



CM-P00072352

ISR PERFORMANCE REPORT

Measurement of  $\gamma_t$  as a Function of Momentum

Run 928 - 11.8 GeV/c - Ring 2 - 13.4.78

Summary

Two independent methods were used to measure  $\gamma_t$  and its variation across the aperture. The results produced from the two methods are consistent and indicate that the A.G.S. predictions are quite accurate on the central orbit but inaccurate at large deviations from there. These results seem to suggest that A.G.S. calculated values should not be trusted at orbits other than at central momentum.

1) Introduction

The measurement of  $\gamma_t$  and its dependence on momentum is important for several reasons:

- To convert measured frequencies into average orbits e.g. bunch frequency measurements, longitudinal Schottky etc.
- To calculate the variation of bucket area across the ISR aperture during phase displacement acceleration and hence estimate the minimum loss per sweep.
- As a check on the accuracy of the A.G.S. program which is used widely for the calculation of all magnetic machine parameters.

In particular the variation of the bucket area predicted from the computed variations in  $\gamma_t$  from A.G.S. would lead to much larger losses per sweep than has actually been obtained.

2) Measurement Techniques

2.1 Variation of Frequency with Radial Position

The variation of frequency with radial position can be expressed as a polynomial in  $\Delta R$  ( $R - R_c$ )

$$f = f_c + D_1 (R - R_c) + D_2 (R - R_c)^2 + \dots \quad (1)$$

$$\frac{df}{dR} = D_1 + 2D_2 \Delta R + \dots \quad (2)$$

$$\frac{\Delta f}{\Delta R} = D_1 + D_2 \Delta R \quad (3)$$

where  $\Delta f = f - f_c$

However the variation of frequency with momentum is given by

$$\frac{df}{f} = -\eta \frac{dP}{P} \quad \text{and} \quad dR = \bar{\alpha}_p \frac{dP}{P} \dots \quad (4)$$

thus  $\frac{df}{f} = -\frac{\eta}{\bar{\alpha}_p} dR$  or  $\frac{df}{dR} = \frac{-\eta f}{\bar{\alpha}_p}$

where  $\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$  and  $\bar{\alpha}_p = \frac{R}{\gamma_t^2}$

hence  $\frac{df}{dR} = \frac{f}{R} \left\{ 1 - \left( \frac{\gamma_t}{\gamma} \right)^2 \right\} \dots \dots \dots \quad (5)$

or  $\frac{df}{dR} \approx \frac{f_c}{R_c} \left\{ 1 - \left( \frac{\gamma_t}{\gamma} \right)^2 \right\}$  (to an accuracy  $\pm 5 \times 10^{-4}$ ) (6)

The measured variation of frequency with radial position (equ (1)) can then be used to evaluate  $\left( \frac{\gamma_t}{\gamma} \right)$  to a high accuracy and hence  $\gamma_t$  can be obtained from an iterative procedure involving  $\bar{\alpha}_p$ .

Using a synthesiser the bunch frequency can be measured extremely accurately ( $\pm$  1Hz). However it is more difficult to measure the average radial position to great accuracy since the existing computer programs make use of the computed values of  $\alpha$  at each pick-up. For this reason another program was written (P. Martucci) which calculated the average radial position of the non  $\alpha_p$  normalised values of position at each pick up. This measurement was tested and found to be independent of the "working condition".

## 2.2 Measurement of the Synchrotron Frequency

The Synchrotron frequency is given by

$$f_p = 1.2688 \times 10^5 \left[ \frac{h V_{RF} \eta \cos \phi_s}{\gamma E_0} \right]^{\frac{1}{2}}$$

Hence for constant  $V_{RF}$  and  $\phi_s$  the variation of  $\sqrt{\frac{\eta}{\gamma}}$  can be evaluated across the aperture by measurement of the synchrotron frequency.

## 3) Results

The results were taken at 11 GeV/c in order to increase the sensitivity of the variations of  $\eta$  across the aperture. An 8C type of working line was used so as to avoid large differences in horizontal orbit distortion across the aperture which may have decreased the accuracy of the measurement of average radial position.

### 3.1 Bunch Frequency as a Function of Radial Position

A beam was injected, kept bunched and accelerated using the manual displacement box to the required orbit. In order to have optimum conditions for the BPMS, the pulse was shaved to  $\sim 100$  mA. It was found that when the R.F. phase lock was left on the frequency of the bunched beam fluctuated by as much as 5 Hz in a slow pulsing manner. For this reason, the phase lock off/hold facility was used and it was found that the bunch frequency was stable to better than .1 Hz. The measurement of bunch frequency was performed by using a synthesiser of the longitudinal Schottky which had been switched on for many days and was extremely stable. The absolute precision of this instrument is checked regularly. The average radial position was measured using the BPMS. The resulting plot of frequency against radial position is shown in Fig. 1. These results were then input to a least squares fitting routine in order to evaluate the constants  $D_1$  and  $D_2$  of equation (1). The results are shown in Fig. 2. From these results the  $\gamma_t$  variation across the aperture was calculated and is shown in Fig. 3. Also shown are the A.G.S. values and the "corrected" A.G.S. values for this working condition.

### 3.2 Measurement of the Synchrotron Frequency

The synchrotron frequency of the bunched beam was measured as a function of radial position. The R.F. voltage was set to 6kV in order that the synchrotron frequency was well away from the 50 Hz which appears on all measuring equipment.

The plot of the synchrotron frequency is shown in Fig. 4. Also shown is the calculated frequencies based on the measurements of  $\gamma_t$  in the previous section. The agreement between the two sets of data is clear. The small offset in the measured synchrotron frequency may be explained by a discrepancy in the RF voltage or by space charge effects.

#### 4) Discussion of results

From the measured values of  $\gamma_t$  on central line it appears that the A.G.S. predicted value is quite precise. However, the predicted slope of  $\gamma_t$  across the aperture is incorrect by nearly a factor of 3!

Using the measured values of  $\gamma_t$  the calculated variation of the RF bucket area during traversal of a stack is

$$\Delta A_b \leq .6\% \quad (\text{from measurements})$$

Whereas using the predicted values from A.G.S. the variation is

$$\Delta A_b \leq 5\% \quad (\text{from A.G.S.})$$

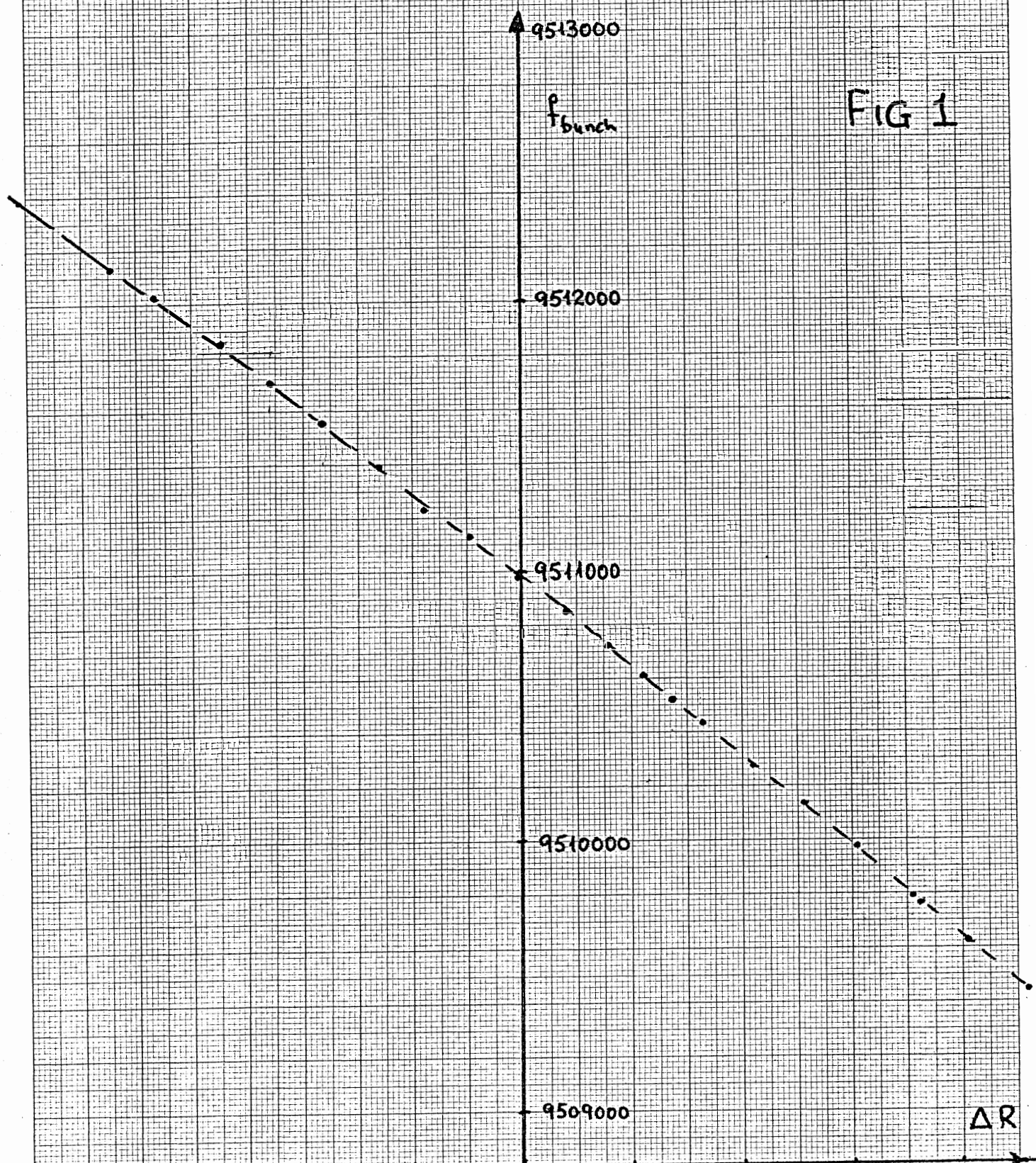
Such a large variation of bucket area across the aperture should cause typical losses per sweep of around 40 mA from a 30 A stack. Much lower losses per sweep have in fact been achieved during acceleration. In fact losses of  $\sim 4$  mA per sweep have been obtained. This seems to confirm the fact that the slope of  $\gamma_t$  is much less than A.G.S. predicts.

#### 5) Conclusions

The variation of  $\gamma_t$  across the ISR aperture can be measured with reasonable accuracy if the experiment is performed at low momentum. The values measured here indicate that A.G.S. is quite inaccurate for off central momenta orbits.

S. Myers

FIG 1



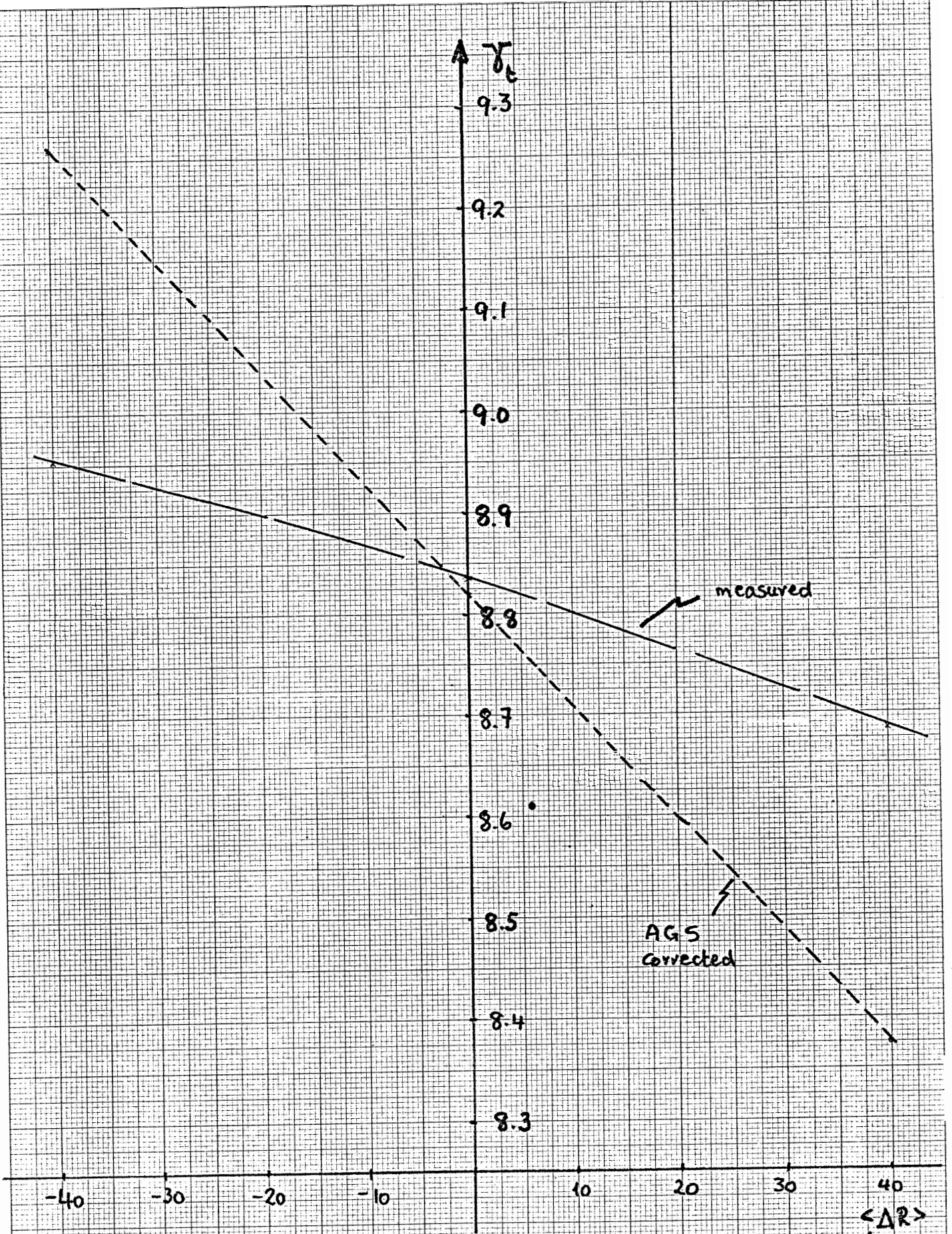
FITTED CURVE  $f = 9510975.6 - 32.2 \Delta R - .028 \Delta R^2$

COEFFICIENTS FOR ORDER = 3

A1 = 10975.594  
 A2 = -32.186  
 A3 = -.028

FIG 2

$\Delta R$ X	$f_{bunches}$ Y	fitte <i>d</i> $f_{bunches}$ ESTIMATE <i>d</i>	% error PERCENT ERROR
-45.6000	9512375.5	12385.8	-.083
-37.2000	12134.9	12134.7	.002
-33.6000	12030.0	12025.8	.035
-27.5000	11846.0	11839.8	.052
-23.1000	11711.3	11704.3	.059
-18.3000	11562.6	11555.3	.063
-13.0000	11395.4	11389.3	.053
-8.9000	11231.8	11259.9	-.250
-4.7000	11132.0	11126.3	.052
-.2000	10984.3	10982.0	.021
4.0000	10848.6	10846.4	.020
8.0000	10724.9	10716.3	.080
11.2000	10613.9	10611.6	.021
13.7000	10524.3	10529.5	-.049
16.5000	10434.0	10437.0	-.029
21.1000	10276.1	10284.2	-.078
25.4000	10138.7	10140.2	-.015
30.6000	9970.2	9964.8	.054
35.5000	9788.1	9798.1	-.103
36.3000	9768.1	9770.8	-.028
40.5000	9629.3	9626.7	.027
46.1000	9442.0	9433.1	.095



$$\frac{\Delta \bar{r}_c}{\bar{r}_c} / \frac{\Delta \bar{r}}{\bar{r}} = -0.727 \text{ (measured)}$$

$$\frac{\Delta \bar{r}_c}{\bar{r}_c} / \frac{\Delta \bar{r}}{\bar{r}} = -2.4 \text{ (AGS)}$$

FIG 3.

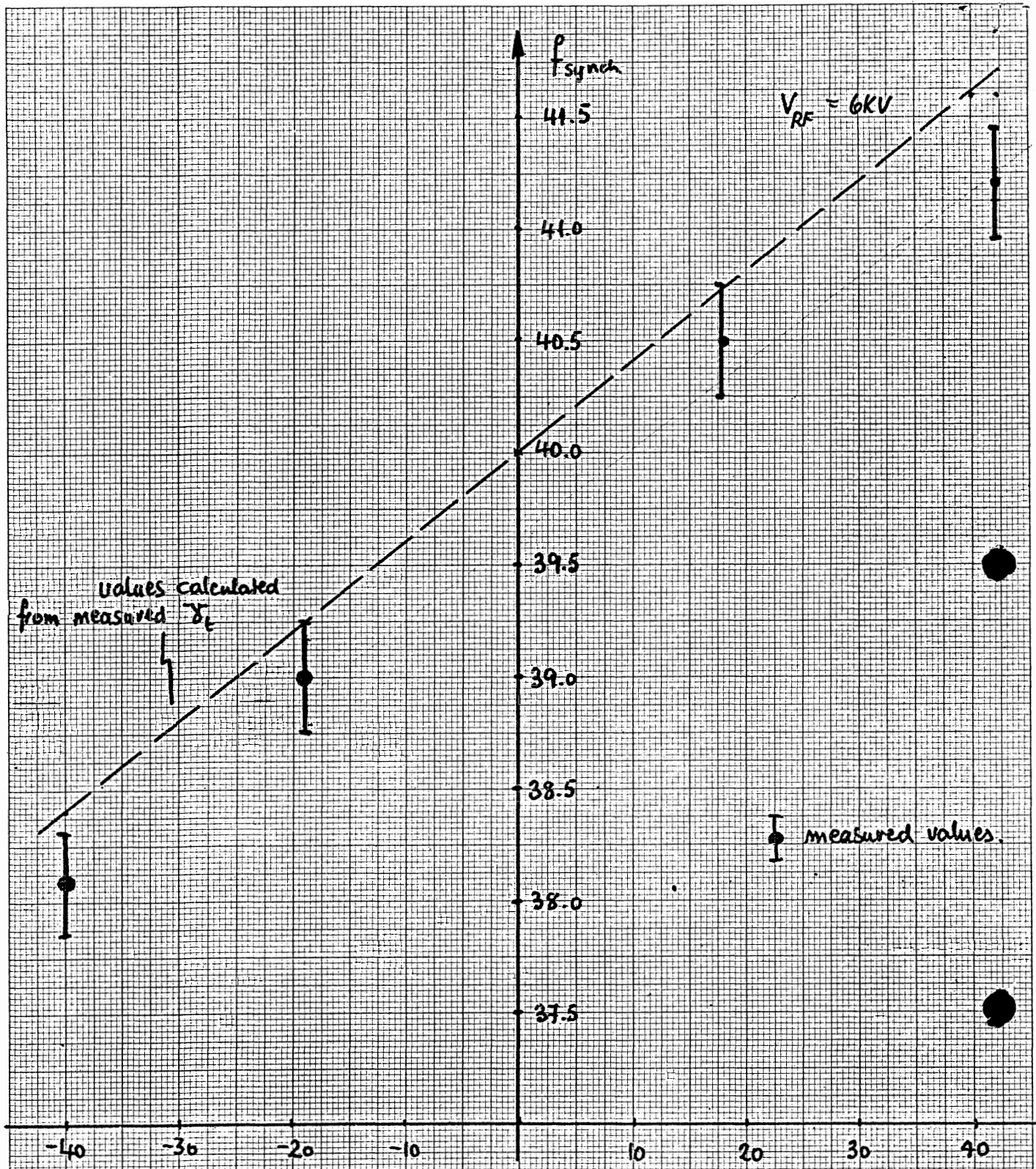


FIG 4.