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MOVING HEAVY LOADS ON CONCRETE FLOORS

F. Krienen

I. INTRODUCTION

Experimental halls are often to be equipped with magnets of hundreds of tons and shielding walls of about ten ton per square meter. The changing arrangement of these loads require a floor which can stand on the average 20 t/m^2, with peak loads on small areas of say, 50 t/m^2. To move the magnets around or to bring them in the beam line, one can choose between rails or a steel floor.

We put against the option "rails" the following:

1) The civil engineering has to be frozen at an early stage, often well ahead of the finalization of the magnet design.

2) Rails are obviously less universal.

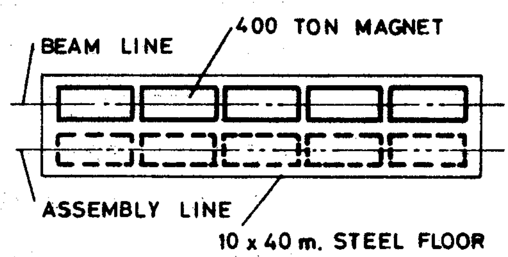
3) The aesthetic aspect is awkward.

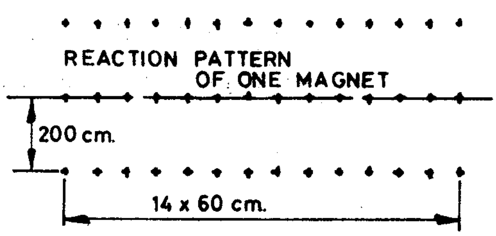
4) Cover plates are needed to move fork trucks around.

Against the option "steel floor" one has presumably the cost. However, we will analyse this problem and apply the analysis to the SPSC/P19 experiment, i.e. the moving of 5 magnets, each of 400 ton.

II. THE SPSC/P19 EXPERIMENT

The magnets, with floor plan of 8 × 4 m, are all aligned on the beam line, but will be assembled on an assembly line and then move sideways in position. The moving will be on roller bearings. Each magnet sits on a carriage, which has 3 longitudinal rows of 15 roller bearings, i.e. about 9 ton per bearing. The pattern of the reaction on the steel floor is a rectangular mesh of about 60 × 200 cm, see figure. The total surface of the steel floor is about $10 \times 40 = 400$ m^2.

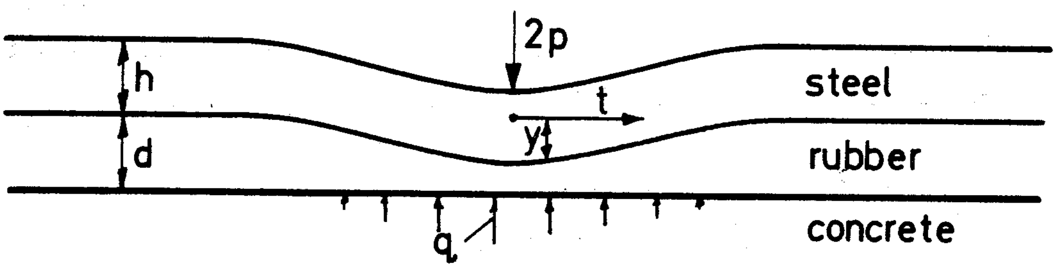




This surface is made of smaller plates, preferably 10 m long and of convenient width, so as not to interfere with the track of the roller bearings.

Between the steel plates and the concrete floor is an elastic layer, a rubber of some sort. The purpose of the rubber is not so much to smooth out the unevenness of the concrete floor, but to spread out the point-like reaction from the roller bearing to a surface-like reaction on the concrete. The two-dimensional problem can be reduced to an one-dimensional if the spacing between the roller bearings in a row is small enough to assume that the load is uniformly spread along the row. The analysis is furthermore simplified if the spacing between rows is so large that the deformations do not interfere. Subsequent analysis shows that the two assumptions are reasonably satisfied in the chosen geometry.

III. THE ELASTIC LINE



The case we have in hand is that of a steel slab, of height h, of width b and of infinite length in the direction of the independent variable t. All dimensions are in cm. The slab is loaded by a force 2P (kg). The force is supposed to be evenly distributed along the width, i.e. perpendicular to the plane of drawing. The steel slab rests on a stiff concrete floor via an elastic rubber layer. This layer has the property that the reaction q (kg/cm) of the floor is proportional to the displacement y (cm) of the elastic line of the steel slab. The proportionality constant k (kg/cm^2) provides the relation

$$q = k\ y$$

with the boundary condition

$$P = \int_0^\infty q\,dt = k \int_0^\infty y\,dt$$

Let the Laplacian of the displacement be:

$$L(y) = Y(s)$$

then, if ' denotes differentiation with respect to t,

$$L(y') = s\,Y(s) - y(o^+)$$

$y(o^+)$ is the symbol for the function t = 0, but on the right hand side.

$$L(y'') = s^2\,Y(s) - s\,y(o^+)\ .$$

Here the symmetry allowed us to put $y'(o^+) = 0$

$$L(y''') = s^3\,Y(s) - s^2\,y(o^+) - y''(o^+)\ .$$

Now the shear force in the steel slab is given by:

$$D = E\,I\,y''' = P - k \int_0^t y\,dt$$

E is the modulus of elasticity of steel and I is the moment of inertia of the steel slab, $I = b\,h^3/12$. Note that D is discontinuous at t = 0. The Laplacian of D is given by:

$$L(D) = (P/s) - (k/s)\,Y(s)$$

hence

$$E\,I\,[s^3\,Y(s) - s^2\,y(o^+) - y''(o^+)] = (P/s) - (k/s)\,Y(s)$$

or

$$[\,s^4 + k/(EI)\,]\;Y(s) = P/(EI) + s^3\,y(o^+) + s\,y''(o^+)$$

Let $k/(EI) = 4\,a^4$ (the dimension of a is in cm^{-1}),

then the inverse transform yields:

$$y = P\,a/k\,[\sin at\ \cosh at - \cos at\ \sinh at] +$$
$$+\ y(o^+)\ \cos at\ \cosh at + y''(o^+)/(2a^2)\ \sin at\ \sinh at.$$

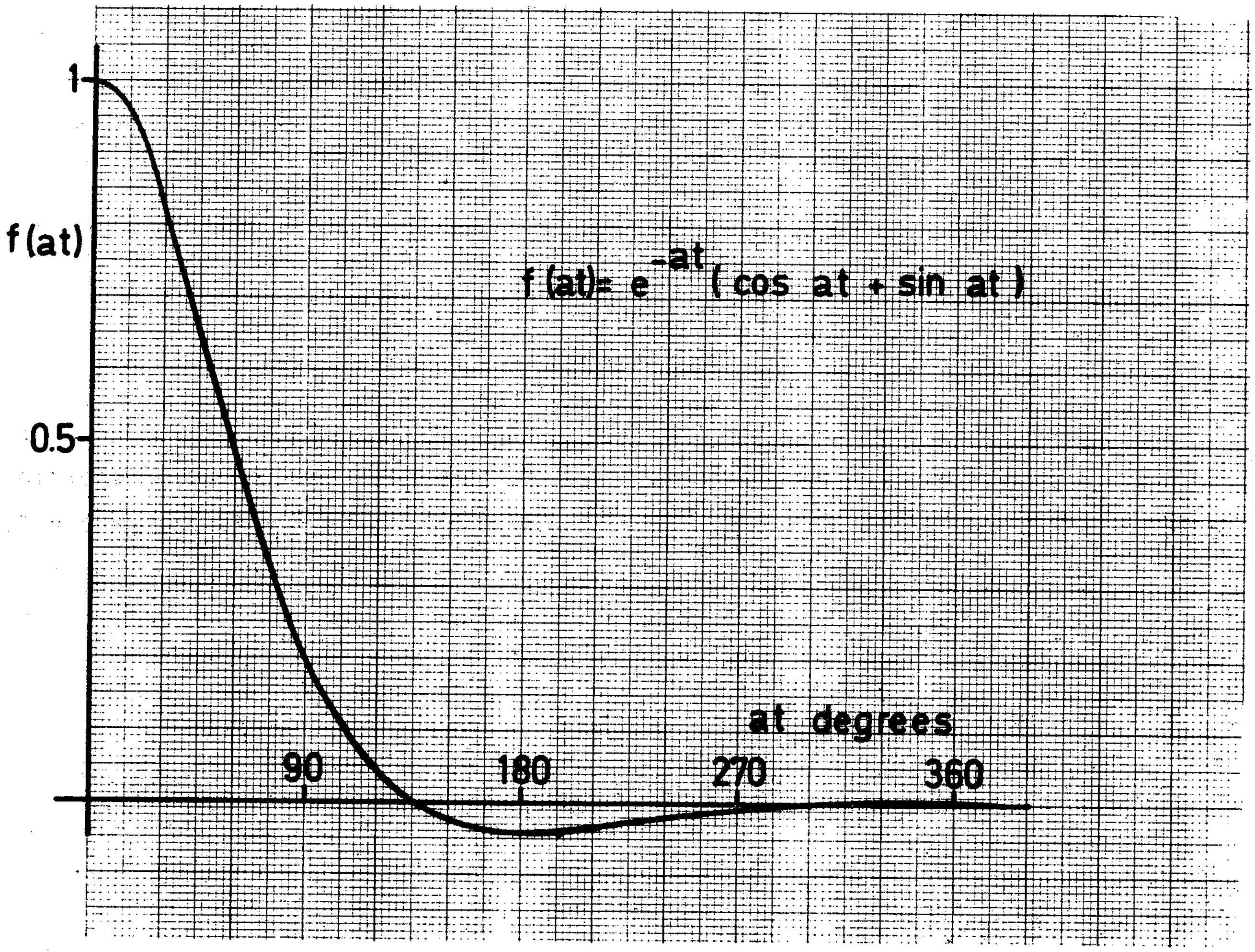
The requirement of a finite solution at $t = \infty$ fixes the relation between P, $y(o^+)$ and $y''(o^+)$, i.e.

$$P\ a/k = y(o^+) = -(\tfrac{1}{2}a^2)\ y''(o^+)$$

hence the equation of the elastic line is given by:

$$y = Pa/k\ e^{-at}\left[\cos at + \sin at\right] = Pa/k\ .\ f(at)\ \text{ for } t \geq 0\ .$$

The elastic line has a universal shape which is independent of load, materials or geometry.



The relation $q = k\ y$ implies that the rubber is bonded to both steel slab and concrete floor. The first zero occurs at $at = 3\pi/4$. The amplitude of the first negative bump is 4% of the maximum deflection, and the area is 7%. Obviously we do not intend to bond the rubber at all and assume that the smallness of these negative effects can be neglected.

The specific reaction R (kg/cm^2) on the concrete floor is given by:

$$R = q/b = ky/b = (Pa/b)\ f(at)$$

Now the average of f(at) over the positive deflection will be

$$(4/3\pi) \int_0^{at=3\pi/4} f(at)\ dt = 0.48.$$

If one associates this average with the average floor load, then the latter is about half the peak specific load and independent of the elasticity of the rubber or the stiffness of the steel slab.

IV. SCALING

The problem is uniquely defined if one gives the load P, the width b over which P is distributed, the maximum allowed reaction $R(o^+) = R_0$ on concrete and the maximum allowed bending stress $\sigma(o^+) = \sigma_0$ in the steel slab. Expressed in those quantities, we obtain:

$$y(o^+) = y_0 = P/(3^{\frac{1}{2}}Eb) \cdot R_0^{-3/2} \cdot \sigma_0^{3/2}$$

$$h = 3^{\frac{1}{2}}\ P/b \cdot R_0^{-\frac{1}{2}} \cdot \sigma_0^{-\frac{1}{2}}$$

$$k/b = 3^{\frac{1}{2}}\ Eb/P \cdot R_0^{5/2} \cdot \sigma_0^{-3/2}$$

$$a = b/p \cdot R_0.$$

It may be useful to consider the scaling laws if one internal parameter is kept constant whilst another is scaled up by a factor Δ. Three of such cases will be tabulated.

Case I Scales the reaction R_0 whilst keeping the bending stress σ_0 constant. This is a direct way to control the peak floor load.

Case II Considers the safety one buys by scaling up the thickness h of the steel slab, whilst leaving the elasticity k constant.

Case III Studies the effect of an uncontrollable variation of the elasticity, whilst keeping the thickness h constant.

Case	I	II	III
R_0	Δ	$\Delta^{-3/4}$	$\Delta^{1/4}$
σ_0	Δ^0	$\Delta^{-5/4}$	$\Delta^{-1/4}$
y_0	$\Delta^{-3/2}$	$\Delta^{-3/4}$	$\Delta^{-3/4}$
h	$\Delta^{-1/2}$	Δ	Δ^0
k	$\Delta^{5/2}$	Δ^0	Δ
a	Δ	$\Delta^{-3/4}$	$\Delta^{1/4}$

V. PARAMETERS OF THE 400 MAGNET SUPPORT

We give three sets of parameters compatible with the 400 ton magnet described earlier. One set aims at a conceivable peak load on the concrete of 50 t/m^2, whilst the two others are calculated for 30 t/m^2 with lower or higher bending stress in the steel.

P	kg	$4\ 10^5/6$	$4\ 10^5/6$	$4\ 10^5/6$
b	cm	800	800	800
R_0	kg/cm^2	3	5	3
σ_0	kg/cm^2	800	1000	1667
y_0	cm	0.105	0.068	0.315
h	cm	2.95	2.04	2.04
k	kg/cm^2	$2.29\ 10^4$	$5.88\ 10^4$	$0.76\ 10^4$
a	cm^{-1}	0.036	0.06	0.036

The elasticity of a solid rubber plate, boxed in between a concrete floor and a steel plate, is far too stiff to suit the above values of k. The compressibility of a rubber is of the order of $50\ 10^{-6}\ bar^{-1}$ ($= (1/v)\ dv/dp$), hence the elasticity constant k of a plate, d cm thick and b cm wide, would be:

$$k = (b/d)\ 2\ 10^4\ kg/cm^2$$ in which b/d is of the order of 10^3.

To remedy this, one could profile the plate, for instance,  or employ an expanded rubber. Also perforated plates or simply narrow strips could be used, presumably at the expense of more rubber weight per unit area. Non-linear behaviour of the rubber would not seem a matter of importance (except for the calculation), since the scaling law, case III, shows that the floor reaction goes with the 1/4 power of the elasticity constant k.

Actually the load P is distributed in 15 mesh points, spaced at a distance of about u = 60 cm. The assumption of uniform distribution is more or less correct if the elastic line due to one mesh point overlaps sufficiently with the elastic line of the neighbouring meshpoint. The criterion would then be:

$$u \leq 3\pi/4a.$$

Since a is proportional to R_0, we see that the above relation is correct for the examples in which the peak load is 3 kg/cm^2, but questionable for the 5 kg/cm^2 case, so that there the actual peak load will be somewhat larger than 5 kg/cm^2. Furthermore the non-interference of the elastic lines of two adjacent rows (200 cm apart) is readily verified.

VI. MISCELLANEOUS

The roller bearings require a rectified floor if all rollers were to carry equally. The shape of the elastic line rules this out. Consequently one should not try to reduce the deflection, by increasing the thickness of the steel plate, so that the roller bearings do carry. Instead,one chooses the next size roller bearing, which can stand the load on the two outer rollers.

The flatness of the floor should be such that the unloaded steel plates follow the waviness everywhere to, say, 1 mm. We assume here that the rubber will take care of this last mm. The sag f (cm) of a 3 cm plate, laid up on two "high spots" at a distance of s cm, is given by: $f = 70\ 10^{-12}\ s^4$. From this can be deduced that the floor is acceptable, if a 2 m straight edge shows everywhere less than 1 mm clearance on the floor. Furthermore, the alignment of the equipment would require that the floor is level to 10 mm peak to peak. It may be possible to relax somewhat on this specification after working out the details.

The carriage must follow the waviness of the concrete floor in the same manner as the steel plates, so that the total load of the magnet is evenly distributed over the roller bearings. The alignment of the various detector parts is quite severe. If now the waviness is excessive, re-alignment must be envisaged of the magnet during the moving. The means for adjustment is supposed to be part of the magnet carriage.

In the calculation we have assumed that the floor is infinitely stiff. However this assumption is not at all necessary. The floor is allowed to bend according to what one would expect if the floor is designed to a certain average floor loading of, say, 20 t/m^2 and to local peak loading of, say, 50 t/m^2. The steel plates will follow the sag of the floor everywhere. But the stresses in the steel may increase somewhat. Note that "Normal" forces, i.e. forces acting along the boundary steel-rubber-concrete are absent. This increase will be negligeable if the sag of the floor is less than 1 mm over a span of 2 m.

The moving of the magnet will be done with hydraulic rams. The friction of the roller bearings is less than 1%, i.e. about 4 ton. Several effects make the required moving force larger. For instance the roller bearings are always in a valley. In the case of perfect elastic rubbers, which have no permanent deformation nor hysteresis, the extra force will be nil. However we will make some contingency for these effects, i.e. assume that the rollers are moving "uphill".

A measure would be the average slope $(4/3\pi)ya$, which in our worked out examples is only a few parts per thousand. The same argument holds for the deflection of the concrete floor : it will be a small effect. Also the waviness or the tilt of the floor could contribute. Finally, the steel plates could flow in the long run under the constant high pressure of the rollers. This we minimize by taking a 50 ton roller for an actual load of 9 ton. So all in all the force which moves the load will be at most 3% of the weight. By applying unqual forces on either side of the magnet carriage, it should be possible to inch the magnet in any position, hence side rollers will not be needed.

VII. COMPARISON WITH RAILS

We have seen that the absence of some elastic element promotes high specific loads on the concrete, since this quantity goes with $k^{1/4}/I$. High specific loads are encountered in the case where the chariot moves on rails. In fact the calculation is not much different. The specific load can be reduced only if one increases the number of roller bearings substantially. An estimate of the cost of a steel floor and that of a system with rails is given, assuming that the concrete floor is the same in both cases.

		kSF.
a)	Steel floor, 400 m^2, 3 cm thick 100 ton	300
	Rubber sheat, 400 m^2, 6 mm thick = 3 ton	30
	Roller bearings, 5 × 45 at 220 SF	50
		380
b)	Rails type A 100, 5 × 15 × 10 m = 60 ton	180
	Roller bearings, 5 × 120 at 180 SF	110
	Side rollers	10
	Rail fixation	40
	Cover plates, 400 m^2 at 50 SF	20
		360

The above cost estimate is more a comparison between two principles than a thoroughly worked out scheme. It may very well be that a simpler system of rollers, and/or a 2 cm steel floor will do.

VIII. CONCLUSION

Steel floors laid on concrete via an elastic layer have all the advantages as outlined in this paper, whereas the cost is quite comparable to the cost of rails.

Hence we recommend the use of steel floors in cases where heavy loads have to be moved around.

The application of an elastic layer seems to be important in reducing the peak floor loading.