Archives

INCOHERENT MULTIPLE COLLISIONS IN HEAVY NUCLEI AND IN THICK TARGETS

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ABSTRACT

For large $p_\perp$ proton production in the forward direction, experiments are proposed that would simulate incoherent multiple collision effects in heavy nuclei by production in thick targets of hydrogen or light nuclei.

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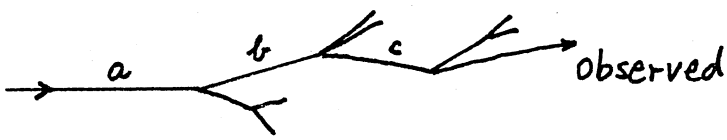
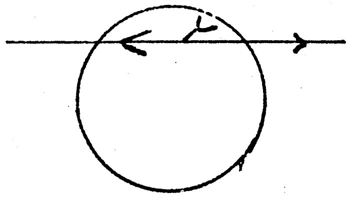
Recently the dependence on target atomic number A of the single particle spectrum from a p - nucleus collision was studied [1] by the Chicago-Princeton collaboration. They found an anomalous A dependence. Part of this anomaly is clearly due to incoherent multiple collision effects, i.e., effects of a showering depicted in Fig. 1 in which several nucleons in the target nucleus



- Figure 1 -

are successively hit. For outgoing particles with small $p_\perp$ it is known experimentally that coherent effects are important, i.e., amplitude, not intensity, superpositions are important. For high $p_\perp$ outgoing particles, however, it seems likely that the incoherent multiple collision process is sufficient to explain the anomalous A dependence. Calculations are underway to check this and will be reported later. In this letter we point out that one can experimentally study the incoherent multiple collision effect by simulating a thin heavy nuclear target with a weighed average of results with "thick" light nuclear targets.

For clarity we shall concentrate on outgoing protons. In the incoherent multiple collision process this observed proton has a series of ancestors, such as a, b, c in Fig. 1. We neglect processes in which any one of the ancestors is not a nucleon. Thus we trace the multiple collision process <u>through the nucleon pedigree</u>. We use the <u>laboratory frame of reference</u> and assume that the fast nucleon in traversing the target nucleus makes K collisions with nucleons, as illustrated in Fig. 2. We shall call K the collision



Collision number
$K = 3$

- Figure 2 -

number. If σ_K is the cross-section for a p-A collision with collision number K, then

$$\sum_{K=1}^{\infty} \sigma_K = \pi R_A^2 , \ (R_A = \text{radius of } A). \tag{1}$$

Notice that we do not include elastic p-A collisions, which is a coherent process, in this consideration. The mean free path λ of a nucleon in nuclear matter is $\lambda = 2.87\times10^{-13}$ cm. if we take the total nucleon-nucleon cross-section to be 40 mb, and

$$R_A = 1.4 \times 10^{-13} (A)^{\frac{1}{3}} \text{ cm}.$$

Assuming the nucleus to be a uniform ball of this radius we find the cross-section at path length L through the nucleus to be $2\pi b \, db$ where $b^2 + (L/2)^2 = R_A^2$. Using $L = \lambda K$ we obtain

$$\begin{aligned} \sigma_K &= \pi\lambda^2 K/2 , & 0 < K \leq 2R_A/\lambda , \\ \sigma_K &= 0 & K > 2R_A/\lambda \end{aligned} \tag{2}$$

which satisfies (1).

Let $R_{K(p)} d^3p$ or $R_{K(n)} d^3p$ be the probabilities for the K^{th} descendant to be a p or n in d^3p :

$$\int [R_K(p) + R_{K(n)}] d^3p = 1.$$

Then

$$\frac{d\sigma_A}{d^3p} = \sum_K \sigma_K R_{K(p)} \cdot \tag{3}$$

By definition, $R_{1(p)}$ is the proton inclusive differential cross-section properly normalized :

$$R_{1(p)} = \frac{d\sigma}{d^3p} \frac{1}{40\,mb} \cdot \tag{4}$$

Similarly $R_{1(n)}$ is the neutron inclusive differential cross-section normalized. To obtain $R_{2(p)}$ we consider a chain of two collisions

$$p \to p \to p \quad \text{and} \quad p \to n \to p.$$

Assuming $p \to n$ and $n \to p$ to have equal forward inclusive distributions, $R_{2(p)}$ can be obtained as a convolution integral of $R_{1(p)}$ on $R_{1(p)}$ plus a similar integral of $R_{1(n)}$ on $R_{1(n)}$ [see Remark (A) below]. $R_{3(p)}$, $R_{3(n)}$, etc., can be obtained in a similar way. In other words, starting with the experimental values of $R_{1(p)}$ and $R_{1(n)}$ one can calculate $R_{K(p)}$ and $R_{K(n)}$ through a stochastic convolution process. Such a calculation is in progress.

Unfortunately data on $R_{1(p)}$ and $R_{1(n)}$ are not very accurate and that hampers the calculation. It may therefore be of interest to simulate experimentally the multiple collision effects within a heavy nucleus by similar effects in a thick hydrogen target.

To this end consider a target of hydrogen of n nuclei $(cm)^{-2}$ in a beam of B protons. Let Xd^3p be the number of outgoing protons in d^3p. For a thin target

$$X \cong Bn\, d\sigma/d^3p\,. \tag{5}$$

For a thicker target, X is not linear in n because of multiple collision effects. It turns out that it is more interesting to study $e^{n\sigma}X$ rather than X, where $\sigma = 40$ mb is the nucleon-nucleon total cross-section. The successive derivatives at $n = 0$ of $e^{n\sigma}X$ will be defined as <u>multiple collision cross-sections</u> :

$$\frac{d\sigma}{d^3p} = \frac{d}{dn}\left(Xe^{n\sigma}B^{-1}\right), \tag{6}$$

$$\frac{d\sigma\text{-}\sigma}{d^3p} = \frac{d^2}{dn^2}\left(Xe^{\sigma n}B^{-1}\right), \tag{7}$$

$$\frac{d\sigma\text{-}\sigma\text{-}\sigma}{d^3p} = \frac{d^3}{dn^3}\left(Xe^{n\sigma}B^{-1}\right), \text{ etc.} \tag{8}$$

Thus

$$X = Be^{-n\sigma}\left[n\frac{d\sigma}{d^3p} + \frac{n^2}{2}\frac{d\sigma\text{-}\sigma}{d^3p} + \frac{n^3}{6}\frac{d\sigma\text{-}\sigma\text{-}\sigma}{d^3p} + \cdots\right]. \tag{9}$$

The value of $d\sigma\text{-}\sigma/d^3p$ is clearly measurable by studying the curvature of the X vs. n curve. Higher multiple collision cross-sections are difficult to measure.

Now the multiple collision process in a thick hydrogen target is also described by the showering process of Fig. 1. Thus the number of outgoing protcns is

$$\Sigma = B e^{-n\sigma} \sum_{m=1}^{\infty} \frac{(n\sigma)^m}{m!} R_{m(p)} \; . \tag{10}$$

To obtain this expression, we remember that the probability that the incoming proton goes through the target without collisions is $e^{-n\sigma}$, with one collision is $e^{-n\sigma}(n\sigma)$, with m collisions is $e^{-n\sigma}(n\sigma)^m/m\,!$. Comparing (10) with (6)-(8), we obtain

$$\frac{d\sigma}{d^3p} = \sigma R_{1(p)} \tag{11}$$

$$\frac{d\,\sigma{\cdot}\sigma}{d^3p} = \sigma^2 R_{2(p)} \text{ etc} \; . \tag{12}$$

(11) agrees with (4). <u>(12) provides a method for measuring</u> R_2.

In principle, $R_{K(p)}$ is measurable and (3) and (2) would lead to a calculation of the total of all multiple collision contributions to $d\sigma_A/d^3p$ for any nucleus A. In other words, the multiple collision effects in A are simulated by similar effects in thick hydrogen targets. In practice, since only $R_2(p)$ is really measurable in thick hydrogen targets, this procedure is only usable for nuclear targets in which the maximum path length inside the nucleus is $\leqq\sim 2.5$ times the mean free path. I.e., $2R_A/\lambda \leqq 2.5$, or $A \leqq 16$. For these light nuclei, only σ_1 and σ_2 are non-vanishing, as Eq. (2) indicates. We take as an approximation

$$\sigma_1(\sigma_1+\sigma_2)^{-1} = \int_0^{1.5} \kappa\, d\kappa \left[\int_0^{2R_A/\lambda} \kappa\, d\kappa\right]^{-1} = 2.3\, A^{-2/3} \; . \tag{13}$$

Thus
$$\sigma_1 = \pi R_A^2 (2.3\, A^{-2/3}) \; ,$$

and
$$\sigma_2 = \pi R_A^2 (1 - 2.3\, A^{-2/3}) \; . \tag{14}$$

Equations (3) and (12) give for $6 \leqq A \leqq 16$,

$$\frac{d\sigma_A}{d^3p} = \pi R_A^2 \left[2.3\, A^{-2/3} \frac{1}{\sigma} \frac{d\sigma}{d^3p} + \left(1 - 2.3\, A^{-2/3}\right) \frac{1}{\sigma^2} \frac{d\sigma \cdot \sigma}{d^3p} \right], \tag{15}$$

where σ and $d\sigma/d^3p$, $d\sigma{-}\sigma/d^3p$ are total and multiple collision cross-sections for hydrogen targets.

REMARKS

(A) - For Eq. (15) to be valid, coherent effects in nucleus A must be absent. This means that one only concentrates on $p_\perp > 2$ GeV/c, say. Furthermore, another important assumption has been made : inside nucleus A, successive collisions take place within a few Fermis of each other, while for the thick hydrogen target, successive collisions take place over spatial separations as long as centimetres or metres. Some hyperon resonances may have a decay path length in between these two orders of magnitude of spatial separations. Their contributions may cause (15) to be invalid. We guess, however, that such contributions are not important and (15) is largely correct for large $p_\perp$.

(B) - In Eqs. (4) and (10), elastic scattering is included.

(C) - In Eq. (2) we have neglected the slight sponginess of the nucleus and regarded the nucleons in the nucleus as packed spheres. The small amplitude oscillations of the nucleons around their packed positions do not lead to any appreciable changes in (2).

(D) - Can one simulate the multiple collision effects inside a target of heavy nucleus A with thick targets of light nuclei of atomic number α, such as $Be(\alpha = 9)$? To analyze this situation we observe that (9), (11) and (12) remain valid, with σ replaced by σ_α, the total inelastic cross-section in p-Be collision. We ignore elastic p-Be collisions altogether since they lead to essentially no deflection of the fast nucleon beam. $(d(\sigma - \sigma)_\alpha)/d^3p$ now describes multiple scattering effects due to collisions of the incoming nucleon with two Be nuclei successively.

For p-A collisions we envisage the heavy A nucleus as consisting of many Be nuclei. Equation (13) then becomes, for $6 \leqq A/9 \leqq 16$,

$$\sigma_1 [\sigma_1 + \sigma_2]^{-1} = \int_0^{1.5} K dK \left[\int_0^{\beta} K dK \right]^{-1}$$

where β is the number of Be constituents along the diameter of A. I.e., $\beta = (A/9)^{1/3}$. Thus

$$\sigma_1 (\sigma_1 + \sigma_2)^{-1} = 2.3 (A/9)^{-2/3} \tag{13'}$$

which happens to be quite similar to (13). We thus have

$$\frac{d\sigma_A}{d^3p} = \pi R_A^2 \left\{ 2.3 \left(\frac{9}{A}\right)^{\frac{2}{3}} \frac{1}{\sigma_\alpha} \frac{d\sigma_\alpha}{d^3p} + \left[1 - 2.3 \left(\frac{9}{A}\right)^{\frac{2}{3}}\right] \frac{1}{\sigma_\alpha^2} \frac{d(\sigma - \sigma)_\alpha}{d^3p} \right\} \tag{15'}$$

for $6 \leqq A/9 \leqq 16$.

This formula is easier to test experimentally than (15).

(E) - How well does a thick target of one material simulate another thick target of a different material if they have the same thickness in gr/cm^2 ? We assume both to be non-hydrogenic. To discuss this, consider a slab of n nuclei/cm^2 of material with atomic number A. The probability of hitting m nuclei in one traversal is a Poisson distribution :

$$e^{-n\sigma_A}(n\sigma_A)^m/m! \quad , \quad (\sigma_A = \pi R_A^2). \tag{16}$$

For one traversal hitting 1 nucleus, the path length L inside the nuclear matter is distributed according to (2) :

$$\begin{aligned} p(L)dL &= (L/2R_A^2)dL , \qquad 0 \leq L < 2R_A , \\ &= 0 \qquad\qquad\qquad\qquad L \geq 2R_A . \end{aligned} \tag{17}$$

For one traversal hitting in nuclei, the same distribution will be called $p_m(L)dL$ where

$$p_m(L) = \int\cdots\int \delta\left(\sum_j^m L_j - L\right) \prod_{j=1}^{m} p(L_j)\, dL_j \; . \tag{18}$$

Thus for the whole slab, the distribution of the total path length inside nuclear matter is $P(L)dL$ where

$$P(L) = \sum_{m=0}^{\infty} e^{-n\sigma_A}(n\sigma_A)^m p_m/m! \tag{19}$$

in which we put

$$p_1 = p , \quad p_0 = \delta(L) . \tag{20}$$

Notice

$$\int_0^\infty P(L)\,dL = 1 \tag{21}$$

We can now compute averages :

$$\bar{L} = \int_0^\infty P(L)\,dL = 4\pi n R_A^3/3 \quad , \tag{22}$$

$$\overline{L^2} - \bar{L}^2 = 3\bar{L} R_A/2 \quad . \tag{23}$$

In these computations we have used

$$\int_0^\infty p_m(L)\,L\,dL = m\,(4R_A/3)\,,$$

$$\int_0^\infty p_m(L)\,L^2 dL = m\,(2R_A^2) + m(m-1)(4R_A/3)^2 \,.$$

Equations (22) can also be written as

$$\bar{L} = (T/42)\,\lambda \tag{22'}$$

where $\lambda = 2.87$ Fermi is the nucleonic mean free path in nuclear matter and T is the thickness in gr/cm^2.

Two slabs of different non-hydrogenic material of the same thickness T thus share the same mean path length $\bar{L}$ inside nuclear matter. Their root mean square deviation of L from $\bar{L}$ are in the ratio

$$(R_A/R_{A'})^{\frac{1}{2}} \qquad (24)$$

according to (23). Equation (24) is independent of the thickness. For example, two slabs of W and Be of the same thickness in gr/cm^2 have the same $\bar{L}$, but their root mean square fluctuation in L is in the ratio $\sqrt{184/9}$. The fluctuation is larger in the W slab because in it the nuclear matter is more bunched. Because of multiple collision effects, the ratio of the yield from the two slabs is unity only if the fluctuation in L becomes unimportant compared with the mean value. This is so only for very thick slabs.

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