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SHAPE OF THE TRANSVERSE MOMENTUM DISTRIBUTION OF SECONDARIES FROM HIGH-ENERGY INELASTIC INTERACTIONS

by

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A B S T R A C T

The shape of the transverse momentum distribution of secondaries from 16 GeV/c π^-p interactions is analyzed in terms of a shape parameter R (eq. (3)). The results show that baryons and mesons have significantly different shapes of the transverse momentum distributions, the former favouring a gaussian distribution while the latter are well fitted by a superposition of two such distributions. They also exclude the CKP (i.e. linear exponential distribution) as a good fit to experimental data.

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1. It has become usual to describe the distribution of transverse momenta p_T of any secondaries from high-energy inelastic interactions by means of the linear exponential or CKP law[1)]

$$f_1(p_T)dp_T = \frac{p_T dp_T \; e^{-p_T/p_o}}{p_o^2} \qquad (1)$$

which written as $d\sigma/d\omega$ yields an exponential differential law.

The "universality" of this law has been used by Hagedorn[2)] as an argument for a "highest temperature" of "boiling hadronic matter".

Recently Cocconi[3)] has put forward the interesting hypothesis that the CKP shape may be connected with nucleon structure, the exponential law appearing as a superposition of a large number of -- suitably weighted -- gaussian distributions. In view of these and other theoretical implications it appears highly desirable to ascertain to what extent -- if any -- the CKP distribution really gives a good fit to the experimental data from inelastic interactions or represents just a rough approximation. As will be shown in the following the former situation appears extremely improbable.

In a previous paper[4)] it has been shown that the experimental data collected from a world survey of empirical p_T values of secondary baryons favour a quadratic exponential law

$$f_2(p_T)dp_T = \frac{p_T dp_T}{\sigma^2} \, e^{-p_T^2/2\sigma^2} \qquad (2)$$

which, again written as $d\sigma/d\omega$ is a gaussian in p_T.

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The pion p_T distributions, however, could be best described by a superposition of two such distributions with parameters varying in such a way as to keep the average transverse momentum almost constant. At that time, lack of statistics and detailed data prevented this type of analysis to be carried out at fixed energy and pion multiplicity.

It might have been argued that such a mixture of data from many different channels may yield a distorted picture of the p_T behaviour.

In the present paper a fixed type of interaction ($\pi^- p$) is studied with respect to the p_T distribution at fixed energy (16 GeV) and at various fixed total pion multiplicities ($2 \div 11$).

As will be seen, even in such severely selected samples, the global features observed in [4] (viz. one gaussian for baryons, superposition of two gaussians for pions) still persist.

2. In most papers on this subject attention is given mainly to the mean value of the transverse momentum. Hereafter we shall be concerned only with the shape of the p_T distribution.

In order to be able to compare different reaction channels we shall make use[4] of an adimensional "shape parameter" R, defined as the ratio of the root mean square deviation σp_T of the p_T values to their expectation value $< p_T >$:

$$R \equiv \frac{\left[< p_T^2 > - < p_T >^2\right]^{\frac{1}{2}}}{< p_T >} \qquad (3)$$

For a number of important distribution laws, R has the useful property of being independent of the mean transverse momentum. Several examples:

a) two-body isotropic decay of a particle ($\delta(p-p^*)dpd\omega$)

$$R = R_o \equiv \sqrt{\frac{32}{3\pi^2} - 1} \simeq 0.281 \qquad (4)$$

b) linear exponential law (eq. (1)), CKP distribution):

$$R = R_1 \equiv \frac{1}{\sqrt{2}} = 0.707 \tag{5}$$

c) quadratic exponential law (eq. (2))

$$R = R_2 \equiv \sqrt{\frac{4-\pi}{\pi}} = 0.523 \tag{6}$$

Thus, for instance if the CKP distribution describes the p_T distributions of all kinds of particles but[5)], the mean $< p_T >$ increases with the mass of the secondaries, baryons and pions should still yield the same value R1 of R given by eq. (5).

Table 1 shows estimates of R for the p_T distribution of secondary nucleons, π^-, π^+ and π^0 from 16 GeV/c $\pi\bar{p}$ interactions measured in the CERN TC Division at pion multiplicities ranging from two to eleven.

Fig. 1 shows a plot of R vs pion multiplicity for secondary baryons (open circles) and the weighted mean of all pions (full dots). It is seen that the pion values lie systematically above the baryon ones.

The weighted mean values of R for each class of particles are given in Table II. They are all in significant disagreement with the CKP distribution which happens to fit only isolated examples (e.g. π^0 at $n\pi = 2$ or $\pi+$ at $n\pi = 3$. The mean value of R for the baryons is, if anything, in better agreement with the value expected for a quadratic exponential (eq. (6)).

A word is here in order as to the statistical significance of the estimates quoted for R. Indeed, from a finite sample of p_T values one obtains only an estimate

$$R^* \equiv \frac{\left[\frac{1}{n}\sum_{i=1}^{n} p_{Ti}^2 - \left(\frac{1}{n}\sum_{i=1}^{n} p_{Ti}\right)^2\right]^{\frac{1}{2}}}{\frac{1}{n}\sum_{i=1}^{n} p_{Ti}} \cdot \left[\frac{n}{n-1}\right]^{\frac{1}{2}} \tag{7}$$

Obviously the denominator and the numerator of this expression are statistically correlated.

A Monte-Carlo calculation has been performed in which R^* values were generated from samples of different size n drawn from populations described by eq. (1), as well as by eq. (2). For sample sizes n exceeding 25, the estimate R* turned out to be practically unbiased, fluctuating about R in a practically gaussian manner with a standard deviation

$$\sigma_{R*} \simeq \frac{0.42}{\sqrt{n}} \qquad (8)$$

for both distributions (eq. (1) and (2)).

The second column in Table II gives the χ^2 values for consistency with a single weighted mean for R values estimated from different multiplicities and/or types of secondaries. Obviously there can be no question of a single shape of the R distribution for all secondaries. Baryons contrast violently with pions. Even the agreement for all pions with the assumption of a single shape is rather poor. However, taken apart the different charge states of the pion do not contradict too badly a single shape. This is especially true of the negative pions. In all cases this shape is significantly different from the CKP distribution. It can, however, always be well fitted with a superposition of two quadratic exponentials. The quality of such a fit is illustrated by the example of Fig. 2*) which shows the integral distribution of p_T plotted as log F (> p_T) vs. p_T^2 . For a quadratic exponential (eq. (2)) this plot should be a straight line. The data refer to

a) protons from $\pi^- p \rightarrow p 2\pi^+ 3\pi^- \pi^0$ and

b) negative pions from the same reaction.

Three important features are obvious viz.:

a) the good agreement of the proton values with the quadratic exponential,
b) the good fit of a decomposition of the π^- distribution into two straight lines and
c) the parallelism of the high p_T "tail" of the π^- distribution to the proton

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*) Very similar results to be described in a forthcoming paper, have been obtained from other $\pi^- p$ reaction channels at the same energy and other energies, as well as from other reactions such as $\bar{p}p$ annihilations.

line. Linear extrapolation of this tail to $p_T = 0$ shows that $\sim \frac{1}{3}$ of the π^- have this behaviour.

Since there were three π^- present in the final state, these results strongly suggest that the gaussian character in the nucleon and pion distributions has a mixed origin. While in the nucleon case the exp $(-p_T^2)$ law is simply the reflection of the exponential t dependence, the pion distributions appear as a mixture of pionization productions and of leading pions which reproduce to a great extent the baryon features.

3. The above analysis, as well as similar results obtained from other reaction channels and at other energies[4)], show convincingly that the transverse momentum distributions cannot be described by a single, simple type of equation.

However, interpretation of this distribution in terms of nucleon structure remains a tempting idea. The high degree of accuracy of presently available data appears to open the possibility of refinements of this kind of approach.

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REFERENCES

1) G. Cocconi, A. Koester, D.H. Perkins, Lawrence Radiation Laboratory, UCID-1444 (1961).

2) R. Hagedorn, Nuovo Cimento, 56A, 1027 (1968) and previous papers quoted within.

3) G. Cocconi, CERN NP Internal Report 68-17 (1968).

4) E.M. Friedländer, Nuovo Cimento 41, 417 (1966).

5) A. Bigi, S. Brandt, A. de Marco Trabucco, Ch. Peyrou, R. Sosnowski, H.B. Wroblewsky, Nuovo Cimento 33, 1249 (1967).

FIGURE CAPTIONS

Fig. 1 Estimates of R (eq. (3)) at various multiplicities for different secondaries from 16 GeV/c π^--p collisions.

a) Secondary baryons, open circles.

b) weighted mean for all kinds of pions, full circles.

Fig. 2 Plot of log F (> p_T) vs. p_T^2 for secondaries from $\pi^- p \rightarrow p 2\pi^+ 3\pi^- \pi^0$.

a) Open circles: protons.

b) Full circles: π^-; the number of events has been decreased 10 times.

c) rectangles: initial part of the pion distribution after subtraction of "high-p_T" tail, again decreased by a factor of 10.

TABLE I

Values of R (eq. 3) (multiplied by 10^3)

type of secondary / Pion multiplicity	Baryon	π^-	π^+	π^o	all π (weighted)
2	519 ± 17	582 ± 17	-	695 ± 17	636 ± 12
3	612 ± 25	631 ± 25	695 ± 18	-	676 ± 15
4	-	614 ± 21	-	642 ± 30	623 ± 17
5	600 ± 16	640 ± 9	626 ± 11	-	634 ± 7
6	564 ± 33	610 ± 19	638 ± 24	685 ± 33	615 ± 14
7	604 ± 90	650 ± 41	559 ± 47	-	615 ± 31
8	495 ± 48	616 ± 24	612 ± 26	502 ± 63	611 ± 17
9	-	-	-	-	-
10	-	762 ± 92	762 ± 94	-	760 ± 66
11	552 ± 79	627 ± 49	589 ± 49	921 ± 104	650 ± 35

TABLE II

Type of secondary	Mean value of R	Deviation of χ^2 from its expectation value in terms of its standard deviation
All secondaries	.609 ± .004	11.2
All pions	.631 ± .005	6.8
Baryons	.564 ± .006	1.56
Negative pions	.611 ± .006	2.04
Positive pions	.631 ± .008	2.75
Neutral pions	.671 ± .015	3.3

Expected R value for CKP : .707

Expected R value for eq. (2) : .523

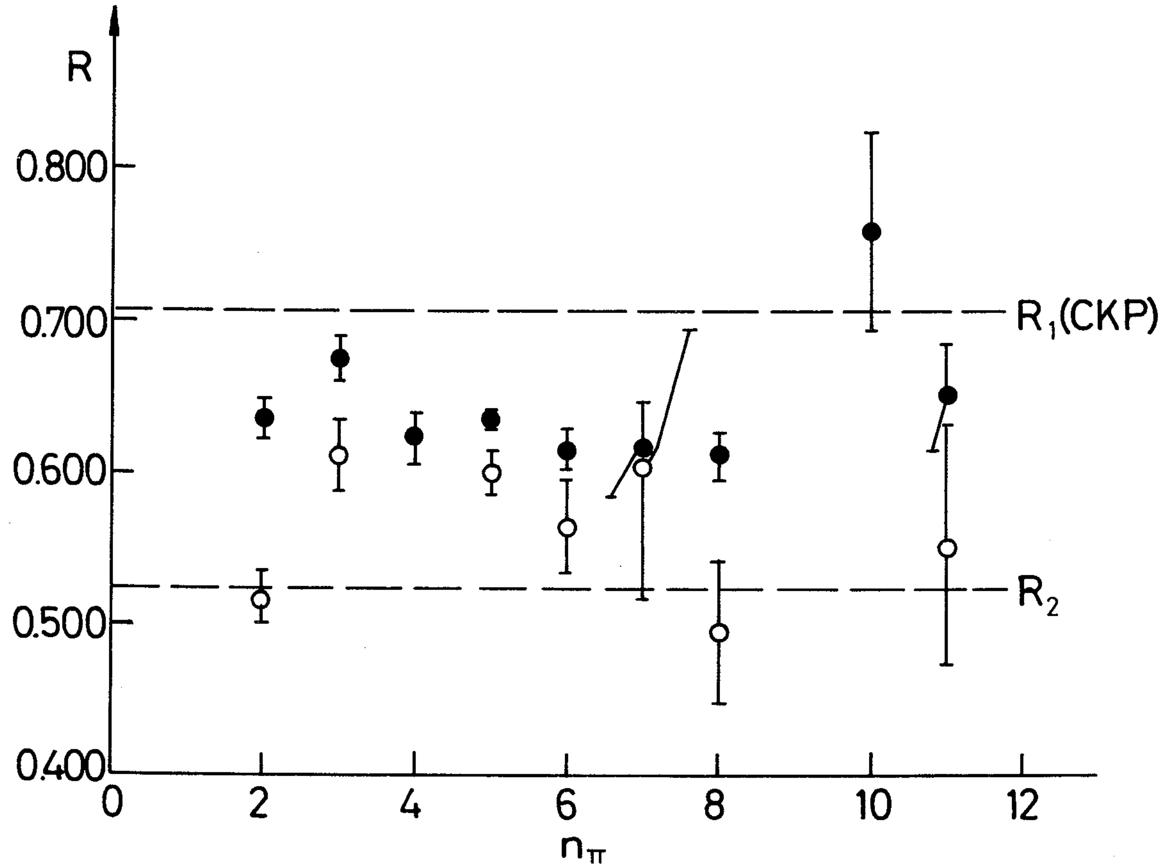
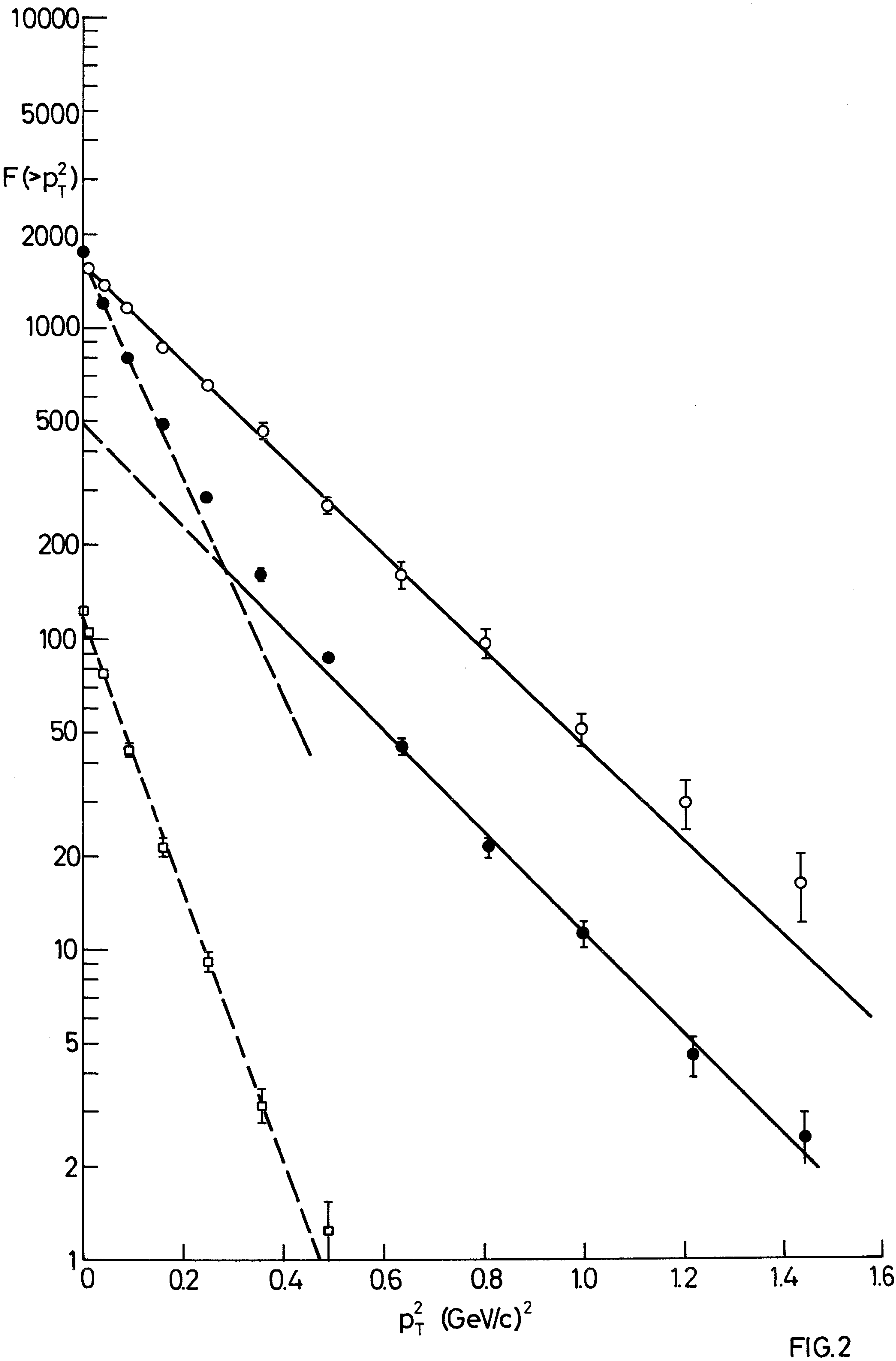


FIG. 1



10000
5000
F(>p_T^2)
2000
1000
500
200
100
50
20
10
5
2
1
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
p_T^2 (GeV/c)^2

FIG.2