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Pion-Nucleon Scattering in the Cloudy Bag Model

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Abstract

Pion-nucleon scattering in the (3,3) resonance region is described using an expanded version of the Brown-Rho bag model. Terms involving iterations of the crossed Born graphs, as well as excitation of the bare (three quark) delta, both give significant contributions to the scattering amplitude. Two parameters, corresponding to the bag size and the bare delta excitation energy, are found by fitting the scattering data.

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Recently Brown and Rho[1] introduced a new bag model of hadronic structure in which the radius is much smaller ($\sim 1.5\ m_\pi^{-1}$) than the MIT bag.[2] In this model the nucleon consists of a small bag of three quarks surrounded by a cloud of pions, which exist only outside the bag. The pions couple to the bag surface with the familiar Yukawa coupling. Thus hadrons can interact with each other through conventional pion exchange mechanisms.

In this note we extend the Brown-Rho (BR) model to describe pion-nucleon scattering in the (3,3) resonance region. The new feature of our model is that two distinct mechanisms combine to produce the observed resonance. First there is a series of terms involving the cloud of pions about the nucleon [see Fig. 1(a)]. With the Yukawa coupling [Eq. (2) below], and a suitable choice of cutoff-function,[3] this series of crossed graphs alone can reproduce the P_{33} resonance. However, in the bag model there is an additional pion nucleon coupling in which the pion field changes the three-quark nucleon bag into a three-quark delta state. This coupling gives rise to the series of graphs shown in Figs. 1(b) and (c). In what follows we sum the graphs of Fig. 1 in the static limit and obtain a good representation of the experimental in the P_{33} phase shifts.

In the BR model,[1] the coupling of the pion field to the surface of the nucleon bag is obtained by assuming $\partial_\mu A_\mu = 0$, where A_μ is the axial vector current. Inside the bag this is carried entirely by the quarks, and outside by the pions. By considering the integral $\int d^4x\ \partial_\mu A_\mu = 0$ over a pill-box through the bag surface, one finds that at the surface S[1,4]:

$$\hat{n} \cdot \underset{\sim}{\nabla}\vec{\phi}_{\pi} \Big|_S = -i(f_\pi^{-1}/2)\bar{q}\gamma_5\vec{\tau}q\Big|_S \ , \tag{1}$$

where f_π is the pion decay constant.

In order to specify completely the meaning of Eq. (1), we observe that the pion interaction at the bag surface is capable of transforming a nucleon into a delta. Hence we rewrite (1) as

$$\hat{n} \cdot \underset{\sim}{\nabla}\vec{\phi}_{\pi}\Big|_S^{NN} = \frac{-i}{2f_\pi} \langle N|\bar{q}\gamma_5\vec{\tau}q\Big|_S|N\rangle \ , \tag{2a}$$

$$\hat{n} \cdot \underset{\sim}{\nabla}\vec{\phi}_{\pi}\Big|_S^{\Delta N} = \frac{-i}{2f_\pi} \langle \Delta|\bar{q}\gamma_5\vec{\tau}q\Big|_S|N\rangle \ , \tag{2b}$$

where $|\Delta\rangle$ and $|N\rangle$ are the delta and nucleon bag states, respectively, and $\vec{\phi}_\pi\Big|_S^{\Delta N}$ represents that contribution of the pion field associated with the transition of a nucleon bag state into a delta bag state.

We could also choose to describe pion-nucleon and pion-Δ interactions through the Lagrangian density

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\left[\left(\underset{\sim}{\nabla}\vec{\phi}_\pi\right)^2 - \left(\frac{\partial\vec{\phi}_\pi}{\partial t}\right)^2 + m_\pi^2\phi_\pi^2\right] \\ & - \frac{f}{m_\pi}\psi^+\vec{\tau}\underset{\sim}{\sigma}\ \psi\underset{\sim}{\nabla}\vec{\phi}_\pi \\ & - \frac{g}{m_\pi}\left\{\psi_\Delta^+\vec{T}\underset{\sim}{\Sigma}\psi\underset{\sim}{\nabla}\vec{\phi}_\pi + \text{h.c.}\right\} \ ,\end{aligned} \tag{3}$$

where ψ and ψ_Δ are field operators for nucleons and bare deltas of finite extension. Equation (3) may be used with the Euler-Lagrange equations to obtain $\nabla^2\vec{\phi}_\pi$ in terms of the nucleon and delta source currents. Integrating over the volume of the nucleon source to obtain $\hat{n} \cdot \underset{\sim}{\nabla}\vec{\phi}_\pi$ at the baryonic surface, we find

$$\hat{n} \cdot \underset{\sim}{\nabla}\vec{\phi}_{\pi}\Big|_S^{NN} = -\frac{f}{m_\pi}\psi^+\hat{n} \cdot \underset{\sim}{\sigma}\vec{\tau}\psi \ , \tag{4a}$$

$$\hat{n} \cdot \underset{\sim}{\nabla}\vec{\phi}_{\pi}\Big|_S^{\Delta N} = -\frac{g}{m_\pi}\psi_\Delta^+\hat{n} \cdot \underset{\sim}{\Sigma}\vec{T}\psi \ . \tag{4b}$$

If the right-hand sides of (4a) and (4b) are the same as the right-hand sides of (2a) and (2b), the Lagrangian density of (3) provides a correct description of the pion field and its interaction with deltas and nucleons. This equality is ensured by simply requiring that the following equations hold:

$$-\frac{f}{m_\pi}\psi^+\underset{\sim}{\sigma}\vec{\tau}\psi \cdot \hat{n}\Big|_S = \frac{-i}{2f_\pi}\langle N|\bar{q}\gamma_5\vec{\tau}q\Big|_S|N\rangle \tag{5a}$$

$$-\frac{g}{m_\pi}\psi_\Delta^+\underset{\sim}{\Sigma}\vec{T}\psi \cdot \hat{n}\Big|_S = \frac{-i}{2f_\pi}\langle \Delta|\bar{q}\gamma_5\vec{\tau}q\Big|_S|N\rangle \ . \tag{5b}$$

The ratio g/f is determined (in principle) from the ratio of the appropriate matrix elements of $\bar{q}\gamma_5\vec{\tau}q$. For the present, we use the ratios as obtained from the static quark model.[5] The use of the Goldberger-Trieman relation (with $g_A = 1$, for reasons discussed in Ref. 1) establishes the equivalence of the right-hand sides of Eqs. (2) and (4) and specifies the Lagrangian density (3) that we use. The terms in $\mathcal{L}$ involving a πN-Δ coupling give rise to the series of graphs of Fig. 1(b).

Next we sum the graphs of Fig. 1. In order to gain a first glimpse of the implications of the new bag model for the pion-nucleon system, we have simplified the calculations by using the static model. In this approximation the crossed Born graph [first in Fig. 1(a)] can be written as v_N/E,[6] where E is the incident pion energy, and v_N an attractive separable potential. Similarly, the lowest order graph involving the Δ [first diagram in Fig. 1(b)] has the form $v_\Delta(E - s_0)^{-1}$, where s_0 is the bare delta excitation energy and v_Δ is also separable.

The mass s_0 is taken as an adjustable parameter in this model. The potential v_N is proportional to f^2 and v_Δ is proportional to g^2. In addition v_N and v_Δ involve phenomenological vertex functions $u_N(p)$ and $u_\Delta(p)$ which in the spirit of the little bag model have equal ranges. We assume that they are both θ-functions with a cutoff at momentum p_M. This is the second parameter in our model, but based on the uncertainty principle we expect p_M to be of the order R^{-1} (R is the bag radius).

The pion-nucleon transition amplitude t(E) corresponding to the sum of all the graphs of Fig. 1 can be shown to have the form

$$t(E) = t_1(E) + t_2(E) \tag{6}$$

where t_1 and t_2 satisfy the coupled equations

$$t_1(E) = \frac{v_N}{E} + \frac{v_N}{E}\frac{E}{h_0}\frac{1}{E^+-h_0}\left(\frac{E}{h_0}\; t_1(E) + t_2(E)\right), \tag{7a}$$

$$t_2(E) = \frac{v_\Delta}{E-s_0} + \frac{v_\Delta}{E-s_0}\frac{1}{E^+-h_0}\left(\frac{E}{h_0}\, t_1(E) + t_2(E)\right), \tag{7b}$$

and h_0 is the pion total energy operator. In the limit where g^2 is zero, Eqs. (6) and (7) reduce to the Chew-Low[7] theory without crossing. When f^2 is zero we ge a pure Δ-model.

There are a number of additional approximations (standard in earlier treatments of πN scattering) implicit in these equations. Terms associated with pion crossing are neglected, and we approximate the crossed Born graph so that there is no pion production cut. These neglected terms provide some attraction in the (3,3) channel,[8] and their inclusion could modify the values of p_M and s_0 that we extract.

Using Eqs. (6) and (7) we find that values of p_M = 900 MeV/c and s_0 = 1400 MeV (dotted line in Fig. 2) give a very good fit to the total cross section (solid curve)[9] in the (3,3) channel. The value of p_M is

consistent with the assumption of a small bag, but s_0 seems very large. The difficulty stems from the large mass of the bare little bag ($\sim 5.4\ m_n$). In the BR paper it was argued that strong self-energy corrections from virtual pion emission and absorption [especially Figs. 3(a) and 3(b)] would bring the bare nucleon mass down to the physical mass. Indeed we have assumed that this renormalization procedure is valid and have used the physical values of m_N and f^2 in evaluating the terms of Fig. 1. On the other hand, while self-energy graphs like Fig. 3(c) can be assumed to have been included in s_0 (i.e., $s_0 = 5.4\ m_N + 3(c) + \ldots$), important self-energy terms for the Δ, such as Fig. 3(d), are included in the solution of Eqs. (6) and (7). Since Brown and Rho found a contribution of $-2.9\ m_N$ from Figs. 3(a) and 3(b) in the nucleon case, our value of s_0 is not unreasonable.

However, there is a question of consistency, because f^2 is a fully renormalized coupling constant, while some parts of the diagrams in Fig. 1(b) may be regarded as renormalizations of the $\Delta N\pi$ coupling. In particular, we expect that $(g/f)^2$ should be given by the quark model only if both f and g are fully renormalized. This has led us to apply standard non-relativistic renormalization techniques[10] to the Δ-Green's function. We make two subtractions on the quantity $[E^+-s_0-\Sigma(E)] \equiv G_\Delta^{-1}(E)$ [see Fig. 3(d)], to guarantee that $\mathrm{Re}[G_\Delta^{-1}(E = s)]$ is zero. Furthermore a renormalization (redefinition of the $\pi N\Delta$ coupling constant) is performed so that the value of $\lim_{E\to s} \mathrm{Re}[G_\Delta^{-1}(E)/(E-s)]$ is unity. The only change in Eqs. (7) is that Eq. (7b) is replaced by

$$t_2(E) = \frac{v_\Delta}{E-s} + \frac{v_\Delta}{E-s}\left(\frac{1}{E^+-h_0}\frac{E}{h_0}\, t_1(E) - \frac{(E-s)^2}{(E^+-h_0)}\left\{\frac{d}{ds}\left(\frac{P}{s-h_0}\right)\right\} t_2(E)\right), \tag{7b'}$$

where P denotes the principal value part of the integral. The potential v_Δ is now proportional to the square of the renormalized $\pi N\Delta$ coupling constant.

Once again we adjust two parameters (a and p_M) so that the solution of Eqs. (6), (7a) and (7b') fits the pion-nucleon data. We find a best fit to the total cross section with p_M = 1280 MeV and s = 950 MeV (dashed curve in Fig. 2). However, fits of comparable quality can be found with a range of values of these parameters. For example, we also show in Fig. 2 (dash-dot curve) the case p_M = 860 MeV and s = 550 MeV. All of these solutions are consistent with the idea of a small bag. In the near future we intend to apply this theory to pion photoproduction with the aim of constraining these parameters further.

Nevertheless, a successful representation of the πN scattering data has been obtained, and the present model offers a novel description of the off-energy-shell behaviour of the πN transition matrix. Pion-nucleon scattering in the (3,3) resonance region is determined by two basic mechanisms. One involves the excitation of the bag-like core from a nucleon to a delta. This process alone has been considered by Brown *et al.*,[11] who found Γ_Δ = 94 MeV, and earlier by Chodos and Thorne[12] (in the MIT bag model) who found Γ_Δ = 32 MeV. The other mechanism involves the pion cloud which surrounds the nucleon core. By itself the latter corresponds to Chew-Low theory.[7]

The cloudy bag model is richer than either the old Chew-Low theory or the pure quark models. Furthermore it provides a calculable framework to study a number of unanswered problems involving widths of resonances.[13]

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Figure Captions

1. Mechanisms for πN scattering in the P_{33} channel involving: a) the nucleon's pion cloud, b) the elementary (3-quark) delta and c) interference terms.

2. Total cross section for πN scattering in the P_{33} channel (see text).
3. Self-energy corrections considered by Brown and Rho[1] for the nucleon [(a) and (b)], and by us [(c) and (d)]. Diagram (d) represents the term $\Sigma(E)$ in the text [above Eq. (7b')].

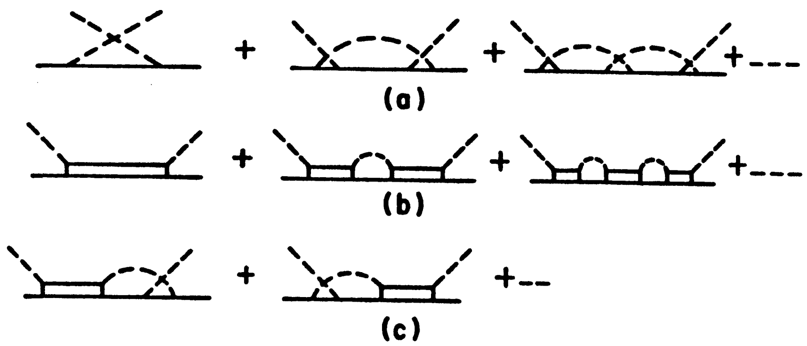

Fig. 1

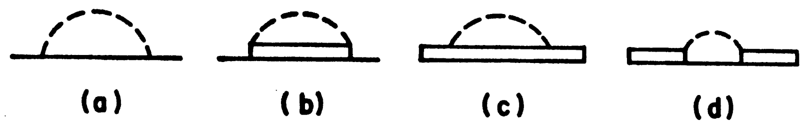

Fig. 3

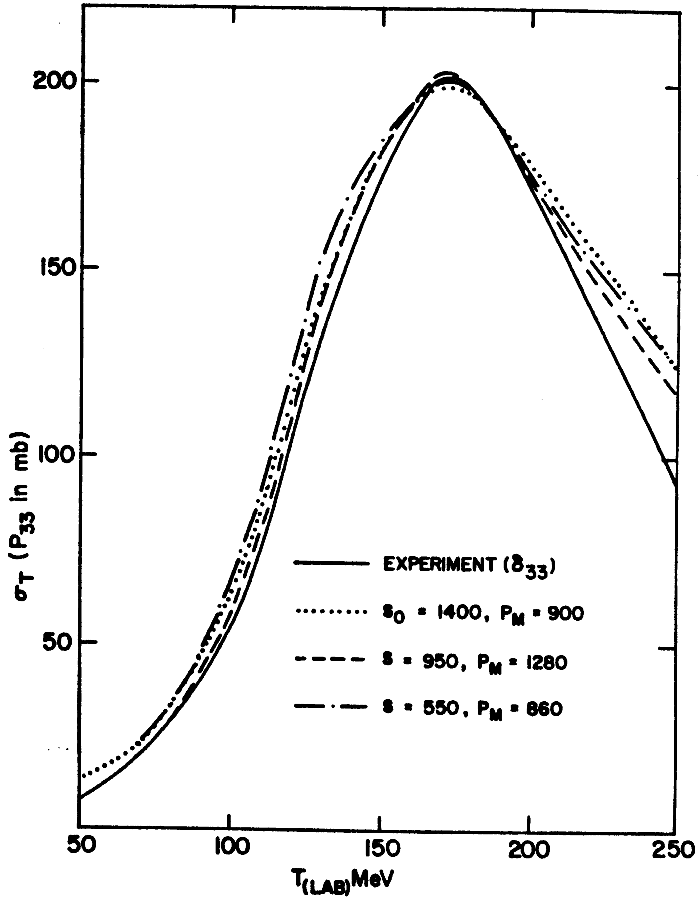

Fig. 2