EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

For internal distribution only

NP Internal Report 68-8
29 February 1968*)

ON A NEW g-2 EXPERIMENT

R.W. Williams

---

*) Note: This report is a revision of an internal group note (RWW) of 8 January 1968 and corrects several errors in that note. The only new points are the problems raised about electron contamination and the eddy currents, and the solution of the digitron least-count problem. A different solution to the problem of a higher precision experiment is considered in the companion report by Brechna on a superconducting storage ring.

I have benefitted from discussions with many colleagues, particularly E. Picasso and H. Brechna. I am indebted to S. Van Der Meer for emphasizing the eddy-current problem, and G. von Bochman for pointing out the random aspect of the least-count problem.

The Muon Storage Ring Group has been discussing for some months the problems of mounting a new experiment aimed at an accuracy of 10-20 parts per million in a = ½ (g-2), that is, about 30 times better than the present result and enough to measure the strong-interaction terms. I present here a rather detailed account of an arrangement which I believe would accomplish this. The storage ring differs from the one we discussed last summer at some length, and which I will call model A: radius equal to present radius (250 cm), aperture much larger, n-value about twice as large. Instead, model B would have a 500 cm radius, a relative aperture smaller in the radial and larger in the vertical direction, and an n-value only a little larger than at present. Both refer to an iron magnet with pions brought in from an external target, but I have tried to indicate a specific practical method for getting the pions in. Since many decisions in such a design are interdependent, I thought it worthwhile to try to present a more or less complete scheme; it should at least stir up controversy, and anyway we need concrete models with which to compare the superconducting-ring design which is under study.

The main challenge to a new design lies in the systematic troubles of the present experiment: (i) the average-radius determination is not good enough; (ii) there appear to be unexplained losses, extending to at least 80 μsec, plus non-rotating and rotating background at early times; (iii) the blast of particles at injection upsets the detection system, and also causes (part of) the background (pile-up). Model B overcomes (i) by permitt a direct measure of the rotation time, before the bunch of particles has filled the ring, (ii) by a combination of more precise field shaping and a new gadget, the "magnetic scraper", and (iii) by an injection scheme which assures that most of the particles go where they are supposed to. The statistical accuracy in ω_{g-2} is obtained by good polarization of the trapped muons, the doubled lifetime of the muons, and a large factor in counting rate.

68/408/5
SIS/sb

1. MAIN FEATURES (Some details are given in Section 2)

A. Storage ring

Iron magnet, B_0 = 15 kG, radius R_0 = 500 cm, aperture full width v = 15 cm vertical, h = 10 cm horizontal. Index n = 0.17. Coils on either side of magnet aperture (to reduce fringing field) with multiple leads arranged to avoid the azimuthal holes in the present magnet.

B. "Magnetic scraper"

A set of single-turn coils in the magnet gap (Fig. 1) which are pulsed after the injected pions have decayed. A single sine wave, of period 2 μsec, giving a peak field B_z = 30 gauss, B_r = 6 gauss in the vacuum chamber, will adiabatically displace the equilibrium orbits ±1 cm horizontally and ±1 cm vertically. The trapped muons which remain do not come closer to the walls than about 1 cm in their subsequent betatron oscillations.

C. Losses

The magnetic scraper is supposed to establish a permanently trapped muon sample, well inside the physical vacuum chamber walls and therefore not subject to slow "scraping" losses we have discussed with the present ring. The building-up of oscillation amplitude due to field imperfections does not, at first, lead to any loss. In addition the field in the chamber must have a very constant gradient, to keep the rate of oscillation growth very low. Losses cause error in two ways: (i) the measured mean radius may be different from the radius appropriate to the ω_{g-2} determination; (ii) if muon polarization is a function of (equilibrium) radius, a radially-selective loss will cause a change in phase which falsifies ω_{g-2}. For both mechanisms a loss of 0.5% could cause errors of ~ 5 ppm in (g-2). A vacuum of 10^{-5} mm Hg leads to a loss under 0.1%.

D. Injection

A pion beam is led through a simple pulsed inflection system to the outer edge of the vacuum chamber, where it is released essentially tangentially (see Fig. 2). It has 1.004 times the central-orbit momentum, ±0.7%, and some of it misses the end of the inflector after the first revolution [because $\lambda_n = (1/\sqrt{1-n}) \times 2\pi R = 1.1 \times (2\pi R)$]. After 3.3 revolutions what remains of

68/408/5
SIS/sb

it encounters the regular exterior vacuum-chamber wall. The section of the vacuum chamber after the inflector has an enlarged outer radius for one-third of the circumference. Only muons with polarization greater than 0.90 (less than 25° in centre-of-mass) will have the correct momentum to be trapped; this should give an asymmetry twice that of the present experiment. The trapping efficiency is improved by the higher storage momentum (smaller $\pi\mu$ decay angles). Numerical estimates look very encouraging.

E. Beam

The pion beam designed by Dahl-Jensen for the Λ-moment experiment is taken as the starting-point. 21 GeV/c protons on a small Pt target gave, at 0°, a beam of 1.05 GeV/c π^- which had: angular divergence ±0.017 rad horizontal, ±0.070 rad vertical; area 4×4 mm²; $(\Delta P/P)$ ±0.7%; and 3×10^7 particles per pulse (20 bunches) of which about half were electrons and half pions. The yield increased with P_π (at least to 1.2 GeV/c, the highest they went) and decreased sharply with P_p (× 1/5 at 10.5 GeV/c). I assume 1 bunch, 21 GeV/c protons, 2.3 GeV/c pions, and a triple focus with much larger area and smaller divergence. I would make a double focus at the momentum slit (Dahl-Jensen has horizontal only) and place there a slab of U about 2 radiation lengths thick (10 g cm^{-2}) to degrade the electrons. As introduced by the deflection system this beam is in a general way comparable with the "beam" we now introduce with the internal target.

F. Mean-radius determination

The main point of the model B design is to permit a direct mean-radius determination by measuring the revolution period before the bunch has begun to overlap itself. With a 10 nsec injection pulse width this design has 45 turns, or 4.7 μsec, before overlap. Allow 0.4 μsec after injection for the pion beam's 3.3 revolutions, 2 μsec for the magnetic scraper, and we have 2.3 μsec or 22 turns to observe the now-stable muon sample. The accuracy to be expected in $\bar{R}$ from an observation of T, the mean rotation time, is worked out in the Appendix on Fast Rotation Statistics. With the expected counting rate (see below) we would have a fantastic accuracy in T, a few parts per million, corresponding to less than 1 ppm in (g-2).

68/408/5
SIS/sb

If losses can be controlled as indicated above, the only remaining serious systematic effect would be the presence of trapped electrons. Radiation loss causes an electron's equilibrium radius to shrink 1.2 mm per turn, so some electrons can be trapped after scattering. These will be present during the fast-rotation period, in fact up to 8 μsec after injection. Their higher velocity (by 0.1%) is not significant, but they will in general have a different average radius. At first it will be larger than that of the muons, by an amount which could be one or two centimetres. If there is a fraction f of the trapped sample which has mean radius differing by ΔR from the rest, the resultant error is $f\Delta R$. With $\Delta R = 2$ cm, an electron contamination of 1% causes an error of 0.2 mm, or 7 ppm in (g-2).

The incident pion beam will be partially cleared of electrons (Section E). The electron contamination can be measured (at small intensity) by a gas Čerenkov counter. The success of the present experiment in removing a very large electron contamination with suitable scrapers suggests that f can be made very small.

G. Detectors

No basic change is contemplated. The energy resolution will be better at double the energy, which helps the asymmetry. The resolving-time curve needs a standard deviation not greater than 2 nsec, assuming the injection pulse has S.D. of ~ 3 nsec. To help achieve this we should separate timing and energy discrimination, using last-dynode pulses for energy measurement. Detectors would be placed around the entire circumference.

H. Electronics and data handling

The digitron principle is retained. We now need a fine-scale digitron for the fast rotation analysis for the first 5 μsec, and a coarse-scale (100 mHz) for ω_{g-2}. The data rate per pulse is supposed to be forty times the present rate, so a buffer and on-line data processing system will be essential. Since the data comes in rather slowly (mean life 46 μsec) it is probably most convenient to have two moderate-sized (for example 100-channel) digitrons which can be strobed alternately into the memory of the on-line computer during data daking, rather than one gigantic digitron. (However, integrated-circuit techniques might make it practical to build a large digitron). The computer would have to accomplish at least part of what our SORT program does. The problem is analogous to wire-spark-chamber data systems, and should not represent new technology.

I. Magnetic field

For a given mean radius the existing system gives the mean field to 10 ppm. One limitation on this system is the field gradient and consequent line-width for a finite sample. Model B will have a smaller field gradient by a factor

$$\frac{R_A}{R_B}\frac{n_B}{n_A} = \frac{250}{500} \times \frac{0.17}{0.13} = 0.65 \ .$$

A system similar to the present one should therefore work better. Higher stability of the iron will probably result from the proposed use of a somewhat smaller field. An improvement of × 2 over present accuracy should be possible.

J. Intensity

Since in model B there is an injected pion beam of well-defined properties it should be possible to calculate the trapping efficiency and polarization much more reliably than in the present (internal-target) system. I have not done this. Instead I make a direct comparison with the present system, arguing that both inject pions at the outer edge of the ring, and that the essential differences can be guessed at. I compare therefore with the μ^- result of Storage 9, that is, 10^6 counts later than 20 μsec, obtained in a four-week run. The factors compared to the present ring are as follows:

i) Beam matching into aperture. Dahl-Jensen's area is a little smaller than the present (old) target size. Instead make vertical focus 4 cm, $\vartheta_V = 0.007$, and horizontal focus 2 cm, $\vartheta_H = 0.0034$. This gains × 10 vertically, perhaps helps horizontally -- there are effects both ways, say factor f = 10.

ii) Solid-angle-momentum acceptance. The factors in $\Omega\Delta p/p$ are

$$\frac{0.0062}{0.0032} \times \frac{0.0091}{0.0150} \times \frac{0.83}{0.87} = 0.7 \ .$$

We get this twice, once for π and once for μ. f = 0.5.

iii) Trapping efficiency. The $\pi\mu$ angle is half as large (f = 2) and the forward muons are more likely to be trapped. f = 4.

iv) Yield from 21 GeV/c protons is much higher; according to Dahl-Jensen, there is a factor 5 at 1 GeV/c. The curves of Jordan (CERN 65-14) suggests less at 2.3 GeV/c, perhaps 2. Take f = 2.

v) Magnetic scraper discards some intensity. f = 0.5

vi) Aperture shape is very favourable for detecting decay electrons, compared to present. f = 2.

vii) One bunch instead of three. f = 0.33.

viii) No particle losses (after magnetic scraper). f = 1.5.

ix) No wrong-momentum π's (which lead to low polarization in present experiment). f = 0.5.

(At this point the rate in an individual counter is up by a factor 10.)

x) Counters all the way around the ring. f = 4.

xi) New, higher CPS cycling rate. f = 2.

xii) Longer data-taking period. f = 4.

xiii) Data starts at 3 μsec, not 20 μsec. f = 2.

The rate per PS pulse is thus up by a factor 40 compared with the Storage 9 experience. The total number of counts is up by f = 640, and we expect a total of 5×10^8 counts.

K. Accuracy

For ω_{g-2} we have the approximate formula derived by Goddard, which agrees well with the error found by the FITG2 program: $\sigma_\omega = \sqrt{2}/(\gamma\ \tau_0\ A\sqrt{N})$. In Storage 9 we had σ_ω/ω = 250 parts per million. Model B has a factor nearly 2 for γ. However, the field is lower, 15 kG instead of 17.3 kG, which lowers both γ and ω. The factor in σ_ω/ω is then $\frac{1}{2} \times (17.3/15)^2 = 0.67$. The factor in A should be $\times$ 2, and in $\sqrt{N}$, $\times$ 25. The error in σ_ω/ω is thus $250 \times 0.67 \times (1/50) = 3.4$ ppm.

Mean radius from fast rotation: as noted in Section F we wait 22 turns and then count in the next 22, before the bunch overlaps itself. With muon radial spread $\Delta R_{r.m.s.} = 2$ cm the bunch has an r.m.s. spread after 22 turns of 7.5×10^{-3} μsec. We therefore use Eq. (A2) (in the Appendix) which gives

$$\frac{\sigma_T}{T} = \frac{\Delta R/R}{\sqrt{N\left[1 - y\left(\frac{\ln y}{y-1}\right)^2\right]}} \quad \text{where } y = \frac{\nu}{\nu_c} = \frac{44}{22} = 2 \; .$$

For model B,

$$\frac{\sigma_T}{T} = \frac{0.021}{\sqrt{N}} \; .$$

With about 5×10^8 counts altogether we have 2.4×10^7 in the fast-rotation observation interval, giving $\sigma_T/T = 4 \times 10^{-6}$. We get $\bar{R}$ from T without difficulty. The contribution to the error in g-2 is $n\sigma_T/T = 0.7$ ppm. This is surely too good. However it appears that if the losses and the backgrounds are indeed controlled, $\bar{R}$ is not a problem.

In Francis Farley's paper "Preliminary error analysis for g-2 experiment", a number of other error sources are considered which are significant at this level. The proton resonant frequency measurements, the ratio of static muon to proton frequencies, the effect of vertical oscillations, all need much attention to be brought below 10 ppm.

On paper, at least, it seems to me that the over-all error in g-2, with model B, can be as low as 10 ppm. This is small enough to constitute a respectable measurement of the strong-interaction effects (predicted to be 50-100 ppm).

L. Cost

The largest item is the big magnet. Part of the cost of a precision magnet is design and labour, so it does not simply scale as the cube of the scale factor. However, the relative aperture is larger vertically, and there would be more copper, so that it is probably fair to assume that this would be about eight times the cost of the existing ring. A rough figure for the MSR is 200,000 Swiss francs, so 1.6 million Swiss francs would be a reasonable guess. It appears practical to use a 3 MW rectifier like the existing ones. A good filter costs 100,000 Swiss francs.

The other large item is the on-line computer. A computer similar to those now used with wire spark chambers would be necessary (unless one had a direct link to the 6600).

2. SOME DETAILS ON THE ABOVE

A. Storage ring

Conductors around the inner radius of the magnet seem to me essential. Detectors do not scale up, so the light pipes would be about the present size, and the fringing field with our existing design would be enormous, especially since the vertical aperture is now relatively twice as large. Also the field shape would be improved by these coils, but they raise the power requirement.

For a scaled-up magnet, power goes up linearly with the scale factor. Thus we need relatively more copper to keep the power within 3 MW.

Modern accelerator construction technology can help. For instance, no vacuum chamber, use the magnet jaws and mylar "walls" well away from the storage region (see Section B).

B. Magnetic scraper

How adiabatic is a 2 μsec displacement? The increase in amplitude of a betatron oscillation of unlucky phase must be small compared to the maximum amount by which the equilibrium orbit is shifted, Z_s. For the worst case a full-wave shift causes a growth of approximately

$$\delta = 2\, Z_s \frac{\omega}{\omega_B} \frac{1}{1 - \left(\frac{\omega}{\omega_B}\right)^2}$$

where ω_B is the betatron frequency and ω is the driving frequency. We have

$$\frac{\omega}{\omega_B} = \frac{0.105}{\sqrt{0.17} \times 2} ,$$

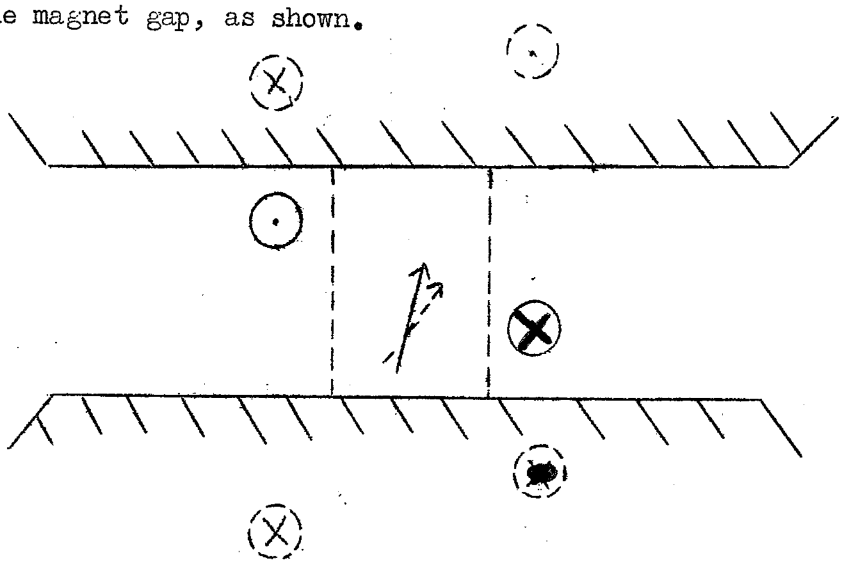
so $\omega/\omega_B = 0.13$ and $\delta = 0.26\ Z_s$ for vertical oscillations, and of course less for horizontal. This is acceptable. The horizontal picture is made a little worse because the outer wall is displaced outward for $^1/_3$ of the circumference.

One pair of these single-turn coils has an inductance about 50 μH, and the currents involved are about 600 A, so this would require a maximum voltage of 95 kV. Perhaps a sawtooth rather than sine wave would suffice -- this would need 60 kV which is more manageable. Considerable study will be necessary to see if this idea works.

The scraper displacement must be large enough so that no subsequent combination of oscillations can cause a particle to touch a wall. This becomes important if we use the (sloping) magnet jaws as the roof and floor of the vacuum chamber.

Eddy currents will be a severe problem. There are two aspects:

(a) During the scraper pulse, surface currents are induced in neighbouring conductors, and these reduce the field of the scraper coils in the storage region, and change its direction. The skin depth in both iron and copper is a fraction of a millimetre, so one has effectively image currents behind any large conducting surface. In particular the jaws of the magnet if not laminated, will have an effect which can be estimated. Consider the two coils located in the magnet gap, as shown.



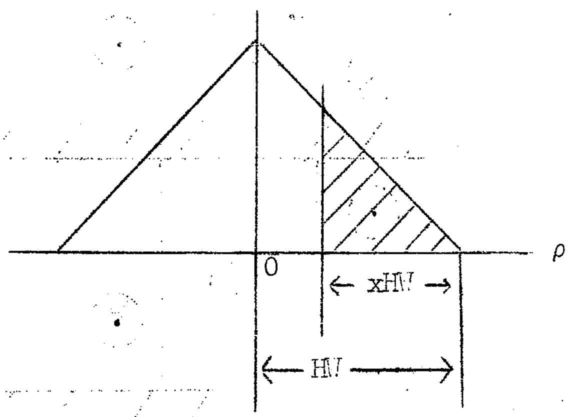
Their plane has a slope of 1:5 and they would produce, in the centre of the vacuum chamber, the required B_t and B_r. The first-order images, shown dotted, cut down the field about 20% and tilt it about 2°. Other nearby metal surfaces will have similar effects, unless laminated. In particular the inflector will be very close to the outer coil, and will introduce an azimuthal bump in the magnetic scraper field. The effect of this on the orbits must be considered at some future time.

(b) After the scraper pulse is over, currents continue to flow in the conductors because some flux has penetrated; these will cause some field in the storage region. I have not succeeded in getting a useful calculation of this. Various attempts show that the characteristic time of decay is the length of the scraper pulse, but that the decay could be slower than exponential. Also, the field is quite small for the kind of pulse considered, which has time-average zero

Laminating the pole tips may cause other problems, possibly increasing the time during which residual field is present. Also, lamination reduces the field and introduces construction problems, and in any case we have the inflector channel which cannot be laminated.

C. Losses

(i) Effect on $\bar{R}$. Assume that the equilibrium-radius distribution is triangular (this is not critical), with half-width HW. A radial distance xHW is removed from one side. How much is the shift in mean radius $\bar{\rho}$, and what fraction, F, is lost? The formulae are simple. Some numerical results



are presented in Table 1, including the effect on the error in g-2 in the present experiment and in model B. The expression evaluated is $(\bar{\rho}/HW) = (3x^2 - 2x^3)/(6 - 3x^2)$.

(ii) Effect of losses on ω_{g-2} by progressive phase shift. As in the above paragraph I am following the thought of Farley's "Preliminary error analysis" [in this case Section 3(iv)] with changes in detail. The polarization of a sample of particles of spin ½ is the incoherent average of their spin directions, $\vec{P} = \langle \vec{\sigma} \rangle$. A sub-sample of muons with equilibrium radius ρ will have some polarization $\vec{P}(\rho)$, and if the direction of $\vec{P}(\rho)$ changes with ρ, then selective elimination of part of the sample, for instance at one edge, will change the direction of $\vec{P}$, since

$$\vec{P} = \frac{1}{j} \sum_j \vec{P}(\rho_j) \ .$$

Of course if $|\vec{P}| = 1$ we know that all $\vec{P}(\rho)$ are in the same direction and there is no worry. However, in the present experiment the observed asymmetry A, which is P times a detection factor, is 0.08 to 0.15 as compared to 0.35 expected for P = 1, and 0.30 observed with the horn. Therefore it appears that $P \leq 0.5$ and we may have trouble at the 500 ppm level; we surely have trouble at the 10 ppm level.

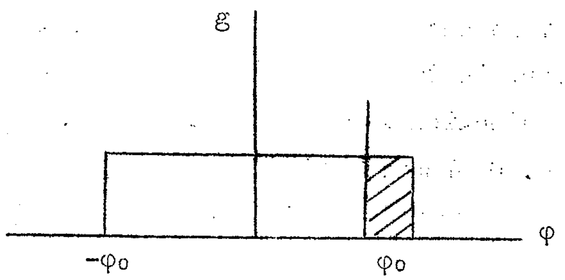
In model B, P has to be high because only those muons that decay forward have enough momentum to be trapped. The design criterion suggested is $\Theta_{c.m.} \leq 25°$ which is P individual ≥ 0.9, so P = 0.95 is reasonable.

Choosing a polar axis in the $\vec{P}$ direction we see that $P = \overline{\cos \Phi}$ where Φ is the polar angle. However, we want to consider the distribution of $\vec{P}(\rho_j)$ as a function of the projected angle φ, the angle, in the horizontal plane, between $\vec{P}(\rho_j)$ and some reference line. It is φ which measures the rotation of $\vec{P}$ in the magnetic field, the g-2 precession.

Assume now that the muon spins all lie in the horizontal plane; this will give the largest possible progressive-phase-shift effect, and it simplifies the analysis. If we measure φ (the projected angle) from the direction of the net polarization we then have $P = \overline{\cos\varphi}$. To get an effect we have to assume that the spin directions are correlated with momentum, or equilibrium radius, ρ. A distribution function $\eta(\varphi,\rho)d\varphi\, d\rho$ will then change with time as the radial distribution changes with time; the polarization distribution $F(\varphi) = \int\eta\, d\rho$ changes with time, causing $\vec{P}$ to change and giving an error. The simplest case is probably good enough to get an idea: say that part of η is uncorrelated and the other part perfectly correlated,

$$\eta(\varphi,\rho) = f(\varphi)h(\rho) + g(\varphi)\delta(\varphi - k\rho) .$$

Let the fraction of the sample which is correlated be ϵ, and let $g(\varphi)$ be a uniform distribution from $-\varphi_0$ to φ_0. Then if we lose a fraction F of the muons, and this loss occurs at one edge of the vacuum chamber, the direction of polarization of the correlated sample is changed by $F\varphi_0/2$. The direction of P, the polarization of the whole sample, is changed by $\epsilon F\varphi_0/2$.



We can estimate φ_0 from the observed magnitude of P, since

$$P = \int_{-\varphi_0}^{\varphi_0} \cos\varphi\, d\varphi \Bigg/ \int_{-\varphi_0}^{\varphi_0} d\varphi = (\sin\varphi_0)/\varphi_0 .$$

For:

$$P \approx 1, \ \varphi_0 = \sqrt{6(1-P)} = 0.55 \text{ radian for } 0.95 .$$

$$P = 0.5, \ \varphi_0 = 1.90 \text{ radian.}$$

68/408/5
SIS/sb

The error in ω_{g-2} caused by a shift $\Delta\varphi$ in the direction of $\vec{P}$, in time t, is $\Delta\varphi/\omega t$. If F is the fraction lost in one lifetime, τ, we have

$$\Delta\omega/\omega = \epsilon \; F\varphi_0/(2\omega\tau) \; .$$

In Table 1 some numerical values are given. For the present experiment we have direct evidence that $P \approx 0.5$, and fairly good evidence that $F \approx 0.035$ for the interval 20-100 μsec (see my note of 18/12/67. There is still no direct proof that the apparent losses are not due to instrumental shifts, though there are arguments against it, given in my note of 16/10/67). A value $F = 0.02$ is reasonable in the present experiment. We have no information on the value of ϵ, it could be anything from 0 to 1, but close to 0 seems more likely to me.

D. Injection

The magnetic channel shown in Fig. 2 is a double strip line or rectangular coax, 2.5 cm separation, 6 cm high, and 2 metres long. It is curved, with a radius 20% larger than the radius of the vacuum chamber, and meets the vacuum chamber with zero slope. The field inside, at the peak of the pulse, is 3 kG, which requires 30 kA. The inductance is 0.5 μH, and it appears that a capacitor bank of 75 kV and 0.5 μF, suitably switched, would make a good 0.5 μsec pulse in it. The field in the vicinity of the channel may be several hundred gauss, probably not serious because it returns to zero in two or three turns.

The central momentum of the pion beam is slightly higher -- a few tenths of a per cent -- than the central momentum of the storage ring, so the chief ray makes a complete turn, reaches its maximum outward displacement one-tenth of a turn beyond the end of the channel, in the enlarged section of the ring, and finally meets the wall after three turns. Part of the beam which goes back toward the channel passes above or below it. This is similar to the behaviour of the correct-momentum pions from the present target, except that at present there is no enlarged section, and there is a scraper upstream from the target for electrons. In model B the electrons are eliminated by hitting the wall at the end of the enlarged section. The electrons decrease in radius by 1.2 mm per turn. To make sure they all hit after 3 turns, the channel wall must be at least 3.6 mm from the radius which represents the normal outer wall of the vacuum chamber.

Muons are injected by $\pi\mu$ decay as in the present ring, but the trapping of forward muons is much better because the $\pi\mu$ angle is smaller and, the vertical aperture is larger.

(Note that in any such injection mechanism other than true trapping of the pions a higher field leads to a smaller decay fraction, so that the accuracy in ω_{g-2} improves as $B^{1.5}$ rather than B^2.)

The field in the channel does not affect the pions much, so it does not have to be shut off in one turn.

The fact that this simple injection scheme resembles the present method in some respects is an advantage in that intensities can be compared rather directly.

E. Beam

From Dahl-Jensen's 4×4 mm^2 and 0.070×0.017 sr we can get 4×2 cm^2 and 0.007×0.0034 sr. This matches the ring well (and also gets down the magnetic channel). The momentum bite, ± 0.7%, is too large to allow a large fraction of the beam to enjoy two or three turns, since the ring has ± 0.83% and only momenta greater than the central ring momentum can be quasi-trapped.

Starting with Dahl-Jensen's 3×10^7 particles I inject (one bunch and few electrons) 8×10^5 pions. This must be far better for the counter system than 10^{11} protons, although the improvement factor is not 10^5 because many (half?) of the counts at injection time are magnet-sensitive. However, even if we assume that a single counter is hit by 10^5 pions simultaneously, and each pion makes 10 photoelectrons on the average, the photo-cathode will not discharge: if its capacity is 5 pF its potential would change by 30 mV. It seems very unlikely that this could cause trouble. Furthermore, the injection dynamics is arranged so that most of the pions that do not decay hit the outside of the vacuum channels. The unstored muons mostly hit the counters but they make very small counts.

68/408/5
SIS/sb

F. Mean radius determination

A clean signal after 2.5 μsec is essential for this measurement. If there are background counts between the maxima we should put scintillators in front of the energy-sensitive counters and demand coincidence.

The digitron least count is 5 nsec which must be compared with the accuracy of 0.0004 nsec claimed above for T. These figures are in fact nearly compatible, because the digitron phase is random with respect to the arrival of the proton bunch at the storage ring, which is the reference for the timing of the decay electrons. For each PS pulse the proton bunch designates a certain cycle of the CW 200 MHz oscillator as zero. Let us calculate the spread which a least count τ introduces into repeated measurements of a precise time interval t. Let the time interval be $t = m\tau + \delta$, $0 \leq \delta \leq \tau$. For n measurements the counts will be divided between two bins, with a fraction $1 - (\delta/\tau)$ in bin m and δ/τ in bin m + 1. The estimate of F is

$$F = \frac{1}{n}\left[\left(1 - \frac{\delta}{\tau}\right) nm\tau + \frac{\delta}{\tau} n \left(m + 1\right)\tau\right] = m\tau + \delta \; .$$

As expected, this is unbiased. Its standard deviation follows from the fluctuation of the number of counts in either bin, which is given by the binomial-distribution formula as

$$\Delta n_1 = \sqrt{\left(1 - \frac{\delta}{\tau}\right)\frac{\delta}{\tau}\, n} \; .$$

The standard deviation in F is thus

$$\sigma_F = \sqrt{\frac{\delta(\tau - \delta)}{n}} \; .$$

Assume now that the n intervals t are measurements of a mean time t_α which itself has a distribution with standard deviation σ_α (as discussed in the Appendix). Then in the limit $\sigma_\alpha >> \tau$, the least-count effect contributes an uncorrelated variance $\langle \delta(\tau - \delta) \rangle$ to each observation. Since $\langle \delta \rangle = \tau/2$

Table 1

Effect on (g-2) accuracy of a loss, at one edge of a fraction F of the muon population.
x is the fraction of the radial half-width which is lost.
"Error" means relative error in (g-2) in parts per million.
For other symbols see Section 2C.

x	0.06	0.1	0.15	0.5	0.7	0.85	1.0
F	0.0018	0.0050	0.011	0.125	0.245	0.362	0.5
$\bar{\rho}$/HW	0.0019	0.0047	0.0068	0.094	0.173	0.246	0.33
Error from radius change, present ring	4	10	14	200	370	520	707
Error from radius change, Model B	3	8	12	160	293	415	565
The figures below are the errors from progressive phase shift caused by loss F, for various values of mean polarization $\bar{P}$ and correlation fraction ϵ. (Here F is per mean life.)							
$\bar{P}$ = 0.5, ϵ = 0.5, present ring	19	53	116	1320	2580	3700	5300
" ϵ = 0.1, " "	4	11	23	260	520	740	1060
" ϵ = 0.5, Model B	13	35	77	870	1700	2500	3500
" ϵ = 0.1, " "	3	7	15	174	340	500	700
$\bar{P}$ = 0.95, ϵ = 0.5, " "	4	10	22	250	500	730	1000
" ϵ = 0.1, " "	1	2	4	50	100	150	200

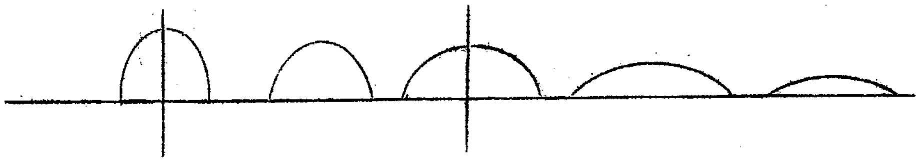
68/408/5
SIS/sb

APPENDIX *)

STATISTICS OF FAST-ROTATION ANALYSIS

1. EARLY TIMES, BEFORE THE DISTRIBUTION BEGINS TO OVERLAP ITSELF

While the muon bunch is still distinct we observe it ν times,



say ν epochs. At each epoch we expect n counts (n is assumed constant), and we find the average time of those counts, t_α for the α^{th} epoch. We want the mean period T. Also we do not know the injection time t_0. (Actually we want $\bar{R}$, and to go from T to $\bar{R}$ we need the distribution, since the velocity depends slightly on R. But this is a very small effect and could be estimated with any reasonable distribution.) Since the expected time at the α epoch is αT, we must minimize (α does not have to start at 1):

$$\chi^2 = \sum_{\alpha=1}^{\nu} \frac{(\alpha T - t_\alpha + t_0)^2}{\sigma_\alpha^{\;2}} .$$

$$\frac{\partial\chi^2}{\partial t_0} = 0 \text{ gives } \sum \frac{(\alpha T - t_\alpha + t_0)}{\sigma_\alpha^{\;2}} = 0$$

$$\frac{\partial\chi^2}{\partial T} = 0 \text{ gives } \sum \frac{(\alpha^2 T - \alpha t_\alpha + \alpha t_0)}{\sigma_\alpha^{\;2}} = 0 .$$

*) Revision of note of 19.11.67.

There are two contributions to σ_α:

i) The injection pulse width, of standard deviation σ_0.

ii) The spreading due to the momentum distribution. This grows with time. If the standard deviation in equilibrium radius is ΔR, then this is $(\Delta R/R)t$ or, say, $\alpha\sigma_p$ with $\sigma_p = T\,(\Delta R/R)$. Thus $\sigma_\alpha^2 = (\sigma_0^2 + \alpha^2\sigma_p^2)/n$.

Solve the two equations to get

$$T = \frac{1}{C - B^2/A}\left(\sum \frac{\alpha t_\alpha}{\sigma_\alpha^2} - \frac{B}{A}\sum \frac{t_\alpha}{\sigma_\alpha^2}\right)$$

$$t_0 = \frac{BC}{AC - B^2}\left(\frac{1}{B}\sum \frac{t_\alpha}{\sigma_\alpha^2} - \frac{1}{C}\sum \frac{\alpha t_\alpha}{\sigma_\alpha^2}\right)$$

where

$$A = \sum \frac{1}{\sigma_\alpha^2}, \qquad B = \sum \frac{\alpha}{\sigma_\alpha^2}, \qquad C = \sum \frac{\alpha^2}{\sigma_\alpha^2}.$$

Since we know the error in t_α, σ_α, we use propagation of errors to find the error in T. This is

$$\sigma_T^2 = \left(\frac{A}{AC - B^2}\right)^2 \sum \left(\frac{\alpha^2}{\sigma_\alpha^2} - \frac{2B}{A}\frac{\alpha}{\sigma_\alpha^2} + \frac{B^2}{A^2}\frac{1}{\sigma_\alpha^2}\right)$$

which gives finally

$$\sigma_T = \sqrt{\frac{A}{AC - B^2}}.$$

Similarly,

$$\sigma_{t_0} = \sqrt{\frac{C}{AC - B^2}} .$$

We can distinguish two simple cases. Call the epoch when the two contributions to σ_α are equal, ν_c. Thus $\nu_c = \sigma_o/\sigma_p$. Then at:

i) very early times $\nu < \nu_c$, σ_o dominates, and we find, using

$$\sum_1^\nu \alpha^2 \approx \frac{\nu^3}{3}$$

and so on,

$$\sigma_T = \frac{2\sqrt{3}\sigma_0}{\sqrt{\nu^3 n}} .$$

But νn is the total number of counts observed, say N, so

$$\sigma_T = \frac{\sigma_0}{\sqrt{N}} \cdot \frac{2\sqrt{3}}{\nu} . \qquad \text{(A1)}$$

ii) At later times, $\nu_c \lesssim \alpha \lesssim \nu$, we ignore σ_0; the spread is dominated by $\alpha\sigma_p$. Use integrals to approximate the sums:

$$\sum_{\alpha=\nu_c}^{\nu} \frac{1}{\alpha} \approx \ln \frac{\nu}{\nu_c} \text{ , etc.}$$

Then with $N = (\nu - \nu_c)n$ as the observed number of counts,

$$\sigma_T = \frac{\sigma_p}{\sqrt{N}} \frac{1}{\sqrt{1 - \frac{y}{(y-1)^2} (\ln y)^2}} , \qquad y = \frac{\nu}{\nu_c} . \qquad \text{(A2)}$$

68/408/5
SIS/sb

Numerical remarks

The initial spread σ_0 is compounded of injection pulse width, counter jitter, counter mistiming, etc. It seems to be equivalent to a square pulse of 0.01 FWHM. The r.m.s. spread is perhaps $\sigma_0 = 0.003$ μsec.

If the momentum distribution is triangular with base corresponding to HW = ±4 cm, $\Delta R_{r.m.s.}$ is 1.6 cm, and $\sigma_p = 3.2 \times 10^{-4}$ μsec. Thus

$$\nu_c = \frac{\sigma_0}{\sigma_p} = 10 \ .$$

The method works only while the bunches are distinct. This stops after

$$\frac{R}{2HW}\left(1 - \frac{\text{pulse width}}{T}\right) = 22 \text{ turns}$$

for the present ring and PW = 0.015 μsec (31 turns if PW = 0).

(a) For our present experiment the regime of Eq. (A1) is excluded, since it takes 14 turns to lose the non-trapped muons and get a stable population. Suppose we could observe from $\alpha = 14$ to 28, i.e. $\nu_c = 14$, $\nu = 28$, $y = 2$ in Eq. (A2) (I am assuming 28 instead of 22 to make the numbers simpler, but it might be true, PW might be 0.010 μsec). Then

$$\frac{\sigma_T}{T} = \frac{\sigma_p}{T}\frac{1}{\sqrt{N}}\frac{1}{\sqrt{0.037}} \ .$$

In Storage 9 there were 10^6 counts starting at 20 μsec, which means that at an early time there would be $N = 5.4 \times 10^4$ counts in 14 turns. Thus $\sigma_T/T = 1.2 \times 10^{-4}$, which would be, for $n = 0.132$, 16 ppm in a!

(b) Implications for new project. It appears that if we could count distinct bunches, and if we had no losses, the mean-radius problem would be solved to the required accuracy, or better, since we expect more counts. There would be some technical problems: any shift in the response of the counting system with digitron time would cause errors.

If the magnet were <u>really</u> good -- equilibrium orbit circular and flat, no target, walls uniform -- the population would be stable after $T_{vertical}/T \approx 2.8$ turns instead of 14, and the (statistical) accuracy for $\bar{\bar{R}}$ would be improved by a factor about $\times$ 1.6 by starting earlier. Of course we need a good magnet to avoid losses.

Fast deflector at late times: suppose we had a deflector which could remove the population in half the ring, at 20 μsec, say. Then we see distinct bunches for 15 turns and we are always in the regime of Eq. (A1), with

$$\sigma_0 = \frac{1}{2}\ T \times \frac{1}{2\sqrt{3}} = 0.0075\ \mu\text{sec}\ .$$

Again assuming 10^6 counts starting at 20 μsec, we observe 1.4×10^4 counts = N,

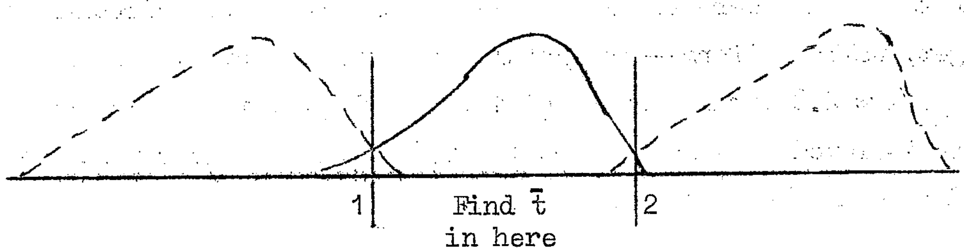
$$\frac{\sigma_T}{T} = \frac{1}{2 \times 2 \times \sqrt{3}}\ \frac{1}{\sqrt{1.4 \times 10^4}}\ \frac{2\sqrt{3}}{15} = 2.8 \times 10^{-4}\ ,$$

or 36 ppm in a. This is not good enough, but with 50 times as many counts it would be. Of course these counts do not contribute to ω_{g-2}. Also it is doubtful that one could make such a deflector; it must be very fast and also its action must be independent of the radius of the particle. It appears more profitable to put the technical effort into obtaining a clean injection situation.

If one had an intense muon beam, one might momentum analyse it before injecting into the ring, selecting Δp smaller than the ring acceptance. The full aperture would be used by betatron oscillations, but with a small Δp the bunches would stay distinct much longer.

2. OVERLAPPED DISTRIBUTION

In contrast to the preceding case there is no direct way to find T after the bunch begins to overlap itself. We can see this by trying to find t_α as before, supposing we somehow know what limits to count between.



We have replaced the contributions 1 and 2 by counts from the preceding and following turns; their average time is in general different from that of 1 and 2, so $\bar{t}$ is wrong. Typically, we would have to know the shape of the distribution to get the answer.

At first the counting-rate peaks are undisturbed. Their spacing gives the maximum in the distribution T_m, which is not what we want -- it is related to the average T by the distribution shape, of course. However, it would at least be something; but as soon as either the preceding or the following turn reaches the maximum point the peak will be shifted and the spacing will not give T_m. How soon this is depends on how far from the centre the maximum is, which of course we do not know.

In other words, it is a very non-linear problem, we need the answer to get the answer, and the only hope is to use self-consistency: to try a hypothesis and see if it fits. There is no way to get an error formula in such a case, because the error depends on the shape of the distribution. In fact I believe, but cannot prove, the following: with a finite number of counts, <u>no</u> procedure can guarantee a statistically good result

$$\left(\bar{R}_{estimate} = \bar{R}_{true} \left[1 + 0 \left(\frac{1}{\sqrt{N}} \right) \right] \right)$$

for a general distribution. One could always find a distribution which would trick the program into giving a biased result.

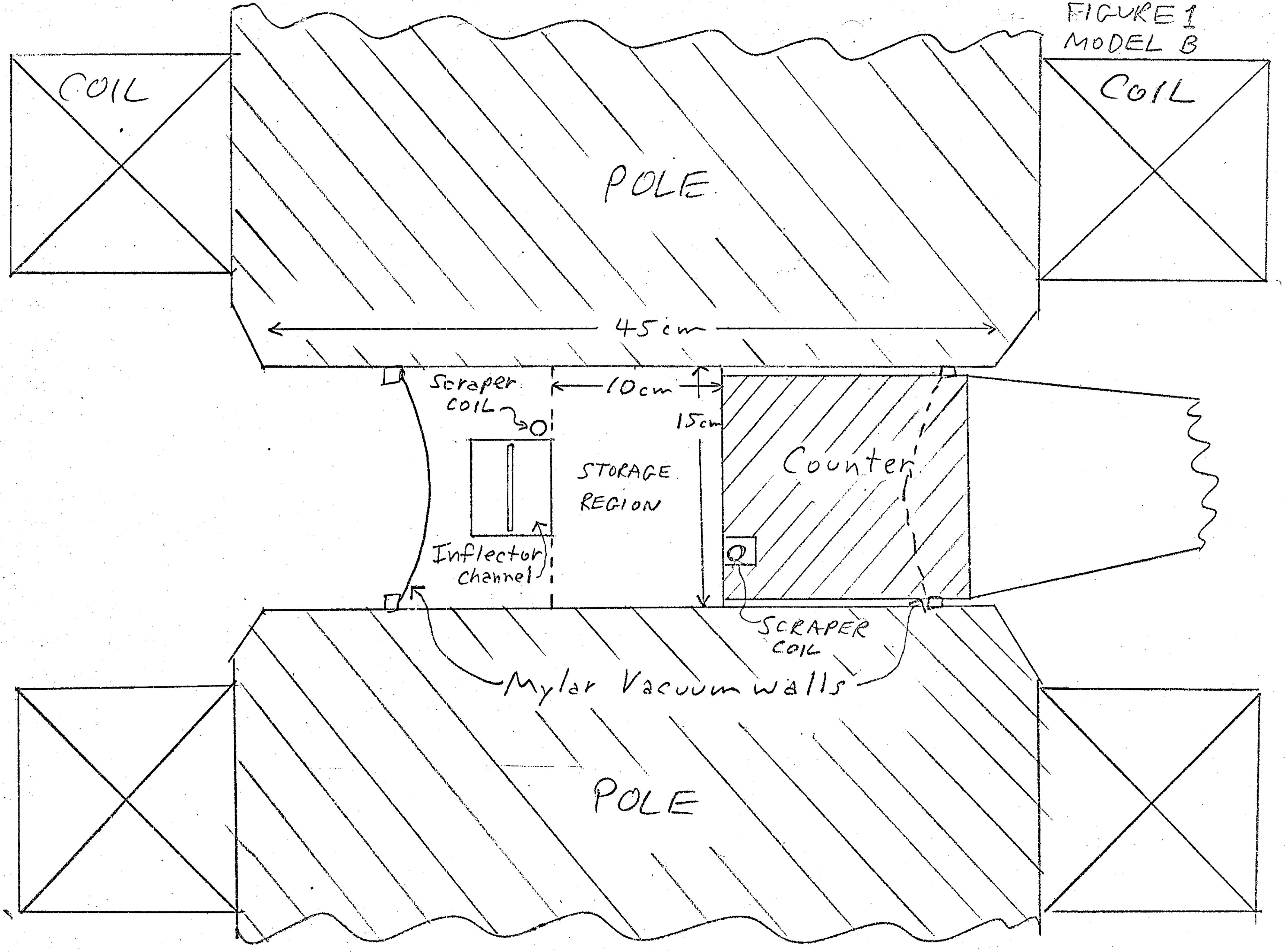
68/408/5
SIS/sb

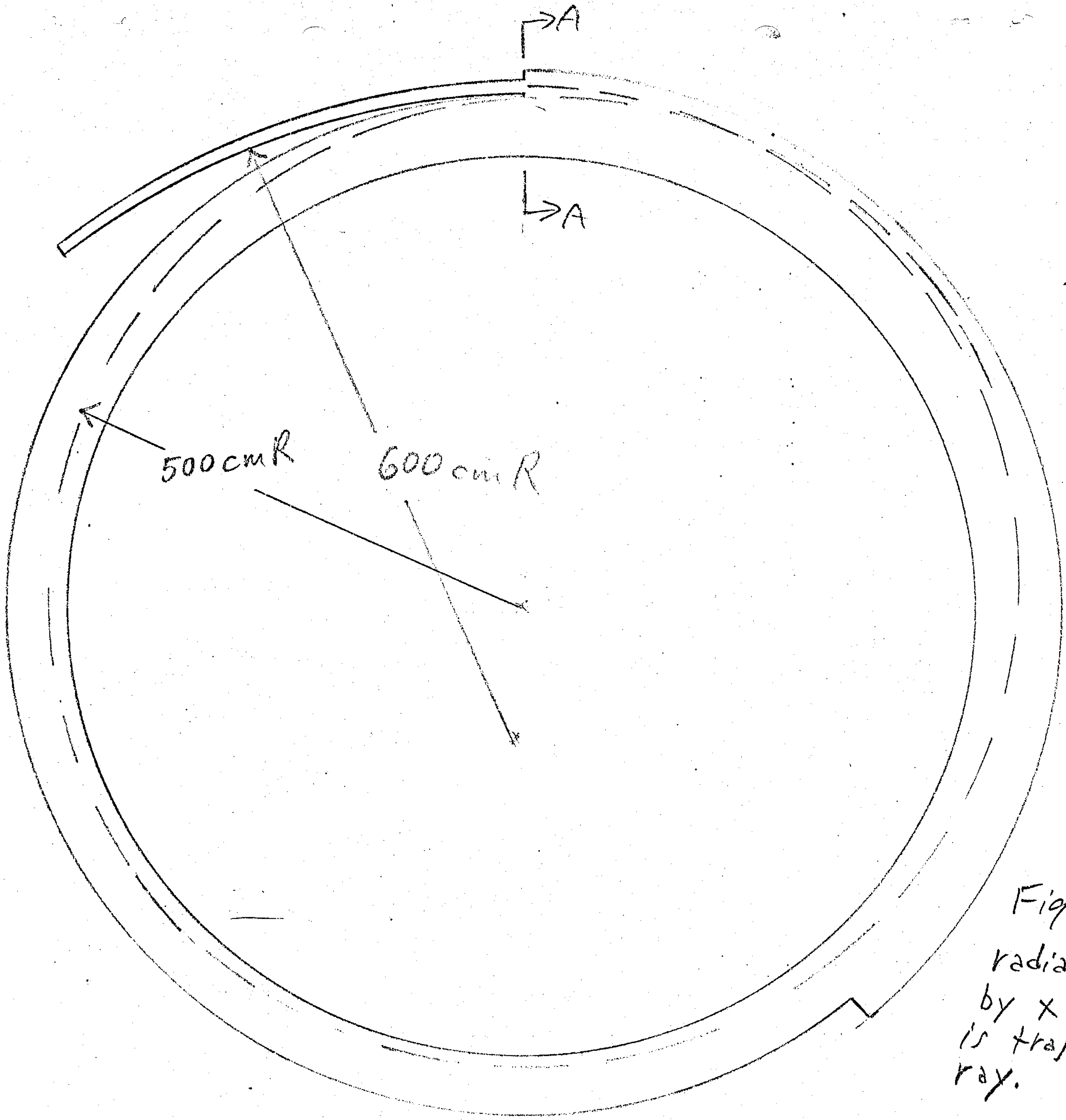
Faced with this, we should give the program all the help possible (which is already obvious from the results of some Monte Carlo studies). That is, make the boundaries coincide with the true boundaries, make the distribution continuous and going smoothly to zero at the boundaries, make it be positive and have only a few maxima, etc. The Van der Meer program does some of these things, but it may be possible to find an improvement. Monte Carlo checks could tell us how we are doing.

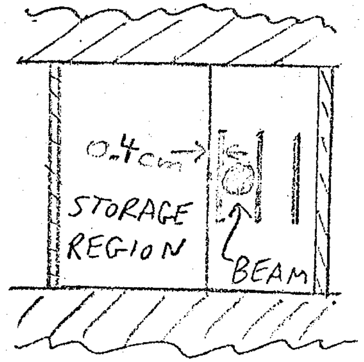
For the new project we need $\bar{R}$ to a few parts in 10^5; to me it seems not too likely that we could get this out of the overlapped distribution.

68/408/5
SIS/sb

FIGURE 1
MODEL B







Section A-A (Enlarged) showing clearance to kill electrons.

Figure 2. Model B, radial scale enlarged by x10. Dashed line is trajectory of chief ray.