

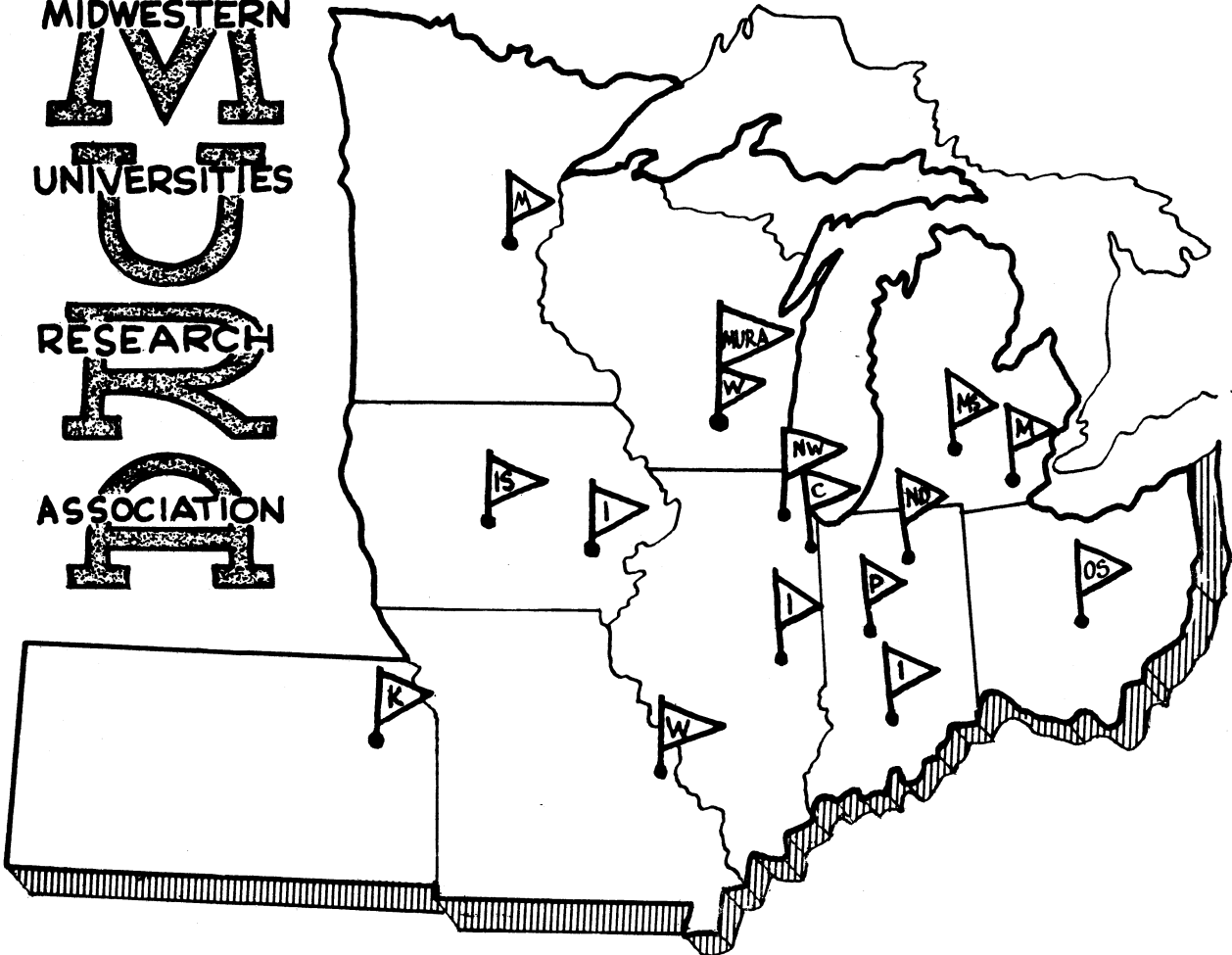
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BACKGROUND IN THE NEIGHBORHOOD OF A COLLIDING BEAM REGION

Don. B. Lichtenberg and Lawrence W. Jones

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BACKGROUND IN THE NEIGHBORHOOD OF A COLLIDING BEAM REGION

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August 26, 1959

ABSTRACT

A calculation is made of the number of background events expected in the neighborhood of two colliding proton beams each of energy 10-15 Bev. Collisions of each beam with the residual gas in a straight section in the target area and collisions of secondary particles in neighboring iron magnets are roughly taken into account. Results are given in Table IV of the report. The calculation breaks down at distances from the beams of the order of the beam diameters.

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I. INTRODUCTION

In every discussion of colliding beams and their experimental utility, the question of the background flux due to collisions with residual gas molecules enters as an important consideration. Previously simple, crude statements about multiplicities and angular distributions have been made in estimating these fluxes. Below is an attempt to formulate the problem more explicitly in order to arrive at somewhat more quantitative flux figures for various kinds of particles in the vicinity of the interaction volume. The results of these calculations indicate that the residual gas pressures and beam currents used in various MURA discussions are realistic and that the background flux is not greater than previously estimated.

II. FORMULATION OF THE PROBLEM

This is a report of a preliminary calculation of the number of background events expected in the neighborhood of a region of high energy colliding proton beams. In these considerations we shall restrict ourselves to the simple geometry shown in Fig. 1.

In Fig. 1, two circulating beams enter a straight section of length $2s$, and collide at its center. In addition to these beam-beam collisions, there are background collisions of the beams with the residual gas in the vacuum tank. We shall ignore the background from these beam-gas collisions around the machine except the background from collisions occurring in the straight section and from collisions occurring within a distance L_i on either side of the straight section, the subscript i indicating that this distance will not be the same for all particles.

The quantity of interest is the number N of background particles per unit area per unit time which pass through a small (spherical) volume in the experimental region. This number depends on the location of the volume, so that we have $N = N(\rho, \xi)$, where ρ is the perpendicular distance from the volume to the vacuum tank, and ξ is the distance parallel to the tank from the volume to the region of interaction (see Fig. 1).

We shall now set up a simple approximate expression for $N(\rho, \xi)$ in terms of the elementary cross sections and the geometry of Fig. 1. Let N_l be the number of interactions per unit length of one beam with the residual gas. Then N_l is given by

$$N_l = I n \sigma / e \quad (1)$$

where I is the current, e is the charge per proton, n is the number of nucleons per unit volume in the gas, and σ is the total cross section per nucleon. In writing formula (1), we assumed that the nucleons in the gas behave as if they are free, and that there is no shielding of nucleons by others in the same nucleus. These assumptions are most valid at high incident energy of the beam and for light target nuclei.

Now n is given by

$$n = \frac{6.02 \times 10^{23}}{22.4 \times 10^3} \frac{p}{760} A \quad (2)$$

where p is the gas pressure in mm. of Hg and A is the average molecular weight of the gas. Then substituting (2) and (1) and putting $\sigma = 25$ mb, we get

$$N_l = 5.6 \times 10^9 I p A$$

or for $A = 28$

$$N_{\ell} = 1.56 \times 10^{11} I p \frac{\text{interactions}}{\text{cm. sec.}} \quad (3)$$

where I is in amperes of circulating current.¹ We also need to know the number n_i of secondary particles of type i produced per primary collision, their energy distribution E_i , and their angular distribution $A_i(\theta)$ where $\int A_i(\theta) d\Omega = 1$. In this simplified treatment, we shall let n_i be the average number of particles produced per collision. Then the average number of particles emitted in a small length $d\ell$ at a distance r from the volume of interest at an angle θ is given by

$$N_{\ell} \sum_i n_i A_i(\theta) d\ell \quad (4)$$

If there are no further interactions of the secondary particles on their way to the volume of interest, then $N(\rho, \xi)$ becomes

$$N(\rho, \xi) = N_{\ell} \sum_i n_i \int A_i(\theta) \frac{d\ell}{r^2} \quad (5)$$

where the integration is over the region of interest and

$$r^2 = \ell^2 + \rho^2, \quad \tan \theta = \rho/\ell \quad (6)$$

(see Fig. 1). Equation (5) applies only to the contribution to $N(\rho, \xi)$ from those interactions which occur in the straight section. The interactions which occur in the region of the tank which has iron magnets around it will give rise to further interactions of the secondaries in the iron. Therefore this region must be treated separately. Putting limits of integration in (5), the contribution to $N(\rho, \xi)$ from the collisions of one beam with the gas in the straight section is

$$N_{\ell} \sum_i n_i \left\{ \int_0^{S-\xi} A_i(\theta) \frac{d\ell}{r^2} + \int_0^{S+\xi} A_i(\pi - \theta) \frac{d\ell}{r^2} \right\} \quad (7)$$

where $\xi \geq 0$ and $\theta \leq \pi/2$. Including the contribution from the beam circulating in the other direction, the total contribution to the background from the straight section is

$$N^{\text{straight}}(\rho, \xi) = N_e \sum_i n_i \left\{ \int_0^{s-\xi} (A_i(\theta) + A_i(\pi - \theta)) \frac{dl}{r^2} + \int_0^{s+\xi} (A_i(\theta) + A_i(\pi - \theta)) \frac{dl}{r^2} \right\} \quad (8)$$

Now we consider the interactions in the part of the tank with iron around it. We shall make the simple assumption that the net effect of the iron is to cause the number of secondary particles to be multiplied by a factor M_i which depends on the distance x the secondaries must travel through the iron. Then letting L_i be a suitable distance from the edge of the iron, the contribution from the region with iron is approximately given by

$$N^{\text{iron}}(\rho, \xi) = N_e \sum_i n_i \left\{ \int_{s-\xi}^{s-\xi+L_i} M_i(x) [A_i(\theta) + A_i(\pi - \theta)] \frac{dl}{r^2} + \int_{s+\xi}^{s+\xi+L_i} M_i(x) [A_i(\theta) + A_i(\pi - \theta)] \frac{dl}{r^2} \right\} \quad (9)$$

III. EVALUATION OF THE INTEGRALS

In order to evaluate the integrals (8) and (9), we need to make some assumptions about A_i and M_i . We shall assume the angular distributions in the laboratory system are the same for all kinds of particles:

$$A_i(\theta) = \frac{f}{2\pi(1 - \cos \theta_m)} + \frac{1-f}{4\pi}, \quad \theta \leq \theta_m$$

$$A_i(\theta) = \frac{1-f}{4\pi}, \quad \theta > \theta_m \quad (10)$$

This angular distribution corresponds to a fraction f of the particles being emitted in a cone of half angle θ_m and the remainder of the particles $(1 - f)$ being emitted isotropically. This angular distribution is not too unrealistic with $\theta_m = 30^\circ$ and $f = 1/2$ for particles produced in collisions where the primary proton has an energy of ~ 10 Bev.^{2 3}

With the angular distribution given by (10), the integrals in (8) are of the standard form

$$\int_a^b \frac{dl}{\rho^2 + l^2} = \frac{1}{\rho} \left(\tan^{-1} \frac{b}{\rho} - \tan^{-1} \frac{a}{\rho} \right) \quad (11)$$

with the restriction $a, b \geq \rho \cot \theta_m$ for the particles in the forward cone.

For the isotropic part of the angular distribution, a and b are simply the limits given in (8). Note that (11) blows up for $\rho = 0$. This is because of our approximate treatment which breaks down unless ρ is appreciably greater than the diameter of the beam.

For the contribution to (8) from the isotropic portion of A_i , we have

$$N_{\text{iso}}^{\text{straight}}(\rho, \xi) = N_e \sum_i n_i \frac{1-f}{2\pi} \frac{1}{\rho} \left(\tan^{-1} \frac{s-\xi}{\rho} + \tan^{-1} \frac{s+\xi}{\rho} \right) \quad (12)$$

The contribution from the forward cone is

$$N_{\text{cone}}^{\text{straight}}(\rho, \xi) = \begin{cases} N_e \sum_i n_i \frac{f}{2\pi(1-\cos\theta_m)} \frac{1}{\rho} \left[\tan^{-1} \frac{s-\xi}{\rho} - \left(\frac{\pi}{2} - \theta_m\right) + \tan^{-1} \frac{s+\xi}{\rho} - \left(\frac{\pi}{2} - \theta_m\right) \right], & \frac{s-\xi}{\rho} \geq \cot\theta_m \\ N_e \sum_i n_i \frac{f}{2\pi(1-\cos\theta_m)} \frac{1}{\rho} \left[\tan^{-1} \frac{s+\xi}{\rho} - \left(\frac{\pi}{2} - \theta_m\right) \right], & \frac{s-\xi}{\rho} \leq \cot\theta_m \leq \frac{s+\xi}{\rho} \\ 0, & \frac{s+\xi}{\rho} < \cot\theta_m \end{cases} \quad (13)$$

A table of values for $N_{\text{iso}}^{\text{straight}}/N_{\ell} \sum_i n_i$ and $N_{\text{cone}}^{\text{straight}}/N_{\ell} \sum_i n_i$ for several values of ρ and ξ and for $f = 1/2$, $\theta_m = 30^\circ$ is given in Table I.

In order to find the contribution from the region of iron, we must specify M_i . We assume M_i is given by a power series in the path length x traveled in the iron where

$$x = \frac{\ell - s + \xi}{\cos \theta} = (\ell - s + \xi) \frac{\sqrt{\ell^2 + \rho^2}}{\ell} \quad (14)$$

Then, keeping only the first three terms

$$M_i = 1 + \alpha_i x - \beta_i x^2 \quad (15)$$

where α_i and β_i are positive constants. Eq. (15) is supposed to be an approximation for a curve which rises at first and then decreases something like an exponential. If the quantity L_i appearing in the limits of integration in (9) is taken to be a small number of interaction lengths and if

$$\frac{s - \xi}{\rho} \gg 1, \quad (16)$$

then the expression (15) for M_i will be positive throughout the range of integration. Using (15), Eq. (9) can be readily integrated to yield:

$$N_{\text{iso}}^{\text{iron}}(\rho, \xi) = N_{\ell} \sum_i n_i \frac{1-f}{2\pi} I_i \quad (17)$$

where

$$I_i = \sum_{j=1}^2 \left\{ \frac{1}{\rho} \left(\tan^{-1} \frac{s_j + L_i}{\rho} - \tan^{-1} \frac{s_j}{\rho} \right) + \alpha_i \left[\sinh^{-1} \frac{s_j + L_i}{\rho} - \sinh^{-1} \frac{s_j}{\rho} + \frac{s_j}{\rho} \left(\text{csch}^{-1} \frac{s_j + L_i}{\rho} - \text{csch}^{-1} \frac{s_j}{\rho} \right) \right] - \beta_i \left[L_i - 2 s_j \log \frac{s_j + L_i}{s_j} + s_j^2 \left(\frac{1}{s_j} - \frac{1}{s_j + L_i} \right) \right] \right\} \quad (18)$$

$$\text{with } s_1 = s - \xi \quad \text{and } s_2 = s + \xi \quad (19)$$

Similarly

$$N_{\text{cone}}^{\text{iron}}(\rho, \xi) = N_0 \sum_i n_i \frac{1}{2\pi(1 - \cos \theta_m)} I_i \quad (20)$$

with the restriction

$$\frac{s - \xi}{\rho} \geq \cot \theta_m \quad (21)$$

If $(s - \xi)/\rho < \cot \theta_m$, then $N_{\text{cone}}^{\text{iron}}$ is given by a similar expression with altered values of some or all of the arguments of the functions. We shall take $\theta_m = 30^\circ$ subsequently, and not consider values of (ρ, ξ) unless they satisfy $(s - \xi)/\rho \geq \cot 30^\circ = \sqrt{3}$, since the approximation (15) breaks down in any case unless (16) holds.

The total number of particles passing through a small volume per cm^2 sec is then in this approximation given by

$$N(\rho, \xi) = N_{\text{iso}}^{\text{straight}} + N_{\text{cone}}^{\text{straight}} + N_{\text{iso}}^{\text{iron}} + N_{\text{cone}}^{\text{iron}} \quad (22)$$

where the quantities on the right are given by (12), (13), (17) and (20).

IV. VALUES OF THE PARAMETERS

In order to compute $N(\rho, \xi)$ we need to specify the quantities f and θ_m , and the quantities n_i , α_i , β_i and L_i for each type of particle i . The values of the parameters are not known very accurately. We shall always choose $f = 1/2$, $\theta_m = 30^\circ$. The types of secondary particles will be protons, neutrons, charged pions, and photons (from the decay of π^0 's). We use for n_i the values found by Fretter⁴ in a survey of cosmic ray events in carbon averaging 10 Bev. He finds the average number of heavy charged particles to be $n_{\text{heavy}} = 0.85$. We shall assume these are all protons and use the figure $n_{\text{protons}} = 1$. We assume the number of neutrons is the

same as the number of protons. The average number of charged pions found by Fretter is $n_{\pi^{\pm}} = 1.5$. We shall assume $n_{\pi^0} = 0.75$. These numbers differ appreciably from the number of charged particles produced by proton collisions in emulsion nuclei as given by Camerini, Lock and Perkins.⁵

The quantities α_i and β_i in the expression for the multiplicity (Eq. (15)) are of course energy dependent. We therefore need to know the energy distribution of the secondary particles in the laboratory system. We shall use only a suitable average value for α_i and β_i .

At a kinetic energy of 10 Bev, in a nucleon-nucleon collision, the available energy (in the center-of-mass system) to produce secondary particles is $E_{cm} = 2.8$ Bev. According to Fretter's results,⁴ about half this energy goes into the production of secondaries, nearly all of which are pions. Using the result that the number of (charged plus neutral) pions is $n_{\pi} = 2.3$, and taking $E = 1.4$ Bev to be the average energy given up to the pions, we get $E_{\pi} = 0.63$ Bev to be the average pion energy (including the rest energy) in the center-of-mass (CM) system. The energy of the pion in the laboratory system can be found from the Lorentz transformation

$$E_{\pi \text{ lab}} = (E_{\pi} + \vec{P}_{\pi} \cdot \vec{\beta}_{cm} c) \gamma_{cm} \quad (23)$$

where \vec{P}_{π} is the momentum of the π in the CM system, $c \vec{\beta}_{cm}$ is the velocity of the CM, and $\gamma_{cm} = 1/\sqrt{1 - \beta_{cm}^2}$.

A typical value for the transverse component of momentum for the pion⁴ is $P_{\pi y} = .3$ Bev/c. For a pion with energy $E_{\pi} = .63$ Bev, the longitudinal component of the momentum is $P_{\pi x} = .56$ Bev/c. The angle of production in the CM system is $\theta^* \approx 28^{\circ}$. In the laboratory system $E_{\pi \text{ lab}} = 2.9$ Bev

for a pion making a 28° angle with the forward direction in the CM system and $E_{\pi \text{ lab}} = 0.31$ Bev for a pion making a 28° angle with the backward direction. The corresponding angles in the laboratory system are given by the following transformation:

$$\gamma_{\text{cm}} \tan \theta = \frac{\sin \theta^*}{\cos \theta^* + c \beta_{\text{cm}} / v^*} \quad (24)$$

where θ is the angle in the lab and v^* is the velocity of the pion in the CM. The angles in the lab are $\theta \approx 6^\circ$ for the pion in the forward direction and $\theta \approx 72^\circ$ for the pion which goes backward in the CM. (The angular distribution is peaked in the forward direction more than the amount given by Eq. (10) with $f = 1/2$, $\theta = 30^\circ$ but we shall continue to use Eq. (10).) Then each of the pions emitted in the forward cone will be assumed to have an energy 2.9 Bev, and each pion emitted isotropically will be assumed to have an energy 0.3 Bev. The total energy taken up by the pions is then 3.6 Bev. The rest of the energy (6.4 Bev) will be divided among the kinetic energy of the two nucleons, with ~ 0.2 Bev going to the nucleon emitted isotropically, and ~ 6.2 Bev going to the nucleon emitted in the forward cone.

We next consider the photons from the π^0 decay. We assume the energy is divided equally between the two photons, so that the photons will each have energy 0.15 Bev from the isotropic π^0 's and 1.5 Bev from the π^0 's emitted in the forward cone. We thus arrive at the numbers of various particles given in Table II.

We obtain α_i and β_i for the photons from Fig. 5.13.2 of Rossi.⁶ This graph gives the number of electrons of energy greater than a certain critical energy (24 Mev for iron) produced by an electron of given energy

as a function of the distance (in radiation lengths) traveled by the electron. The α_i and β_i for photons given in Table III are found by roughly fitting the curves in Fig. 5.13.2 of Rossi at two points. No attempt was made to obtain the best two-parameter fit. The number of photons is assumed equal to the number of electrons. The value L_i given in Table III is chosen so that M_i never becomes negative. (The negative term $-\beta_i x^2$ would dominate if we let x become too large.)

To obtain reasonable values for α_i and β_i for the nucleon cascade is more complicated, as the secondaries are not all emitted in the forward direction. We shall make only a crude estimate of these values. From Fig. 4 of reference 3, we find that a proton with kinetic energy ~ 6 Bev makes six "fast" secondaries and eight "slow" ones⁷ in a collision in an emulsion nucleus. We shall arbitrarily take the multiplicities from the fast (2.9 Bev) pions to be the same as for these protons so that one α and β will serve for both. We shall assume the fast particles are emitted forward, and shall ignore the slow ones which are emitted isotropically. This means essentially that we ignore the flux of slow neutrons entering the small volume under consideration. (Slow neutrons will enter this volume from all directions.) We shall assume the number of fast secondaries persists for one more interaction length⁸ and then decays. We then obtain the values for α_i and β_i for nucleons and charged pions given in Table III.

V. RESULTS

With the information in Tables I, II and III, we can compute the number of particles entering a small volume at (ρ, ξ) . The results are given in

Table IV including the contribution from a 10-meter long straight section and the contribution from the region with iron. We have assumed a circulating current of 10 amps and a vacuum of 10^{-8} mm of Hg in computing these numbers. Values for other currents and pressures can be found by multiplying the numbers in Table IV by $10^7 I p$. If the residual gas is primarily hydrogen rather than nitrogen, the numbers are a factor ~ 10 too large. For values of (ρ, ξ) not given in the table, the formulas (12), (13), (17) and (20) should be used.

Near the edge of the iron, (16) is violated and the formulas are no good. However, one can guess that right at the edge of the iron ($\xi = 500$ cm), the number of particles may be up by as much as a factor 5 compared with the number in the straight section when ρ is small. On the other hand, the number would be smaller than the number in the middle of the straight section for large ρ . We may have underestimated the flux from the iron in any case, since the $1/r^2$ attenuation factor which we used is probably too great for particles scattered in the iron.

One important defect of this calculation is that it breaks down when ρ is of the order of the diameter of the beam. If it should become necessary to know the background fluxes at distances of the order of 1 cm. from the beam, the calculation should be repeated using a tube of current rather than a line current. Another defect in the calculations is that the angular distributions were greatly simplified so that the integrations could be carried out analytically. If a more accurate result is desired, angular distributions can be computed from the data of Fretter⁴ and others, and the integrations can be done numerically.

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TABLE I

Number of secondary particles/cm² sec passing through a small volume whose coordinates are (ρ, ξ) , for each secondary produced per cm. Here $2s$ is the length of the straight section from which the background arises. See text and Fig. 1 for other definitions.

ρ/s	ξ/s	$N_{iso}/N_g \sum_i n_i$	$N_{cone}/N_g \sum_i n_i$
.02	0	12.3/s	30.0/s
.02	.5	12.3/s	29.6/s
.1	0	2.3/s	5.0/s
.1	.5	2.3/s	4.9/s
.4	.0	.5/s	.4/s

TABLE II

Particles produced in a single interaction at 10 Bev.

Type of Particle	Energy Bev	n_i (iso) No. Emitted Isotropically	n_i (cone) No. Emitted in Forward Cone
Protons ^a	6.2	0	.5
Protons ^a	.2	.5	0
π^\pm	2.9	0	.75
π^\pm	.3	.75	0
Photons ^b	1.45	0	.75
Photons ^b	.15	.75	0

^aThe number of neutrons is assumed to equal the number of protons.

^bFrom decay of π^0 's.

TABLE III

Values of parameters appearing in Eqs. (15) and (18).

	Protons and Charged Pions	Photons
Attenuation Length λ_i (g cm ⁻²)	220 ^a	14.1 ^b
λ_i for iron in cm	28	1.8
$\alpha_i \lambda_i$ $\left\{ \begin{array}{l} \text{cone} \\ \text{isotropic} \end{array} \right.$	5.3 $\sim 0^c$	3.8 $\sim 0^c$
$\beta_i \lambda_i^2$ $\left\{ \begin{array}{l} \text{cone} \\ \text{isotropic} \end{array} \right.$	1.4 $\sim 0^c$	0.38 $\sim 0^c$
$\frac{L_i}{\lambda_i}$ $\left\{ \begin{array}{l} \text{cone} \\ \text{isotropic} \end{array} \right.$	3.3 3.3 ^d	8.7 8.7 ^d

^aThis is the value of the attenuation length used in reference 2.

^bThis is the value of the radiation length given in reference 6.

^cThe particles emitted isotropically have relatively low energy (~ 0.2 Bev), so that the multiplicities are small compared with the multiplicities of the forward particles. Here we arbitrarily set $\alpha_i = \beta_i = 0$ for these low energy particles, i. e., we assume the multiplicities are unity. This approximation somewhat underestimates the flux.

^dThe values for L_i for the isotropically emitted particles are arbitrarily assumed to be the same as for the particles emitted forward. This approximation is in the direction of overestimating the flux.

TABLE IV

Number of particles entering a small volume per cm^2 per sec at (ρ, ξ) . The coordinates are measured from the center of a 10-meter straight section. A circulating current of 10 amps and a vacuum of 10^{-8} mm Hg are assumed, if the residual gas is nitrogen. If the gas is hydrogen, these numbers apply for a circulating current of 140 amps.

ρ cm	10	10	50	50	200	
ξ cm	0	250	0	250	0	
Charged Pions						
+ Protons + Neutrons*	670	670	130	130	25	
Photons	290	290	50	50	10	$N_{\text{iso}}^{\text{straight}}$
Total	960	960	180	180	35	
Charged Pions						
+ Protons + Neutrons*	1600	1600	270	250	25	
Photons	700	700	120	110	10	$N_{\text{cone}}^{\text{straight}}$
Total	2300	2300	390	360	35	
Charged Pions						
+ Protons + Neutrons*	1	3	1	3	1	
Photons	0.1	0.3	0.1	0.2	0.1	$N_{\text{iso}}^{\text{iron}}$
Total	1	3	1	3	1	
Charged Pions						
+ Protons + Neutrons*	45	110	45	90	40	
Photons + Electrons**	8	40	10	30	10	$N_{\text{cone}}^{\text{iron}}$
Total	50	150	60	120	50	
Charged Pions						
+ Protons + Neutrons*	2300	2400	450	470	90	
Photons + Electrons***	1000	1000	180	190	30	Total
Total	3300	3400	630	660	120	

*Of these, $\sim 40\%$ are pions, $\sim 30\%$ are protons and $\sim 30\%$ are neutrons.

**Of these, $\sim 50\%$ are photons and 50% are electrons.

***Most of these are photons.

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1. MURA-479 (Internal), 1959.
2. A. Citron and W. Gentner, CERN PS/WG3 (Interim report), 1953.
3. Camerini, Lock and Perkins, Progress in Cosmic Ray Physics Vol. I, 1952, p. 5ff.
4. Private communication from W. D. Walker. After the calculation was finished, we saw a preprint by Louisa Hansen and W. B. Fretter in which the numbers were slightly different. However, the numerical work was not repeated.
5. Reference 3, p. 11, Fig. 4.
6. B. Rossi, High Energy Particles, 1952, p. 258, Fig. 5.13.2.
7. "Fast" particles are the thin plus gray tracks of reference 3; "slow" ones are the black tracks.
8. We shall actually use attenuation lengths rather than interaction lengths in the calculation. The attenuation length is the "effective" length for the decay of particles in a nuclear shower, and is longer than the interaction length because of the buildup of new particles.

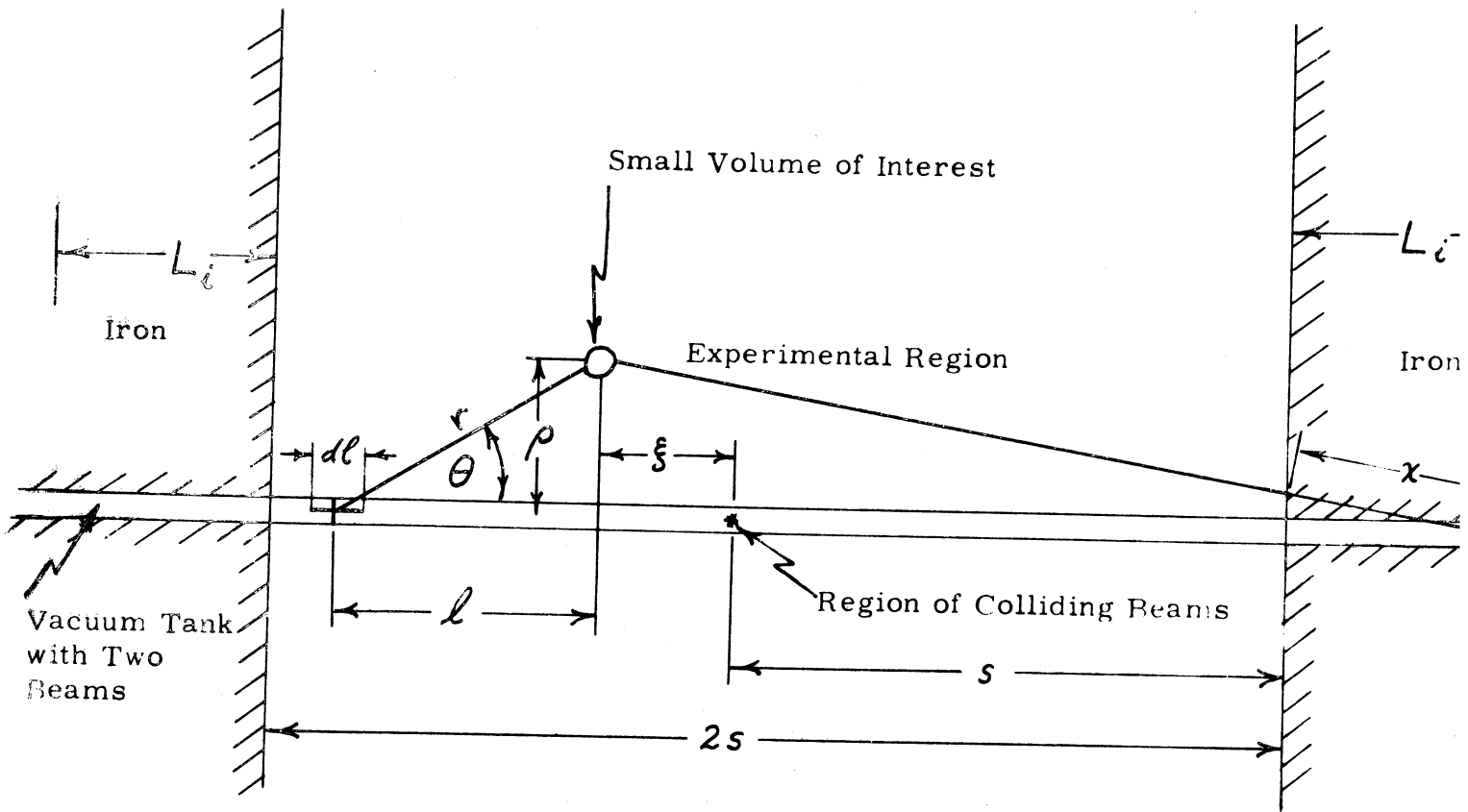


Fig. 1