ISR-TH/AH/KH/a   21.3.1974

E-6

ISR PERFORMANCE REPORT

RUN 427, 6.3.1974
RUN 430, 9.3.1974

Ring 1, 11.8 and 26 GeV

Space charge effect for bunched beams

1. Introduction

The longitudinal space charge force causes a shift of the phase oscillation frequency for bunched beams. We tried here to measure this frequency shift using two methods:

a) The first method, which has been suggested by H.G. Hereward, uses the fact that the space charge frequency shift is different for the dipole and quadrupole mode oscillation. The quadrupole mode oscillation frequency ω_2 is no longer exactly twice the frequency ω_1 of the coherent dipole mode but for a bunch with parabolic longitudinal particle distribution ¹⁾:

$$\omega_2 = 2\omega_1 + \tfrac{1}{2}\,\Delta\omega_{sc}$$

where $\Delta\omega_{sc}$ is the so-called dipole space charge frequency shift 2). For the case of a stationary bucket in the ISR, it can be calculated with the handy formula

$$\frac{\Delta\omega_{sc}}{\omega_1} = 4.07 \cdot 10^{-8}\,\frac{N\,g_o}{\gamma^2\,V_{rf}\,\theta^3}$$

with

θ = half-bunch length (at the base) expressed in RF phase angle

N = number of protons per bunch

V = RF voltage in volts

g_o = coupling coefficient (∿ 4) for the ISR vacuum chamber 3)

B) A second method to measure $\Delta\omega_{sc}$ has been suggested by F. Sacherer. The space charge frequency shift can move the coherent dipole mode frequency outside the incoherent frequency distribution of the particles in the bunch. In this case no Landau damping is present and a small wall impedance can cause an instability. By reducing the bunch current we can find the threshold of this instability where the Landau damping is about equal to the sum of the frequency shifts $\Delta\omega_{sc}$ due to space charge and $\Delta\omega_{in}$ due to the impedance. At the threshold we have approximately

$$\frac{s}{4} = |\Delta\omega_{sc}| + |\Delta\omega_{in}| \qquad \text{(see Ref. 4)}$$

where s is the phase oscillation frequency spread in the bunch. The $\Delta\omega_{in}$ at the threshold can be estimated from a measurement of the risetime τ of the instability at a current I_1 which is far above the threshold current I_{th}. The $|\Delta\omega_{in}|$ at threshold is then about

$$|\Delta\omega_{in}| \sim \frac{1}{\tau}\,\frac{I_{th}}{I_1}$$

This is of course not quite correct since the instability can also cause a real frequency shift which should be included above but is not known. However, at the threshold these frequency shifts caused by the impedance are probably much smaller than $\Delta\omega_{sc}$, and their exact knowledge is not important.

2. Experiment A, comparing the phase oscillation frequency of two modes

With the parameters used: $E = 11.8$ GeV, $I \sim 68$ mA, $V_{rf} = 16$ kV and bunchlength = 16.2 ns, we expect $\Delta\omega_{sc} \sim 2\pi \cdot 2.4\ s^{-1}$. To get quadrupole oscillations, no matching was used. By setting the RF frequency slightly wrong, dipole oscillations were obtained. S. Hansen pointed out to us that this last procedure also provided some acceleration of the beam to a position of ∿ - 10 mm. After ∿ 200 ms the RF phase lock was turned off and the bunches could execute free phase oscillations. Most of the time bunch 1 was chosen for observation because it should be least affected by beam induced wake fields. The bunch was displayed on the "mountain range" display. The scope was triggered every revolution such that many sweeps where superimposed on the same trace before one jumped to the next trace. In this way one obtained a superposition of several phase oscillations on each trace. Depending on the phase relation between the quadrupole and the dipole mode oscillation, this superposition looks different. If these two frequencies are not different by an exact factor of 2, this picture changes periodically with a frequency f_{sp} (Fig. 1):

$$f_{sp} = \frac{1}{2\pi} \left|(\omega_2 - 2\omega_1)\right| = \frac{1}{2} \left|\frac{\Delta\omega sc}{2\pi}\right.$$

From many such pictures we found an average experimental value for the space charge frequency shift

$$\Delta\omega_{sc} = 2\pi \cdot 10 \ s^{-1}$$

This value is about a factor of 4.2 times larger than expected.

3. Experiment B, measurement of the instability threshold

This experiment was done during Run 430 with a beam of 26 GeV. For a current of ∿ 80 mA and a bunchlength of 16.2 ns we expect a theoretical space charge frequency shift $\Delta\omega \sim 2\pi\ 0.55$.

The rise of the well-known dipole mode instability was observed on the "mountain range" display. Again many sweeps were used on the same trace which allowed to watch the development of the instability of a single pulse for a rather long time; Fig. 2 and 3. Smaller currents were obtained by scraping vertically. The measured rate of rise $\Delta\omega_{in}$ as a function of beam current is shown on Fig. 4 a). The crosses on the baseline indicate measurements where no instability was observed on the picture. This does not necessarily mean that no instability occurred; it could well have appeared later, beyond the time span covered by the picture. From Fig. 4 a) we get a threshold current I_{th} somewhere between 0 and 18 mA, depending on the interpretation of the points x which showed no instability.

From this range of possible thresholds we can at least give a lower limit for the space charge frequency shift, provided that the bunchlength is independant of the beam current. In this case

$$\Delta\omega_{sc} > 0.8 \cdot 2\pi \ s^{-1} \quad \text{for} \quad I = 80 \text{ mA}$$

This would mean that the space charge frequency shift is larger than expected.

However, there is an indication that the bunchlength was shorter for smaller currents. In Fig. 4 b) the bunchlength, as measured on a scope, is plotted against beam current. There seems to be strong dependance of the bunchlength on the beam current (obtained by scraping vertically). L. Vos pointed out to us that for smaller currents the tails of the observed bunchform disappear more and more in the noise, an effect which looks like a shortening of the bunch with decreasing current. It is not clear how much of our observation of the bunch shortening could be explained by this instrumental effect. For the moment, however, we have to consider the possibility that the bunch is really shorter for the smaller currents. In this case the space charge frequency shift (which depends strongly on the bunchlength) as well as the frequency spread S are complicated functions of the beam current. With this taken into account no conclusion about the magnitude can be drawn from this experiment.

4. Conclusions

By comparing the phase oscillation frequencies of the dipole and quadrupole modes, we derived a number for the space charge frequency shift

$$\Delta\omega_{sc}\ (\mathrm{exp}) = 2\pi \cdot 10\ s^{-1}$$

while the theory predicts a value

$$\Delta\omega_{sc}\ (\mathrm{theory}) = 2\pi \cdot 2.4\ s^{-1}$$

The experimental value found here is therefore about 4.2 times too large. It is possible that we have impedances in the ring (like the one which drives our instability) which produce real frequency shifts, different for the two modes. It is, however, difficult to imagine that they are large enough to explain our observed effect. It may be interesting to remember that already on earlier experiment 5), using the same method but done at a different energy and in the other ring, gave a number for the space charge effect which was 5 times too large. But more measurements have to be made before any conclusions can be drawn.

The second experiment, which tried to get information about the space charge frequency shift from a measurement of the instability threshold, was inconclusive. The observation that vertical scraping reduces the bunchlength, needs further investigations.

A. Hofmann K. Hübner

References

1) Note by A. Sørenssen; 1970 - see also:
F. Sacherer; Methods for Computing Bunched Beam Instabilities, CERN/SI-BR/72-5

2) H.G. Hereward; Estimates of Bunch Lengths and Longitudinal Space-Charge Forces in the CPS, MPS/DL-Int. 66-3

3) A.G. Ruggiero and V.G. Vaccaro; ISR-TH/68-33

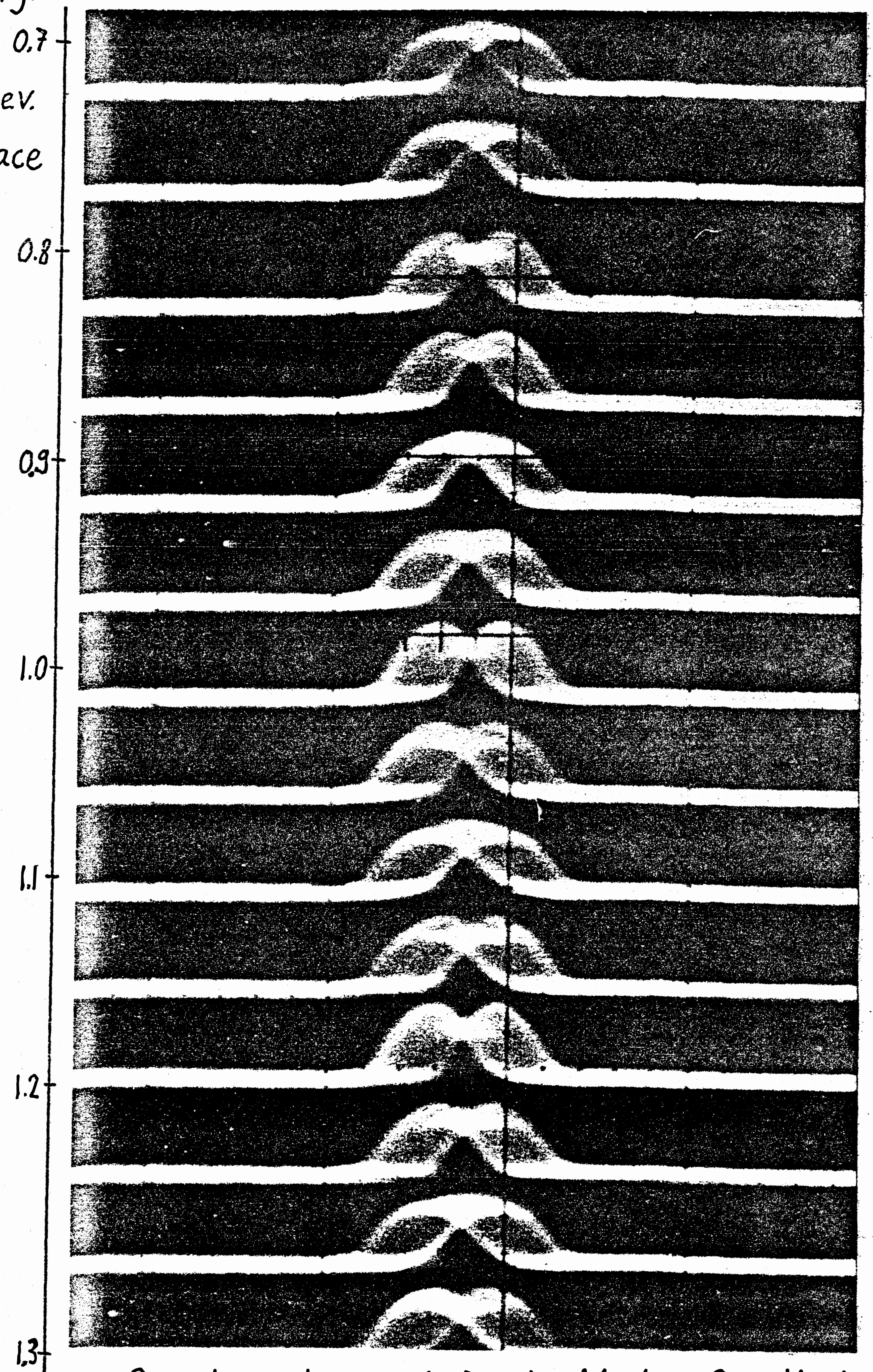
4) F. Sacherer; A Long. Stability Criterion for Bunched Beams, CERN/MPS/Int-BR/73-3

5) S. Hansen, A. Hofmann; Perf. Report, Run 373, 6.11.1973.

E = 11.8 GeV, I = 67 mA

time T
after inj.

15000 rev.
per trace



Quadrupole and Dipole Mode Oscillations

E = 26 GeV   I = 76 mA   Longitudinal Instability

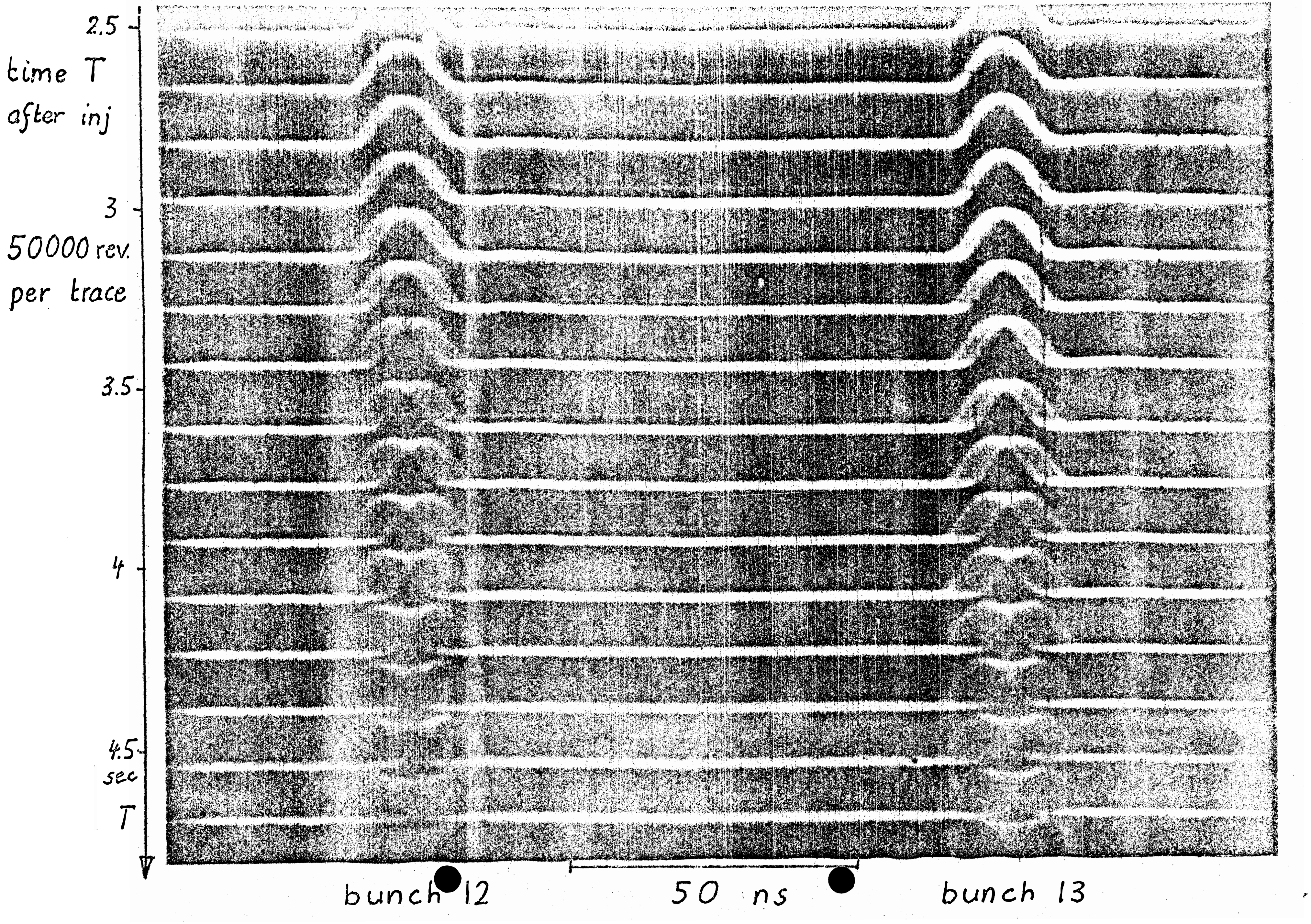
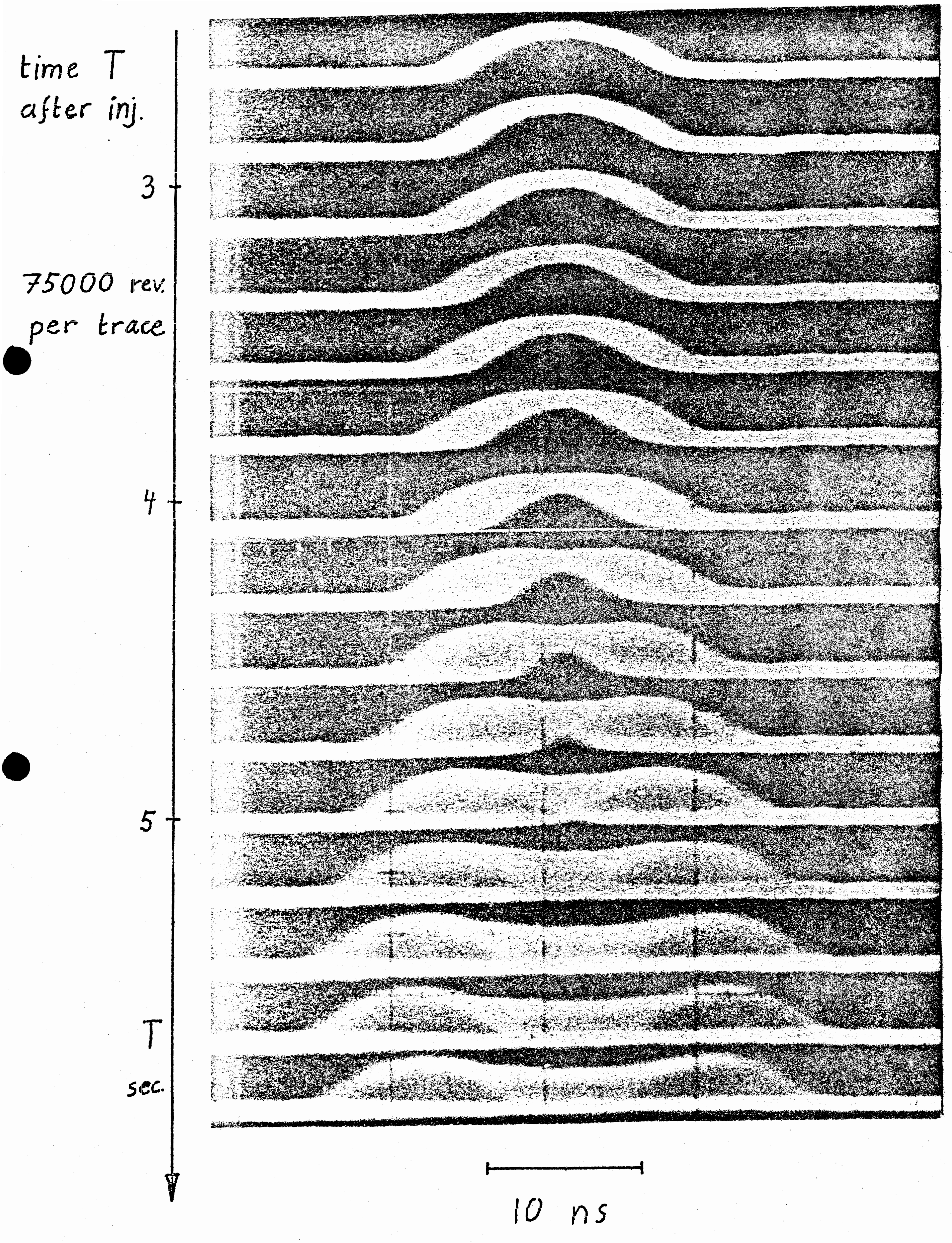


Fig. 3

E = 26 GeV, I = 60mA   Longitudinal Instability



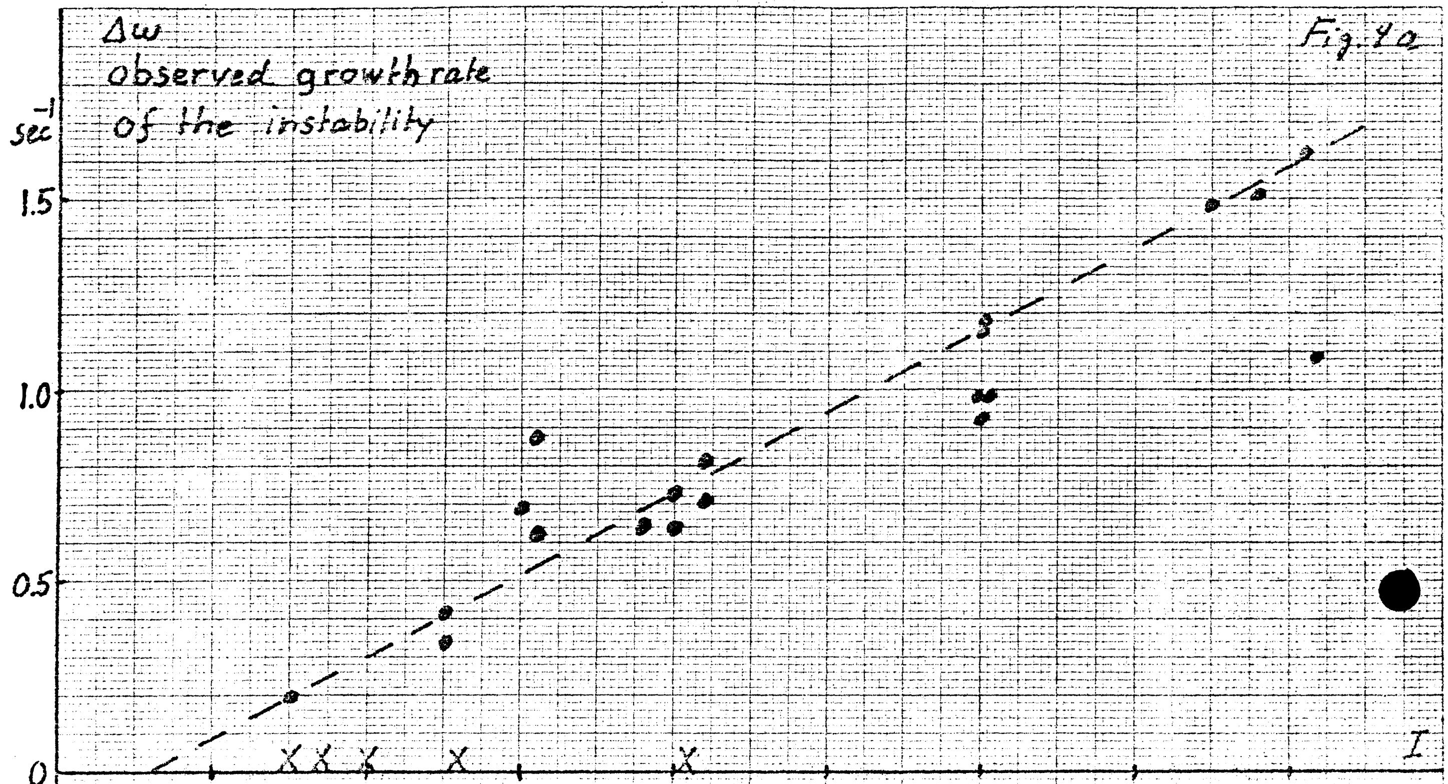


Fig. 4a

Δω
observed growthrate
of the instability
sec⁻¹
1.5
1.0
0.5
0
X X X X X
I

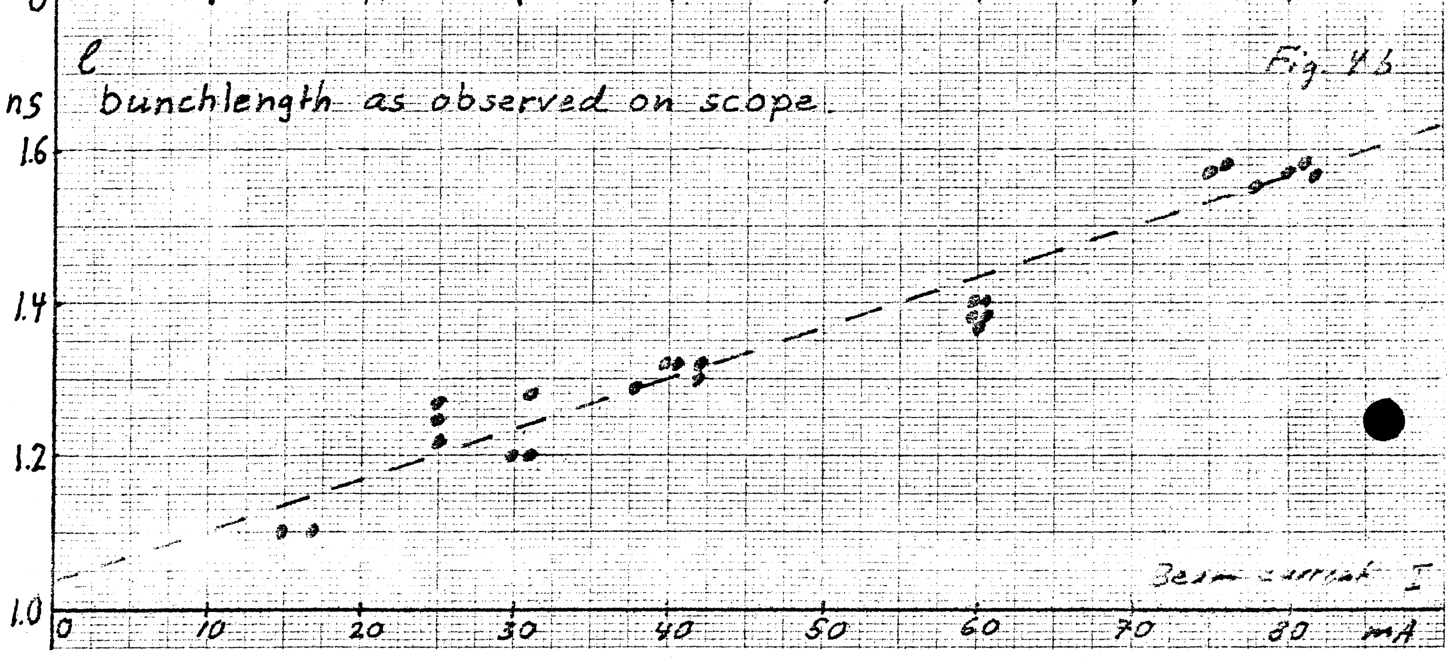


Fig. 4b

ℓ
bunchlength as observed on scope
ns
1.6
1.4
1.2
1.0
0
10
20
30
40
50
60
70
80
mA
Beam current I