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Large Electroweak Logarithms in Heavy Quark Decays

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Abstract

Electroweak radiative corrections to heavy quark decays are analysed and evaluated within the leading logarithmic approximation at the first order in the electroweak coupling α_W. It is found that, owing to uncancelled double logarithms, large contributions to the decay rates appear to augment usual radiative contributions. We show in a test channel that sizeable effects could take place at the energies attainable at the Large Hadron Collider (LHC).

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1 Introduction

Decays of heavy quarks play a relevant role at hadronic colliders such as the Tevatron and will play a leading part also in the physics at the forthcoming Large Hadron Collider (LHC) [1]. A considerable fraction of the events at the Next Linear Collider (NLC) will also be constituted by states made by highly virtual heavy flavours. At first sight, it could appear that electroweak radiative corrections to the processes involving heavy flavours would play a secondary role and will have a smaller impact on the physics outputs when compared with strong interactions ones.

The aim of this work is to show that a new class of electroweak radiative effects has a sizeable effect on the decays of heavy quarks at energy ranges like those of the LHC thanks to a conspiracy of large uncompensated logarithmic corrections. The origin of those uncancelled corrections can be attributed to be breaking of the $SU(2)$ invariance when a specific initial or final flavour is selected. Consequently, the sum over the weak-isotopic spin doublets cannot be performed any longer. The Bloch - Nordsiek and Kinoshita - Lee - Nauenberg singularity cancellation theorems [2, 3, 4] are no longer applicable since there is an uncompensated correspondence between real and virtual diagrams. This mechanism was first observed by the authors of Ref. [5] for the case of inclusive cross-sections in e^+e^- annihilation processes, when radiative electroweak corrections of W and Z^0 bosons were applied. We briefly outline here a novel proof of this mechanism for the specific case of the heavy quark decays by using the same approach of Feynman cut diagrams as developed by Kinoshita in his seminal paper [3]. The same proof can be extended to the analogous cases as, for example, the one discussed in Ref. [5].

We leave to a forthcoming work a detailed derivation of the results presented here together with a detailed phenomenological analysis [6].

2 Discussion

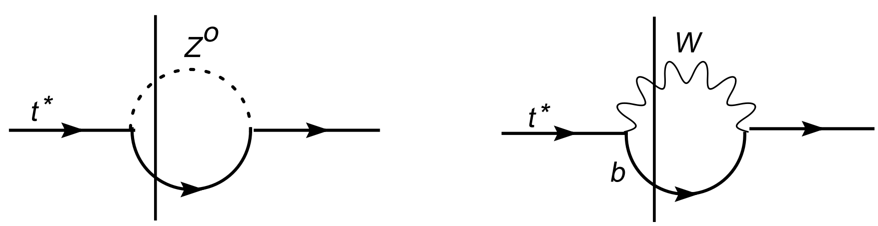

Figure 1: Born cut diagrams for the decays $t^* \to tZ^0$ and $t^* \to bW^+$

Let us consider the decay of t and b quarks at high energies. For both decays the contributing diagrams will contain real and virtual W and Z bosons. At Born level, diagrams contributing to the amplitude are those in Fig. 1. The transition amplitude to a given final state, as first discussed by Kinoshita [3], is represented by the diagram cutting the corresponding final state. The total transition rate, at a given order in the perturbative expansion, is obtained by applying cuts to the set of corresponding amplitudes that are then summed up and squared [3]. We assume, as will be the case at high energies at the LHC as well as at the NLC, that the t quark will have a

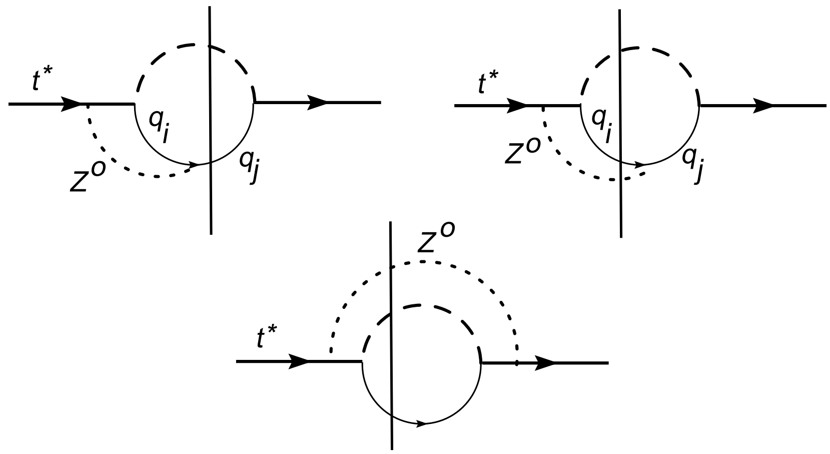

Figure 2: Radiative Z^0 corrections for the processes $t^* \to q_j$, $W(Z^0)$ i.e. to the diagrams of Fig. 1. The dashed line represents a W or a Z^0 boson and q_j a b or a t quark correspondingly.

virtuality Q^2 larger than any other scale within the process: $Q^2 \gg m_t^2, M^2$, with m_t the t quark mass and $M = M_{Z^0} \simeq M_{W^\pm}$ the Z^0 or $W^\pm$ gauge boson mass. Since $m_t \sim M \gg m_b$ the b-quark can be taken to be massless. The boson mass provides a physical cut off at the infrared singularities [3, 4] which then forbids bosons to be infrared or collinear[2].

Let us consider first Z^0 radiative corrections to the Born amplitudes in Fig. 1[3]. Diagrams that contribute are shown in Fig. 2. The dotted lines represent Z^0 and the dashed ones Z^0 or W, as in Fig. 1 where q_i lines correspond to the t and b- flavour[4]. All diagrams in Fig. 2 give logarithmic contributions that become larger as Q^2 increases. In Feynman gauge the first and the second diagram of the first row have a double logarithm of the form $\ln^2 Q^2/M^2$. Double logarithms are known as Sudakov logarithms [7] and their appearance is related to the infrared and collinear behaviour of the theory. The third, the *rainbow diagram*, gives a single logarithm $\ln Q^2/M^2$. We will work within the double logarithmic or leading logarithmic approximation (LLA). We will then neglect single logarithms as well as other subleading terms.

In the evaluation of the diagrams an additional scale appears: the t quark mass m_t. We may define two adimensional parameters: $\mu = Q^2/m_t^2$ and $\lambda = Q^2/M^2$ with the approximation $\mu \simeq \lambda$. Subleading logarithmic terms in μ and λ can be neglected as well. Within the chosen accuracy we have:

$$\ln\mu \;\ln\lambda = \ln^2\lambda + O\left(\ln\frac{\mu}{\lambda}\ln\lambda\right)$$

$$\ln^2\mu = \ln^2\lambda + O\left(\ln\frac{\mu}{\lambda}\ln\lambda\right)$$

where $\ln\frac{\mu}{\lambda} \simeq O(1)$ and therefore negligible too[5].

[2]Only for asymptotically large energies can vector bosons be effectively considered as massless and the usual terminology of soft and collinear does make sense.

[3]We neglect photon brehmsstrahlung. The effects of photon radiation can be described by using QED, whose infrared behaviour is well known. Diagrams are the same as those of the Z^0.

[4]We neglect all self-energies containing diagrams and their cuts as subleading contributions.

[5]Let us note that this ordering arises because of the relations between the hard scale Q^2, the infrared scale

Let us consider the amplitude for $t^* \to tZ^0$. At Born level the only contributing diagram is the first of Fig. 1. To the first perturbative order in the electroweak coupling $\alpha_{\text{weak}} = \alpha_W$, where $\alpha_W = \frac{g^2}{8\, 4\pi}$ with g the electroweak gauge coupling, the Z^0 radiative corrections are given by the diagrams in Fig. 2. Here the long-dashed lines represent the Z^0- boson and $q_{i,j}$ represent t quarks.

The contributions to the total inclusive rate, i.e. to matrix elements squared and integrated over longitudinal and transverse degrees of freedom, are from the virtual diagram in Fig. 2:

$$V^{tt}{}_{\text{virt}} = -\, V_0^{tt} C_{Z^0}^{tt} \ln^2 \lambda + O(\ln \lambda) + O\left(\ln \frac{\mu}{\lambda} \ln \lambda\right) \tag{1}$$

where [8]

$$C_{Z^0}^{tt} = \frac{\alpha_W}{2\pi\ \cos^2 \theta_W} \left[\left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right)^2 + \frac{1}{4} \right] \tag{2}$$

with θ_W the Weinberg weak angle; V_0^{tt} gives the Born level inclusive rate for the production of a t quark out of a highly off-shell virtual t quark. Real Z^0 emission gives the contribution:

$$V^{tt}{}_{\text{real}} = \ V_0^{tt} C_{Z^0}^{tt} \ln^2 \lambda + O(\ln \lambda) + O\left(\ln \frac{\mu}{\lambda} \ln \lambda\right) \tag{3}$$

The sum of Eqs. (1) and (3) shows the cancellation of the Sudakov logarithms among real and virtual diagrams. Both Z^0 and γ radiative corrections satisfy the overall cancellation of logarithmic contributions as advocated by the Kinoshita - Lee - Nauenberg (KLN) [3, 4] for inclusive quantities[6].

The same pattern of logarithmic cancellation can be shown to take place in the $t^* \to bW$ radiatively Z^0 corrected amplitude[7] obtained from the Born second diagram of Fig. 1.

To complete the electroweak corrections, we consider W insertions in the Born amplitude, second diagram of Fig. 1.

Radiative corrections to $t^* \to t$ involving real and virtual W emission, within the LLA, require one single diagram: the first of Fig. 3 with the cut on the final t.

The virtual contribution in Fig. 3 produces the term

$$V^{tt}{}_{\text{virt}} = -\, V_0^{bb} C_W^{tb} \ln^2 \lambda + O(\ln \lambda) + O\left(\ln \frac{\mu}{\lambda} \ln \lambda\right) \tag{4}$$

where V_0^{bb} is the Born level inclusive rate for a production of a b quark and

$$C_W^{tb} = \frac{\alpha_W}{2\pi} \| v_{tb} \|^2 \tag{5}$$

μ^2 and the parameters, μ and λ, the last two being of the same order. At the leading level we use λ as argument of the logarithms.

[6]Additional diagrams of the kind of those in Fig. 2, containing a virtual photon and a real Z^0 and vice versa, add up to the decay channels $t^* \to t$ and $b^* \to b$ as electroweak isotriplet and isosinglet contributions respectively. These terms do not give double logarithmic contributions. We do not include them in the evaluation of the decay amplitudes. However these channels will enter in the evaluation of the total decay rates. As a phenomenological consequence these additional diagrams in the amplitudes for the flavour diagonal channels, therefore, will smear out the enhanced contribution coming from the $W^{\pm}$ containing diagrams. For the logarithmic contributing off-diagonal flavour channels $t^*(b^*) \to b(t)$ the double logarithmic effect will be more manifest.

[7]t^* represents a quark with a large virtuality.

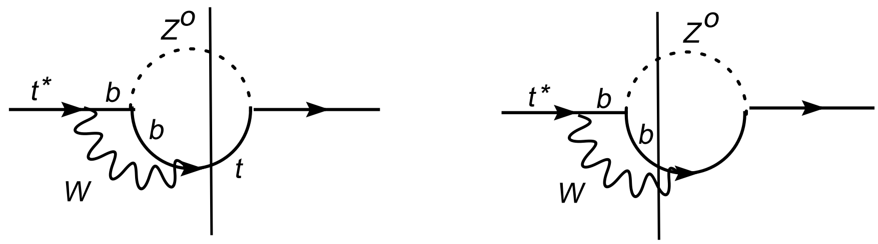

Figure 3: Cut diagrams for the $O(\alpha_W)$ radiative corrections due to W emission

where $|v_{tb}\|^2$ is the squared Cabibbo - Kobayashi - Maskawa quark mixing matrix element for the transition $t \to b$ coming from the vertex correction. ${V^{tt}}_{\rm virt}$ is not cancelled by a corresponding real diagram. For the amplitude of the process $t^* \to t$ there is no diagram containing a real W-boson emission. The second diagram in Fig. 3 describes the transition $t^* \to b$ and reads:

$$ {V^{tb}}_{\rm real} = V_0^{bb} C_W^{tb} \ln^2 \lambda + O(\ln \lambda) + O\left(\ln \frac{\mu}{\lambda} \ln \lambda\right) \tag{6} $$

It appears that, contrary to what happened before for the Z^0- boson emission, this quantity does not have a corresponding virtual contribution. Therefore cancellation, also in this case, cannot take place. The rates associated to the processes of interest $t^* \to t$ and $t^* \to b$ are respectively:
- $t^* \to t$. The sum of Born contribution, of Z^0 radiative corrections, and of Eq. (4) is

$$ \begin{aligned} V^{tt} &= V_0^{tt} - V_0^{tt} C_{Z^0}^{tt} \ln^2 \lambda + V_0^{tt} C_{Z^0}^{tt} \ln^2 \lambda - \\ &\quad - V_0^{bb} C_W^{tb} \ln^2 \lambda = V_0^{tt} - V_0^{bb} C_W^{tb} \ln^2 \lambda \end{aligned} \tag{7} $$

- $t^* \to b$. The sum of Born contribution, of Z^0 corrected expressions, and of Eq. (6) is

$$ \begin{aligned} V^{tb} &= V_0^{tb} - V_0^{tb} C_{Z^0}^{tt} \ln^2 \lambda \\ &+ V_0^{tb} C_{Z^0}^{tt} \ln^2 \lambda + V_0^{bb} C_W^{tb} \ln^2 \lambda \\ &= V_0^{tb} + V_0^{bb} C_W^{tb} \ln^2 \lambda \end{aligned} \tag{8} $$

It appears from Eqs. (7) and (8) that non cancelled double logarithms arise here in *timelike* processes in the same way as recently advocated in Refs. [9, 5]. Comparing (7) and (8) with (4) and (6), it turns out that these residual logarithms are related to real and virtual W contributions associated to the Fig. 3.

Now let us allow the final state to be either a t quark or a b quark, considering the transition $t^* \to b, t$. This corresponds to sum (7) and (8). The sum gives the total width i.e. the sum over the weak isospin doublet, showing the complete cancellation of the infrared logarithms:

$$ \begin{aligned} V^{t(b,t)} &= V_0^{tt} - V_0^{bb} C_W^{tb} \ln^2 \lambda + V_0^{tb} + V_0^{tt} C_W^{tb} \ln^2 \lambda \\ &= V_0^{tt} + V_0^{tb} \end{aligned} \tag{9} $$

The same cancellation may be also obtained by summing over the initial flavour in $V^{(b,t),t}$ or $V^{(b,t),b}$, by keeping the final quark fixed and, as above, the total inclusive rate is free from infrared logarithms. The KLN theorem [3, 4] would be again satisfied.

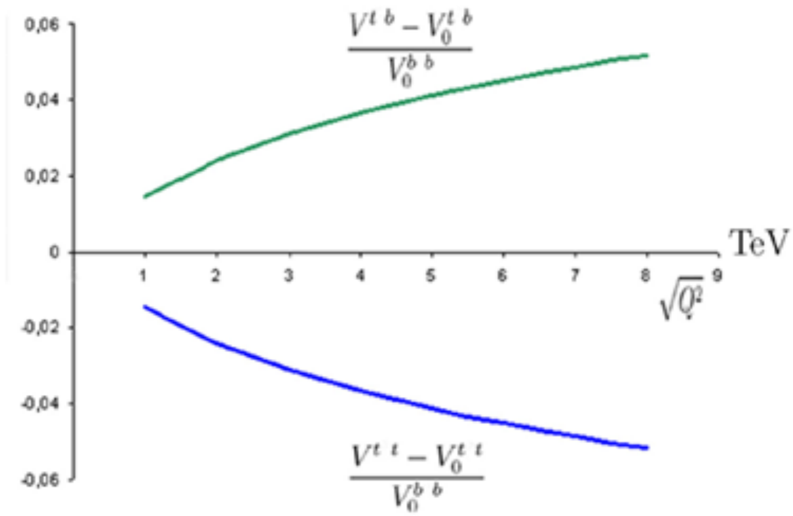

Figure 4: Plots of the ratios of $(V^{tt} - V_0^{tt})/V_0^{bb}$ (blue line) and $(V^{tb} - V_0^{tb})/V_0^{bb}$ (green line) as functions of Q between 1 and 8 Tev.

It can be shown [6] that a one to one correspondence can be established, among the diagrams contributing to the $t^* \to t, b$ in the t^* - decays and the $e^- \to e^-, \nu_e$ $O(\alpha_W)$ amplitudes of the e^-e^+, scattering considered in Refs. [5, 9]. Double logarithmic structure does not depend on the kinematics of the process but it's a property related to the evaluation of the matrix elements, to the $SU(2)$ group structure of the Electroweak theory and to weak isospin invariance[8].

At LHC these factors can give in principle sizeable corrections. For energies that vary between the values of $1 \leq \sqrt{s} \leq 8$ TeV we considered the ratio of $(V^{tt} - V_0^{tt})/V_0^{bb}$ and $(V^{tb} - V_0^{tb})/V_0^{bb}$ as given in the Eqs. (7) and (8). Fig. 4 shows the contribution to the inclusive rates of the logarithmic factors in the channels considered above[9]. It is apparent that, by varying the energy within the range reachable at LHC, the ratios are affected by the corrections for an amount between 0.014 and 0.051 for the $t^* \to t$ decay as well as for the $t^* \to b$ a variation of the approximately the 4% in the chosen range of energies. At lower energies of $\sqrt{s} = 1$ TeV as the range of a possible Next Linear Collider the corrections, corresponding to a virtualness of about 350 GeV for the quarks, become of approximately 0.004 which corresponds to the order of 0.4%.

We may consider, as an example, the case of a pair of quarks $q^a\bar{q}^a$, of a given flavour a, produced by an hard process as, for example, a gluon fusion process or quark annihilation in a hadron collider. The produced quarks, as highly virtual states, will decay either by a strong or a weak interaction mechanisms. The total decay width will be, perturbatively described by a series expansion of the form

$$\hat{\sigma}_F = \hat{\sigma}^{ij}(x_1 x_2 S)(1 + c_1^S \alpha_S + \ldots)\,(1 + c_1^w \alpha_W + \ldots) \tag{10}$$

[8] An analogous non cancellation mechanism has been also observed, for instance, also in the Quantum Chromodynamics $SU(3)_c$ case for colored quark amplitudes in Ref. [10].

[9] The impact of the logarithmic contributions to the physical cross-section will be obtained by adding the logarithmic non enhanced isosinglet channels.

where S is the total center of mass energy squared for the proton-proton system and $x_1 x_2 S = s = Q^2$ is the center of mass energy squared for the heavy quark pair. The strong interactions do not change the flavour quantum numbers of the involved quarks. The emission of the charged $W^\pm$ will produce quarks of different flavour: $Q^a \to W^\pm Q^b$. By taking the case of a final state with a chosen flavour as $t\bar{b}$ and assuming that this combination is accompanied by the emission of a W^- with the $O(\alpha_W)$ corrections including the corresponding virtual diagrams the KLN cancellation cannot take place and the final distribution is logarithmically enhanced as:

$$\hat{\sigma}^{t\bar{b}} = \hat{\sigma}_0^{f\bar{f}}\ \alpha_W\ \ln^2 \frac{Q^2}{M_W^2} \tag{11}$$

where $f = b, t$.

A detailed phenomenology related to these logarithmic enhancing factors has still to be developed as well as their quantitative relevance at high energies [6].

3 Conclusions

In quark decays, when the initial flavour is kept fixed and the final flavour is chosen, we observe, in inclusive distributions, the appearence of double logarithms of the ratio of the process hard scale with the intermediate boson mass.

The impact of such contributions can be significant for $s \gg M_W^2$. Under these conditions these effects must be included in the evaluation of the matrix elements for quark decays.

The enhancement, as shown in the examples given here, confirms the non cancellation of large electroweak logarithms in flavour dependent amplitudes as already noticed, for a different class of processes, in Refs. [5, 9].

4 Acknowledgments

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