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DISCONTINUOUS BEHAVIOUR IN LARGE ANGLE PROTON - PROTON
ELASTIC SCATTERING AT HIGH ENERGIES

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ABSTRACT

Measurements of elastic proton-proton differential cross sections for angles between 65° and 90° c.m.s. have been made at 8, 9, 10, 11, 14, 15 and 21 GeV/c. The shape of the angular distribution is found to change suddenly between 8 and 11 GeV/c. An interpretation of this discontinuous behaviour in terms of the reactive effects of baryon-antibaryon pair production is proposed.

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1-7-77

Dear Mr. [Name]:

I have received your letter of [Date] regarding [Subject].

I am sorry that I cannot provide a more definitive answer at this time.

The information you requested is currently being reviewed.

I will contact you again once a final decision has been reached.

Sincerely,

[Name]

Very truly yours,

[Name]

[Title]

cc: [Name]

Differential cross sections for elastic proton-proton scattering in the large angle region ($65^\circ - 90^\circ$ c.m.s.) have been measured at incident proton momenta of 8, 9, 10, 11, 14, 19 and 21 GeV/c. These measurements were stimulated by the result of an experiment¹⁾ performed at 16.9 GeV/c which searched for Ericson fluctuations²⁾ in the angular distribution. In that experiment, no evidence for fluctuations was found, the angular distribution between 67° and 90° c.m.s. being smooth and well represented by the formula

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} \propto \exp(-p \sin \Theta / b) \quad (1)$$

where p and Θ are the momentum and the scattering angle in the centre-of-mass system. The parameter b was found to be 225 ± 4 MeV/c.

Since other measurements at 3, 5 and 7 GeV/c, performed at Berkeley³⁾, could also be fitted with an exponential in transverse momentum but with rather different values for the parameter b , the experiment reported here was undertaken specifically to make a systematic study of the energy dependence of b and, more generally, to measure the form of the angular distribution over as wide an energy range as possible.

The experimental layout and technique were essentially the same as for the 16.9 GeV/c experiment¹⁾, in which both protons emerging from a liquid hydrogen target, bombarded by a long pulse extracted proton beam ($\sim 3 \cdot 10^{11}$ protons per pulse in ~ 50 ms), were momentum analysed and detected by two scintillation counter telescopes. At 8, 9 and 10 GeV/c, however, targets of CH_2 (3 cm thick) were used rather than liquid hydrogen.

The monitoring of the beam intensity was done, as before, using a secondary emission chamber and two counter telescopes for relative measurements. The absolute calibration was taken from the amounts of ^{24}Na and ^{22}Na activities produced in thin aluminium foils placed in the beam. The activation cross sections used were obtained from reference 4. The intensities obtained from the ^{24}Na and from the ^{22}Na activities agreed within the statistical accuracy of the calibration ($\pm 5\%$).

The rate of registering elastic scattering events ranged from about 10 per hour at 21 GeV/c to about 1500 per hour at 9 GeV/c. The very low cross sections at the highest momentum prevented extension of the experiment to still higher momenta.

The measured differential cross sections are given in Table 1. The correction for empty hydrogen target background was negligible, while for the Carbon in the CH₂ target it was about 2%. Accidental coincidences and dead time losses gave rise to corrections usually less than 5%. Corrections for absorption of the protons in the target, in the air and in the detectors were about 10%.⁵⁾ The errors quoted in the table result from the combination of statistical errors (~ 2-15%) and systematic errors from point to point in each angular distribution (~ 1-5%). Relative uncertainties from one momentum to another due to the monitor calibration are about 3%. The absolute scale is subject to an error of ± 7% due to the uncertainty in the ²⁴Na cross section⁴⁾.

Fig. 1 shows a logarithmic plot of $s(d\sigma/d\Omega)_{c.m.}$, where s is the square of the total c.m.s. energy, as a function of the transverse momentum $p \sin \Theta$. It is evident that the angular distributions deviate from the dependence which Orear⁶⁾ found to give a reasonable overall representation of the Cornell-BNL wide angle measurements⁷⁾, namely

$$s \left(\frac{d\sigma}{d\Omega} \right)_{c.m.} = A \exp(-p \sin \Theta / b) \quad (2)$$

with $A = 595 \pm 135$ (GeV)² mb/sr and $b = 158 \pm 3$ MeV/c.

The angular distributions at each momentum have been fitted by $(d\sigma/d\Omega)_{c.m.} \propto \exp(-p \sin \Theta / b)$ taking b as a parameter. The best fit values of b from the present experiment are given in Table 2, together with that from Allaby et al.¹⁾, those from the data of Clyde et al.³⁾ and from the new data of Ankenbrandt⁸⁾, the last two for c.m. angles larger than 50°. The exponential fits gave good χ^2 probability levels except in the region of 8-11 GeV/c where the χ^2 probability was ~ 1%. In this latter momentum range, a linear dependence of $\log(d\sigma/d\Omega)$ on the transverse momentum would seem to be unlikely. The rather large uncertainties assigned to b (Table 2) between 8 and 11 GeV/c arise partly from the poor fit.

The inset to Fig. 1 shows b as a function of the incident proton momentum (laboratory system). The rapid change of b between 8 and 11 GeV/c, corresponding to a flattening of the angular distribution⁹⁾ is striking.

This sharp change in b is related to a phenomenon found recently by Akerlof et al.¹⁰⁾ who measured p - p elastic differential cross sections at 90° c.m.s. for incident momenta between 5.0 and 13.4 GeV/c. They found that their data could be fitted by the formula

$$\frac{d\sigma}{dt} \propto \exp(-Dp^2) \quad (3)$$

with $D = 3.29 (\text{GeV}/c)^{-2}$ up to an incident momentum of about 8 GeV/c ($p^2 = 3.4 (\text{GeV}/c)^2$) beyond which the best fit was obtained for $D = 1.51 (\text{GeV}/c)^{-2}$. It seems clear that the change in behaviour of the 90° elastic p - p cross section at around 8 GeV/c is related to the change in slope of the angular distributions (50° - 90° c.m.s.) between 8 and 11 GeV/c.

In view of the considerable amount of accurate data now available it is interesting to search for some empirical functional dependence which can represent all of the results. After many trials it was found that a useful kinematic variable is

$$s \sin \Theta = 4(p^2 + m^2) \sin \Theta. \quad (4)$$

Fig. 2 shows the data above 50° from Clyde et al.³⁾, Ankenbrandt⁸⁾, Allaby et al.¹⁾ and from the present experiment, together with the 90° data of Akerlof et al.¹⁰⁾, plotted on a logarithmic scale against $s \sin \Theta$. The experimental points, which span an incident momentum range from 3 to 21 GeV/c and cover almost 8 decades of cross section, fall on two curves¹¹⁾, each of which can be well represented by the formula

$$\frac{d\sigma}{dt} = B \exp(-s \sin \Theta / g). \quad (5)$$

The best fit values for g and B are as follows;

$$s \sin^2 \theta \leq 16.0 \text{ GeV}^2; \quad g = 1.24 \pm 0.01 \text{ GeV}^2, \quad B = 134.6 \pm 11.7 \text{ mb}(\text{GeV}/c)^{-2}$$

$$s \sin^2 \theta \geq 20.0 \text{ GeV}^2; \quad g = 2.77 \pm 0.02 \text{ GeV}^2, \quad B = 56.4 \pm 3.4 \mu\text{b} (\text{GeV}/c)^{-2}$$

It should be remarked that, according to considerations of the analyticity and boundedness of the scattering amplitude¹²⁾, the relation (5), for fixed scattering angle, cannot be satisfied indefinitely and a slower fall off with energy should prevail as $s \rightarrow \infty$.

Neither the significance of the dependence on the variable $s \sin^2 \theta$ nor the reason for the break in the curve of Fig. 2 are clear. With regard to the latter and to the related change of slope of the angular distributions an interesting connection with the onset of antibaryon production may be remarked. The threshold for nucleon pair production is at about 6.5 GeV/c incident proton momentum but between 8 GeV/c and 10.9 GeV/c many new channels (28 taking account of charge multiplicity) open for the pair production of the baryons with mass between that of the Λ and of the $N(1400)$. Such baryon pair production processes near threshold are surely the result of low impact parameter collisions and their reactive effects should be such as to change the low partial waves of the elastic scattering, thus modifying the angular distributions at large angles.

The change of slope in Fig. 2 has been discussed by Akerlof et al.¹⁰⁾ in terms of diffraction from a nucleon "core" or "inner shell". The effects of baryon pair production is obviously another, and more specific, way of describing "core" processes. Furthermore, Akerlof et al. have commented on the sharpness of the break in Fig. 2, noticing that if the situation were to be described by two amplitudes, then considerable destructive interference between them would be required to obtain such a violent change of slope. In the picture described above, the reactive threshold effect simply changes, suddenly, the partial wave constitution of the scattering amplitude.

If these ideas are meaningful, the change in the elastic angular distributions at wide angles can, perhaps, give some measure of the total baryon-antibaryon production and also some information on the relative amounts of real and imaginary parts of the scattering amplitude.

We are greatly indebted to H. Ravn and Mme W. Stötzel for the measurements of the activities of the aluminium monitor foils. Thanks are due to the members of the MPS Division (Proton Synchrotron) who operated the machine and the beam extraction equipment during the experiment. Excellent technical assistance was provided by R. Donnet, M. Ferrat, and C.A. Ståhlbrandt.

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The cross section used for ^{24}Na was 8.6 ± 0.5 mb and $^{22}\text{Na}/^{24}\text{Na} = 1.23 \pm 0.05$.
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8. C.M. Ankenbrandt, U.C.R.L. 17257 (Ph.D.thesis).
9. The parameter b^{-1} gives a measure of the slope of the angular distribution in the range $50^\circ - 90^\circ$ as

$$b^{-1} = \left[- \frac{d}{dp_{\perp}} \log (d\sigma/d\Omega) \right]$$

where $p_{\perp} = p \sin \Theta$. Thus rapid changes in b reflect rapid changes in the slope of the angular distribution.

10. C.W. Akerlof, R.H. Hieber, A.D. Krisch, K.W. Edwards, L.G. Ratner and K. Ruddick, Phys.Rev.Letters 17, 1105 (1966) and paper submitted for publication.
11. The range of angles for which cross sections fall on the curve of Fig. 2 is energy dependent. For example, the data of Ankenbrandt⁶) (3-7 GeV/c) down to 30° fall on the curve, but at high momenta (> 20 GeV/c) the Cornell-BNL data⁷) fit the curve only for angles larger than 60° .
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TABLE CAPTIONS

Table 1 : Measured values of the differential cross section for p-p elastic scattering at various incident momenta. The values at 16.9 GeV/c are from Allaby et al.¹⁾ corrected by ~ 10% for absorption.

Table 2 : Values of the parameter b, in the relation $(d\sigma/d\Omega)_{C.M.} \propto \exp(-p \sin\theta/b)$, obtained from the best fits to large angle p-p elastic scattering data at various incident momenta.

TABLE 1

Incident Momentum [GeV/c]	Θ c.m. [degrees]	$\left(\frac{d\sigma}{d\Omega}\right)$ c.m. [cm ² sr ⁻¹]
8.1	68.8	$(5.43 \pm 0.16) \times 10^{-31}$
	72.2	$(4.55 \pm 0.20) \times 10^{-31}$
	76.2	$(3.23 \pm 0.10) \times 10^{-31}$
	82.2	$(2.14 \pm 0.07) \times 10^{-31}$
.....		
9.1	68.1	$(1.88 \pm 0.05) \times 10^{-31}$
	74.2	$(1.18 \pm 0.03) \times 10^{-31}$
	82.2	$(0.94 \pm 0.03) \times 10^{-31}$
.....		
10.0	67.0	$(10.62 \pm 0.17) \times 10^{-32}$
	70.0	$(7.91 \pm 0.17) \times 10^{-32}$
	75.0	$(5.89 \pm 0.15) \times 10^{-32}$
	83.0	$(4.94 \pm 0.11) \times 10^{-32}$
.....		
11.0	73.0	$(3.60 \pm 0.09) \times 10^{-32}$
	78.0	$(2.96 \pm 0.07) \times 10^{-32}$
	86.0	$(2.69 \pm 0.09) \times 10^{-32}$
.....		
14.25	67.0	$(7.58 \pm 0.23) \times 10^{-33}$
	71.0	$(5.86 \pm 0.19) \times 10^{-33}$
	77.0	$(4.36 \pm 0.14) \times 10^{-33}$
	90.0	$(3.31 \pm 0.09) \times 10^{-33}$
.....		
16.9	67.0	$(2.01 \pm 0.04) \times 10^{-33}$
	70.0	$(1.58 \pm 0.06) \times 10^{-33}$
	72.0	$(1.43 \pm 0.05) \times 10^{-33}$
	75.0	$(1.21 \pm 0.04) \times 10^{-33}$
	77.0	$(1.03 \pm 0.03) \times 10^{-33}$
	80.0	$(0.92 \pm 0.04) \times 10^{-33}$
	82.0	$(0.86 \pm 0.03) \times 10^{-33}$
	85.0	$(0.81 \pm 0.04) \times 10^{-33}$
	90.0	$(0.75 \pm 0.05) \times 10^{-33}$
.....		
19.3	64.0	$(6.82 \pm 0.29) \times 10^{-34}$
	69.0	$(4.19 \pm 0.26) \times 10^{-34}$
	75.0	$(2.70 \pm 0.20) \times 10^{-34}$
	90.0	$(1.88 \pm 0.17) \times 10^{-34}$
.....		
21.3	66.0	$(2.20 \pm 0.18) \times 10^{-34}$
	70.0	$(1.35 \pm 0.11) \times 10^{-34}$
	75.0	$(1.01 \pm 0.08) \times 10^{-34}$
	87.0	$(0.61 \pm 0.09) \times 10^{-34}$



TABLE 2

Incident momentum [GeV/c]	b [MeV/c]	Reference
3.0	251 ± 12	3
3.0	179 ± 4	8
4.0	120 ± 6	8
5.0	147 ± 3	3
5.0	158 ± 6	8
6.0	152 ± 6	8
7.1	154 ± 6	3
7.1	132 ± 12	8
8.1	118 ± 13	present work
9.1	172 ± 27	" "
10.0	191 ± 21	" "
11.0	294 ± 50	" "
14.25	240 ± 2	" "
16.9	225 ± 5	1
19.3	224 ± 9	present work
21.3	209 ± 17	" "

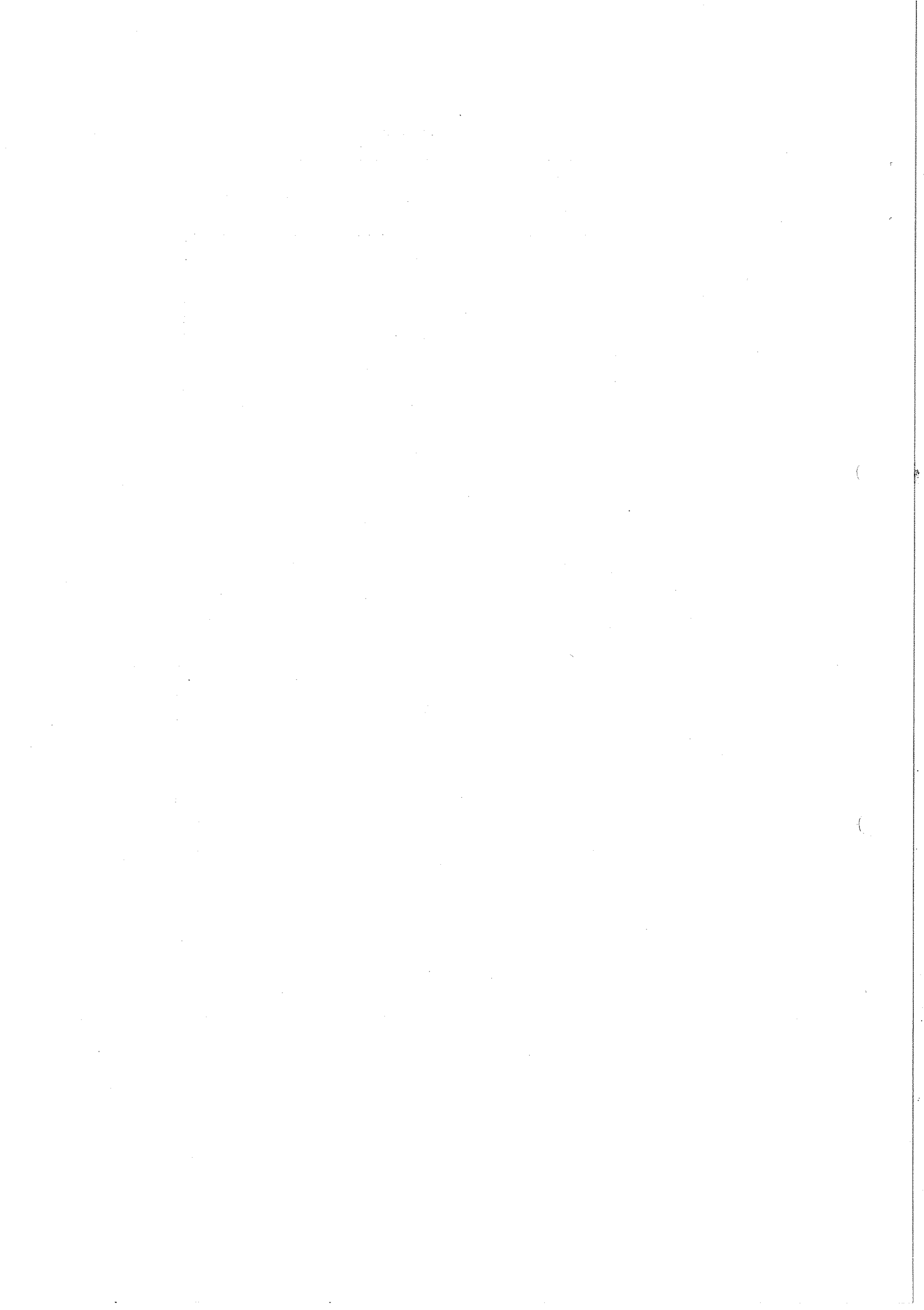


FIGURE CAPTIONS

Figure 1 : Logarithmic plot of $s(d\sigma/d\Omega)_{c.m.}$ as a function of $p \sin\theta$. The angular distributions at various incident proton momenta are from Clyde et al.³⁾ and from the present experiments. The broken line represents Orear's fit⁶⁾ [equation (2)] to previous data. The inset shows values of the parameter b obtained from a fit to the angular distributions by $(d\sigma/d\Omega)_{c.m.} \propto \exp(-p \sin\theta/b)$. Results from the data of Ankenbrandt⁸⁾ are also included.

Figure 2 : Logarithmic plot of $(d\sigma/dt)$ as a function of $s \sin\theta$. The data are from Clyde et al.³⁾, Ankenbrandt⁸⁾, Akerlof et al.¹⁰⁾ and the present experiments. The lines in the figure result from a fit to the points by $(d\sigma/dt) \propto \exp(-s \sin\theta/g)$. The inset gives values of g obtained from the individual angular distributions, the two horizontal lines indicating the values obtained from the overall fit shown in the Figure.

PROBLEM 11

Let $f(x) = x^2 + 2x + 1$. Find $f'(x)$ using the definition of the derivative.

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = (x+h)^2 + 2(x+h) + 1 = x^2 + 2xh + h^2 + 2x + 2h + 1$

$f(x) = x^2 + 2x + 1$

$f(x+h) - f(x) = (x^2 + 2xh + h^2 + 2x + 2h + 1) - (x^2 + 2x + 1) = 2xh + h^2 + 2h$

$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$

$f'(x) = \lim_{h \rightarrow 0} (2x + h + 2) = 2x + 2$

Let $f(x) = x^3 - 2x^2 + 5x - 7$. Find $f'(x)$ using the definition of the derivative.

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = (x+h)^3 - 2(x+h)^2 + 5(x+h) - 7 = x^3 + 3x^2h + 3xh^2 + h^3 - 2(x^2 + 2xh + h^2) + 5x + 5h - 7$

$f(x) = x^3 - 2x^2 + 5x - 7$

$f(x+h) - f(x) = (x^3 + 3x^2h + 3xh^2 + h^3 - 2x^2 - 4xh - 2h^2 + 5x + 5h - 7) - (x^3 - 2x^2 + 5x - 7) = 3x^2h + 3xh^2 + h^3 - 4xh - 2h^2 + 5h$

$\frac{f(x+h) - f(x)}{h} = \frac{3x^2h + 3xh^2 + h^3 - 4xh - 2h^2 + 5h}{h} = 3x^2 + 3xh + h^2 - 4x - 2h + 5$

$f'(x) = \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2 - 4x - 2h + 5) = 3x^2 - 4x + 5$

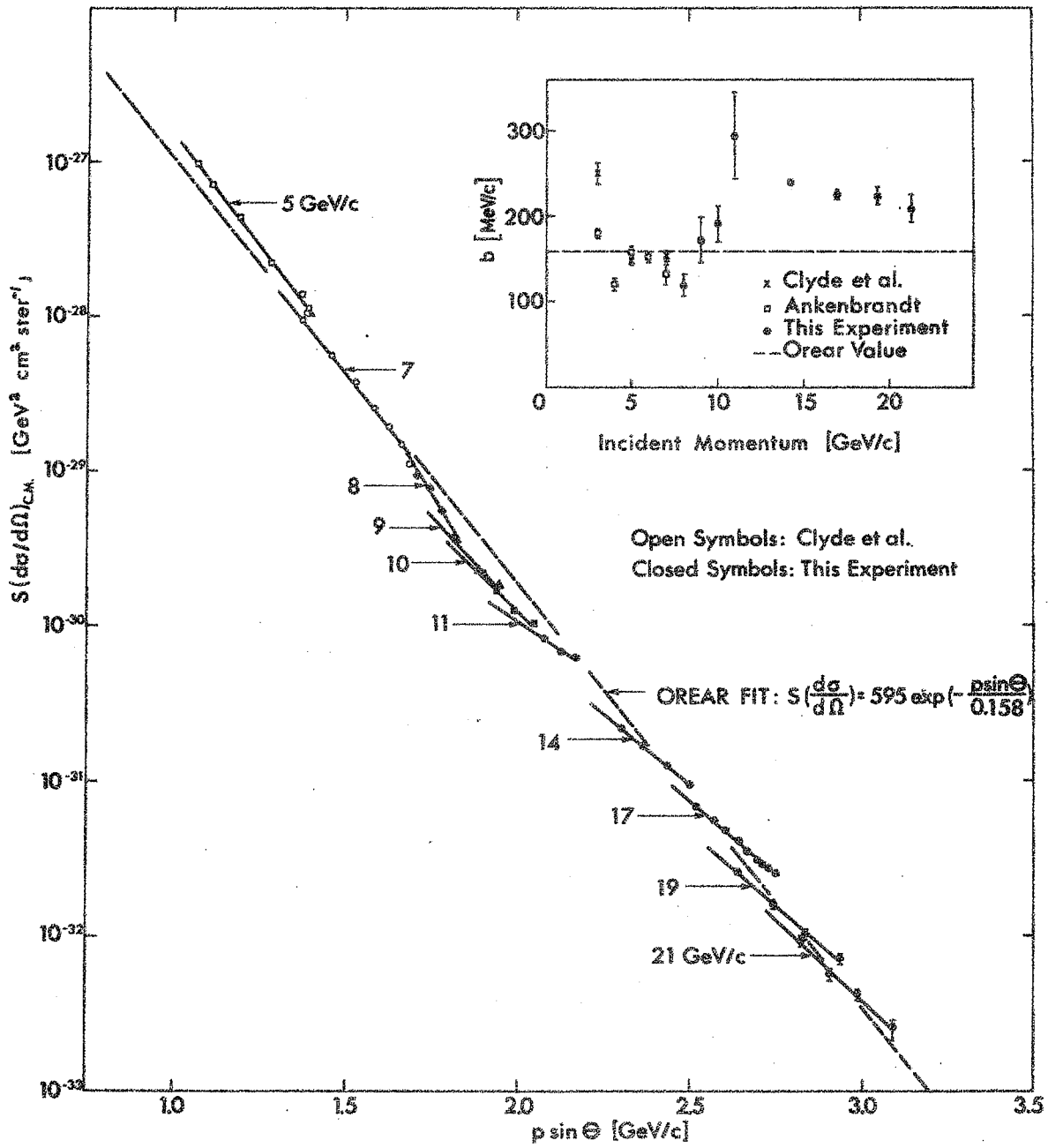
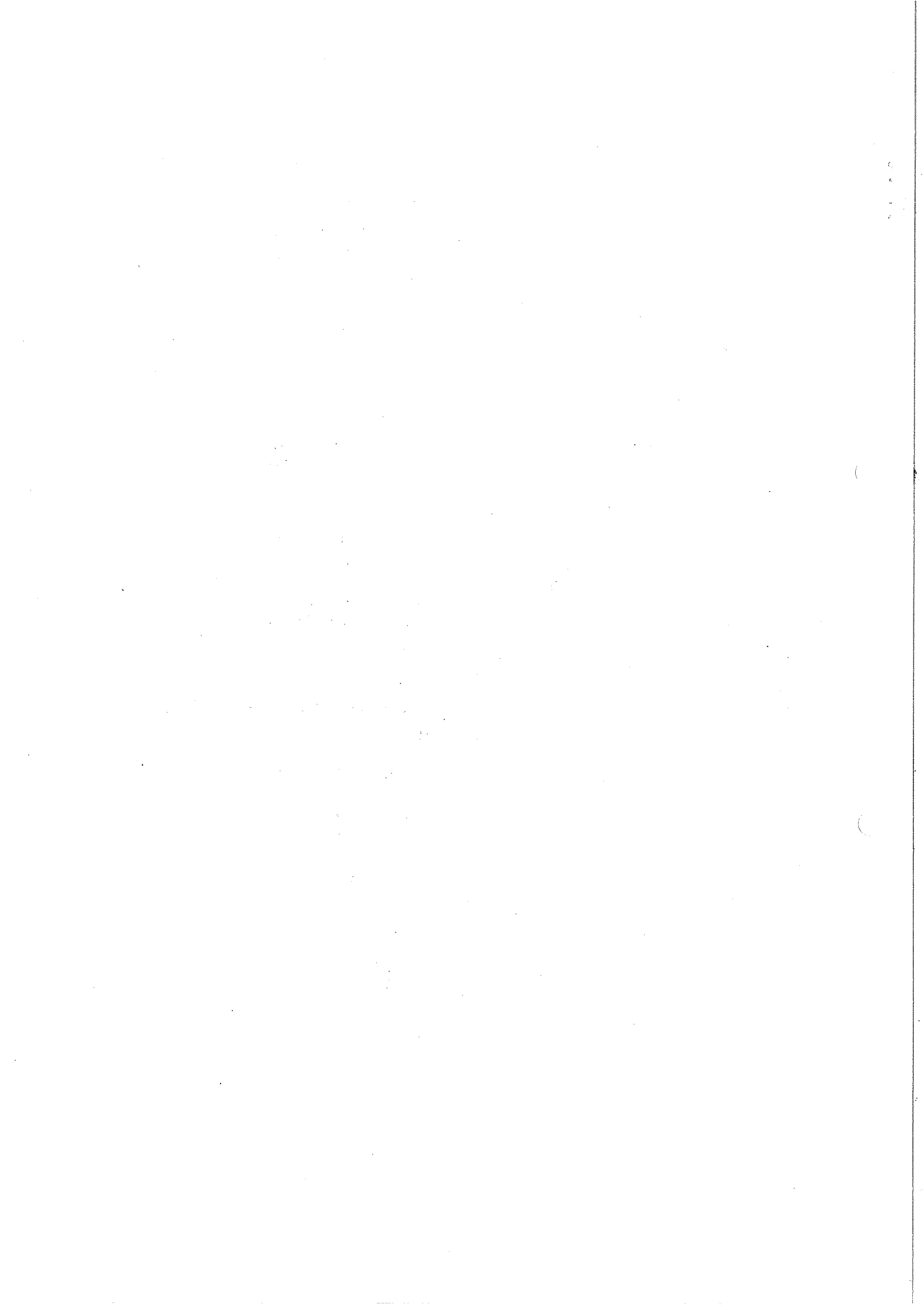


Fig 1



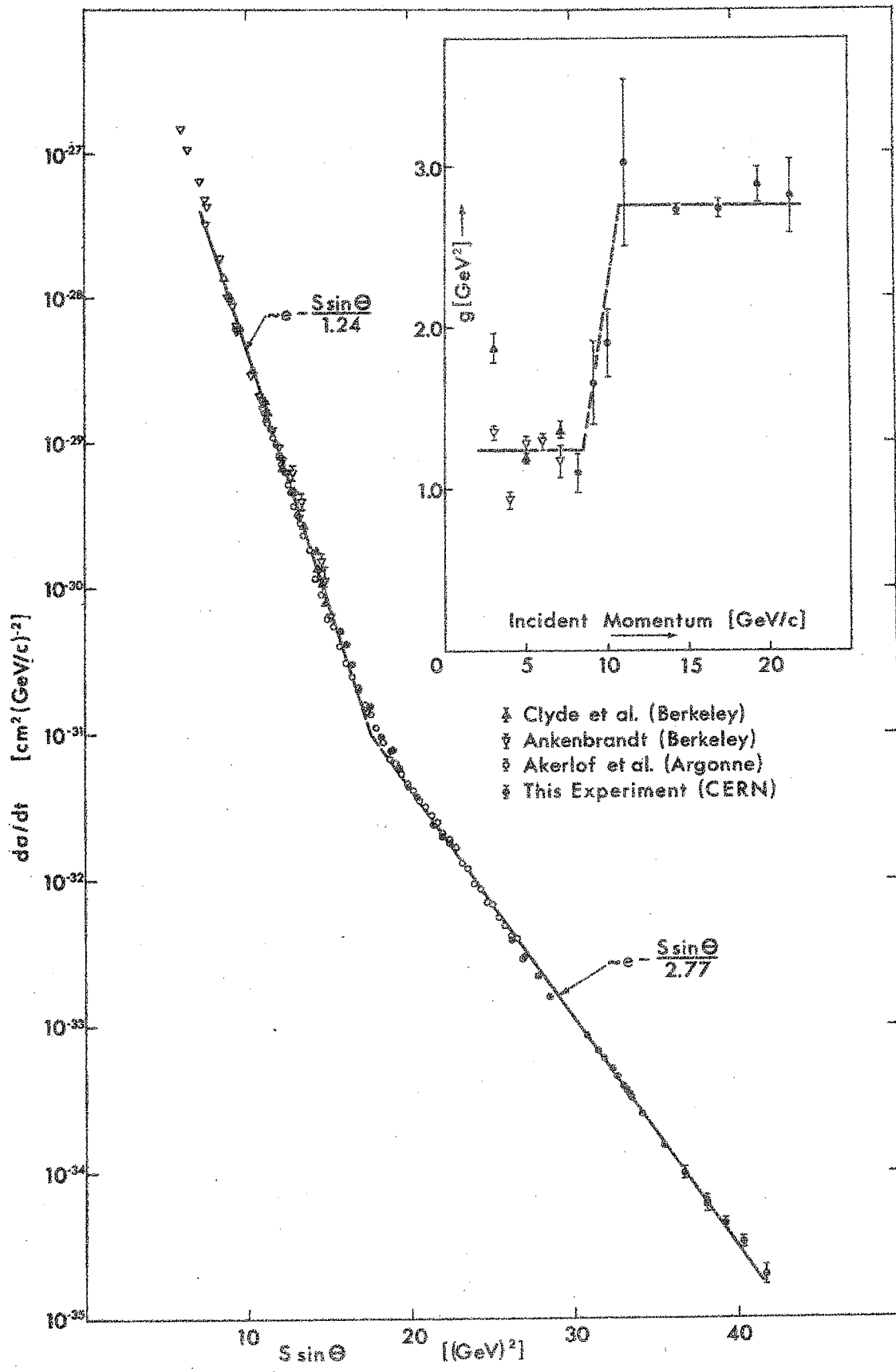


Fig 2

