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A RING CYCLOTRON KAON FACTORY

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Abstract

A two stage isochronous ring cyclotron is proposed for accelerating a 100 μA proton beam to 8.5 GeV. The first stage of 15 sectors and 10 m radius would take a 450 MeV beam from TRIUMF to 3 GeV, the acceleration being completed by a second stage of 30 sectors and 20 m radius. Superconducting magnets would be used, the weight of steel being estimated to be 2000 m tons for the first stage and 1800 m tons for the second. Numerical orbit tracking through simulated magnetic fields has confirmed that the focusing properties of the design are satisfactory and has emphasized the importance of using small pole-gaps to prevent fringing field effects weakening the edge focusing. Steel is provided outside the coils on the focusing edge to help keep it hard and increase the flutter. The accelerating system consists of SIN-style cavities, with flat-topping provided by operating some at the second harmonic (first stage) or third harmonic (second stage). The phase compression effect is also utilized to allow higher fundamental frequencies to be used on successive stages.

Introduction

The possibility of building an accelerator to provide several hundred times the currents (≃0.3 μA) available from present accelerators in the GeV range is a challenging one. Such a machine could open up the whole field of kaon physics in the same way that the meson factories have done for pion physics. The threshold for kaon production is ≃1.1 GeV, but to produce intense and clean beams higher energies are needed. Berley[1] has analysed various kaon beams, showing that their intensities rise very strongly with incident proton energy at first, but flatten off at ~7 GeV for K^+ and ~9 GeV for K^-. A high intensity accelerator operating near these energies could produce not only intense secondary beams of kaons (themselves a source of Σ- and Λ- hyperons) but also non-strange beams of protons, neutrons, pions, muons and neutrinos. A wide range of problems in nuclear and particle physics could be studied (a useful reference for kaon physics is the 1976 Brookhaven Summer Study on Kaon Physics). Some topics of particular interest are

1) K^--N interaction - the phase shift analysis is "still very confused" and unconfirmed resonances abound.
2) K^+-nucleus scattering - the K^+ is the only strongly interacting probe weakly absorbed by the nucleus.
3) Hypernuclei - intense beams permit counter experiments and spectroscopy of hypernuclear excited states.
4) (K^-,K^+) reactions - could give stable baryonic systems with strangeness -2 such as $(\Lambda\Sigma^-)$ and $^6He_{\Lambda\Lambda}$.
5) Kaonic and hyperonic atom X-rays - give information on K^- and Σ^--nucleus interactions.
6) K, Σ, Λ-decays - several possible channels involving neutral currents have yet to be observed.
7) Neutrino-induced reactions at several hundred MeV.
8) K^o regeneration on nuclei.
9) p-p scattering - polarized beam would allow spin-dependent effects to be explored.
10) π-N and π-nuclear scattering and reactions.
11) Electromagnetic breeding of nuclear fuel by spallation neutrons - a possible practical application.

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With 100 μA currents already available, the meson factories are in an unrivalled position to act as injectors to a future generation of high-current accelerators in the GeV range. In the case of TRIUMF, with its capability for simultaneous extraction of several proton beams, the problem of splitting off a portion of the main beam for further acceleration is a particularly simple one. It should be possible to accelerate three or four hundred microamperes of H^- ions up to the onset of significant electric stripping at 450 MeV without exceeding the radiation spill limits. A 100 μA beam of good quality could readily be extracted there by inserting an additional stripping foil and dispatched to a higher energy accelerator without causing any interference with the 200-500 MeV experimental programme.

High Energy Cyclotrons

The high intensities achieved by cyclotron meson factories are in large measure attributable to their CW operation. To match them to the (pulsed) synchrotrons conventional for accelerating protons to GeV energies poses considerable problems, one of the most vexatious being how to stack the CW beam from the cyclotron into different areas of phase space for each of many hundred successive turns in the synchrotron. We have therefore considered the possibilities of isochronous cyclotrons for accelerating protons to 8.5 GeV, an energy close to the shoulder of the cross-section for kaon production. Cyclotron designs in the GeV range have previously been considered by Sarkisyan,[2] Gordon,[3] Mackenzie[4] and Joho.[10]

The chief problems in designing a high energy cyclotron are of course the rapid rise of average field with radius ($d\bar{B}/dr \sim \beta\gamma^3$) needed to maintain isochronism, and the consequent axial defocusing. The flutter $F^2 \equiv \overline{(B/\bar{B} - 1)^2}$ and spiral angle ε needed to keep $\nu_z^2 > 0$ therefore rise drastically with energy. Using the rough approximation

$$\nu_z^2 \simeq -\beta^2\gamma^2 + F^2(1 + 2\tan^2\varepsilon) \qquad (1)$$

we see that as $\beta \to 1$ we require

$$\sqrt{2F^2}\,\tan\varepsilon \gtrsim \gamma\ . \qquad (2)$$

To avoid excessive spiral it is therefore vital to have a large flutter ($F^2 \gg 1$). In this respect the design criteria lead naturally to the choice of a ring cyclotron with separated sector magnets. The magnetic field is then restricted to the hill regions, giving in hard edge approximation[5]

$$F^2 = \ell_v/\ell_h\ , \qquad (3)$$

ℓ_v and ℓ_h being the orbit lengths in valley and hill respectively (Fig. 1). Obtaining sufficient flutter is thus dependent on arranging sufficient separation between the magnets. In the designs described below we have chosen $F^2 \simeq 2$ at maximum energy, so that for example at γ = 10 (8.5 GeV) Eq. (2) requires a spiral tan ε ≥ 5 - a value still within the bounds of practical possibility.

Separated sector machines have other important design advantages:

1) Large magnetic field-free regions between the magnets where the injection, extraction, pumping, diagnostic and acceleration systems may be located (with all the advantages of separated function design).
2) The high ℓ_v/ℓ_h ratio increases the cyclotron radius and hence the turn separation (making extraction easier) and reduces the radial derivatives of flutter, spiral

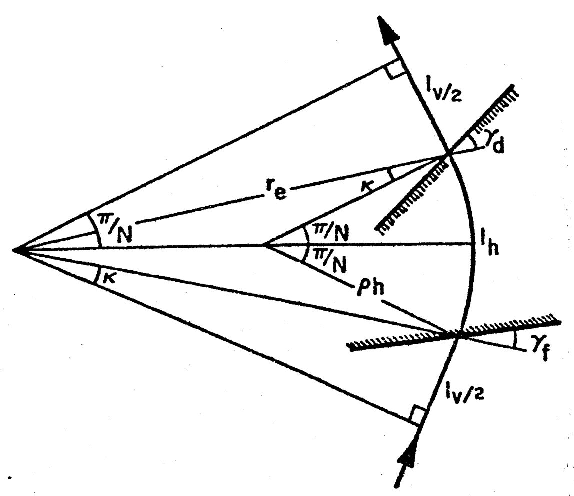

Fig. 1. Orbit geometry for one sector

and average field (making the magnets easier to construct).

Of course, wider valleys and larger radii also imply higher costs in equipment and buildings. This argument was perhaps crucial until the advent of reliable superconducting magnets over the past few years. With the factor 2 gain in hill field which these provide the machine radius required is halved and the costs drastically reduced—in the case of the magnet steel by about a factor 8. That the savings in capital costs and power bills more than offset the extra cost of refrigeration is of course the reason for the growing wave of interest in cyclotrons with superconducting magnets.[6]

Supposing then that we can obtain a hill field $B_h = 5.0$ T, how large will the machine be? In hard edge approximation, isochronism requires

$$\overline{B} = \frac{B_h}{F^2+1} = \gamma B_c \tag{4}$$

where the "central field" B_c is related to the cyclotron radius r_c and the angular frequency ω_p of the proton (charge e, mass m) by $eB_c/m = \omega_p = c/r_c$. For $\gamma = 10$ and $F^2 = 2$ Eq. (4) gives $B_c = 0.17$ T. However, our choice of B_c is not entirely free; to ensure an integral number of proton bunches per turn we must keep B_c commensurate with its value in TRIUMF, namely 0.30 T. We therefore choose to give B_c and ω_p half their TRIUMF values (i.e. 0.15 T and 2.305 MHz), making the cyclotron radius twice as large, namely 20.6 m (and raising F^2 to 2.3).

If a single machine were used to accelerate protons at 450 MeV ($\beta = 0.71$) from TRIUMF to 8.5 GeV ($\beta = 0.995$) the sector magnets would still be undesirably large—the gain in radius being ~6 m. However, by designing the machine in two stages, the lower energy one with a smaller value of r_c, substantial saving can be achieved, together with greater versatility in the shape of beams of intermediate energy—advantages which should outweigh the complication of additional extraction and injection systems. Choosing the same cyclotron radius (10.3 m) and frequency (4.61 MHz) as TRIUMF the lower energy orbits are halved in size. For a maximum γ of 4 ($\beta = 0.968$, $T \simeq 3$ GeV) the gain in radius is only 2.4 m (similar to SIN) while Eq. (4) shows that the flutter factor $F^2 = 3.2$.

Betatron Oscillations and Fringing Field Effects

To proceed further and decide on the optimum number of sectors (N) and variation of spiral angle with radius, we need to know fairly accurately how the radial and axial tunes ν_r and ν_z depend on the machine parameters. Continuing to put our faith in the hard edge approximation we have utilized the expressions for ν_r and ν_z derived by Schatz[7] for a separated sector cyclotron with spiral magnets and uniform hill field B_h = constant (the restriction to uniform fields turns out not to be too important, as will become apparent).

In the case of radial motion, ν_r grows much faster than the simplest approximation $\nu_r \simeq \gamma$ when the flutter is high, and the π-stop band is reached well before $\gamma = N/2$. For $F^2 \simeq 2$ we have found ν_r reaches N/2 for $\gamma \simeq N/3$ in the cases studied. (Numerical orbit studies - see below - confirm the accuracy of the hard edge formulae.) It is therefore neccessary to choose the number of sectors $N \geq 3\gamma_{max}$. Thus we have chosen N = 15 for the first stage ($\gamma = 4$) and N = 30 for the second stage ($\gamma = 10$).

For axial motion the usefulness of hard edge theory is rather limited, the reason being the growing importance of fringing field effects as the spiral angle increases, for pole gaps of realistic height. Enge[8] shows that while the radial focusing strength is (to first order) unaffected by the fringing field, the axial focusing strength is reduced by two effects—firstly the bend is incomplete at the point of maximum field gradient so that the effective crossing angle γ is reduced, and secondly there is a thick lens effect. Altogether

$$\Delta\tan\gamma = -gI_2(1+\sin^2\gamma)/\rho_h \cos^3\gamma \tag{5}$$

where g is the pole gap, ρ_h the radius of curvature in the hill (see Fig. 1) and $I_2 \approx 0.5$ is a dimensionless parameter depending on the shape of the field edge. For large γ, $\Delta\tan\gamma/\tan\gamma \sim \tan^2\gamma$ so the effect increases rapidly with spiral angle. Moreover the net focusing effect of the two edges depends on the square of $\tan\varepsilon$, doubling the magnitude of the effect. For g = 6 cm, ρ_h = 6 m and $\tan\varepsilon = 3$ the loss would be $\simeq$20%. For greater spirals the effect is even more serious, and it is important to keep the pole gap as small as possible. In the magnet design presented below g = 2.5 cm.

To explore the orbit properties in the proposed machines more precisely we have tracked protons through a simulated magnet field using the equilibrium orbit code CYCLOPS. The field is presented as a polar grid rather than in Fourier components; B_h may be given any desired radial variation but is set constant along the orbits—except near the edges, where B is given a Woods-Saxon (tanh) shape

$$B = B_h[1 + \exp(2.36\ s/g)]^{-1}\ , \tag{6}$$

s being measured at right angles to the edge. Starting from the hard edge expression the spiral profile $\tan\varepsilon(r)$ was adjusted iteratively until the desired value of ν_z^2 ($\simeq$3) was obtained at all radii. Figure 2 shows the results for a γ = 4 to 10 cyclotron with

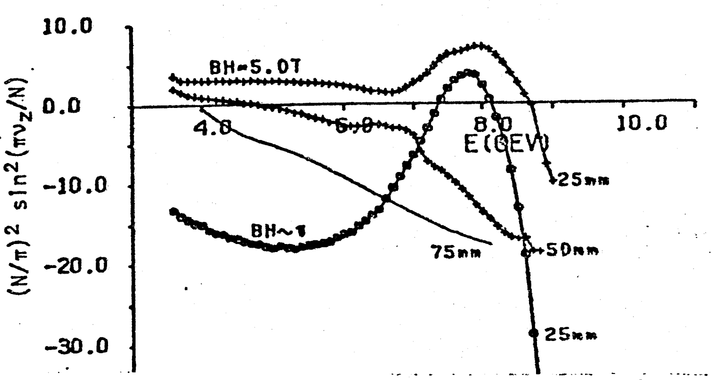

Fig. 2. Axial focusing strength for different radial variations of hill field and different pole gaps, but the same spiral edge shape ($N = 30$, $\omega_p = 2.305$ MHz)

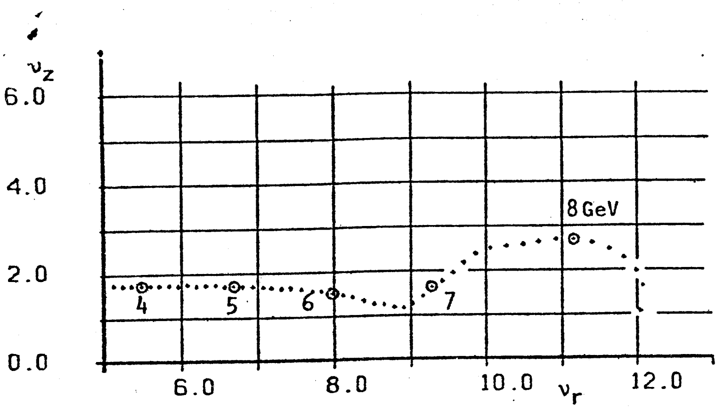

Fig. 3. Focusing frequencies for N = 30 cyclotron

$N = 30$, $r_c = 20.6$ m, a uniform field $B_h = 5.0$ T and a pole gap $g = 25$ mm. The spiral required rises smoothly to a maximum $\tan \varepsilon = 6.5$, and the resulting pole shape is shown in Fig. 6 below. Also shown are two cases where the pole gap has been increased to 50 and 75 mm, keeping the same $\tan \varepsilon (r)$. The curves demonstrate the importance of the fringing field effect, and confirm its growth with increasing spiral and the linear dependence on g predicted by Eq. (5) above.

In addition, results are plotted for $B_h = (\gamma/10) \times 5.0$ T (zero flare) for the same $\tan \varepsilon (r)$ and uniform 25 mm gap. Clearly the flared magnet with uniform B_h has better axial focusing properties than the magnet of constant angular width. The reason for this is also thought to be associated with the fringing field. The magnet with zero flare has the same spiral angle at both edges. The flared magnet, however, has a reduced spiral at the focusing edge and increased spiral at the defocusing edge. Consequently the fringing field effects will be larger at the defocusing than at the focusing edge, and the net effect is to give stronger focusing.

The motion of the working point in the resonance diagram is illustrated in Fig. 3 for the higher energy N = 30 cyclotron. We avoid crossing the dangerous $\nu_z = 1$ resonance up to 8.6 GeV; with a little more adjustment to the edge shape near maximum radius it should be possible to avoid crossing the $\nu_z = 2$ resonance also. The crossing of the integer and half-integer radial resonances should cause no serious problems with sufficient energy gain per turn.

Magnet and RF Design

There is an intimate relationship between the design of the magnets and the RF system, particularly when one is trying to achieve final energies between 5 and 10 GeV. For example, the fact that one needs 15 sectors to reach $\gamma = 4$ and 30 sectors to reach $\gamma = 10$ in order to avoid the π stop band in the radial focusing, precludes the use of "dee" type RF structures (because of lack of space) and forces the use of SIN type cavities. Conversely, the length of the magnets in the radial direction, together with the wavelength of the RF to be used, dictates the use of two cyclotron stages, instead of one, in accelerating to $\gamma = 10$.

Proposed arrangement of sectors and cavities for the two cyclotrons is shown in Figs. 4 and 5. In the 8.5 GeV machine the cavities are arranged in three groups of seven in order to keep groups of three neighboring valleys clear for injection and extraction systems.

In considering the radio frequency to be used in coupled cyclotrons the conventional wisdom states that the frequency should be the same in the two stages. This requirement would present a serious problem in our case since TRIUMF operates at 23 MHz and SIN type cavities at this frequency are very large and expensive and would require large amounts of RF power. The reason for the requirement is that a phase spread of ±14° in TRIUMF would become ±28° at 46 MHz, resulting in an increase in the spread in energy gain per turn from 3% to 12% over the phase interval. However, the following table shows that 25% 2nd harmonic (92 MHz) would result in a spread in energy gain per turn of less than 1%—at the cost of a reduction of 25% in peak energy gain per turn.

Table I

α_1	$V/V_m(1)$	α_2	α_4	$V/V_m(2+4)$
5°	0.9962	10°	20°	0.99985
8°	0.9903	16°	32°	0.9990
10°	0.9848	20°	40°	0.9976
12°	0.9782	24°	48°	0.9950
14°	0.9703	28°	56°	0.9909
16°	0.9613	32°	64°	0.9846
18°	0.9511	36°	72°	0.9757

α_1 is the ion phase in TRIUMF (23 MHz).
α_2 is the corresponding ion phase at 46 MHz.
α_4 is the corresponding ion phase at 92 MHz.
$V/V_m(1)$ = relative energy gain in a 23 MHz cyclotron.
$V/V_m(2+4)$ = relative energy gain in a cyclotron operating at 46 MHz + 25% 92 MHz.

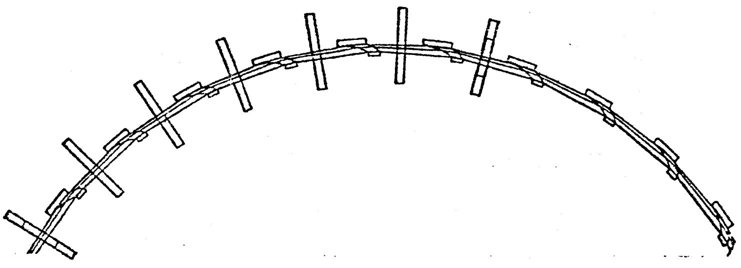

Fig. 4. 10-sector superperiod of N = 30 cyclotron showing 5 69 MHz and 2 207 MHz cavities, and the 3, 5 and 8.5 GeV orbits. (r_c = 20.6 m).

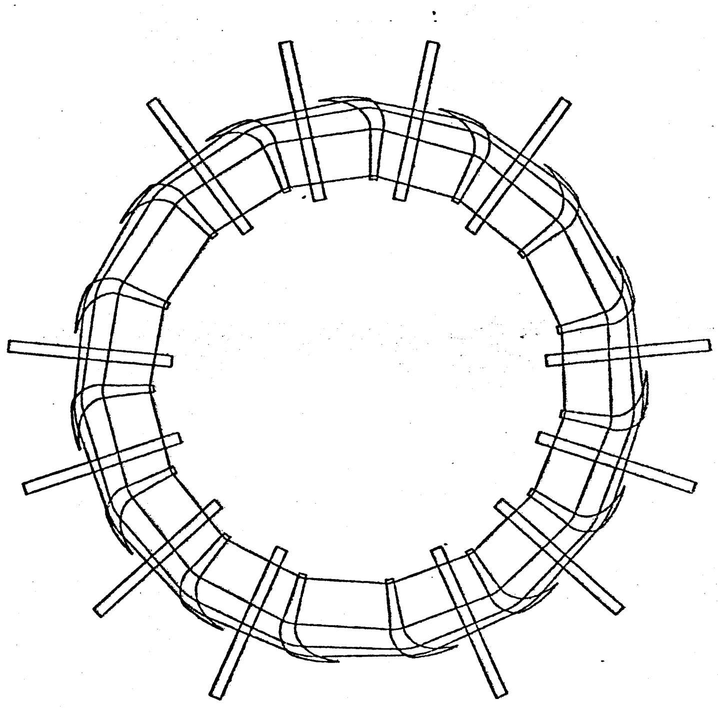

Fig. 5. The 15-sector first-stage cyclotron showing the 12 accelerating cavities and the 0.45, 1, 2 and 3 GeV orbits (r_c = 10.3 m).

In this case we are using "flat topping" to make the transition from the low TRIUMF frequency to a higher frequency in the second cyclotron without increasing the spread in energy gain.

It is also possible in a two-stage post-accelerator to use the first stage cyclotron as a phase compressor by arranging that the cavity voltage increase with radius.[9] If the first stage accelerates from $\gamma = 1.5$ to $\gamma = 4$ with a cyclotron radius $r_c = c/\omega_p = 10.3$ m, the increase in radius is 2.3 m. An RF cavity can be arranged so that its peak electric field coincides with the final orbit, and if its dimensions are 5.9 m (horizontal) by 3.6 m the accelerating voltage will increase by a factor of 3 from initial to final orbit, resulting in a phase compression by a factor of 3. The beam can now be injected into the second stage cyclotron with a phase spread for an accelerating frequency of 69 MHz just equal to that on leaving TRIUMF at 23 MHz. Third harmonic cavities (207 MHz) can be installed in the second stage to give "flat topping" there—at a loss of 1/9 of the peak energy gain.

An important possibility in magnet design for superconducting ring cyclotrons is the use of part of the return flux along a channel or "gully" between a hill and valley to increase the flutter. This gully would have a reverse field of 12-20 kG, be parallel to the edge of the hill and increase the flutter by some 30-50%. In making the extremely crude estimate of magnet weight shown in Table II it has been assumed that enough iron will be provided to provide a complete return flux in iron (except for the gullies).

In conclusion we have aimed to show that it is not only technically feasible to accelerate a high intensity beam of protons to many GeV in a cyclotron, but that with the help of superconducting technology it is economically feasible also. Provided pole gaps are kept small and separated sector magnets and gullies are used to obtain high flutter, axial focusing can be maintained to 8.5 GeV with not unreasonable spiral angles. The use of 2nd and 3rd harmonic cavities and phase compression can reduce the energy spread to 1% or better. The major question remaining to be tackled is that of extraction. If the full 3π mm-mrad emittance of TRIUMF were injected the incoherent radial amplitude of the beam would be 1.1 mm at 3 GeV and 0.7 mm at 8.5 GeV—several turns in each case. A 1% energy spread corresponds to 10 turns. With the help of radial resonances ($\nu_r = 6$ at 3 GeV and $\nu_r = 12$ at 8.5 GeV) and some reduction in energy spread and emittance, reasonably efficient extraction would seem to be within reach.

Table II

	First Stage	Second Stage
Injection energy (MeV)	450	3000
Extraction energy (MeV)	3000	8500
$r_c = c/\omega_p$ (m)	10.3	20.6
Number of sectors	15	30
Primary cavities	8 at 46 MHz	15 at 69 MHz
Harmonic cavities	4 at 92 MHz	6 at 207 MHz
Approx. dimensions of		
primary cavities (m^2)	5.9 × 3.6	4 × 2.6
secondary cavities (m^2)	5.9 × 1.6	4 × 1.5
Total RF power (MW)	2.0	1.7
Peak energy gain/turn (MeV)		
at injection	1.2	7.9
at extraction	3.6	7.9
$\Delta E/\Delta r$ (MeV/mm)		
at injection	0.23	1.9
at extraction	5.6	30
Radius gain per turn (mm)		
at injection	5.3	4.2
at extraction	0.64	0.26
Crude estimate of magnet weight (m tons)	2000	1800
Approx. number of turns	900	700

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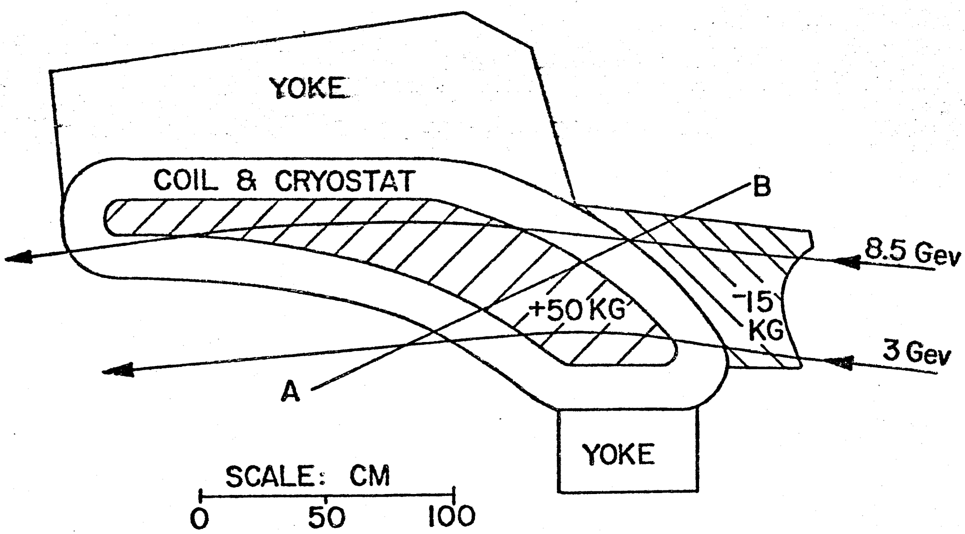

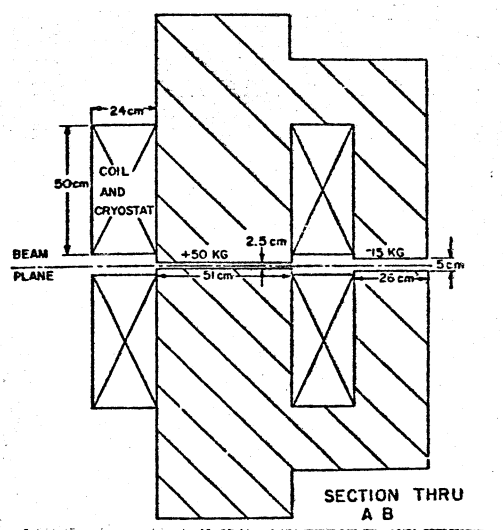

Fig. 6. Superconducting magnet design for second stage 3 to 8.5 GeV 30 sector cyclotron