E-6

ISR-RF/WS/ps *15th June 1978*

ISR PERFORMANCE REPORT

Measuring nanoamperes by Schottky scans

In this note I shall try to estimate the lower limit of unbunched beam current that can be observed by straight longitudinal Schottky scans. A single high-impedance pick-up cavity is assumed to be used.

The r.m.s. "signal" current is given by[1]

$$I_{rms} = \sqrt{2ef_r\delta I} \qquad (1)$$

where e is the elementary charge, f_r the average revolution frequency and δI the fraction of total circulating current carried by particles whose revolution frequency is within the resolution, δf, of the spectrum analyzer. Assuming a rectangular distribution of full width $\Delta f/f$ in frequency and $\Delta p/p$ in momentum, one obtains:

$$I_{rms} = \sqrt{\frac{2eIp\delta f}{\eta\Delta ph}} \; , \qquad (2)$$

where I is the total circulating current, h the harmonic number and

$$\eta = \left| \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \right| \; . \qquad (3)$$

The available "signal" power from a pick-up cavity of unloaded shunt impedance R (peak voltage gain squared over two times power-dissipation) is given by

$$P_s = \frac{I_{rms}^2 R}{4} \qquad (4)$$

This has to be compared with the available thermal noise power from the same source

$$P_n = kT\delta f \qquad (5)$$

with $kT = 4 \times 10^{-21}$ Ws at room temperature. Hence, the "signal"-to-noise power ratio is given by

$$\varepsilon = \frac{P_s}{FP_n} = I \frac{eR}{2kTFh} \frac{p}{\eta \Delta p} , \tag{6}$$

where F is the noise factor of the amplifier; and

$$I = \varepsilon \frac{2kTFh}{eR} \eta \frac{\Delta p}{p} \tag{6a}$$

is the minimum detectable current for given ε.

The "signal" is statistical noise just like the thermal and amplifier noise, but it is concentrated in bands $h \eta \Delta p/p$ so that the spectrum analyzer will show a hump of fractional amplitude $\sqrt{\varepsilon}$ on top of a pedestal of equipment noise. Averaging over a few minutes - the Fast Fourier Transform device should be much superior to the sweeper - one may hope to recognize quite small values of ε, say $\varepsilon = 0.01$ or a 10% hump in amplitude.

For antiprotons filling the radial aperture of the ISR at 3.5 GeV/c

$$\frac{\Delta p}{p} \sim 0.05$$

$$\eta = 0.055$$

i.e. the particles cover the unusually large frequency range $\Delta f/f = 2.75 \times 10^{-3}$, making real high-Q cavities problematic.

The harmonic number h in the numerator of (6a) suggests that a ferrite "cavity" (or resonant transformer) at $f_r = 318$ kHz could be advantageous. A core built of blocks of low-frequency ferrite ($\mu \sim 3000$) with 1 m length, 40 mm core thickness and about 1 mm air gap, might reach 40 μH inductance and a Q-factor of 100, hence a shunt impedance R = 8 kΩ. With $\varepsilon = 0.01$ and F = 10 one finds a minimum detectable current of

$$I = 1.72 \text{ nA} .$$

As the device measures density, not current, the beam can be scraped down to a smaller momentum spread, giving a narrower hump on the spectrum analyzer, without loss of sensitivity.

Using an ISR stacking cavity is another possibility. This would mean $h = 30$ and, approximately, $Q = 1200$, $R = 20\ k\Omega$ (information from H. Frischholz and S. Hansen), perhaps $F = 5$, and hence

$$I = 10.3\ \text{nA}$$

if scaled to the full 5% momentum spread. In reality the beam can and should be scraped down to bring $\Delta f/f$ inside the cavity bandwidth. Alternatively the cavity could be tuned across the beam during the measurement.

A higher energy would be desirable because of the rapidly decreasing value of η.

As an extreme example of both bunched and unbunched beams one may consider a single particle. The peak RF current at each revolution harmonic,

$$I_{RF} = 2ef_r \quad ,$$

equals 10^{-13} A. Now - before resorting to superconductivity - one could at least use a real high-Q accelerating structure made of copper, of the kind that is used in electron accelerators. For instance a five-cell, π-mode, iris loaded structure at 350 MHz will yield 10 MΩ/m (as defined above), or $R = 20$ MΩ. The Q-factor will be about 30,000, so that the structure will have to be mechanically tuned across the aperture to search for the particle. Assuming $F = 2$ and $\delta F = 10$ Hz (with the Fast Fourier Transform device, so that all particles circulating within the bandwidth of the cavity can be seen simultaneously) one obtains a signal-to-noise power ratio

$$\varepsilon = \frac{(2ef_r)^2 R}{8FkT\delta f} = 0.32$$

i.e. a 57% amplitude hump that should be clearly visible. It is true that no such accelerating structure is available at the ISR at present.

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Reference

1) J. Borer *et al.* Proc. IXth Int. Conf. High Energy Acc. 1974, p. 53.