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Measurement of beta function and betatron phase advance by means of the beam transfer function

Run 950, 15.6.1978, Ring 2, 26 GeV

Abstract

Vertical betatron oscillations were excited with noise covering a bandwidth which includes the modes (27+Q) and (45-Q). These two modes were observed on the beam position monitors around the ring and the relative amplitude and the phase advance measured (beam transfer function between two monitors). The relative value of the vertical betatron functions at the monitors is obtained from the square of the observed amplitudes. From the phase of the observed oscillations the betatron phase advance between the monitors is obtained. By observing two sidebands (fast and slow wave) the influence of the length of the signal cables is greatly reduced. The results are compared with AGS[1] calculations. The agreement is quite good. For the determination of the beta function from the amplitude of observed oscillation the relative rms disagreement with AGS is 16%. These measurements depend of course on the calibration constants. The measurement of the betatron phase advance does not depend on any calibration and gives an rms error of only 5.3%.

1. Introduction

The measurement of the relative amplitude and the phase of a betatron oscillation at different position monitors was carried out a long time ago within the investigation of the beam transfer function[2] but has never been written up as such. The method has been used once to check a high beta configuration with calculations[3]. To evaluate the usefulness of this method for measuring lattice functions in LEP the old measurements have been analysed and are presented here.

2. Theory

2.1 Bunched beams

We consider M equal and equidistant bunches (Fig. 1) which represent for an observer at $\Theta = 0$ a current

$$I(t) = \sum_{k=-\infty}^{+\infty} I_b\left(t - \frac{kT_o}{M}\right) \tag{1}$$

where $I_b(t)$ is the bunch form expressed as the instantaneous current and T_o is the revolution time. To avoid overlap between bunches we demand $I_b = 0$ for $|t| > T_o/2M$. The current $I(t)$ is periodic with the period T_o/M and can be expressed as a Fourier series:

$$I(t) = \sum_{p=0}^{\infty} \tilde{I}(pM\omega_o)\cos(pM\omega_o t)$$

with

$$\tilde{I}(pM\omega_o t) = \frac{2M}{T_o} \int_{-T_o/2M}^{T_o/2M} I_o(t)\cos(pM\omega_o t)dt, \quad \tilde{I}(o) = I_o$$

where we assumed symmetric bunches

$I(t) = I(-t)$ for simplicity; $\omega_o = \dfrac{2\pi}{T_o}$ = revolution frequency.

The contribution of an individual bunch k to the Fourier series is

$$I_k(t) = +\frac{1}{M} \sum_{p'=0}^{\infty} \tilde{I}(p'\omega_0)\cos\left(p'(\omega_0 t - k2\pi/M)\right).$$

For an observer at a distance s from the reference point $\Theta = 0$ the signal is delayed by $\delta t = \frac{s}{\beta c} = \frac{\Theta}{\omega_0}$ and the observed current is:

$$I_k(t,\Theta) = \frac{1}{M} \sum_{p'=0}^{\infty} \tilde{I}(p'\omega_0)\cos\left(p'(\omega_0 t - \Theta - k2\pi/M)\right). \qquad (2)$$

We assume now that the bunches execute a coupled betatron oscillation which could be described by an observer at $\Theta = 0$ as

$$y_k(t) = y_o \cos(Q\omega_o t + kn2\pi/M)$$

where n is the coupled bunch mode number, $0 \leq n \leq M-1$, defined so that $n2\pi/M$ is the phase difference between the oscillations of adjacent bunches. If the same oscillation is observed at a location Θ the beta function $\beta(\Theta)$ and the betatron phase difference $\phi(\Theta)$ have to be considered

$$\phi(\Theta) = \int_o^{\Theta} \frac{1}{\beta(\Theta)} R d\Theta$$

when R is the average radius, $R\Theta = s$.

The oscillation becomes

$$y_k(t,\Theta) = y_o \sqrt{\frac{\beta(\Theta)}{\beta(o)}} \cos\left(Q\omega_o t + \phi(\Theta) - Q\Theta + kn2\pi/M\right). \qquad (3)$$

A position-sensitive monitor gives a signal U(t) which is proportional to the displacement y(t) times the current I(t).

$$U_k(t,\Theta) \;=\; gy_k(t,\Theta)I_k(t,\Theta) \;=$$

$$= \frac{gy_o}{M}\sqrt{\frac{\beta(\Theta)}{\beta(o)}}\sum_{p'=0}^{\infty}\tilde{I}(p\omega_o)\cos\big(p'(\omega_o t - \Theta - k2\pi/M)\big)\cos\big(Q\omega_o t + Q(\Theta) - Q\Theta + kn2\pi/M\big)$$

with g being a calibration factor.

To obtain the signal due to all bunches we have to sum over k which leads to the result

$$U(t,\Theta) \;=\; gy_o\sqrt{\frac{\beta(\Theta)}{\beta(o)}}\;\sum_p\Big[\tilde{I}(pM+n)\cos\big((pM+n+Q)(\omega_o t-\Theta) + \phi(\Theta)\big)\Big]$$

$$+\sum_p\Big[\tilde{I}(pM-n)\cos\big((pM-n-Q)(\omega_o t-\Theta) - \phi(\Theta)\big)\Big] \qquad (4)$$

The first sum (fast wave) goes over all values of p for which $pM+n \geq 0$ and the second term (slow wave) over all values of p for which $pM-n \geq 0$.

The spectrum (4) has the well-known upper and lower betatron sidebands illustrated in Fig. 2.

2.2 Unbunched beams

A single particle of charge q is, according to (1), observed with an intensity monitor at $\Theta = 0$ as a current of the form

$$I(t) \;=\; q\sum_{k=-\infty}^{+\infty}\delta(t-kT_o) \;=\; \frac{q\omega_o}{2\pi}\Big(1 + 2\sum_{p'=1}^{\infty}\cos(p'\omega_o t)\Big)$$

A continuous beam can be divided into infinitesimal pieces of length $ds = Rd\alpha$, each producing a current $dI(t)$ at $Q = 0$

$$dI(t) \;=\; \frac{I_o}{2\pi}\Big[1 + 2\sum_{p'=1}^{\infty}\cos\big(p'(\omega_o t-\alpha)\big)\Big]d\alpha$$

We assume now that the particles execute a betatron oscillation observed at $\Theta = 0$

$$y(t) = y_o \cos(Q\omega_o t + n\alpha)$$

The mode number n gives the number of periods this mode represents for a fixed value of t. For the continuous beam n can take positive and negative values depending on whether the particle in front of a test particle has a phase advance or a retardation. Observed at a location Θ the oscillation is given by

$$y(t,\Theta) = y_o \sqrt{\frac{\beta(\Theta)}{\beta(o)}} \cos\left(Q\omega_o t - Q\Theta + n\alpha + \phi(\Theta)\right)$$

The signal in a position-sensitive monitor is

$$dU(t,\Theta) = \frac{gI_o}{2\pi} y_o \sqrt{\frac{\beta(\Theta)}{\beta(o)}} \left(1 + 2 \sum_{p'=1}^{\infty} \cos\left(p'(\omega_o t - \alpha - \Theta)\right)\right.$$

$$\left. \cdot \cos\left(Q\omega_o t - Q\Theta + n\alpha + \phi(\Theta)\right)\right) d\alpha$$

Integrating this over α and using

$$\int_o^{2\pi} \cos(N\alpha)d\alpha = \begin{cases} 2\pi \text{ if } N = 0 \\ 0 \text{ if } N \neq 0 \end{cases}$$

$$\int_o^{2\pi} \sin(N\alpha)d\alpha = 0$$

we obtain for the signal

$$U(t,\Theta) = gI_o\, y_o \sqrt{\frac{\beta(\Theta)}{\beta(o)}} \cos\left((n + Q)\omega_o t + \phi(\Theta) - (n + Q)\Theta\right) \tag{5}$$

Since n takes positive and negative numbers the expression $(n + Q)$ can be positive (fast wave) or negative (slow wave).

3. Measuring phase advance using two neighbouring frequencies belonging to a fast and a slow wave

We investigate the phase measurement for the unbunched beam (5) first. We consider a set-up as shown in Fig. 1 where the phase difference of the signals of the two pick-up stations lcoated at $\Theta = 0$ and Θ is measured for a fast and a slow wave. We take first the case where the two cables from the pick-up stations to the control room are exactly the same.

We select for comparison a fast and a slow wave given by (5) which have nearly the same frequency

$$(n_1 + Q) \approx -(n_2 + Q) \tag{6}$$

We split the tune into an integer and a fractional part

$$Q = Q_{int} + \Delta Q$$

Assuming first $|\Delta Q| < 1/4$ we can write condition (6) as

$$n_1 + Q_{int} = -(n_2 + Q_{int})$$

and get for the signals of the two waves observed at Θ

$$U_f t,\Theta) = gI_o\, y_o \sqrt{\frac{\beta(\Theta)}{\beta(o)}} \cos\left((n_1+Q_{int}+\Delta Q)\omega_o t+\phi(\Theta) - (n_1+Q_{int}+\Delta Q)\Theta\right)$$

$$U_s t,\Theta) = gI_o\, y_o \sqrt{\frac{\beta(\Theta)}{\beta(o)}} \cos\left((-n_1-Q_{int}+\Delta Q)\omega_o t+\phi(\Theta) - (-n_1-Q_{int}+\Delta Q)\Theta\right)$$

$$= gI_o\, y_o \sqrt{\frac{\beta(\Theta)}{\beta(o)}} \cos\left((n_1+Q_{int}+\Delta Q)\omega_o t-\phi(\Theta) - (n_1+Q_{int}-\Delta Q)\Theta\right)$$

Comparing the phases of the signals we get

$$\xi_f' - \xi_s = 2(\phi(\Theta) - \Theta\Delta Q)$$

If the signals are observed in the control room with cables of different lengths L_1 and L2 an additional phase shift occurs. However this phase shift involves only the difference of the cable length and only the usually small difference of the two frequencies $2\Delta Q\omega_0$. The final difference in phase is

$$\boxed{\xi_f - \xi_s = 2\left(\phi(\Theta) - \Theta\Delta Q + \frac{L_1 - L_2}{c'} \omega_0 \Delta Q\right)} \tag{7}$$

where c' is the signal velocity in the cable. This equation permits the determination of the betatron phase advance from a comparison of the phase of the signals observed in two pick-up stations. As seen from (7) there is an ambiguity of + k 180°. We go back to (6) and consider the case where $\Delta Q > 1/4$. The closest frequencies are centred around a half revolution harmonics. If we measure those we introduce

$$\delta Q = \Delta Q - \frac{1}{2}$$

and obtain for (6)

$$(n_1 + Q_{int} + \tfrac{1}{2}) = -(n_2 + Q_{int} + \tfrac{1}{2}$$

In the final result (7) the quantity ΔQ has to be replaced by δQ in this case.

For the case of bunched beams we take expression (4) and select a fast and a slow side band with approximately the same frequency so that

$$pM + n_1 + Q_{int} = pM - n_2 - Q_{int}$$

and obtain the same expression (7) as for the case of a coasting beam.

4. Experiment

During MD run 950 a small stack was made in the centre of the vacuum chamber. This beam was excited vertically at ~11 MHz with noise covering a limited bandwidth which covered the modes (27 + Q) and (45 - Q) with Q = 8.882. The oscillation was measured on all the beam position monitors. Monitor 504 was used as a reference and the signals from all other monitors were compared with this reference signal. This was done with the FFT where amplitude and phase of the two modes were measured directly on the screen without taking a picture. A few stations gave signals of very different amplitudes for the two modes and were ignored. The estimated reading error for the phase measureement was about 3°. The results are shown in Table 1. The amplitude measurement depends on the calibration constant of each monitor; the phase measurement, however, is independent of such a constant. The cable length is the same for each monitor. According to (7) even a difference in cable length as large s 10 m would only introduce an error of about 0.5°. The measurements are compared with the beta function and the phase advance calculated with AGS for the same conditions. Since the measurements of the beta function give only relative values the average has been adjusted to the expected average value. The rms relative deviation of the measured beta values from the calculated values is 16%. In the case of the phase advance the rms error is 5.3° mainly due to one bad measurement in monitor 728.

5. Conclusions

The measurement of the beam transfer function between different pick-up stations of betatron oscillations can give valuable information about the lattice functions. In particular the phase measurement is very accurate and does not depend on monitor constants and only very little on cable length. In the ISR the spacing between the monitors includes many focussing elements so that the knowledge of the betatron phase advance between monitors is not sufficient to determine the beta function everywhere. However in a separated function machine such as LEP with monitors at each quadrupole the complete beta function could be measured and focussing errors localized. The measurement may be accurate enough to determine to off-energy beta function and check the chromatic correciton.

Similar measurements have also been carried out in other machines[4)].

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References

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pick up	beta function (m)			phase advance (deg.)		
	AGS	measured	diff.	AGS	measured	diff.
504	30.19	-	-	0.0	-	-
512	27.85	29.51	1.67	24.75	24.14	-0.61
520	8.00	8.60	0.60	77.61	77.58	-0.03
528	17.80	17.85	0.05	163.29	162.42	-0.87
536	31.93	30.46	-1.47	192.96	193.35	0.39
548	9.37	10.46	1.09	251.53	253.15	1.62
556	17.52	17.13	-0.39	339.03	335.80	-3.23
564	34.86	33.88	-0.98	370.21	368.95	-1.26
604	38.88	46.95	8.07	434.73	430.87	-3.86
620	21.23	19.34	-1.89	525.91	519.68	-6.23
632	14.64	14.72	0.08	555.03	557.72	2.69
644	30.05	32.88	2.83	609.79	608.07	-1.72
660	31.13	29.05	-2.08	694.45	692.45	-2.00
712	25.20	28.12	2.92	800.77	798.80	-1.97
720	6.55	6.69	0.14	868.63	867.96	-0.67
728	23.25	22.09	-1.16	943.25	968.61	25.35
736	32.82	35.40	2.58	968.54	965.76	-2.78
748	7.30	7.61	0.31	1040.99	1036.86	-4.13
756	23.30	24.16	0.86	1120.02	1119.52	-0.50
800	31.05	17.79	-17.26	1225.05	1214.96	-10.09
820	27.36	26.32	-1.04	1305.27	1305.60	0.33
832	14.71	15.39	0.68	1330.60	1331.72	1.12
844	23.13	22.49	-0.64	1398.74	1396.37	-2.37
120	6.30	6.92	0.62	1711.49	1710.06	-1.43
128	23.40	25.01	1.61	1790.06	1794.96	4.90
136	34.88	33.38	-1.50	1814.46	1812.36	-2.10
148	7.15	7.38	0.23	1883.63	1885.71	2.08
156	23.25	22.90	-0.35	1966.83	1963.86	-2.37
204	30.10	28.12	-1.98	2067.79	2065.71	-2.08
220	27.68	23.57	-4.11	2152.06	2152.20	0.14
232	15.68	12.72	-2.96	2176.37	2176.44	0.07
244	22.78	21.53	-1.25	2241.34	2238.81	-2.53
248	15.72	13.34	-2.38	2296.05	2291.81	-4.24
260	41.49	36.70	-4.79	2323.39	2318.19	-5.20
304	26.65	44.93	18.28	2403.98	2411.67	7.69
320	8.72	6.65	-2.07	2499.03	2495.98	-3.05
328	18.98	19.97	0.99	2566.74	2562.87	-3.87
336	24.79	25.27	0.48	2599.69	2598.27	-1.42
348	9.55	7.56	-1.99	2673.25	2673.84	0.59
356	19.54	19.97	0.43	2742.83	2742.99	0.16
420	22.14	25.71	3.57	2930.25	2922.35	-7.90
444	29.63	34.15	4.52	3031.60	3037.83	6.23

Table I: Beta function and betatron phase advance at the pick-up stations; comparison between the AGS calculations and the measurements

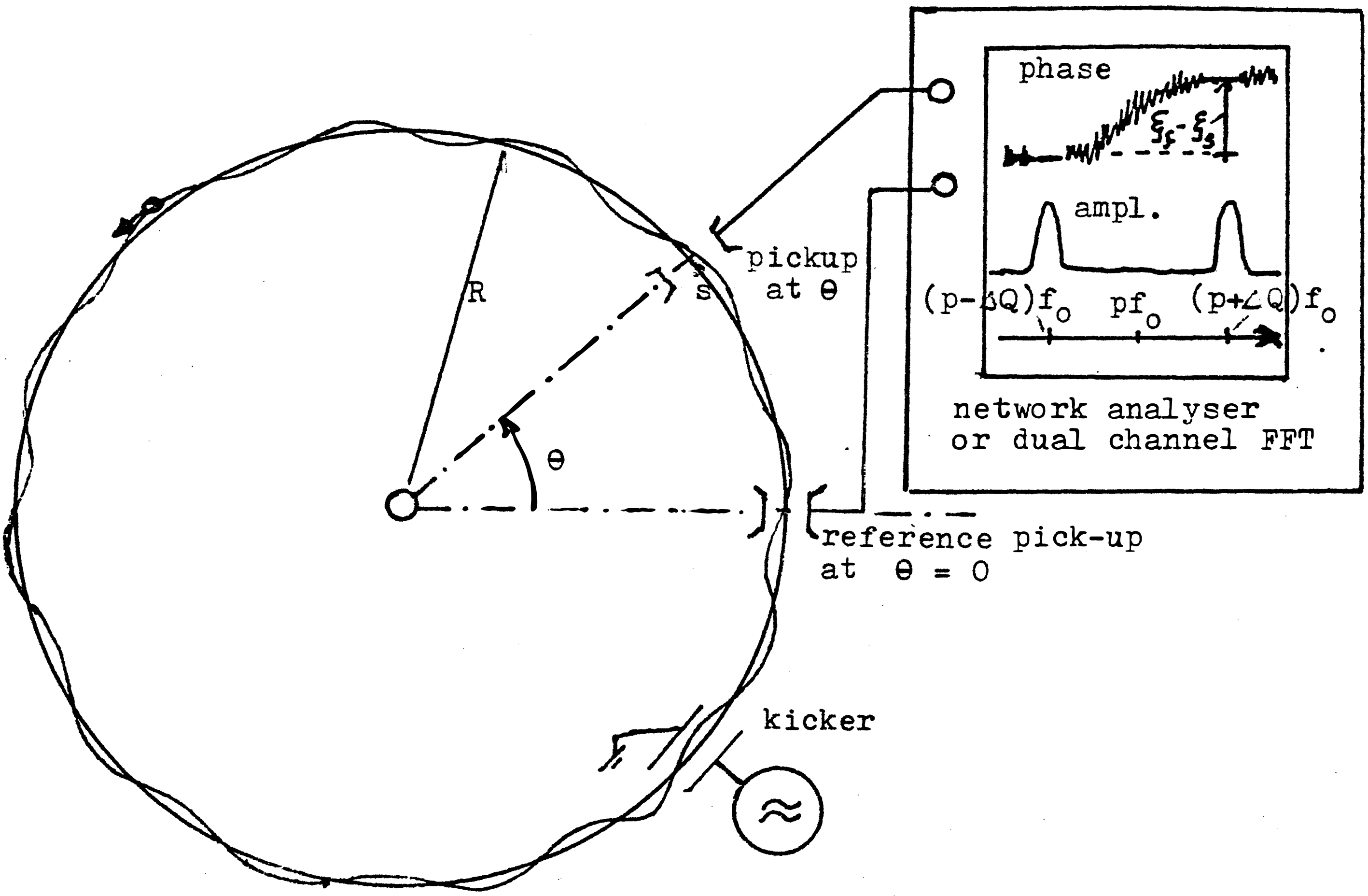

Fig. 1. Experimental layout

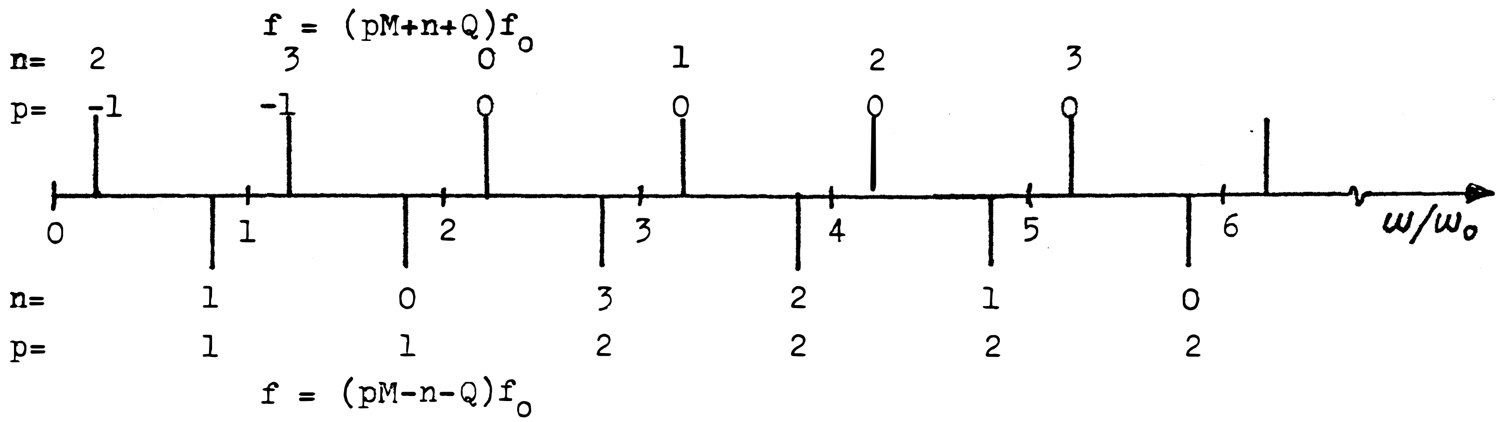

Fig. 2. Betatron oscillation spectrum for $M = 4$, $Q = 2.2$