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Computational Design of Deployable Auxetic Shells

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Abstract

We propose an interactive computational design method for deployable auxetic shells. We realize deployable auxetics as triangular linkages that can be actuated with simple expansive mechanisms to assume a desired freeform target shape. The core feature of these structures is that the target shape is directly and uniquely encoded in the 2D linkage layout. As a consequence, the structure can be fabricated and assembled in the plane and automatically deployed to its 3D target configuration without the need for any scaffold, formwork, or other temporary support structure. We focus on automatic deployment via inflation or gravitational loading for which a rigorous theoretical analysis has been given in prior work. Our paper builds upon these results and presents optimization-based direct manipulation tools to edit and adapt an auxetic linkage structure to effectively explore design alternatives. In addition, our solution enables simulation-based form-finding, where the desired target surface is interactively constructed using the deployment mechanism as a form-finding force. We present several design case studies that demonstrate the effectiveness of our approach and highlight potential applications in architecture.

Key words: auxetic structures, deployable shells, form finding, computational design, numerical optimization

1 Introduction

Architectural structures are commonly composed of multiple elements that are assembled on-site. Construction is executed by incrementally placing components at their target 3D location, using scaffolding or other support mechanisms to guide element positioning and maintain structural stability during intermediated stages of the assembly. Especially for intricate free-form geometry, the complexities of this process can pose severe challenges. Deployable structures offer an interesting alternative for construction. They typically can be assembled in a significantly simpler state and then deform to the desired target shape. A prominent example is elastic grid shells that can be assembled on the ground and mounted into a double-curved form, see Lienhard (2014) .

We propose a computational design system for a new type of deployable structure based on a triangular auxetic linkage. Our structures can be fabricated and assembled in the plane and deployed to their target position using either inflation or gravity. No additional guiding scaffold is required because the target shape is *directly encoded* in the planar assembly. The key concept is a spatially graded auxetic pattern, where individual triangular elements are scaled to *program* the maximal local expansion factor required to achieve the global target shape. Paired with an area-expanding deployment, such as air-inflated cushions or gravitational loading, this yields a simple and robust way to realize double-curved surface structures.

Deployable auxetics offer a number of benefits:

- Form-defining deployment: The double-curved target shape is automatically achieved via expansive deployment from a planar configuration. Inflation or gravitational loading (for height field geometry) can be used to maximally stretch the material everywhere, which then constrains the surface to the desired target configuration.
- Simple fabrication: The geometric simplicity of the auxetic linkage directly transfers to fabrication. Variable-sized triangles can easily be cut using CNC fabrication technology from a wide variety of approximately inextensible base materials, such as fabrics, wood, metals, or plastics. Mass fabrication of joints is possible since all node connections are identical.
- Rich geometry: Deployable auxetics admit a rich and well-defined design space, enabling new forms beyond the existing classes of structures deployable from planar rest states.

This paper complements the work of Konaković-Luković et al. (2018) who proposed a post-rationalization process to find a deployable auxetic linkage for a fixed input design surface. While post-rationalization is an important design tool, it offers limited support for evaluating design alternatives or engaging in material- and construction-aware exploration. The functional and aesthetic properties of the resulting auxetic linkage are difficult to anticipate when designing the required

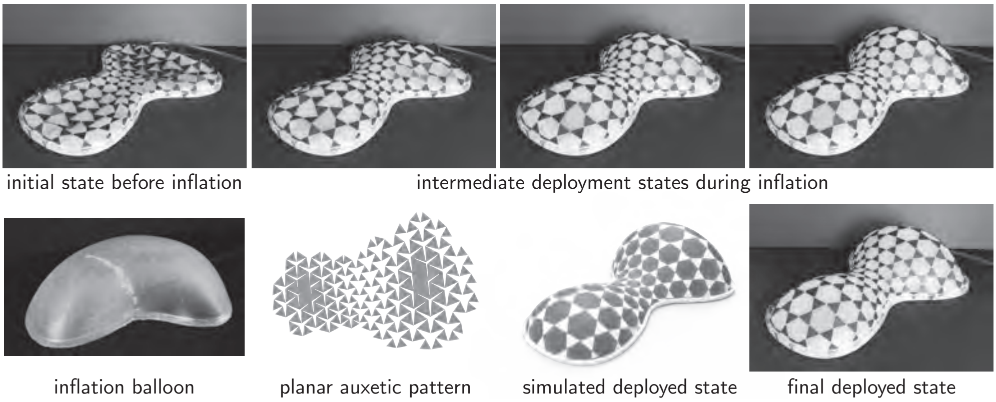

Figure 1: Physical prototype with inflation deployment, Konaković-Luković et al. (2018). The graded auxetic pattern has been laser cut, mounted onto a support frame and inflated with a generic rubber balloon.

reference geometry. In particular, the sizing of triangles and specific boundary alignment result from a global optimization that does not necessarily yield easily foreseeable results. It is therefore beneficial to provide direct manipulation tools to further edit and adapt the optimized structure to better meet the design goals. Our work introduces such direct editing operations. The presented computation-assisted design system allows for effective design space exploration of deployable auxetic structures and gives the designer full control of the final deployed surface geometry. In addition, our approach provides tools for computational form-finding, where the desired target surface is interactively constructed using the deployment mechanism as a form-finding force.

2 Related Work

To put our work into context, we briefly review related work on deployable structures and auxetic materials. We refer the reader to Konaković-Luković et al. (2018) for additional discussion of prior work, particularly methods for material-aware post-rationalization in computer graphics and digital fabrication.

The concept of kinematic deployment is well studied in architecture. For large-scale structures, elastic grid shells are probably the most prominent example. Composed of interconnected elastic beams, an elastic grid shell achieves its desired target shape by active bending, Lienhard (2014). Common methods of erection include lifting with cranes or various types of scaffolding or mechanical formwork. Erection of elastic grid shells via inflation has been discussed in Quinn and Gengnagel (2014), where the authors identify a number of potential benefits in terms of safety, construction time, and cost. Form-finding for elastic grid shells is also an active topic in material science; see, for example, the recent work of Baek et al. (2017). Deployable structures are also used for various building components. For example, Hannequart et al. (2018) investigate the use of shape memory alloys for deformable facade shading devices.

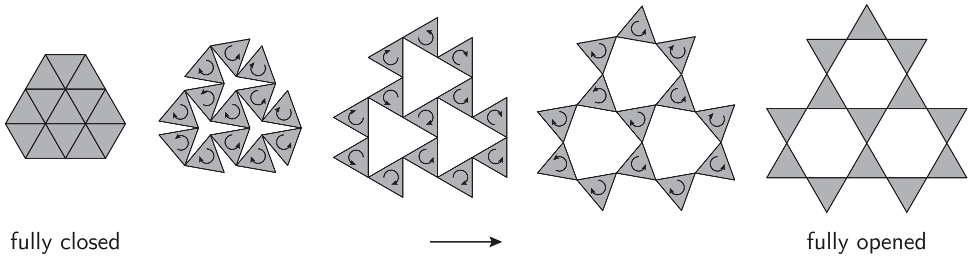

Figure 2: Our auxetic linkage is defined as a tri-hexagonal pattern. A uniform linkage can transition in the plane between a fully closed state (left) and a fully opened state (right) by rotating triangles around their connecting vertices. This expansion increases total area by a factor of four, which corresponds to a scaling of length by a factor two.

Auxetic meta-materials have been extensively studied in material science; see Saxena et al. (2016) for a comprehensive review. Konaković et al. (2016) proposed an optimization method for designing curved target surfaces that can be fabricated by deforming flat auxetic sheets. In this method, the auxetic structure is assumed to be spatially homogenous and have the same physical properties everywhere. Deforming such a 2D sheet material to the desired 3D shape is a complex manual process that requires a guiding surface or scaffold. Uniform auxetic materials have also been studied in Naboni and Sortori Pezzi (2016) to design bending-active grid shells. Spatially graded auxetics have been explored for freeform reinforced concrete components by Friedrich et al. (2018). They introduce an iterative evolutionary optimization process to find a planar pattern that conforms to a given target shape when expanded fully. The idea of optimizing the spatial layout of flat-produced patterns has also been studied by La Magna and Knippers (2018). They investigate how to induce controlled curvature through elastic bending of spatially graded cellular structures.

3 Programmable Auxetics

In this section, we describe the basic principles of deployable auxetic linkages and briefly review the post-rationalization approach presented in Konaković-Luković et al. (2018). Auxetic linkages are initially planar assemblies of rigid triangles that connect at hinge vertices in the specific arrangement shown in Figure 2. This arrangement allows the triangles to freely rotate around the hinge points to form openings, uniformly expanding the structure in all directions while resisting shear deformations. This uniform expansion behavior indicates the pattern has an effective Poisson's ratio of -1 (making it an *auxetic* structure) and offers a key advantage for architectural applications: it allows the linkage to be shaped into double-curved surfaces, unlike inextensible sheets of material, which can only bend into developable surfaces.

As the linkage progressively expands, eventually its openings become regular hexagons, and its pattern of rigid triangles and holes forms a trihexagonal tiling known as a

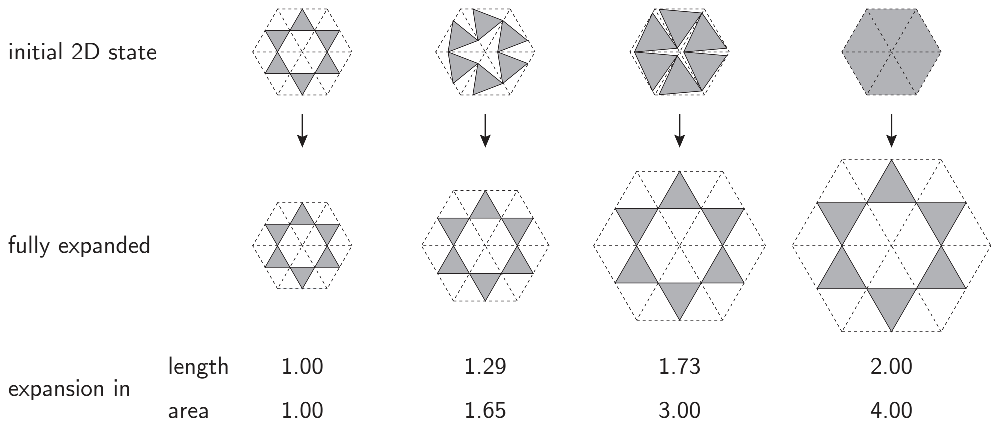

Figure 3: To vary the maximal possible expansion rate we can pre-stretch the linkage in the initial 2D state by shrinking and rotating the triangles appropriately. This allows controlling the deployed expansion factor within the range of one, when already fully opened in the rest configuration (left), to two in length resp. four in area, when fully closed in the rest configuration (right).

Kagomi lattice, see Grünbaum and Shephard (1986). In this fully opened configuration, the linkage has stretched from its closed configuration by a length scaling factor of two, and further expansion is blocked.

The *deployable auxetics* introduced in Konaković-Luković et al. (2018) leverage this fully expanded state as a mechanism for rapid deployment (see Figure 1). Observing that applying a specific spatially-varying stretch λ to a flat sheet forces it to assume a unique shape (up to isometric deformation), the authors propose a spatially graded linkage that reaches its fully extended state exactly when stretched by λ. The key idea is to fabricate a planar linkage that is already partially opened: pre-opening the linkage by different amounts λ_{pre} at each point effectively programs a spatially varying maximum stretching factor $\lambda_{\text{max}} = 2/\lambda_{\text{pre}}$ (see Figure 3). If we program a planar linkage with the specific scaling field λ_{max} corresponding to some desired curved shape and subsequently apply an expansion-driven deployment process like inflation or gravitational loading, the process will automatically terminate when this scaling limit is hit; the resulting fully opened deployed linkage will form a trihexagonal tiling of the desired 3D surface. Note that pre-opening the linkage by different amounts in different regions requires varying the linkage's triangle sizes (see Figure 3).

Konaković-Luković et al. (2018) have shown that a large and well-defined class of surfaces can be rationalized with deployable auxetics. Specifically, they prove that a stretch-limited surface can be deployed with inflation if and only if the target surface has positive mean curvature $\frac{1}{2}\nabla \cdot N$ everywhere (where N is the outward-pointing normal vector). Similarly, a *height field* surface can be deployed via gravity if and only if it has positive mean curvature. Surfaces not meeting these requirements can be projected to the nearest positively curved surface with a mean curvature

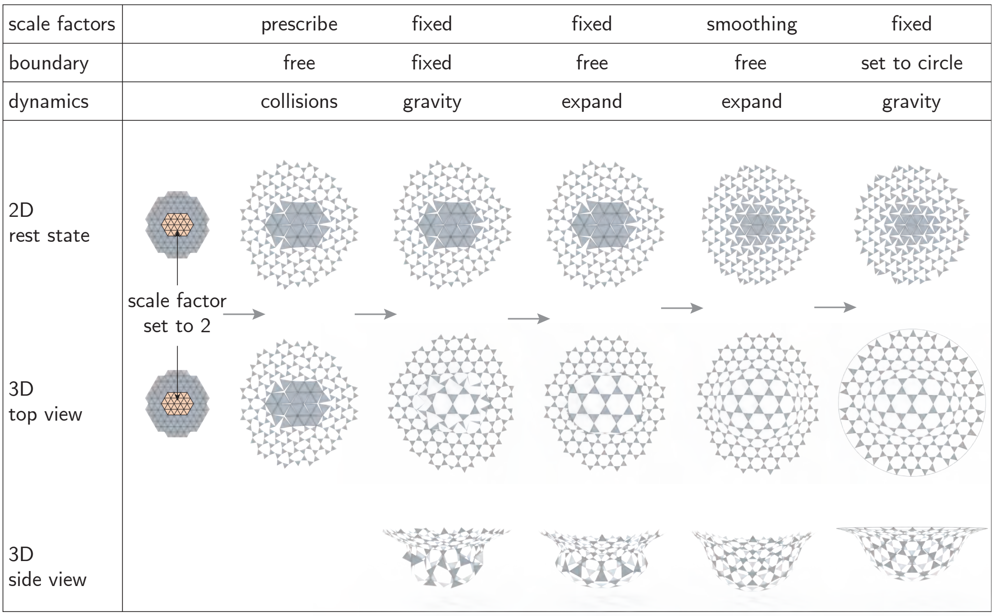

Figure 4: A simple form-finding example to illustrate our atomic editing operators and their effect on the auxetic structure. After prescribing scale factors, we resolve collisions which expands the material in the plane. Applying gravity forces pushes the linkage to a deployed state. However, when applying full expansion, we observe that the surface cannot be realized as a height field, mainly due to the sharp transition in scale factors. After smoothing the scale factors and letting the boundary evolve freely, we obtain a consistent height field surface. Finally, we show how to constrain the boundary onto a circle curve.

flow process described in the paper. There are additional mechanical restrictions imposed by the linkage pattern: the range of length scale factors should fit between one and two.

The post-rationalization pipeline proposed in Konaković-Luković et al. (2018) builds on the close relationship between the auxetic linkage pattern and conformal maps. Like auxetic linkages, conformal maps permit uniform scale distortion but prohibit shearing deformations. Consequently, the map from the linkage's planar configuration to the deployed 3D surface is nearly conformal, and a conformal map from the plane to the desired curved surface can be approximated by a linkage (provided its conformal scaling factors fall within the permissible range). This motivates the use of a discrete conformal map to initialize a joint 2D/3D optimization to find the parameters of the auxetic linkage that best approximates the design surface when maximally stretched everywhere. For more details on this optimization, we refer the reader to their paper.

4 Design Space Exploration and Form-Finding

As discussed above, the desired target shape in the deployed state can be programmed into the auxetic structure by optimizing for suitable maximal expansion factors across the linkage, which in turn determine the spatial layout and sizing of linkage triangles. The indirect nature of this post-rationalization provides only limited support for exploring design alternatives or discovering new forms that are directly informed by the material and deployment mechanism. More direct manipulation is required to offer interactive design control in a tight feedback loop.

However, trying to manipulate the deployed geometry by directly displacing linkage vertices is not appropriate since the consistency of the design cannot be easily maintained. Linkage vertices would need to be moved in a coordinated way to respect the complex global coupling imposed by the material structure and deployment mechanism, which becomes virtually impossible without computational support.

This is why we propose interactive, optimization-assisted design operators. Specifically, we allow the designer to directly modify the maximal scale factors of the linkage and impose design-specific geometric constraints. We then apply optimization to jointly determine the 2D rest shape and the 3D deployed shape. Since this optimization can be executed at interactive rates, the designer gets immediate feedback on her edits, while being freed of the complexities of maintaining consistency of the structure. We found that the following editing operators yield an effective toolbox for design space exploration:

- *Prescribing scale factors:* We provide a painting interface where the designer can directly prescribe the desired maximal scale factors in the allowable range [1,2]. Increasing scale factors allows the material to stretch more under deployment, while reducing scale factors locally shrinks the deployed surface.
- *Smoothing scale factors:* Sharp transitions in scale factors can lead to non-smooth surface appearance and, in extreme cases, surface wrinkles. Spatially averaging the scale factors evens out these variations and generally leads to smoother deployed surfaces. Controlling the amount of scale factor smoothing yields different design alternatives.
- *Boundary control:* The user can directly edit the 3D boundary curves of the design and control the behavior of boundary linkage vertices, which can slide along boundary curves. Since the boundary has a strong influence on the overall shape of the deployed surface, we also allow boundary linkage triangles to deviate from equilateral shape, which can improve the overall surface quality.
- *Geometric constraints:* The user can further control the geometry of the deployed surface by imposing additional geometric constraints, for example on the planarity of certain edge curves, symmetry of selected vertices, or smoothness of the surface.

We also provide a separate form-finding optimization for the boundary curves. This can be helpful when the total area of the chosen linkage is not well-suited for the imposed boundary curve, e.g., when there is too much material or too little for the surface to conform to the boundary. In such cases, we apply an expansion

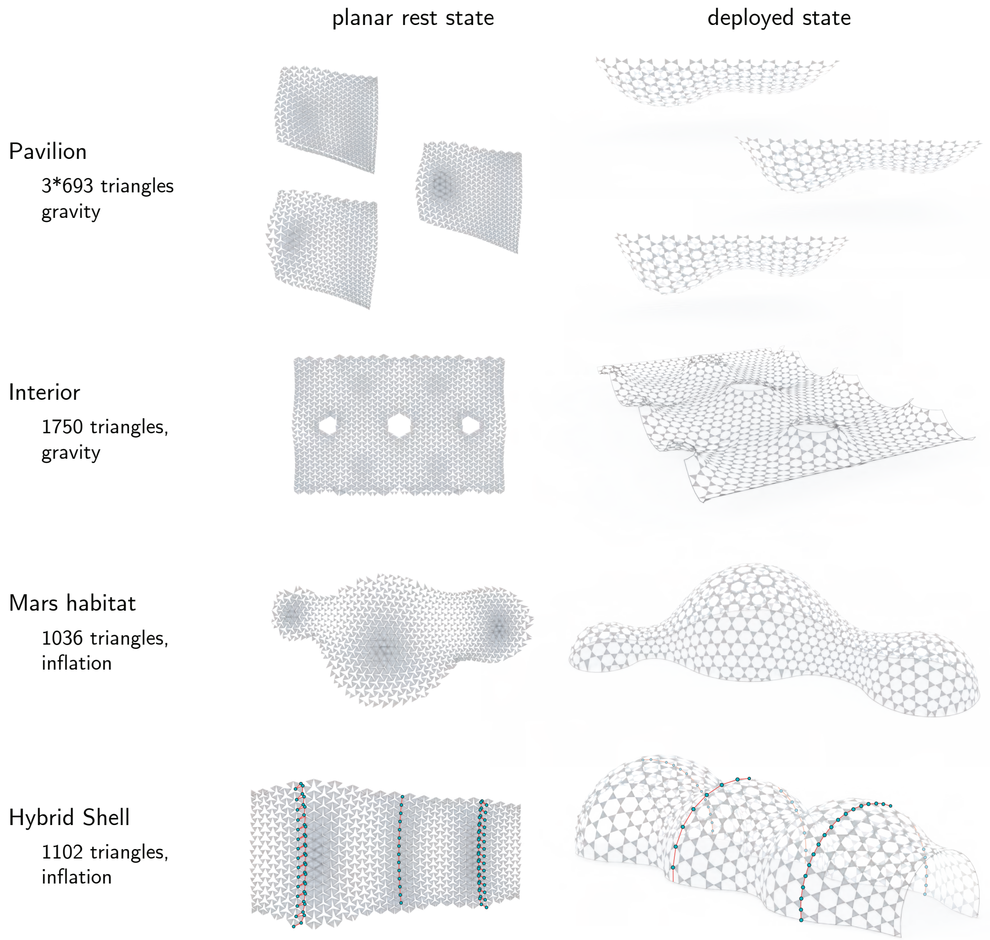

Figure 5: Four design examples shown in planar rest configuration and final deployed state. The number of auxetic linkage triangles and deployment method is indicated. In the bottom row, the highlighting shows three sets of vertices and edges that are each constrained to lie on a plane in the deployed 3D model to create planar support beams. See Figures 6 to 9 for detailed renderings.

force on the linkage to fully expand the hexagonal openings and let the boundary vertices move freely to their preferred positions. Figure 4 illustrates how these design operators can be employed in an interactive form-finding design.

4.1 Algorithm

Our interactive design system runs a constraint-based optimization algorithm to provide direct visual feedback on the flat and deployed state of the auxetic linkage. This optimization is based on the projective approach of Bouaziz et al. (2014); Deuss et al. (2015) that allows combining different geometric constraints to model the material behavior and the dynamics of the deployment mechanism.

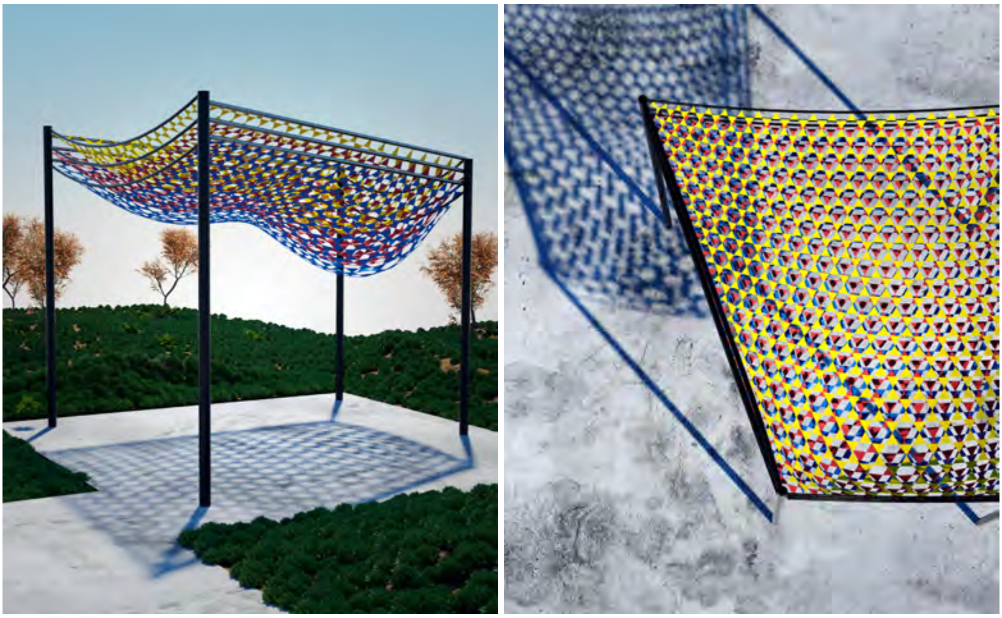

Figure 6: Multi-layer shading pavilion deployed by gravity.

Painting or smoothing scale factors provides constraint targets for the triangle edge lengths. We apply point-to-curve constraints to limit the movement of boundary vertices to the boundary curves. When optimizing for the boundary, we apply circle constraints on the hexagonal openings to expand the surface, as the maximal area is achieved when all hexagon vertices lie on a circle, see Niven (1981), page 236. Additional geometric constraints, e.g., planarity of user-selected edge curves in the deployed state, can easily be formulated on the linkage vertices. Gravitational deployment is modeled with a constant downward force, while inflation is approximated by outward-pointing normal forces. These forces are converted into geometric constraints in an implicit time integration solver as discussed in Bouaziz et al. (2014).

During editing, the constraint-based optimization solves for the linkage vertex positions in the flat 2D state and the deployed 3D state to provide immediate visual feedback on the performed edits. For more implementation details and an open-source library of the projection-based solver, we refer to `www.shapeop.org`.

5 Application Case Studies

We illustrate the potential of our computational design approach with a number of application case studies for deployable auxetic structures. The design process starts with an initial 2D triangular linkage, either obtained by the post-rationalization process of Konaković-Luković et al. (2018) or simply created as a uniform triangle pattern when designing from scratch. We then apply a series of editing operations as described above to explore design alternatives. The final output of this interactive form-finding process is a specific triangular linkage with spatially varying triangles

Figure 7: Interior decorative cladding. This hanging structure has been optimized to align with the boundary constraints imposed by the ambient space. The designer controls the shape by interactively modifying scale factors.

that can be fabricated and assembled in the plane and deployed automatically to the desired target shape. Four example designs are summarized in Figure 5 and described in more detail below.

Figure 6 shows a design study of a shading pavilion, realized as a linkage of inextensible fabric triangles that are connected with ring joints at the triangle vertices. As a hanging structure, the surface deploys under gravity to its desired double-curved target state. This design has been created in an interactive form-finding process from a uniform auxetic linkage that is subsequently manipulated using our design operators to create three design variations. These are combined in a multi-layer structure, which allows designing spatially varying opacity to optimize the shading performance of the structure for the anticipated use case.

Figure 7 shows another gravity-deployed structure in an interior space, with potential use cases of acoustic dampening or decoratively masking of functional components such as AC pipes or wirings. This example shows how manipulating scale factors in combination with detailed boundary control offers effective ways to integrate a deployable auxetic structure into an existing space with precisely defined boundary constraints.

Figure 8 shows a speculative design study for habitats on Mars. Since the atmospheric pressure on Mars is 100 times lower than Earth's, the interior must be pressurized. This motivates the use of inflatable structures that can be efficiently erected from flat configurations, offering the additional benefits of low weight and compact storage. Our deployable auxetics offer a rich design shape space, so we can optimize the shape of the freeform domes to match interior space objectives.

Figure 8: Inflatable freeform dome for a potential Mars habitat.

In Figure 9, we demonstrate how we can incorporate additional geometric constraints to optimize the design. In this example, we impose planarity constraints on selected edge and vertex curves of the auxetic linkage to form structural arches that can reinforce the inflated shell. The planarity of these arches significantly simplifies their fabrication.

6 Conclusion

We have shown how optimization-assisted shape exploration yields an effective method for designing deployable structures based on auxetic triangular linkages. By directly manipulating the form-defining geometric properties, i.e., the material scaling and the surface boundary, the designer obtains full control of the deployed shape while being shielded from the complexity of maintaining consistency between the 2D assembly state and the 3D deployed state. Automatic deployment via inflation or gravity allows transforming compact flat assemblies into freeform surfaces without the need of any supporting structures or complex construction process. Fabrication requires only 2D technologies such as sawing or laser cutting to produce the triangular panels. Despite this inherent simplicity, expressive freeform surfaces can be realized for a variety of different use cases.

A number of open questions offer numerous opportunities for future work. So far, we did not address questions of structural integrity in a systematic way, nor did we incorporate performative objectives into the optimization. For example, light transmission of the shading pavilion or the acoustic dampening for the interior cladding design study could be directly integrated into the form-finding method to yield a more informative shape exploration process. Another important topic for future work is the design of joint connections, in particular ones that lock into

Figure 9: A hybrid shell structure integrates planar support arches in the interior into a deployable auxetic surface.

a stable state when deployed to the final target configuration. Finally, we see interesting research potential in exploring other expansive deployment mechanisms, for example based on material swelling, motorization, or pre-stressing.

Acknowledgments

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