ISR-TH/EK/ls 17th May, 1971

ISR RUNNING-IN

Beam loss by intra-beam scattering II

1. The rate of change of the mean square momentum error was calculated in my memorandum dated 8th March, 1971. It is, with the assumptions made there :

$$\frac{d}{dt} \left\langle \frac{\Delta p^2}{p} \right\rangle = 3.75 \times 10^{-11} s^{-1} \qquad (1)$$

2. We now solve the diffusion equation

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t} \qquad (2)$$

with a new set of initial conditions. We assume that for t = 0, $V = {}^1/d$ for $0 \leqslant x \leqslant d$, and that V = 0 for all $t \geqslant 0$ and $x < 0$ and $x > d$. This means that we start with a rectangular pulse and have resonances which remove all particles at either end of the stack.

The most general solution of (2) which is symmetric, with $V(x) = V(d - x)$, is

$$V(x,t) = \sum_{m=0}^{\infty} A_m \sin \frac{(2m + 1)\pi x}{d} \exp\left(- \frac{(2m + 1)^2 \pi^2 t}{d^2}\right) \qquad (3)$$

The coefficients A_m are determined by the initial conditions. With the above conditions we find :

$$V(x,t) = \sum_{m=0}^{\infty} \frac{4}{(2m + 1)\pi d} \sin \frac{(2m + 1)\pi x}{d} \exp\left(- \frac{(2m + 1)^2 \pi^2 t}{d^2}\right) \qquad (4)$$

This function is shown in Fig. 1 for various normalised times t/d^2.

The fraction of surviving particles is

$$A(t) = \int_0^d V(x,t)dx \qquad (5)$$

and this becomes

$$A(t) = \sum_{m=0}^{\infty} \frac{8}{(2m + 1)^2 \pi^2} \exp(- \frac{(2m + 1)^2 \pi^2 t}{d^2}) \qquad (6)$$

A(t) is shown in Fig. 2.

For large t only the m = 0 term remains and we find approximately

$$A(t) \approx \frac{8}{\pi^2} \exp (-\pi^2 t/d^2) \qquad (7)$$

This corresponds to an exponential decay with a time constant

$$T = d^2/\pi^2 \qquad (8)$$

The asymptotic loss rate becomes

$$\dot{A}/A = - \pi^2/d^2 \qquad (9)$$

Since we are interested in the solution of the equation

$$a^2 \frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t} \qquad (10)$$

we have to introduce a change in time scale, and obtain for the loss rate

$$\dot{A}/A = - a^2 \pi^2/d^2 \qquad (11)$$

from which we can calculate the distance between resonances which would explain observed loss rates. We find with a^2 being one half of the figure given in (1) :

$\dot{A}/A$ [mm^{-1}]	$- 10^{-3}$	$- 10^{-4}$	$- 10^{-5}$
d	3.3×10^{-3}	1.1×10^{-2}	3.3×10^{-2}
<r> [cm]	0.61	2.0	6.1

These distances between resonances are much more plausible than those given earlier on.

The normalised times t shown in Fig. 1 and 2 correspond to the real times in minutes shown in the table below, for various decay rates $\dot{A}/A$:

	$\dot{A}/A$ [min^{-1}]		
t	10^{-3}	10^{-4}	10^{-5}
0.001	0.17	1.7	16.7
0.002	0.33	3.3	33.3
0.005	0.83	8.3	83.3
0.01	1.7	16.7	166.7
0.02	3.3	33.3	333.3
0.05	8.3	83.3	833.3
0.1	16.7	166.7	1666.7

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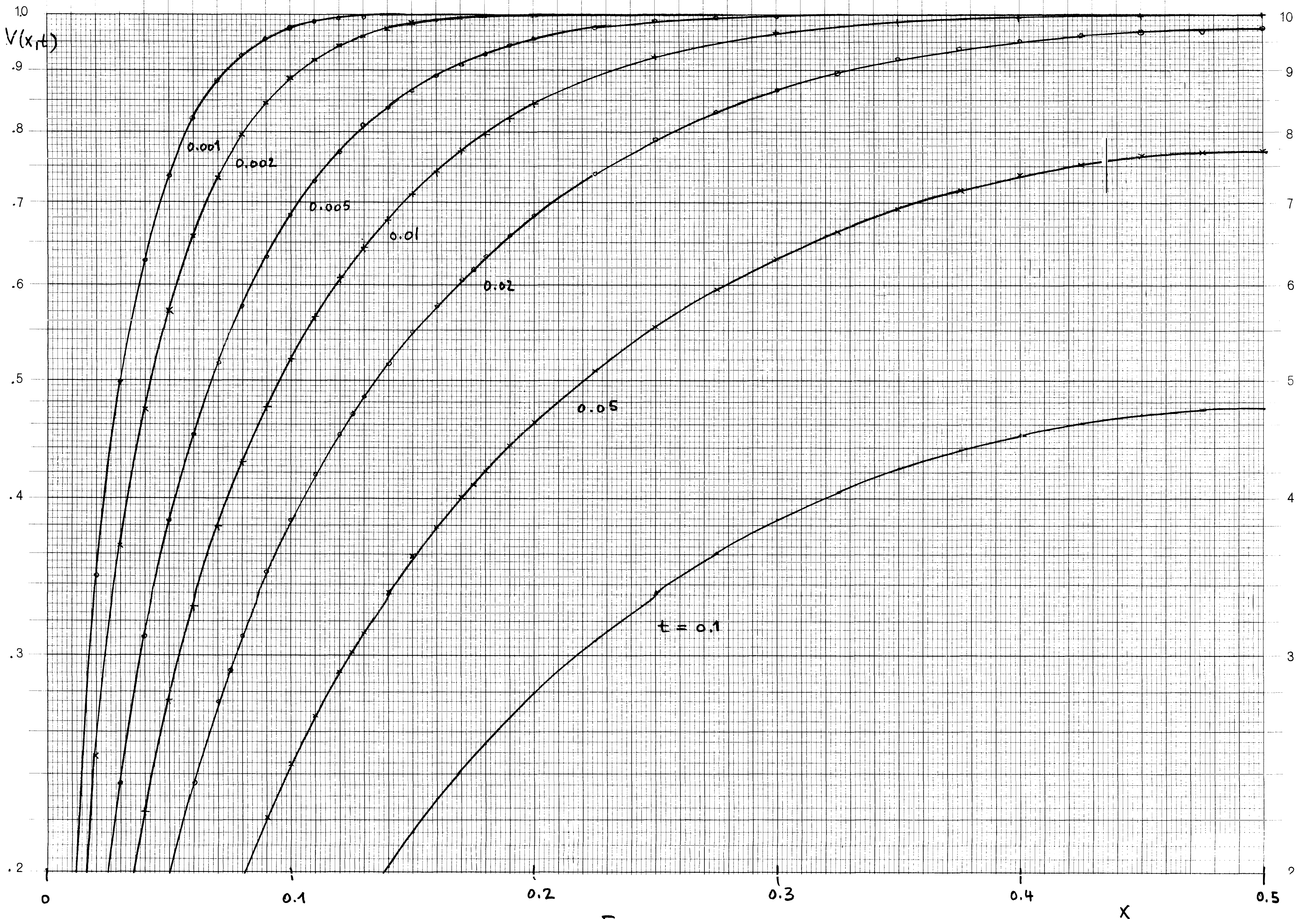

Fig. 1

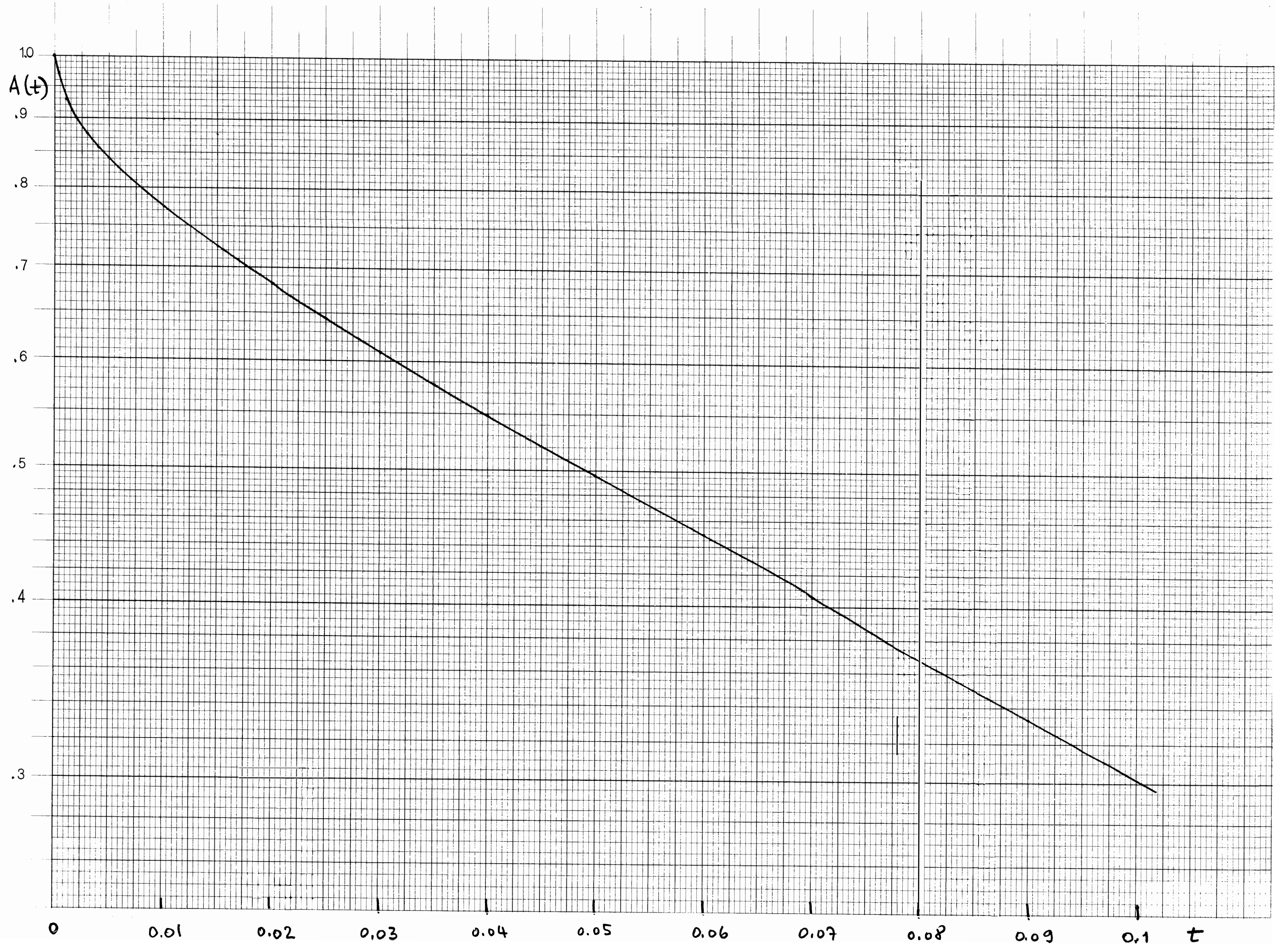

Fig. 2