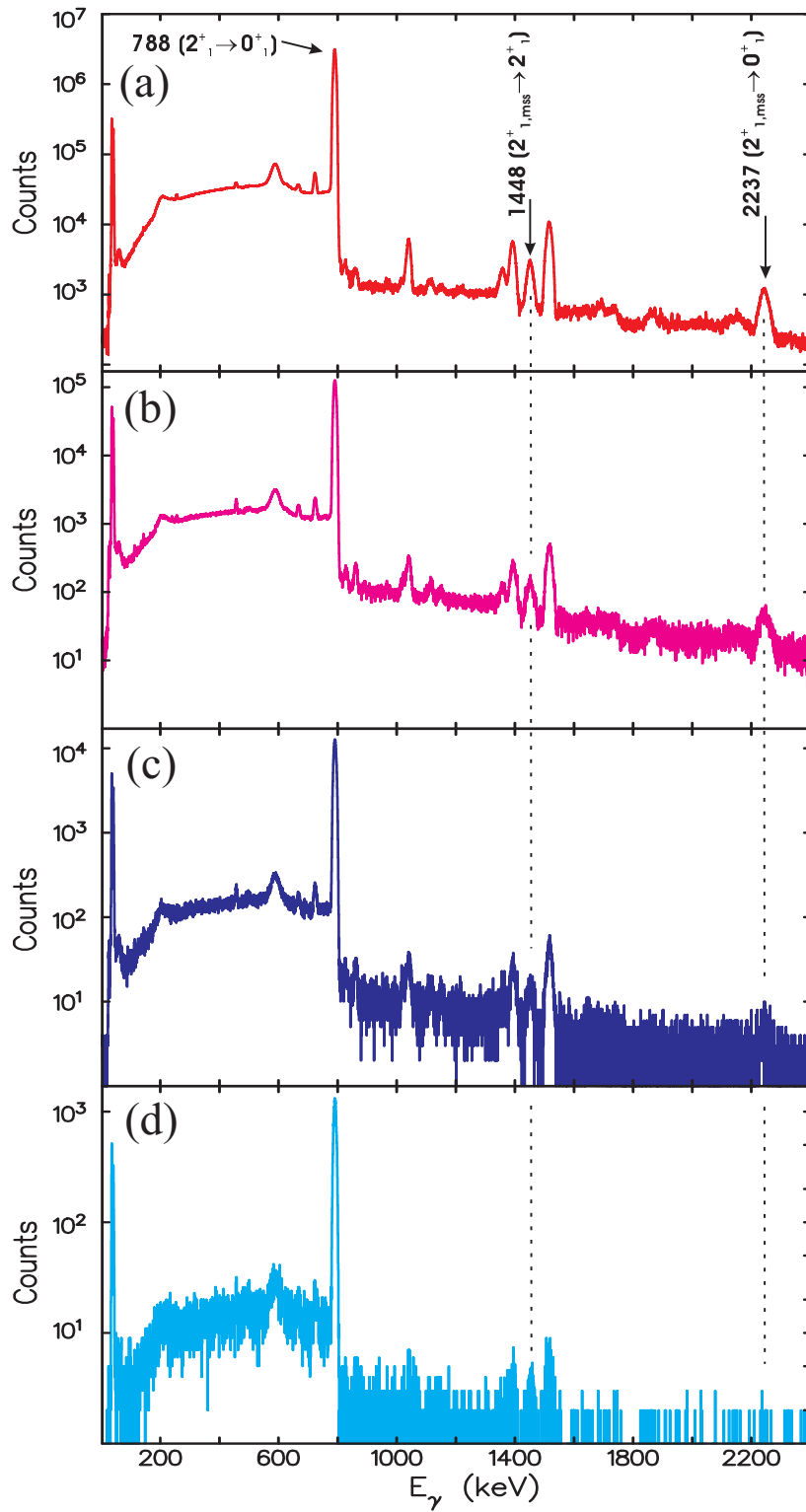


FIG. 1: Background-subtracted, Doppler-corrected γ -ray spectrum in ^{138}Ce observed with Gammasphere after Coulomb excitation on a carbon target for 14 hours at a beam intensity of 6×10^9 pps (a) (From Ref. [21]). The same spectrum obtained from a part of data which corresponds to a running time of two weeks at beam intensities of 10^7 pps (b), 10^6 pps (c) and 10^5 pps (d). The γ rays deexciting the $2^+_{1,ms}$ are indicated.

- Development of the radioactive beams ^{140}Nd and ^{142}Sm .
- Determination of the absolute $B(E2; 2_1^+ \rightarrow 0_1^+)$ values for ^{140}Nd and ^{142}Sm . These values will be used in the next experiments at HIE-ISOLDE for normalization of the CE cross-sections. Note, that the experiments at HIE-ISOLDE will be CE reactions on light targets like carbon which excludes the possibility to use the target excitations for normalization of the CE cross-sections.

With the present proposal we ask for beam time as it was suggested by INTC in their report on our LoI INTC-2007-028/INTC-I-073. The Nd and Sm nuclei of interest can potentially be produced at ISOLDE with sufficiently high primary intensity ($> 10^7$ ions/s) which can provide at least of about 10^5 ions/s on target for γ -ray yield measurements. A beam of ^{140}Nd has not been produced before but some lighter and heavier Nd isotopes have been produced by using the Ta target. A beam of ^{142}Sm has been produced by using the GdLa target. At present, both nuclei of interest can be made out of a ThOx or Ta target [30]. Both, ^{140}Nd and ^{142}Sm , have stable isobars - ^{140}Ce and ^{142}Nd , respectively. It might be expected that these isobars will constitute major contaminations in the beams of interest. To avoid this problem we intend to use selective laser ionization. The ionization schemes for the Nd and the Sm elements exist but have never been tested at RILIS [31]. One of the aims of the present proposal is to check the applicability of the ionization schemes and to determine the suppression factor for the stable contaminants. The required charge state to accelerate the beams of ^{140}Nd and ^{142}Sm up to 2.85 MeV/u will be obtained with REX-EBIS charge breeder. Accelerated beams will be delivered to MINIBALL target position where the nuclei of interest will be Coulomb excited in inverse kinematics using the secondary ^{48}Ti target. The energy of the first excited state of ^{48}Ti is $E_{2_1^+} = 983$ keV which is away from the γ -ray energies of interest ($E_{2_1^+} = 773$ keV and $E_{2_1^+} = 768$ keV for ^{140}Nd and ^{142}Sm , respectively). MINIBALL Ge-detector array, with a photopeak efficiency of $\approx 7\%$ for 1.3 MeV γ rays, will detect γ rays de-exciting the first 2^+ states of both the target and the projectile nuclei. The recoiling target nuclei will be detected with the CD detector covering the forward angular range of 16 to 53 degree. It can be seen in Fig. 2 that the ^{140}Nd nuclei will be scattered in a narrow cone after the target with a maximum scattering angle of 20° , *i.e.* atoms detected in the CD detector covering 20° to 53° will be only scattered ^{48}Ti atoms. This information will be used for complete kinematic reconstruction which will provide respective Doppler corrections for the γ -rays de-exciting the levels of interest. The estimated yields for ^{140}Nd for Coulomb excitation at a center-of-target energy of 371 MeV, for a 1 mg/cm² thick ^{48}Ti target are presented in Table II. The cross-sections for Coulomb excitation were estimated with the multiple CE code [32], the available experimental information for the energies of the excited 2^+ states and the results for the absolute $B(E2)$ obtained within the framework of the QPM model [28] (see Table I). The γ -ray yields were estimated assuming a beam current of 10^5 pps on the target. The yields for ^{142}Sm are expected to be similar. The cross-section for CE of the first 2^+ in ^{48}Ti is calculated to be 158 mb which yields about 1197 counts in the 983 keV ($2_1^+ \rightarrow 0_1^+$) line of ^{48}Ti . Since the $B(E2; 0_1^+ \rightarrow 2_1^+)$ value for ^{48}Ti is known with a high precision [33], the main uncertainty in the $B(E2; 0_1^+ \rightarrow 2_1^+)$ values for ^{140}Nd and ^{142}Sm will originate from the statistical errors in γ -rays yields. Therefore, if the required parameters for the radioactive beams are achieved it can be expected that $B(E2; 0_1^+ \rightarrow 2_1^+)$ values for ^{140}Nd and ^{142}Sm will be determined with a sufficient precision of about 5 %.

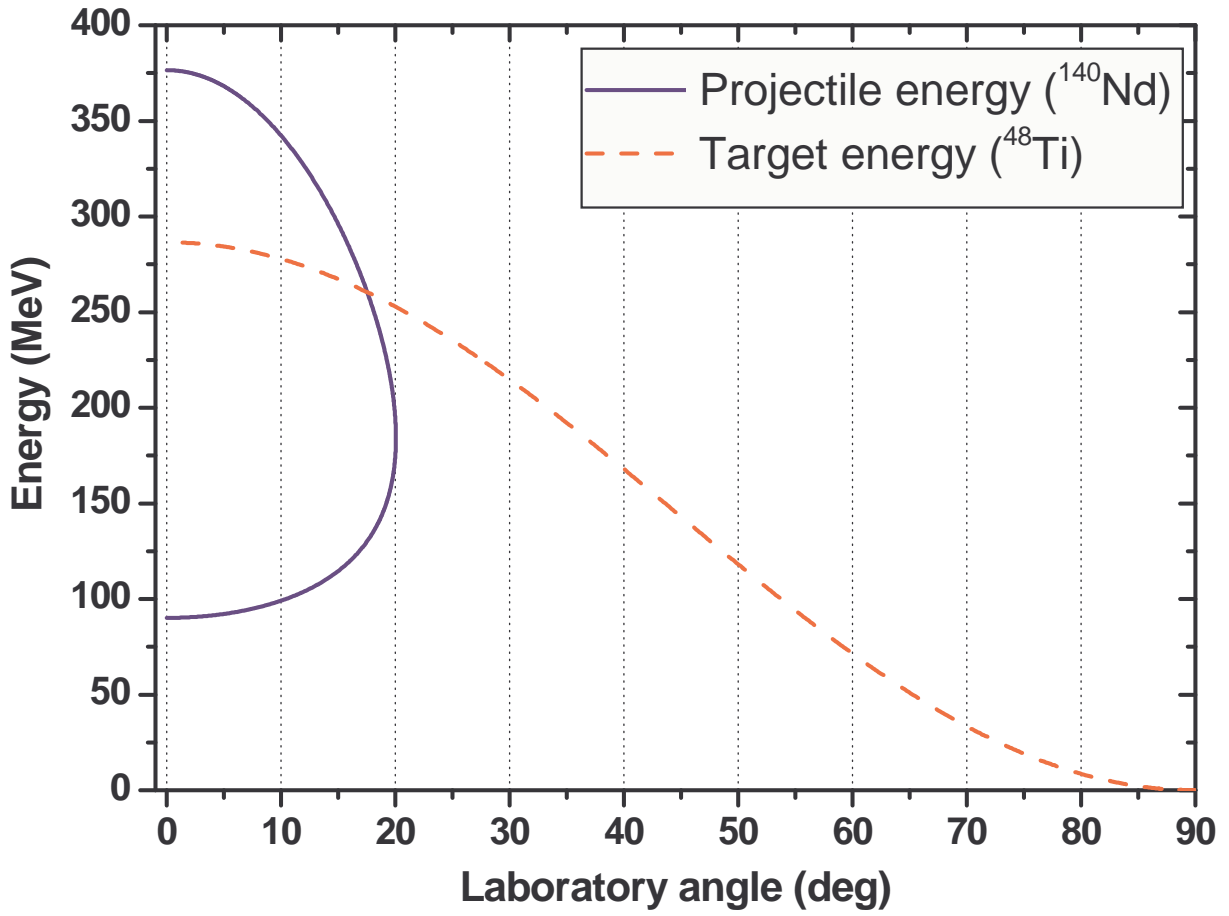


FIG. 2: Kinematics for Coulomb excitation of ^{140}Nd on ^{48}Ti at 400 MeV.

TABLE II: Coulomb excitation yields for ^{140}Nd calculated using the code CLX [32]. The CE cross-sections result from integration over the angular range of interest (20° to 53°). γ -ray yields are presented for 3 shifts (1-day) running time.

| E_{level} (keV) | J^π | σ (mb) | γ -ray yields |
|--------------------------|---------|---------------|----------------------|
| 773 | 2_1^+ | 202 | 1530 |
| 1490 | 2_2^+ | 1.7 | 13 |
| 2140 | 2_3^+ | 0.48 | 3.6 |
| 2259 ^a | 2_4^+ | 0.40 | 3 |
| 2399 ^a | 2_5^+ | 0.02 | 0.15 |

^aQPM predictions for the level energies.

IV. BEAM TIME REQUEST

The beam time request is based on the assumption that about 5 shifts per isotope will be needed to develop and deliver the ^{140}Nd and ^{142}Sm beams on the target. The CE yields measurements will in essence be a check for beams quality and the efficiency of the laser ionization. In addition, these measurements could provide an important physics information,

namely the $B(E2; 0_1^+ \rightarrow 2_1^+)$ values for ^{140}Nd and ^{142}Sm . To obtain the necessary statistics about 4 shifts will be needed for data taking in combined laser on/off mode. In summary, we apply for:

- 5 shifts for developing and delivering ^{140}Nd beam on the target position at 2.85 MeV/u;
- 4 shifts for CE yields measurement of ^{140}Nd beam in combined laser on/off mode;
- 5 shifts for developing and delivering ^{142}Sm beam on the target position at 2.85 MeV/u;
- 4 shifts for CE yields measurement of ^{142}Sm beam in combined laser on/off mode;

Total Request: 9 shifts per isotope - in total 18 shifts

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