ISR-TH/EK/1s 8th March, 1971

ISR RUNNING-IN

Beam loss by intra-beam scattering

1. Coulomb scattering of protons within the same beam results in small changes of the betatron amplitude and of the momentum spread of the stacked beam. This phenomenon is a generalisation of the well known Touschek effect in electron storage rings. Its effect on the overall dimensions of the ISR beam was calculated by Pellegrini (LNF-68/1) and found to be harmless.

Here we want to invoke this effect in a different way as a possible explanation of fairly slow beam losses. We assume that we have a stack sitting fairly close to a resonance which we take to be a perfect sink for protons. We want to calculate the distance ℓ between the stack and the resonance which explains experimentally observed beam losses. This is shown in Fig. 1.

2. According to Pellegrini the fastest intra-beam scattering process is a change of momentum. The rate of this is

$$\frac{\partial}{\partial t} \langle\delta^2\rangle = \frac{\pi^{\frac{1}{2}} N\, r_o^2 c}{V \langle x'^2\rangle^{\frac{1}{2}} \gamma^3 \beta^3} F(\lambda_e) \qquad (1)$$

where $\delta = \Delta p/p$, r_o is the classical proton radius, V is the volume of the beam, $\langle x'^2\rangle^{\frac{1}{2}}$ is the rms betatron angle, and $F(\lambda_e)$ is a numerical factor which can be taken as 65 for a wide range of values of λ_e.

In the ISR part of the beam size comes from betatron oscillations and part from the momentum spread. We take V to be the betatron volume, and assume that on average particles with a momentum difference corresponding to one horizontal beam radius can be scattered on each other. This determines N.

We take the rms beam sizes from the emittance measured by B.W. Montague in Run 23, and find $<x^2>^{\frac{1}{2}} = 4$ mm, $<z^2>^{\frac{1}{2}} = 3$ mm.

The beam radius is then 8 mm, which corresponds to 0.5 % momentum error. Since the current one can stack is typically 3.6 A in 2 %, we find for the effective current 0.9 A or $N = 1.8 \times 10^{13}$. The betatron angle is $<x'^2>^{\frac{1}{2}} = <x^2>^{\frac{1}{2}} Q/R = 2.3 \times 10^{-4}$.

Putting all this into (1) yields with $\gamma = 15$, $V = 2\pi R \cdot 2 <x^2>^{\frac{1}{2}}$ $2 <z^2>^{\frac{1}{2}} = 0.048$ m^3

$$\frac{\partial}{\partial t} <\delta^2> = 0.375 \times 10^{-10} \text{ s}^{-1} \tag{2}$$

3. The behaviour of (1) - a constant time derivative of a mean square quantity - is typical for a diffusion equation.* We therefore try to describe this process by a diffusion equation of the form

$$\frac{\partial V}{\partial t} = a^2 \frac{\partial^2 V}{\partial x^2} \tag{3}$$

where V is a distribution function. If we start off with a δ-function at $x = \ell$ at $t = 0$, (3) has the normalised solution :

$$V = \frac{1}{2a\sqrt{\pi t}} \left[\exp - \frac{(x - \ell)^2}{4a^2 t} \right] \tag{4}$$

and we find that

$$<(x - \ell)^2> = \int_{-\infty}^{+\infty} (x - \ell)^2 \, V(x) \, dx \tag{5}$$

becomes

$$<(x - \ell)^2> = 2a^2 t \tag{6}$$

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* I understand that B.W. Montague has a properly derived differential equation for the distribution function in the presence of intra-beam scattering. This might be used instead of (3) which is obtained by intuition.

Or

$$\frac{\partial}{\partial t} \langle (x - \ell)^2 \rangle = 2a^2 \qquad (7)$$

This last equation gives the relation between the diffusion constant a^2 and (1).

4. A solution of (3) with a resonance at $x = 0$ and the beam starting as a δ-function at $x = \ell$ is obtained by imposing the boundary condition $V = 0$ at $x = 0$ for all t. This is achieved by writing

$$V(x) = \frac{1}{2a\sqrt{\pi t}} \left\{ \exp\left(- \frac{(x - \ell)^2}{4a^2 t} \right) - \exp\left(- \frac{(x + \ell)^2}{4a^2 t} \right) \right\} \qquad (8)$$

The fraction A of particles which survive a certain time t is given by

$$A = \int_0^\infty V(x)\, dx = \mathrm{erf}\, (\ell/2a\sqrt{t}) \qquad (9)$$

The rate of particle loss is

$$\dot{A} = - \frac{1}{\sqrt{\pi}} \left(\frac{\ell}{2a\, t^{3/2}} \right) \exp\left(- \frac{\ell^2}{4a^2 t} \right) \qquad (10)$$

Introducing the "characteristic time" of the diffusion process $\tau = 4a^2 t/\ell^2$ gives the universal loss rate $dA/d\tau$ shown in Fig. 2 :

$$\frac{dA}{d\tau} = - \frac{1}{\sqrt{\pi}}\, \tau^{-3/2} \exp\,(-1/\tau) \qquad (11)$$

This expression has a maximum at $\tau = 2/3$ with the value $dA/d\tau = -0.23$, and $A^{-1}\, dA/d\tau = -0.25$.

5. In the ISR, the observed decay rates are of the order of $\dot{I}/I = -10^{-6} \ldots -10^{-5}\ s^{-1}$. Comparing this to the decay rate $A^{-1}\, dA/d\tau = -0.25$ leads to the conclusion that $a^2/\ell^2 = 10^{-6} \ldots 10^{-5}$. From (2) and (7) we know that $a^2 = 0.187 \times 10^{-10}\ s^{-1}$. Hence we conclude that ℓ must fall within the limits

$$\ell = (1.4 - 4.3)\ 10^{-3} \qquad (12)$$

This means that in order to explain the observed beam losses in the ISR by intra-beam scattering into a fatal resonance we need one such resonance at about 2.6 ... 8.1 mm distance in average radius from the stack; the average distance between adjacent fatal resonances must therefore be between 0.5 and 1.5 cm. This does not seem at all unlikely.

It should be noticed that this effect has a fairly strong γ dependence which might be used for experimental verification. Also making rather dilute 4-bunch stacks produces less intra-beam scattering and smaller loss rates as observed in practice.

E. Keil

Distribution

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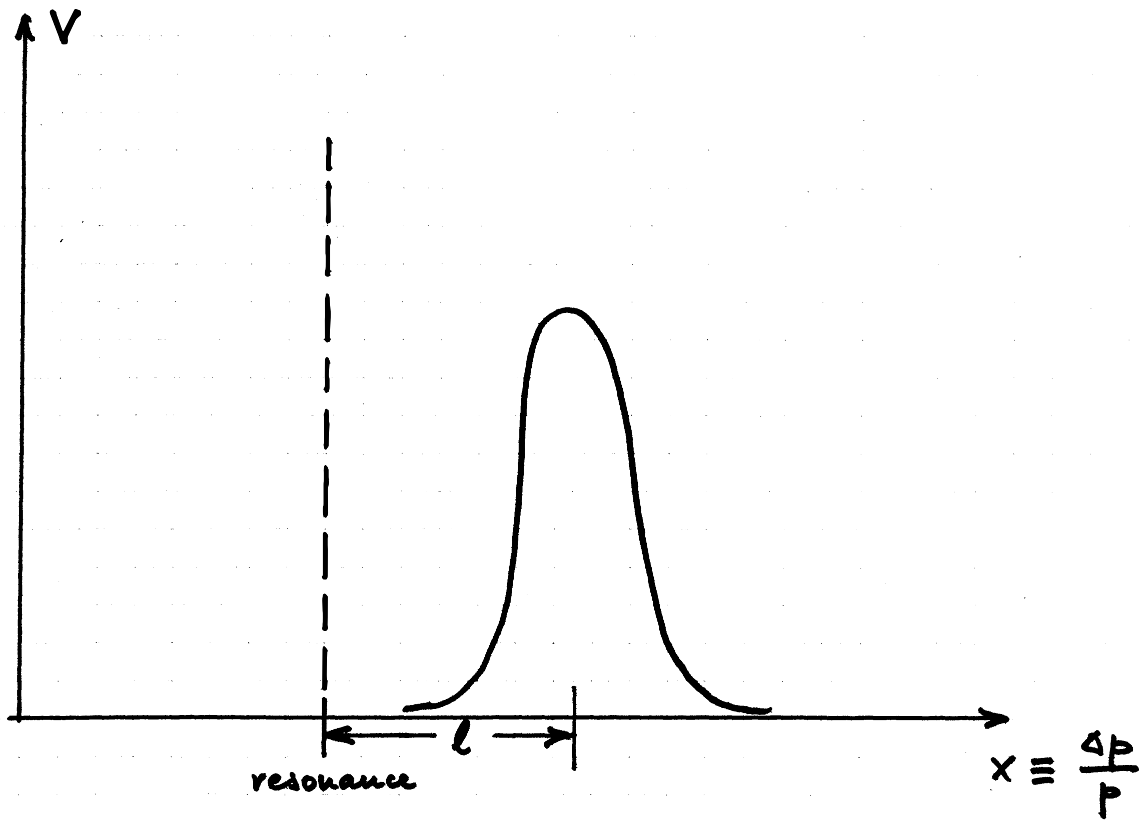

Figure 1

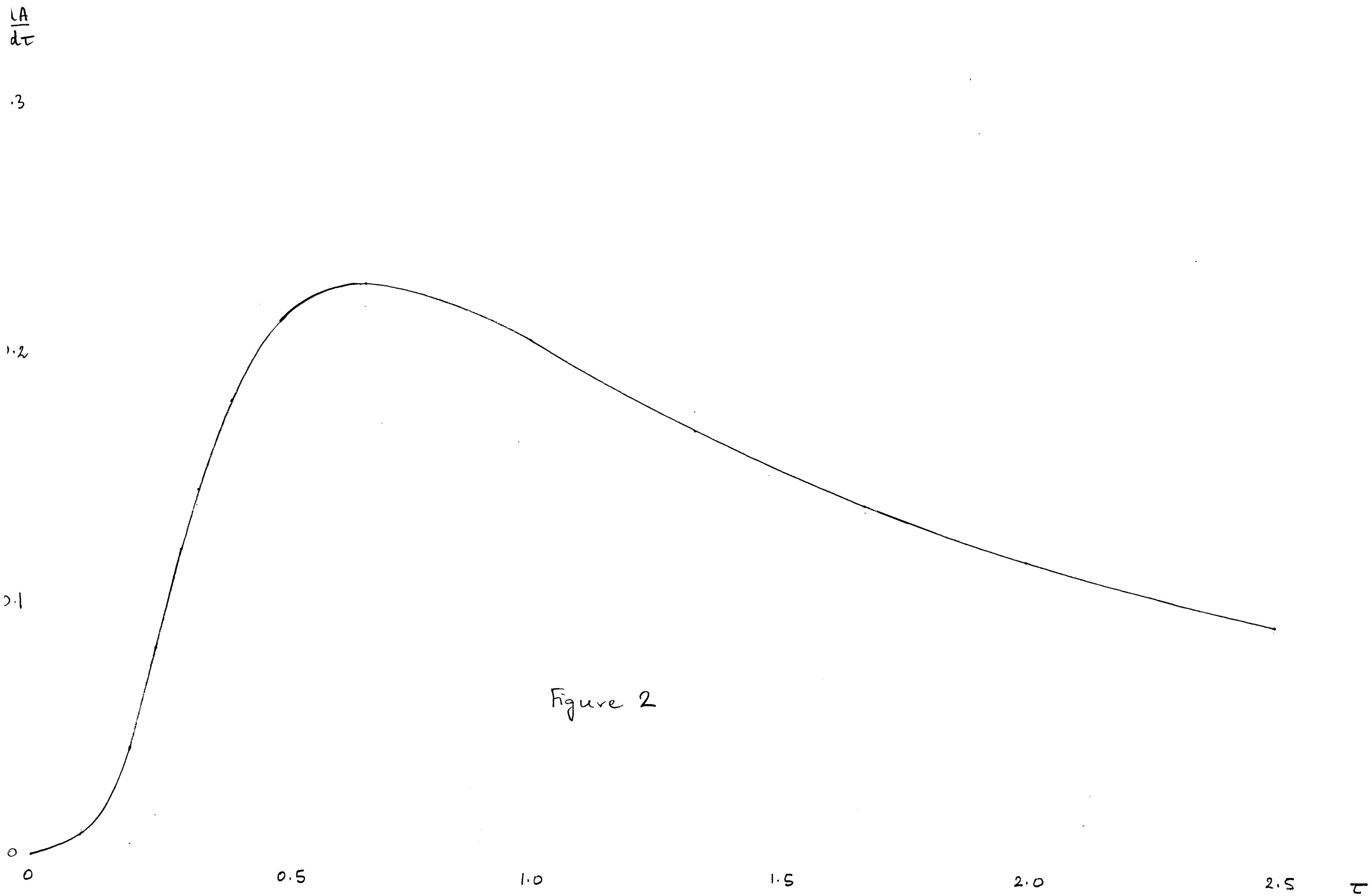

Figure 2