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MESON SPECTROSCOPY AND e^+e^- ANNIHILATION INTO HADRONS IN A GAUGE THEORY WITH LINEAR BINDING POTENTIAL

Z. Kunszt *)

CERN - Geneva

INTRODUCTION

The recent discovery of two neutral heavy bosons stimulated [1] much theoretical work to understand their nature, origin and relation to the previously developed phenomenology. The four quark (charm) model, especially its colour version [2], seems to be in acceptable agreement with the data on $\psi(3.1)$, $\psi'(3.7)$, dimuon production by neutrinos, possible existence of new baryons, etc. [3].

Preceding the discovery of the ψ's, a new concept, namely the quark confinement has been partly developed. According to this idea the quarks at short distances have weak forces, but the long range part of their interactions is strong, leading to quark trapping in hadrons.

The $SU(4) \otimes SU(3)$ coloured gauge model seems to be consistent with this idea. The corresponding field theory is asymptotically free (explaining scaling in electron and neutrino production) at short distances, nevertheless at large distances it has serious infra-red divergences the nature of which is not yet understood. Several arguments have been, however, constructed to suggest that these infra-red divergences may be responsible for quark confinement (infra-red slavery).

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*) On leave from Department of Atomic Physics, L. Eötvös University, Budapest, Hungary.

Since the infra-red divergences prevent to perform systematic field-theoretic calculations, a phenomenological scheme has been developed for the ψ's : the charmonium [2]. In a recent attempt, De Rujulà, Georgi and Glashow [4] proposed to extend the charmonium idea to the charmed and uncharmed spectroscopy as well. In order to have quantitative application they have assumed that : i) the effective long range binding potential is spin-independent ; ii) a non-relativistic description with first order $(1/c^2)$ corrections, as determined by one gluon exchange, is adequate approximation in calculating the hadron masses. SU(4) breaking is taken into account by allowing different masses for the various quark types (p, n, λ, c). These assumptions are partly motivated by the success of the $SU(6) \otimes O(3)$ classification. As to assumption i) we think that its consequences are worth while to explore although we are aware of the fact that its validity is an open question in the underlying field theory. Concerning assumption ii), it has an important practical consequence, namely the one-gluon exchange diagram, up to first order in $1/c^2$, leads to the standard Breit-Fermi Hamiltonian (H_{BF}), therefore in a perturbative treatment, the relativistic corrections are determined by the matrix elements of H_{BF} (computed by use of non-relativistic Schrödinger wave functions).

To explore the consequences of that hierarchy of assumptions two different lines can be followed.

One can parametrize the matrix elements of the Breit-Fermi Hamiltonian in a general way assuming degenerate multiplets and deduce mass splittings independent of the explicit form of the binding potential and the wave functions. Alternatively assuming a specific potential the necessary matrix elements can explicitly be calculated and up to first order relativistic corrections a complete diagonalization can be performed.

The first line of approach, pursued by De Rujulà, Georgi and Glashow with good results, is more general, but it has the shortcomings that many parameters should be introduced : the bound states containing a charmed quark are essentially uncorrelated with the usual hadrons because of the large charmed-uncharmed quark mass differences ; furthermore, the multiplets of different orbital momenta must be treated separately.

The second method, although it is less general in character, but it involves less parameters, correlates the heavy and light mesons and allows us to see more clearly whether a non-relativistic method is applicable or not. Furthermore, having explicitly the different wave functions, various decay modes can also be estimated.

In this talk I report on results obtained in collaboration with Barbieri, Gatto and Kögerler, pursuing the second line of approach [5]. We have used a linear binding potential [6),7] and calculated the light and heavy meson masses, their leptonic width, mixing effects and we have investigated the consistency of the non-relativistic approximations.

The results show a remarkable agreement with the experimental data for meson masses $\gtrsim$ 1 GeV, whereas the breakdown of the non-relativistic expansions puts limits on the analysis for the lighter particles.

The ratio $R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ has also been estimated at and above the charm threshold assuming the dominance of the ψ, ψ', ψ'', ψ''' vector meson poles and that the two-body final states of the charmed particles account for the structure of the ratio $R(s)$ in the region $\sqrt{s} \simeq 3.5 - 5.5$ GeV. Important interference contributions have been found which may be used to obtain an explanation of the experimental data in this region.

An essential feature of the calculation is that it involves altogether only four free parameters.

MASS SPECTRUM

We have chosen to work with the following potential

$$V(r) = \lambda r - V_0 - \frac{4}{3}\alpha_s/r \qquad (1)$$

An r behaviour seems to be suggested by field theoretic investigations and bag models as well [6),7)]. The $1/r$ term is given by the one-gluon exchange ; the constant V_0 determines how to interpolate between the r and $1/r$ asymptotic terms. The $4/3$ factor in front of the $1/r$ term is given by the $SU(4) \otimes SU(3)$ group structure. As to α_s appearing in (1), we have made the simplifying assumption that it is the strong fine structure constant, corresponding to a bound state of squared invariant mass M^2 ($\alpha_s = \alpha_s(M^2)$). The mean momenta carried by the constituent quarks (and so the mean momenta flowing through the gluon line) are decreasing with their masses. Therefore, using asymptotic freedom we obtain increasing values of $\alpha_s(M^2)$ for bound states of decreasing mass. Typically we have taken for a $c\bar{c}$ bound state $4/3\,\alpha_s^c = 0.27$, for a $\lambda\bar{\lambda}$ bound state $4/3\,\alpha_s = 0.36$ and for a $d\bar{d}$ bound state $4/3\,\alpha_s^d = 0.42$. As to the quark masses, the values of m_d and m_λ are taken from the general analysis of baryons by De Rujulà, Georgi and Glashow [4)] ($m_d = 0.34$ GeV, $m_\lambda = 0.54$ GeV). The value of the charmed quark mass, furthermore the parameters λ and V_0, have been fitted to account correctly for $m(\psi) = 3.095$ GeV, $m(\psi') = 3.684$ GeV and the leptonic width of the ψ particle $\Gamma(\psi \to e^+e^-) = 5.2$ keV.

We solved the non-relativistic Schrödinger equation with potential (1) numerically, then a perturbation calculation has been performed by the Breit-Fermi Hamiltonian given by the one-gluon exchange diagram as order $O(1/c^2)$ corrections. The Hamiltonian has the form

$$H = \frac{\vec{p}^2}{2m} + V(r) + 1/c^2\, H_{BF} + O(1/c^4)$$

The $O(1/c^2)$ corrections are determined by the matrix elements of H_{BF} between the non-relativistic wave functions.

Since H_{BF} contains spin-spin and spin-orbit forces, the relativistic corrections lead to splittings between the spin singlet and spin triplet states, furthermore between states of different angular momentum. The Breit-Fermi Hamiltonian has non-vanishing off diagonal matrix elements between the 1^{--} (s wave) and 1^{--} (d wave) states as well as between the charged elements of the 1^+ (p wave, spin triplet) and 1^+ (p wave, spin singlet) 16 plets, leading to non-trivial mixing effects.

This perturbative calculation has been performed for different reduced masses and different α_s, corresponding to $c\bar{c}$, $\lambda\bar{\lambda}$, $c\bar{\lambda}$, $c\bar{d}$, λd and dd type bound states.

The obtained energy levels for the S wave spin triplet vector meson states (M_V) and for their pseudoscalar partners (M_P) are given in Table I. For the $c\bar{c}$ bound states the ground states and the first three radial excitations are calculated. For the other type of bound states, only the ground states and the first radial excitation have been considered. In the column $E_{NR}+V_o$ the non-relativistic energies (without the m_1+m_2 rest mass contributions), in the column E_{RC} the full relativistic corrections, (including the spin-independent corrections as well) and in the column E_{BF} the spin-dependent relativistic corrections are quoted. In the last column, we give the values of the parameter

$$\xi = \frac{\left\langle \frac{1}{8}\left(\frac{1}{m_1^3}+\frac{1}{m_2^3}\right)(\vec{p}^{\,2})^2\right\rangle}{\left\langle \frac{1}{2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\vec{p}^{\,2}\right\rangle}$$

which characterize the system with respect to the validity and consistency of the non-relativistic approximation.

From the numbers given in Table I, we can conclude that, although the relativistic corrections are not negligible (especially for the radial excitations) the cc, $\lambda\lambda$, cλ type bound states may be regarded as non-relativistic systems. For bound states made up only of charmed quarks, the results are in agreement with previous purely non-relativistic calculations. The strange quark bound states (φ and x^{O}) fit also fairly well with the calculation.

The agreement with the experimental masses of the remaining low-lying vector mesons (ρ, ω, K*) is also acceptable. But we should recall that the relativistic corrections to all the bound states containing at least one d type quark are quite large, the value of the ξ parameter is near 1 and it usually exceeds 1 for the first radial excitations. This indicates that despite acceptable agreement, the non-relativistic description is presumably not adequate for these bound states. A Dirac equation description could be more relevant for the cλ and cd type bound states, where recoil effects are negligible as compared to the relativistic motions of the λ or d quarks.

In Table II, the energy levels of the P wave ($J^{PC} = 2^{++}$, $1^{++}, 1^{+-}, 0^{++}$) bound states are given. In addition, the D wave excitations of the $c\bar{c}$ bound states are also calculated. We should emphasize that at this stage of calculation, no new free parameter is introduced. Again the relativistic corrections to the states containing at least one d type quark are large. The agreement with the experimental data (when available on a sufficiently sound basis) is acceptable.

We remark, in particular, the good agreement for the f' - φ "hyperfine" splitting

$$(f' - \varphi)_{th} \simeq (f' - \varphi)_{exp} \simeq 490 \text{ MeV}$$

as well as for the $K^{**}-K^*$ splitting

$$(K^{**}-K^{*})_{th} \simeq (K^{**}-K^{*})_{exp} \simeq 530 \text{ MeV}$$

As to the mixing, we obtained that the $\psi_D - \psi'_S$ mixing is negligibly small. The mixing of the charged P wave spin triplet - spin singlet mesons is quite important, however, the experimental information here is not accurate enough to have any test of this prediction.

DECAY WIDTHS

In a non-relativistic approximation the various decay matrix elements are given by the proper overlap integrals of the initial and final state non-relativistic wave functions. We can then calculate the leptonic, semi-leptonic, radiative and two-body decay matrix elements.

In Table III, the different leptonic widths are given. The e^+e^- decays of the neutral vector mesons have been calculated by use of the formula

$$\Gamma(V \to e^+e^-) = 3e_V^2 \alpha^2 \frac{16\pi}{3} \frac{|\psi(0)|^2}{m_V^2}\left(1 - \frac{16}{3\pi}\alpha_s(m_V^2)\right)$$

where the last factor in parentheses accounts for a first order radiative corrections.

As to the weak decays of the s wave mesons, we assume point-like couplings without axial vector renormalization, therefore we have used the formula

$$\Gamma(M \to \ell\nu) = 3\,\frac{G^2}{\pi}|\psi(0)|^2\left(1 - \frac{m_\ell^2}{M^2}\right)^2 * \left[\frac{2M^2}{3}\left(1 + \frac{m_\ell^2}{2M^2}\right)\frac{\langle \vec{S}^2\rangle}{2} + m_\ell^2\,\frac{\langle 2 - \vec{S}^2\rangle}{2}\right]$$

where g should be red as $g_F \cos\Theta$ for pn and cλ type states and $G_F \sin\Theta$ for pλ and cn type bound states, M is the meson mass, $< \vec{S}^2 > = S(S+1)$ where $S = 1$ for spin triplets and $S = 0$ for spin singlets. We would like to remark that the first term is proportional to M^2 and the second term to M_e^2. Therefore, the vector mesons like F^+ have by a factor of ~400 larger leptonic decay than the F mesons and $\sim 10^4$ times less than the D mmesons. If it would happen that $F^* - F$ mass splitting is smaller than the predicted one (< 5 MeV), the $F^* \to e^- e$ decay and high number $e^- \mu^+$ events would be recorded. Such a situation could account for the rate of the $e^- \mu^+$ events

As to the semi-leptonic and non-leptonic weak decays, here we only make the remark that as far as the non-relativistic description is more adequate for the charmed mesons, there is no natural argument for the dynamical enhancement of the non-leptonic decay mode of the charmed mesons [8].

Concerning the two-body hadronic decays we restrict our investigations only to the decay modes of the $c\bar{c}$ type bound states at and above the charmed threshold. Here, our main purpose is to investigate whether the mesured shape and
annihilation near
body charmed meson final states, assuming that the form factors of the charmed mesons are dominated by the vector meson poles (ψ, ψ',...). In a non-relativistic framework, the three-meson vertices ($\psi \to DD$, $\psi' \to DD$, $\psi \to FF$, etc.) can be calculated by use of a generalized version of the quark pair creation model [9] where the meson vertices are defined by the quark diagram of Fig. 1. The bubbles denote the products of spin SU(4) wave functions and of the spatial non-relativistic wave functions. The three meson couplings will involve an over-all universal coupling strength γ for the creation of the light quark pair in a spin triplet p wave state of zero total angular momentum.

The three meson couplings are proportional to the overlap integrals which are given as follows

$$I_{ABC} = \int_0^\infty r_1^2 dr_1 \left(2\psi_A(r_1) + \frac{r_1}{3}\psi'_A(r_1)\right) \cdot \int_0^\infty r_2^2 dr_2 \, \psi_B^*(r_2) *$$

$$* \int_{-1}^{1} dz \, \psi_C^*\left(\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 z}\right)$$

We should emphasize that we have to introduce only one new free parameter, namely γ, to calculate all the meson vertices. For sake of illustration, we quote the formula for the vector → 2 vector decay mode

$$\langle V_2(k_2, e^{(2)}) V_3(k_3, e^{(3)}) | V_1(k_1, e^{(1)}) \rangle =$$

$$= \frac{\pi^{3/2}}{\sqrt{2}} \gamma \, m_{V_1}^{3/2} \cdot I_{V_1 V_2 V_3} * \left[2(e^{(2)} e^{(3)})(e^{(1)} k_3) - \right.$$

$$\left. - (e^{(1)} e^{(3)})(e^{(2)} k_1) + (e^{(1)} e^{(2)})(e^{(3)} k_2) \right]$$

The effective three-meson coupling and the meson mass spectrum having been calculated, the various partial widths of the ψ, ψ', ψ'', ψ''' resonances and the shape of the ratio

$$R(s) = \frac{\sigma(e^+e^- \longrightarrow \text{hadrons})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)}$$

can be given in terms of the only unknown parameter γ. In computing R a constant background B_{eq} due to uncharmed hadron production is added with a value B_{eq} 2.5 which is suggested not only by the experimental value of R at $\sqrt{s} < 3.7$ GeV, but also by the light quark parton model (including asymptotic freedom corrections).

In the usual applications of the VDM, the interference between the different vector meson poles is negligible. Conversely, the validity of the VDM is questionable and its application is not free from ambiguities if the interference is not negligible. Since the proper unitarization of the problem (by a multichannel Omnès equation approach to the form factors of the final state particles) seems to be practically untreatable, we have taken a purely phenomenological attitude. In order to simulate the interference of the weak meson pole contributions of the vector meson poles bothaamong each other and with the non-resonating background, we have introduced three extra phase factors. In the amplitude, the Breit-Wigner poles corresponding to the three radial excitations $\psi'(3.7)$, $\psi''(4.1)$, $\psi''(4.45)$ get multiplied by $\exp(i\varphi')$, $\exp(i\varphi'')$, $\exp(i\varphi''')$, respectively.

In Fig. 2, the curve (A) was obtained by $\gamma = 0.45$ and by neglecting the interference effects. The curve (B) on the other hand, represents a best fit to the data by four free parameters, allowing for the above-mentioned phases (the fitted values are $\gamma = 0.57$, $\varphi' = 2.7$ rad, $\varphi'' = 1.5$ rad, $\varphi''' = 3.76$ rad). The curve (A) in Fig. 3 was calculated putting all phases equal to zero which amounts in taking strictly the prediction of the non-relativistic model even for the interference terms. Finally, the curve (B) in Fig. 3 was obtained by requiring maximum constructive interference (with $\gamma = 0.40$).

We can see that even though the leptonic width of ψ'' is only about 2 keV in this calculation, it is easy to make a fit to the data, if we interpreted them as sum of several resonances interfering with the background and each other.

	M_V (1^{--})	$M_V^{exp}(1^{--})$	M_P (0^{-+})	$M_P^{exp}(0^{-+})$	$E_{NR} + V_o$	E_{RC}	E_{BF}	ξ
cc	3095	ψ (3095)	3041	-	607	- 29	6	0.17
(cc)'	3680	ψ'(3684)	3635	-	1255	- 94	- 16	0.19
(cc)"	4111	ψ''(4100)?	4061	-	1766	- 175	- 28	0.25
(cc)'''	4471	-	4435	-	2239	- 287	- 58	0.30
cλ	2105	-	2034	-	832	- 144	23	0.60
(cλ)'	2607	-	2543	-	1616	- 427	- 12	0.75
cd	1912	-	1842	-	995	- 301	33	0.90
(cd)'	2200	-	2137	-	1879	- 898	10	1.44
$\lambda\lambda$	1131	φ (1020)	959	X^o(958)	978	- 163	11	0.53
($\lambda\lambda$)'	1685	-	1541	E (1416)?	1883	- 517	- 79	0.70
λd	918	K*(892)	678	K (494)	1083	- 281	18	0.90
(λd)'	1315	-	1118	-	2060	- 864	- 85	1.13
dd	716	ω (783) $\vec{\rho}$ (770)	448	η (549) $\vec{\pi}$ (140)	1168	- 370	- 19	0.90
(dd)'	1033	$\vec{\rho}'$(1250)?	755	-	2215	-1100	-143	1.30

TABLE I : Masses (MeV) of the S wave bound states.

	$M(0^{++})$	$M^{exp}(0^{++})$	$M(1^{++})$	$M^{exp}(1^{++})$	$M(1^{+-})$	$M^{exp}(1^{+-})$	$M(2^{++})$	$M^{exp}(2^{++})$	E_{NR}	ξ
$(cc)_P$	3425	-	3466	-	3482	-	3503	-	1025	0.11
$(c\lambda)_P$	2385	-	2456	-	2488	-	2528	-	1324	0.49
$(cd)_P$	2125	-	2205	-	2246	-	2294	-	1539	0.93
$(\lambda\lambda)_P$	1326	-	1480	D (1285)	1540	-	1619	f' (1516)	1540	0.45
$(\lambda d)_P$	1030	K (~1300)?	1220	Q_A(1242)?	1297	Q_B(~1300)?	1400	K**(1420)	1680	0.70
$(dd)_P$	781	S*(993)? $\vec{\delta}$ (976)	1039	$\vec{A}_1$(1100)	1141	$\vec{B}$ (1237)	1274	f (1270) $\vec{A}_2$ (1310)	1791	0.72
	$M(1^{--})$	$M(2^{--})$	$M(2^{-+})$	$M(3^{--})$						
$(cc)_D$	3778	3794	3798	3811					1351	

TABLE II : Masses (MeV) of the P and D wave bound states.

	ψ	ψ'	ψ''	ψ'''	φ	ρ	ω	F	D	K	π
Γ_e	5.2 keV	3.1 keV	2.7 keV	1.7 keV	2.1 keV	10.6 keV	1.2 keV	2.7×10^{10}	9.1×10^{8}	6.7×10^{8}	8.0×10^{9}
Γ_e^{exp}	5.2 keV	2.2 keV	-	-	1.44 keV	6.0 keV	0.8 keV	-	-	5.13×10^{7}	3.84×10^{7}
$4\pi\|\psi(0)\|^2$ $(GeV)^3$	0.808	0.679	0.751	0.538	0.209	0.110	0.110	0.335	0.201	0.110	0.110

TABLE III : Leptonic widths of the S wave bound states (weak decays in sec^{-1}).

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FIGURE CAPTIONS

Figure 1 The three-meson vertex.

Figure 2 The ratio $R(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. The curve (A) (full line) has been calculated without the interference contributions (with $\gamma = 0.45$). The dotted curve (B) has been obtained by allowing for three arbitrary relative phases between the resonances ψ, ψ', ψ'', ψ''' and performing a least square fit ($\gamma = 0.57$).

Figure 3 R(s) in the VDM model. The curve (A) (full line) is the strict prediction of the non-relativistic model (real coupling constants, destructive interference, $\gamma = 0.40$). The dotted curve (B) has been computed with maximum constructive interference (with $\gamma = 0.40$).

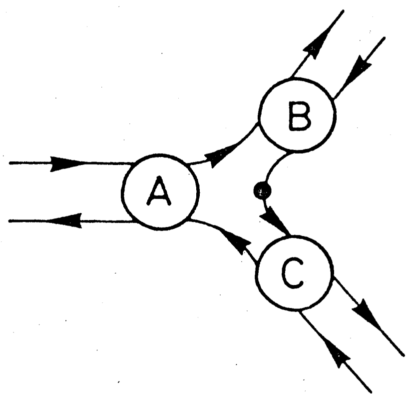

FIG. 1

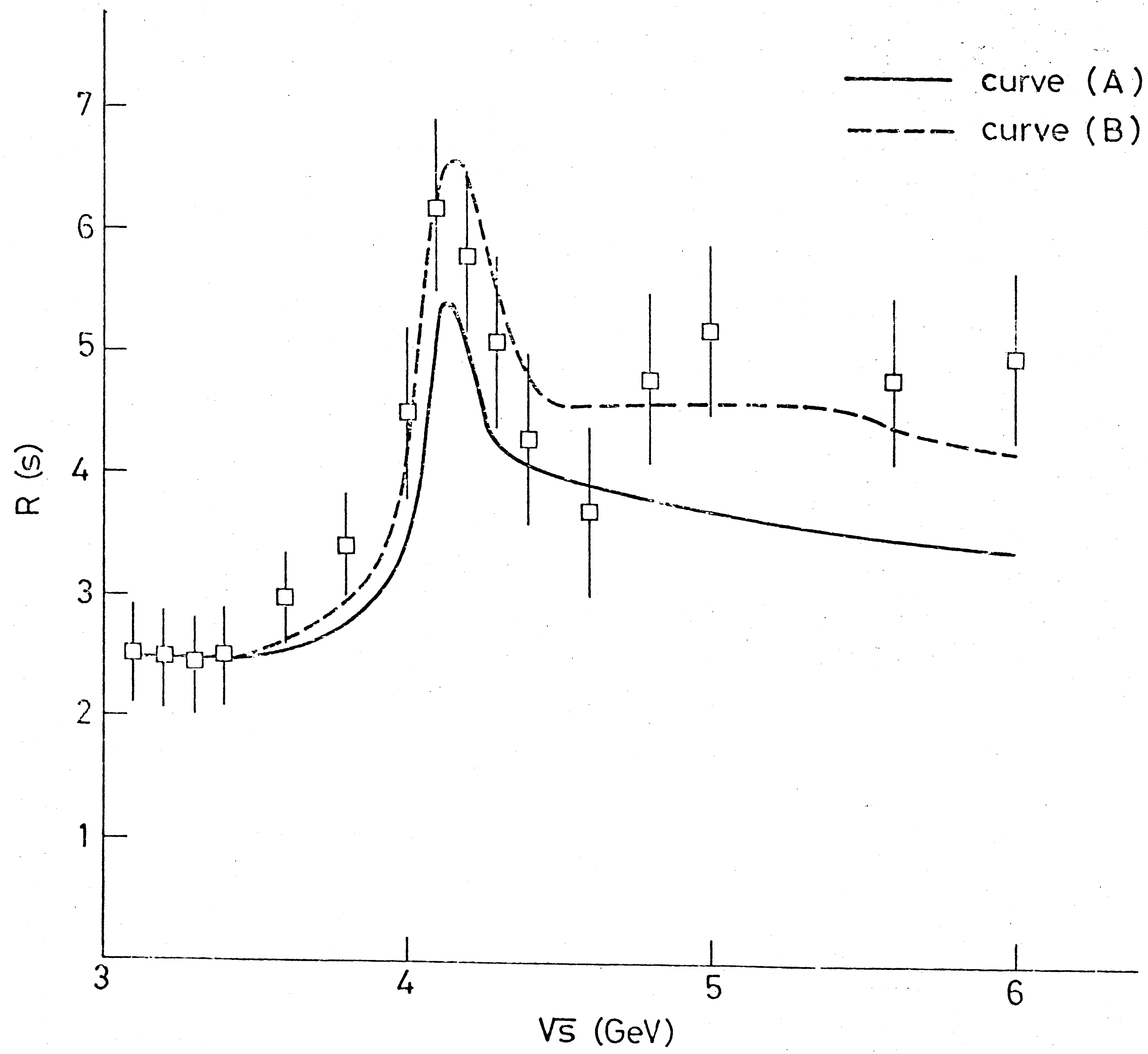

FIG. 2

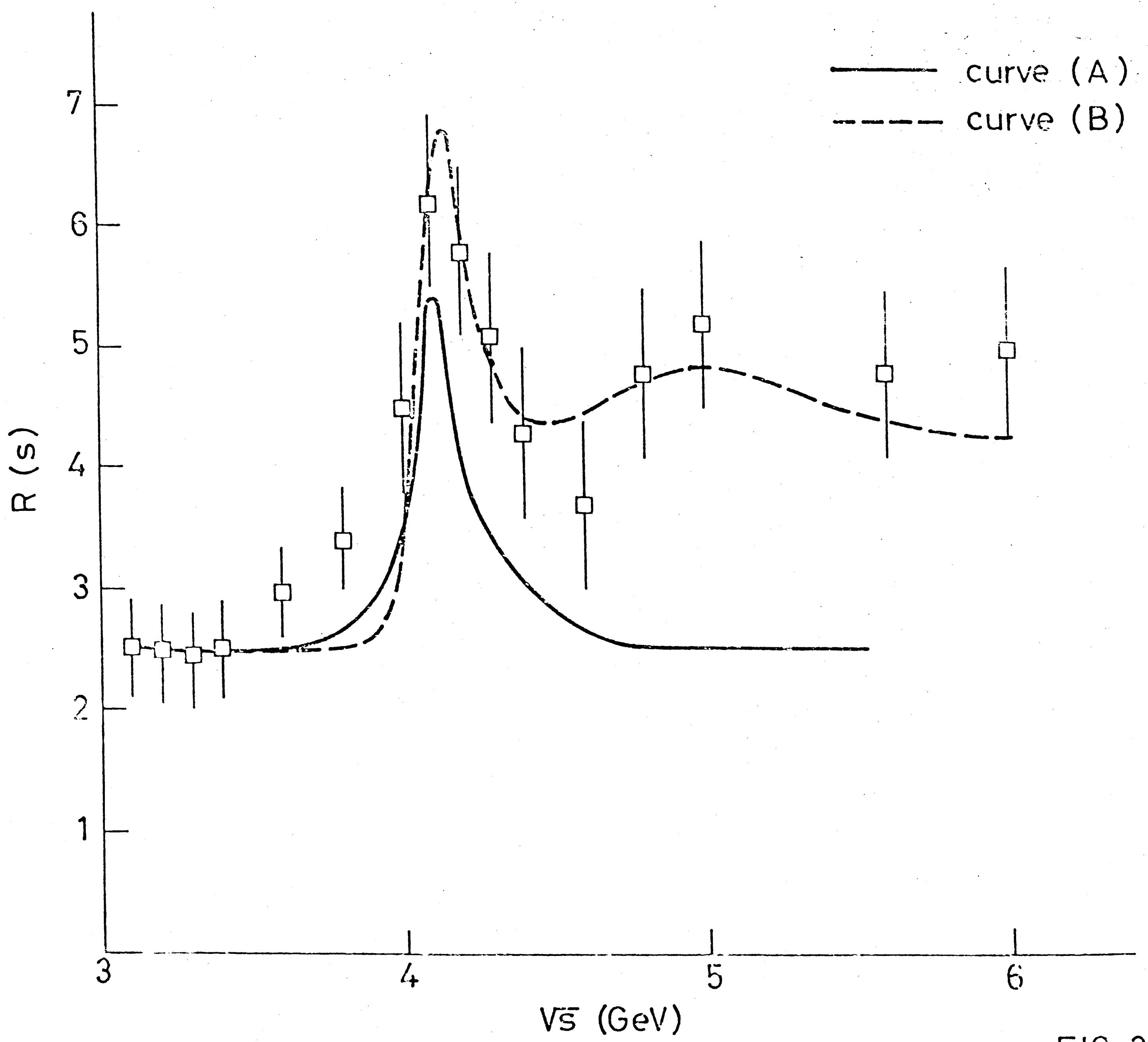

FIG. 3